PhD Junior Seminar
20th November 2012

Coupling independent compartments of the cardiovascular system: fluid-fluid interaction

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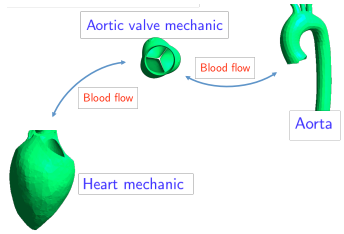
Outline

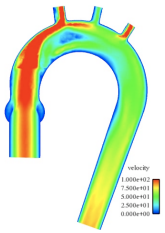
1. Motivations
   - Cardiovascular models



2. Application to blood flow
   - Navier - Stokes equations

$$\begin{aligned} \partial_t \mathbf{u}_1 - \nabla \cdot \boldsymbol{\sigma}(\mathbf{u}_1, p_1) &= 0 \\ \nabla \cdot \mathbf{u}_1 &= 0 \end{aligned}$$

3. Simulations samples
   - Coupled and reference solutions comparisons



4. Conclusions
   - Current and further investigations

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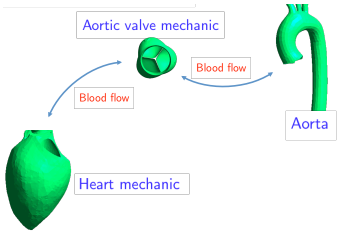

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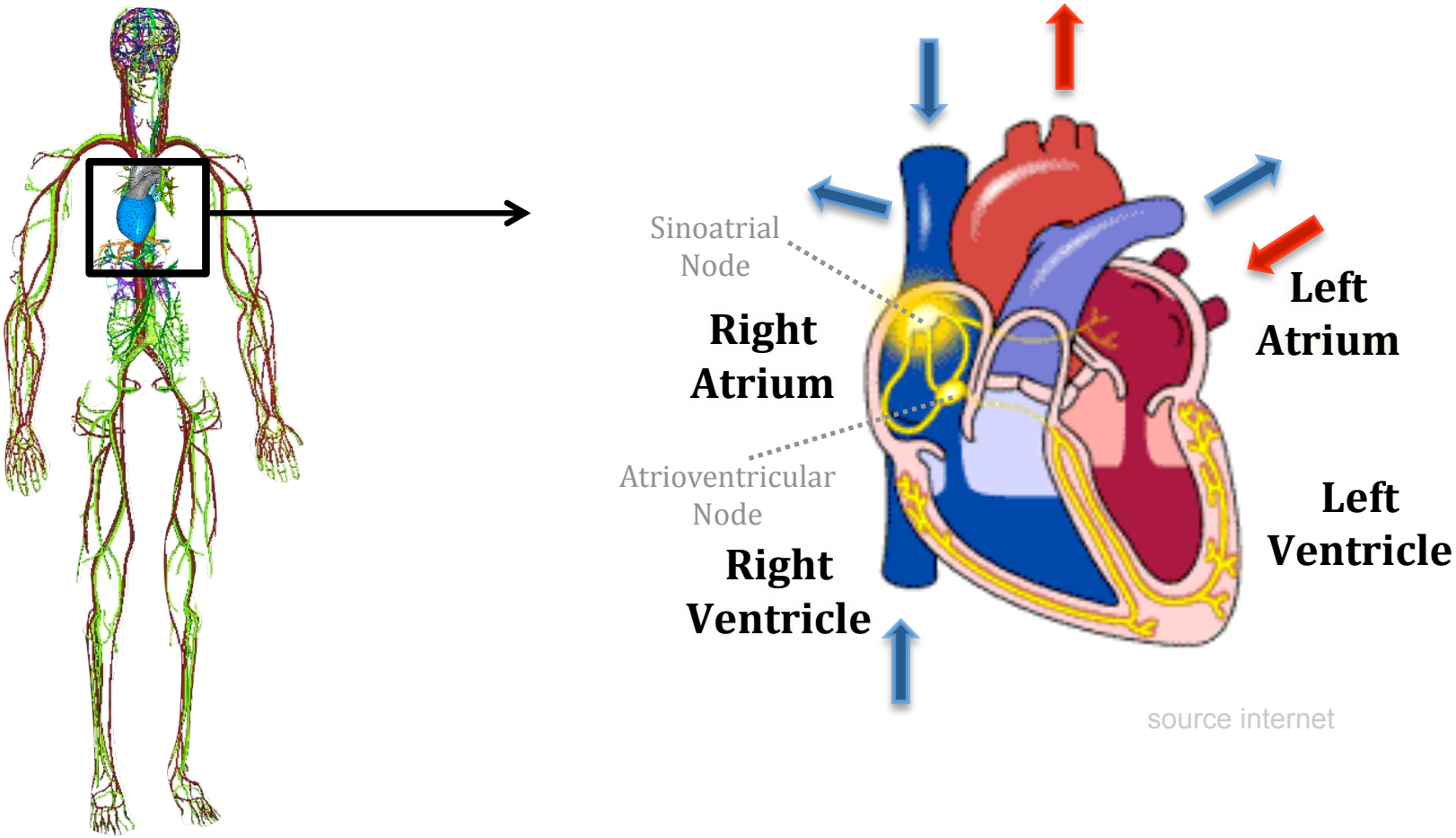
Motivation

- Challenge: modelling the whole cardiovascular system.

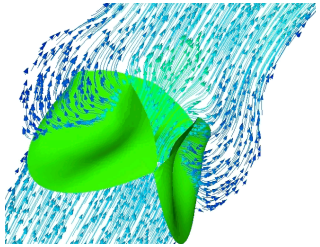


- Monolithic algorithms: simultaneous solution for all compartments;

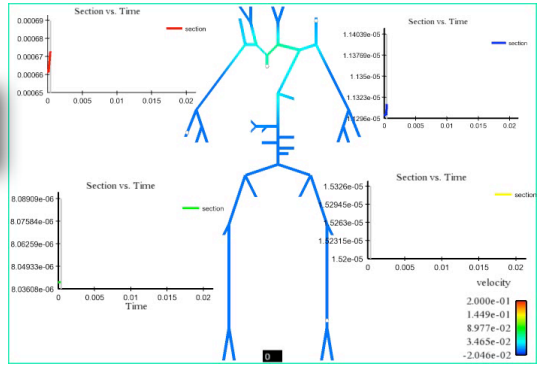
- Partitioned algorithms: separate solutions for each compartment.

CardioVascular Models

[D.Lombardi, '12]



[Astorino & Diniz dos Santos, '09]



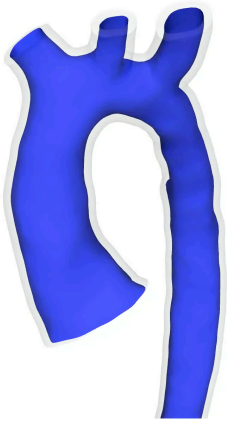
Blood circulation

Aortic valve mechanics

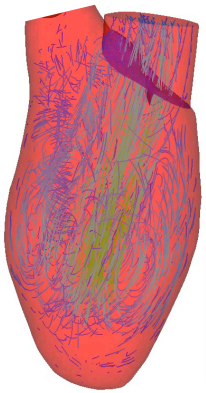
Coupling Code [CVGraph]

[Gerbeau, Fernández, Smaldone]

FSI in the aorta



[Bertoglio et all, '12]



Cardiac mechanics

[M. Astorino, '09]

Cardiac electrophysiology

[Boulakia, Fernández, Cazeau, JFG, Zemzemi, '10]

Aim: Split cardiovascular system complexity, starting from coupling blood flow in different and independent compartments.

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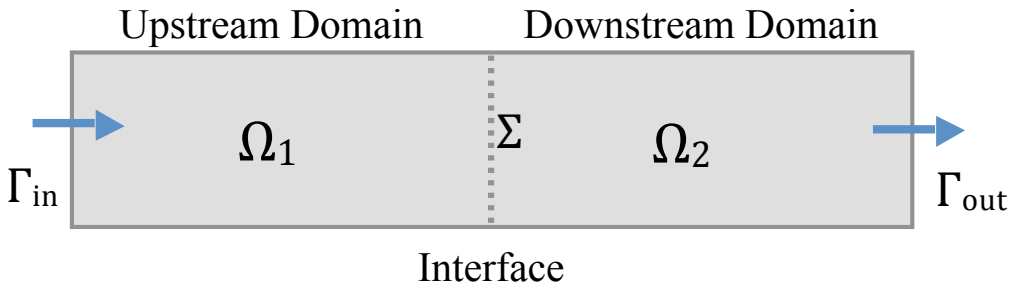
Blood models: Navier-Stokes equations



$\mathbf{u}_1^n$ = velocity $\quad p_1^n$ = pressure

$\mathbf{u}_2^n$ = velocity $\quad p_2^n$ = pressure

Upstream Domain: Fluid 1

$$\begin{cases} \rho\partial_\tau \mathbf{u}_1^n + \rho(\mathbf{u}_1^{n-1}\nabla)\mathbf{u}_1^n - \nabla\cdot\boldsymbol{\sigma}(\mathbf{u}_1^n, p_1^n) = 0 \\ \nabla\cdot\mathbf{u}_1^n = 0 \end{cases}$$

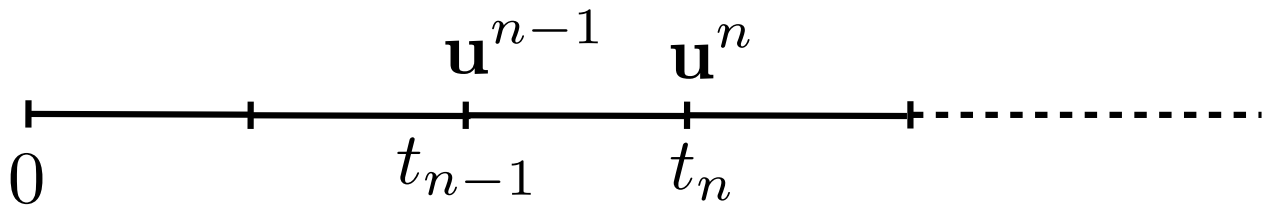
Downstream Domain: Fluid 2

$$\begin{cases} \rho\partial_\tau \mathbf{u}_2^n + \rho(\mathbf{u}_2^{n-1}\nabla)\mathbf{u}_2^n - \nabla\cdot\boldsymbol{\sigma}(\mathbf{u}_2^n, p_2^n) = 0 \\ \nabla\cdot\mathbf{u}_2^n = 0 \end{cases}$$

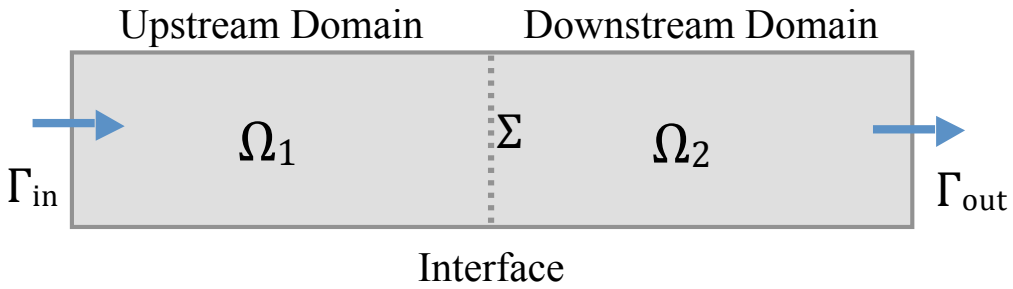
$\mathbf{u}_1^{n-1} = \mathbf{u}_1(t_{n-1})$ previous time step

$\mathbf{u}_1^n = \mathbf{u}_1(t_n)$ current time step $\quad p_2^n = p_2^n(t_n)$

time discretization



Blood models: Navier-Stokes equations



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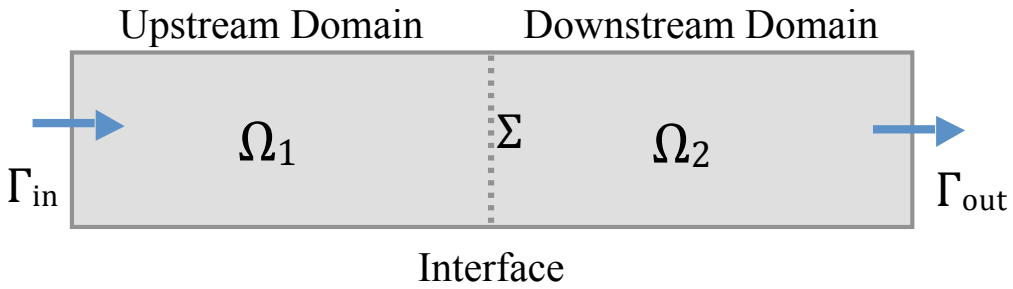
Downstream Domain: Fluid 2

$$\begin{cases} \rho\partial_\tau \mathbf{u}_2^n + \rho(\mathbf{u}_2^{n-1}\nabla)\mathbf{u}_2^n - \nabla\cdot\boxed{\boldsymbol{\sigma}(\mathbf{u}_2^n, p_2^n)} = 0 \\ \nabla\cdot\mathbf{u}_2^n = 0 \end{cases}$$

$$\boldsymbol{\sigma}(\mathbf{u}, p)$$

Stress tensor = measure of the forces acting on an inner surface in the fluid

Blood models: Navier-Stokes equations



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Downstream Domain: Fluid 2

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Transmission Conditions on Σ

$$? = ?$$

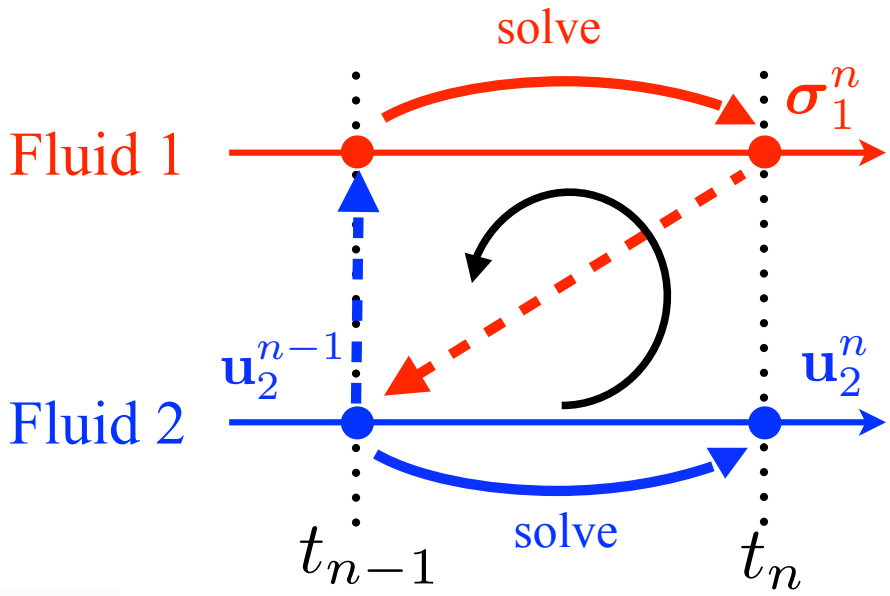
$$? = ?$$

Coupling techniques: Classical method

- Dirichlet-Neumann transmission conditions:

Transmission conditions on Σ

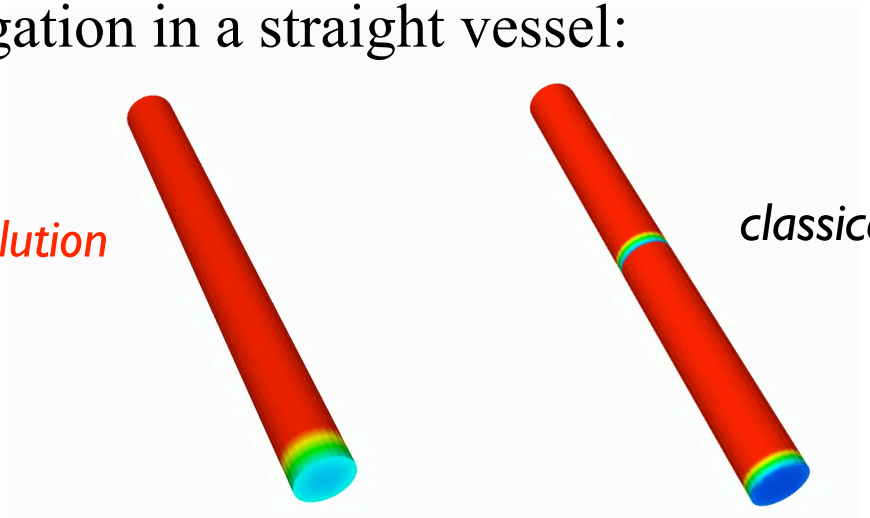
$$\begin{cases} \mathbf{u}_1^n = \mathbf{u}_2^{n-1} \\ \boldsymbol{\sigma}(\mathbf{u}_2^n, p_2^n) \cdot \mathbf{n}_2 = -\boldsymbol{\sigma}(\mathbf{u}_1^n, p_1^n) \cdot \mathbf{n}_1 \end{cases}$$



- Loosely coupled scheme (explicit).

- Pressure propagation in a straight vessel:

reference solution



classical coupling method

- Standard treatments :
  - Strongly coupled schemes (implicit): iterate until convergence, expensive;
  - New loosely coupled methods: stable and computationally cheap.

Coupling techniques: New method

- Robin-Robin transmission conditions derived from Nitsche's method:

Transmission conditions on Σ

$$\begin{cases} \boldsymbol{\sigma}(\mathbf{u}_1^n, p_1^n)\cdot\mathbf{n}_1 + \alpha\mathbf{u}_1^n &= \alpha\mathbf{u}_2^{n-1} - \boldsymbol{\sigma}(\mathbf{u}_2^{n-1}, p_2^{n-1})\cdot\mathbf{n}_2 \\ \boldsymbol{\sigma}(\mathbf{u}_2^n, p_2^n)\cdot\mathbf{n}_2 + \alpha\mathbf{u}_2^n &= \alpha\mathbf{u}_1^n + \boldsymbol{\sigma}(\mathbf{u}_2^{n-1}, p_2^{n-1})\cdot\mathbf{n}_2 \end{cases}$$

- Nitsche's penalty parameter: $\alpha > 0$ to control velocities on the interface;

Fluid 1 -

Fluid 2 -

Relaxation method to treat interface conditions for coupled problems.
Used in Fluid - Structure problems.

[Burman, Fernández, 09]
[Hansbo, '05]

t_{n-1} t_n

Coupling techniques: New method

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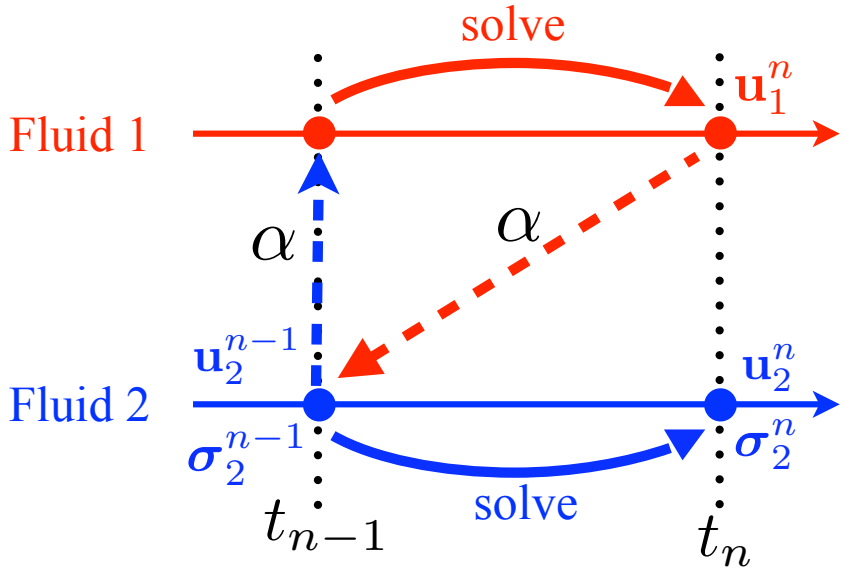


- Nitsche's penalty parameter: $\alpha > 0$
  to control velocities on the interface;

- Add in the downstream Fluid 2:

Stabilization term on Σ

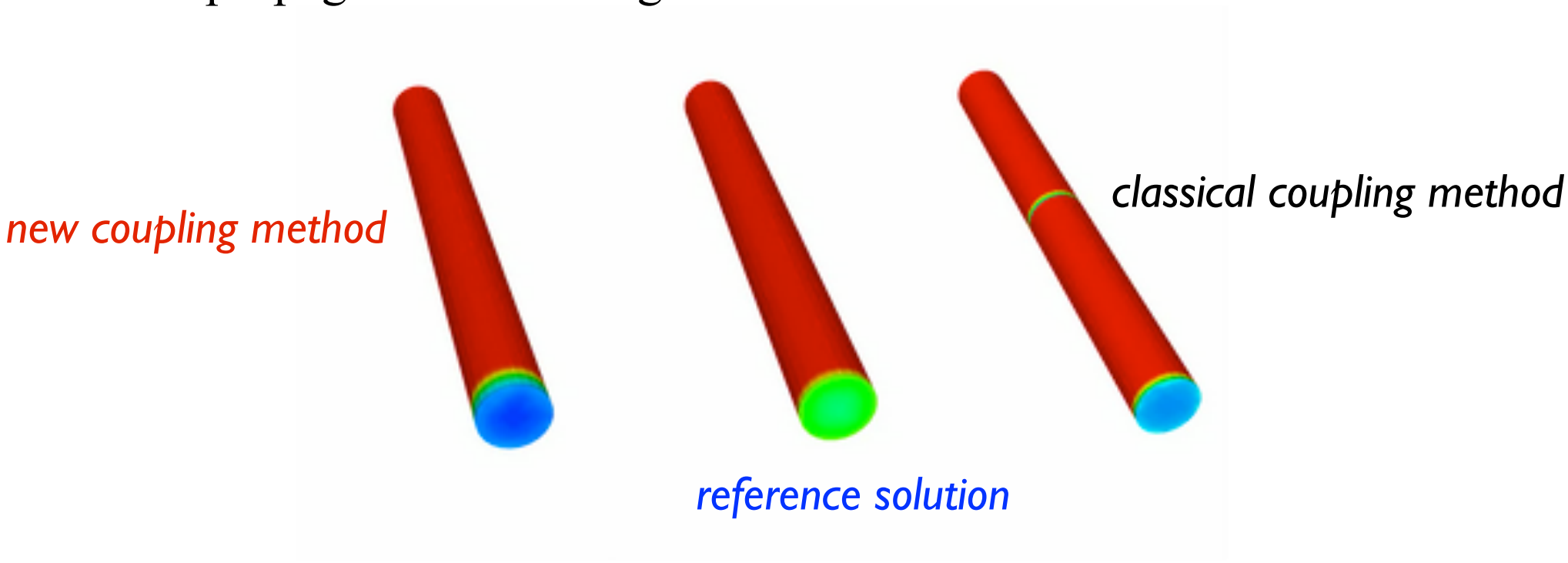
$$S(p_2^n, p_2^{n-1}) = \beta \int_\Sigma (p_2^n - p_2^{n-1}) q_2$$

- Stabilization parameter: $\beta > 0$
  to controll pressures on interfaces.

Coupling techniques: New method

- Taking appropriate values of α and β the Fluid-Fluid coupled model becomes stable.

- Pressure propagation in a straight vessel:



- Pressure wave instabilities disappeared with Robin-Robin method.

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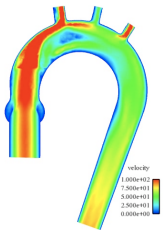

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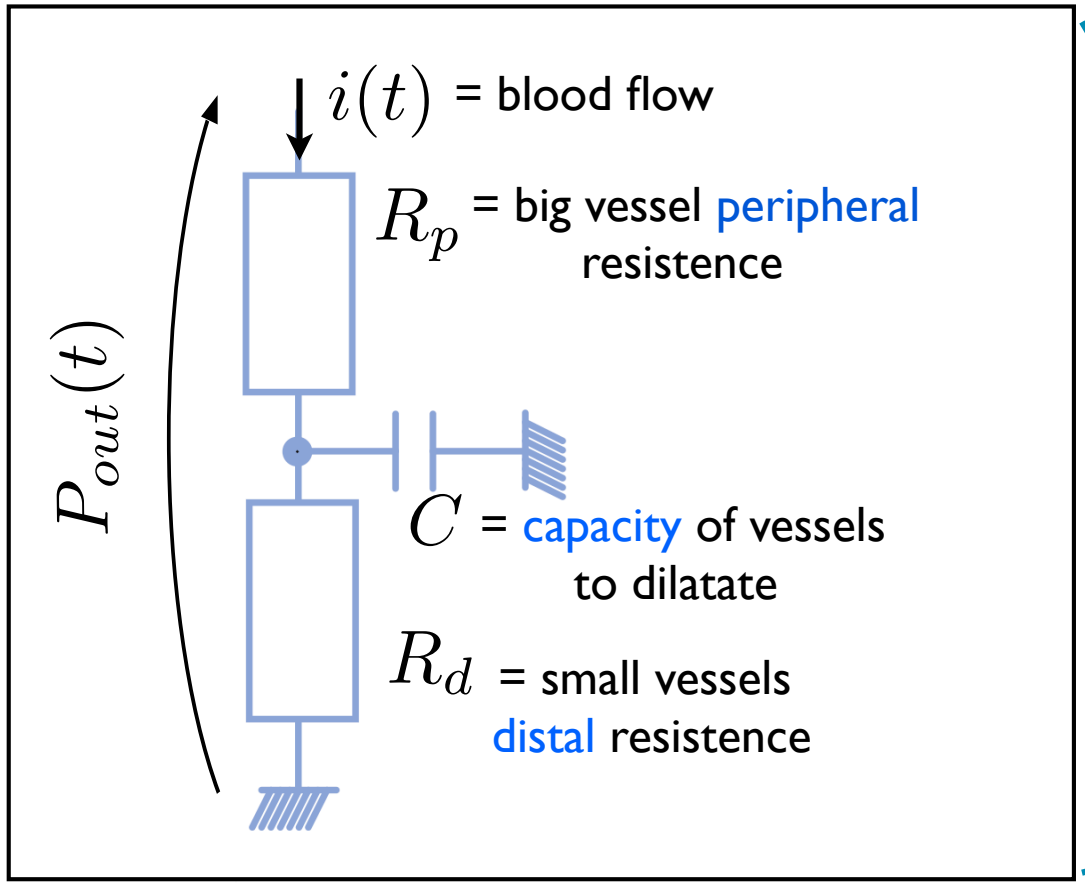
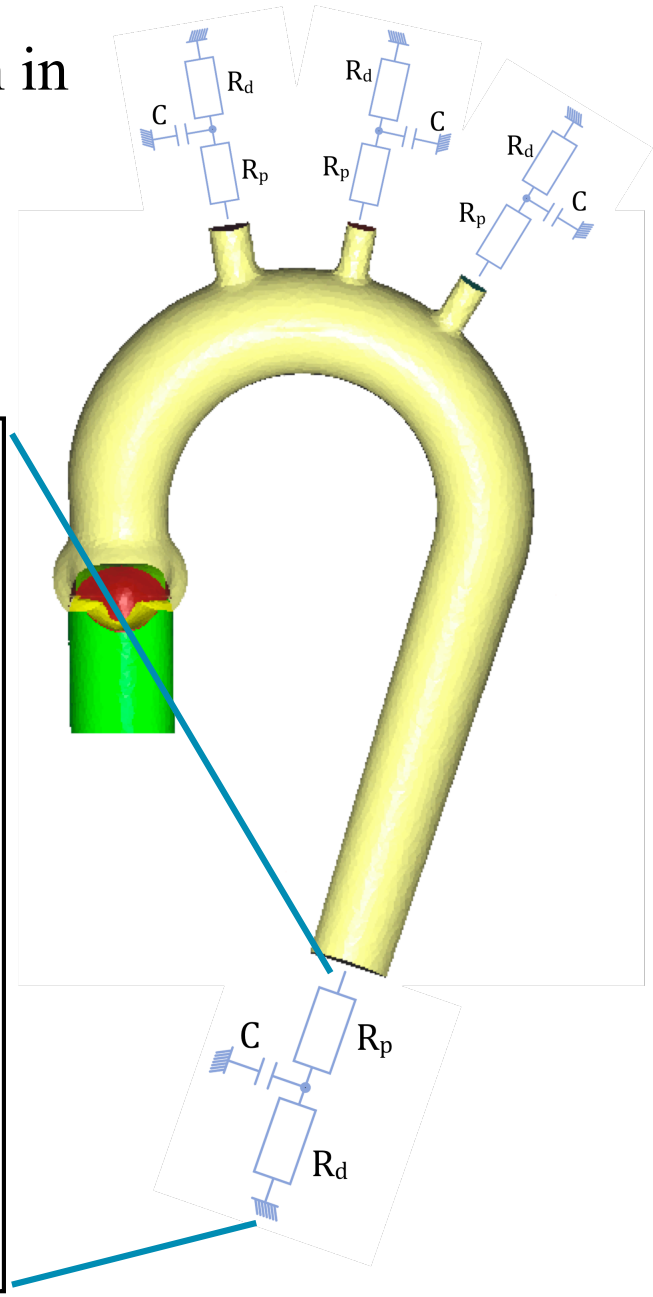
Haemodynamics application: aorta model

- Comparisons:
  Fluid-Fluid Model vs. Reference Solution in a Template of an Aorta;

- Windkessel 0D model in the outputs;





Haemodynamics application: aorta model

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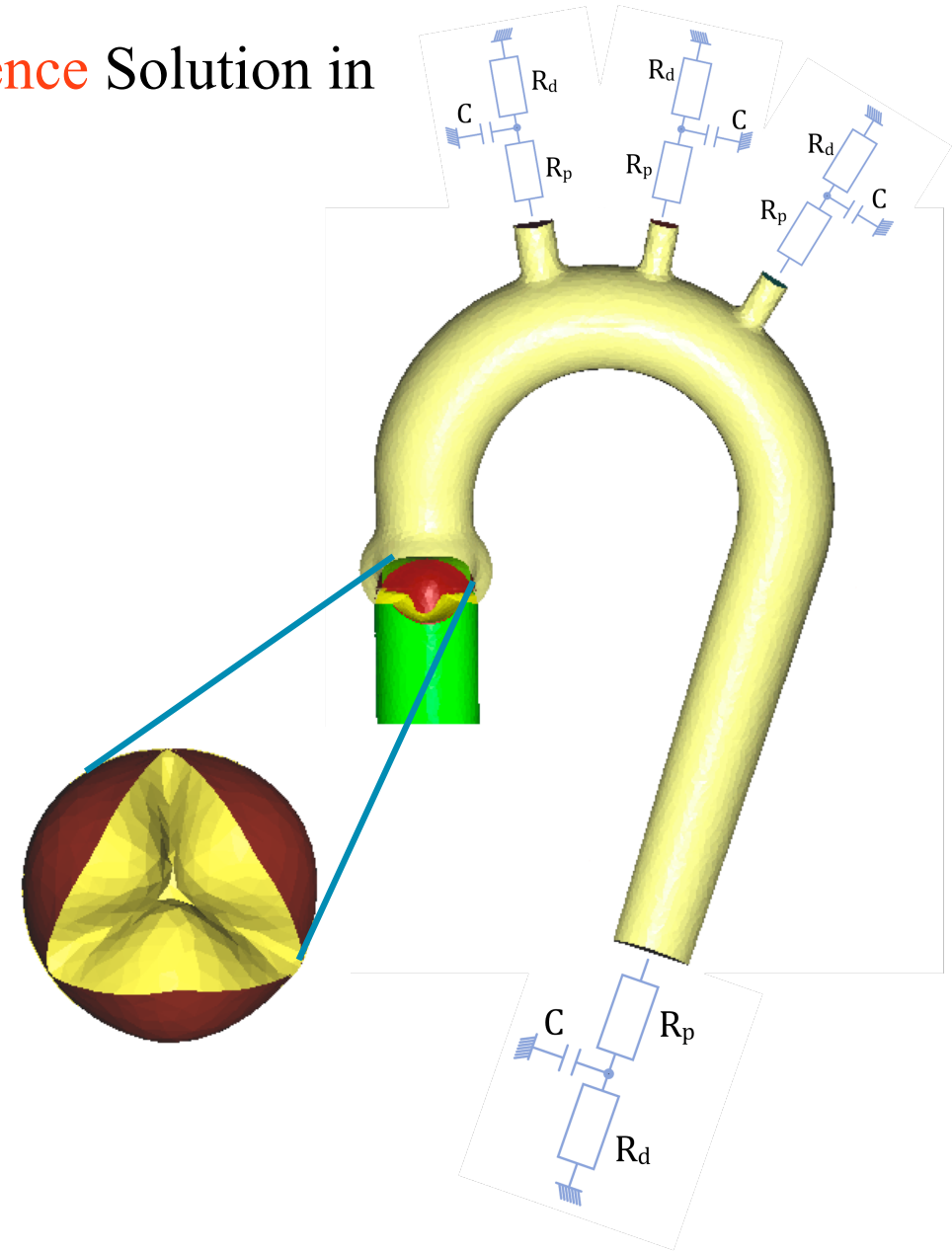

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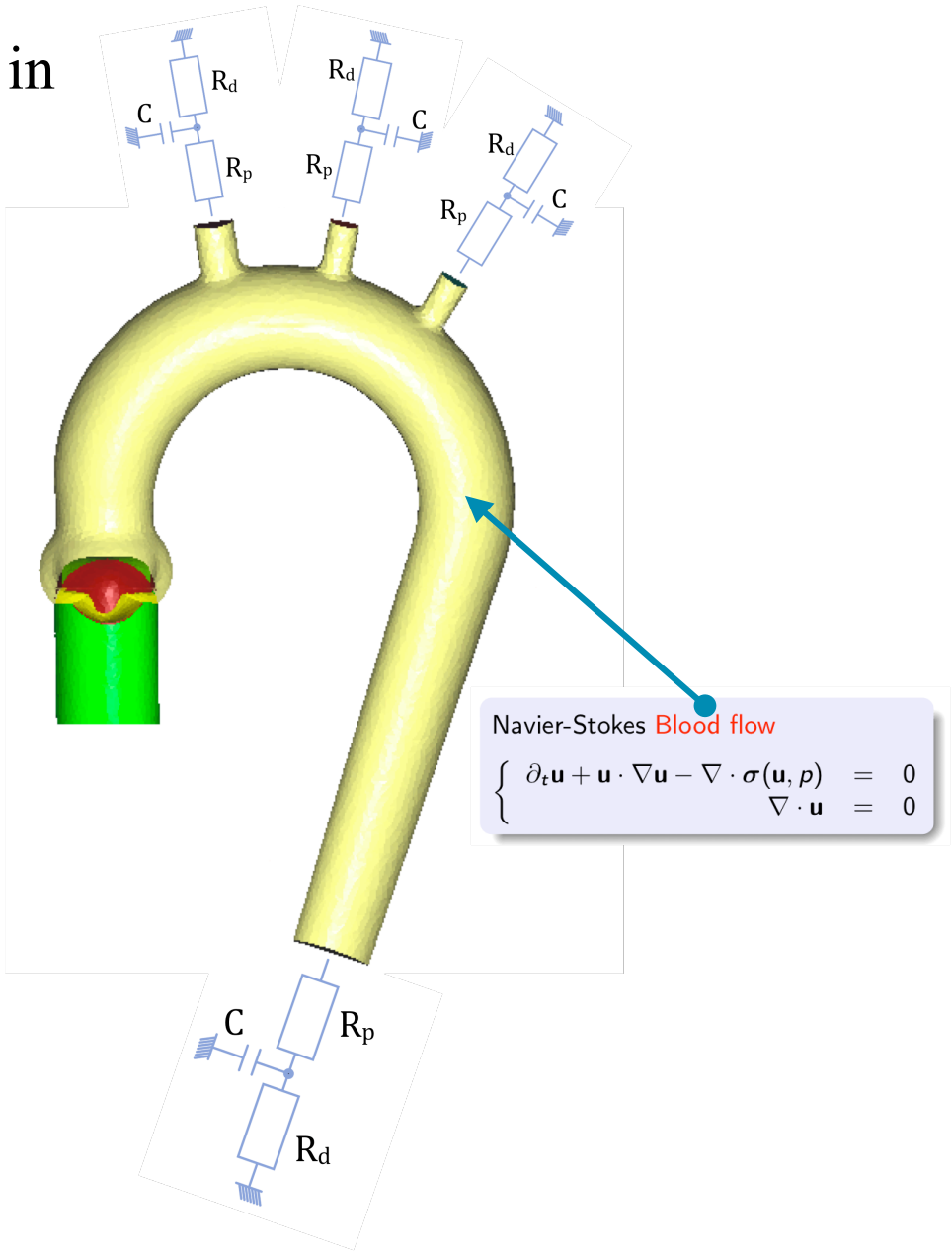

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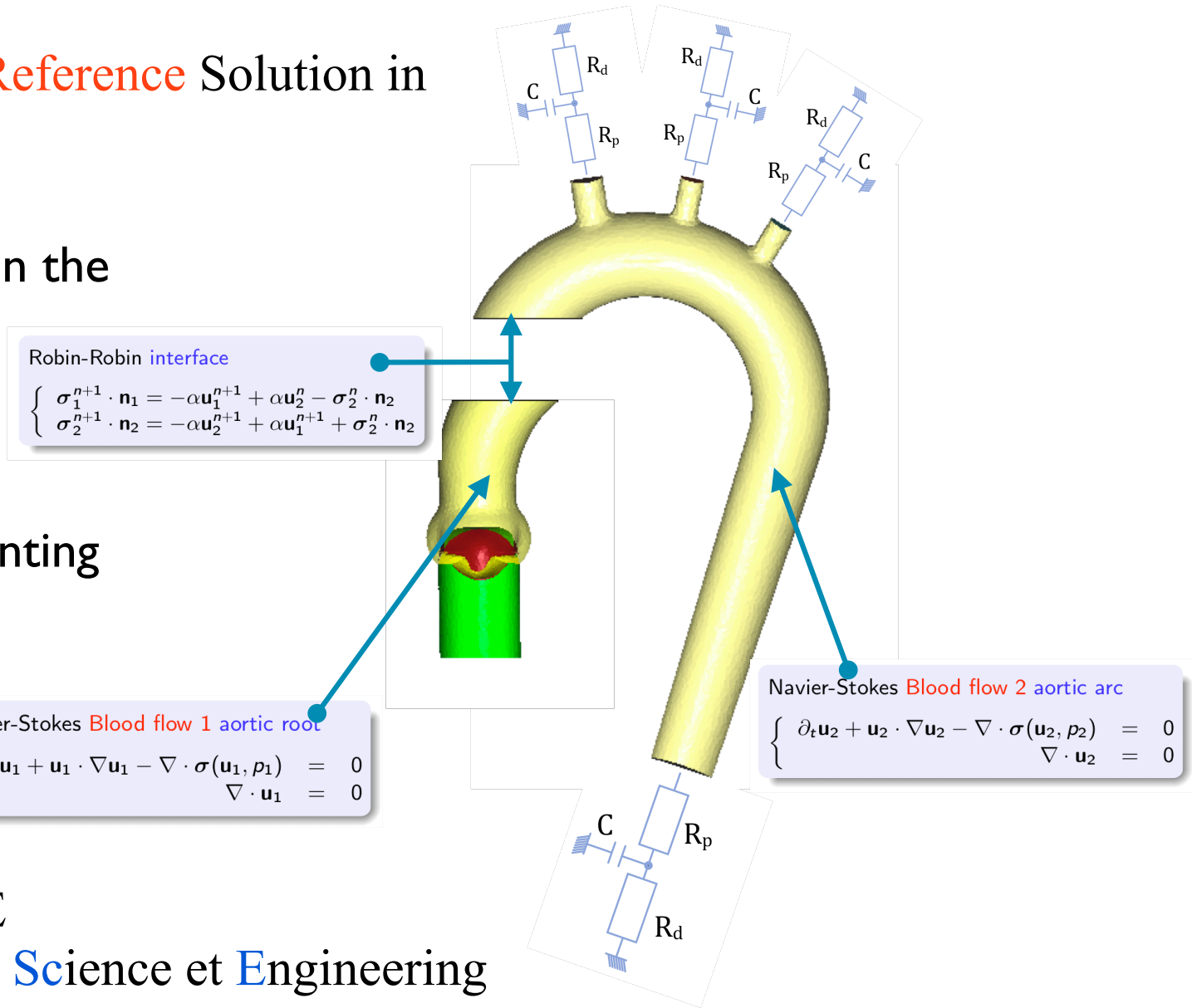
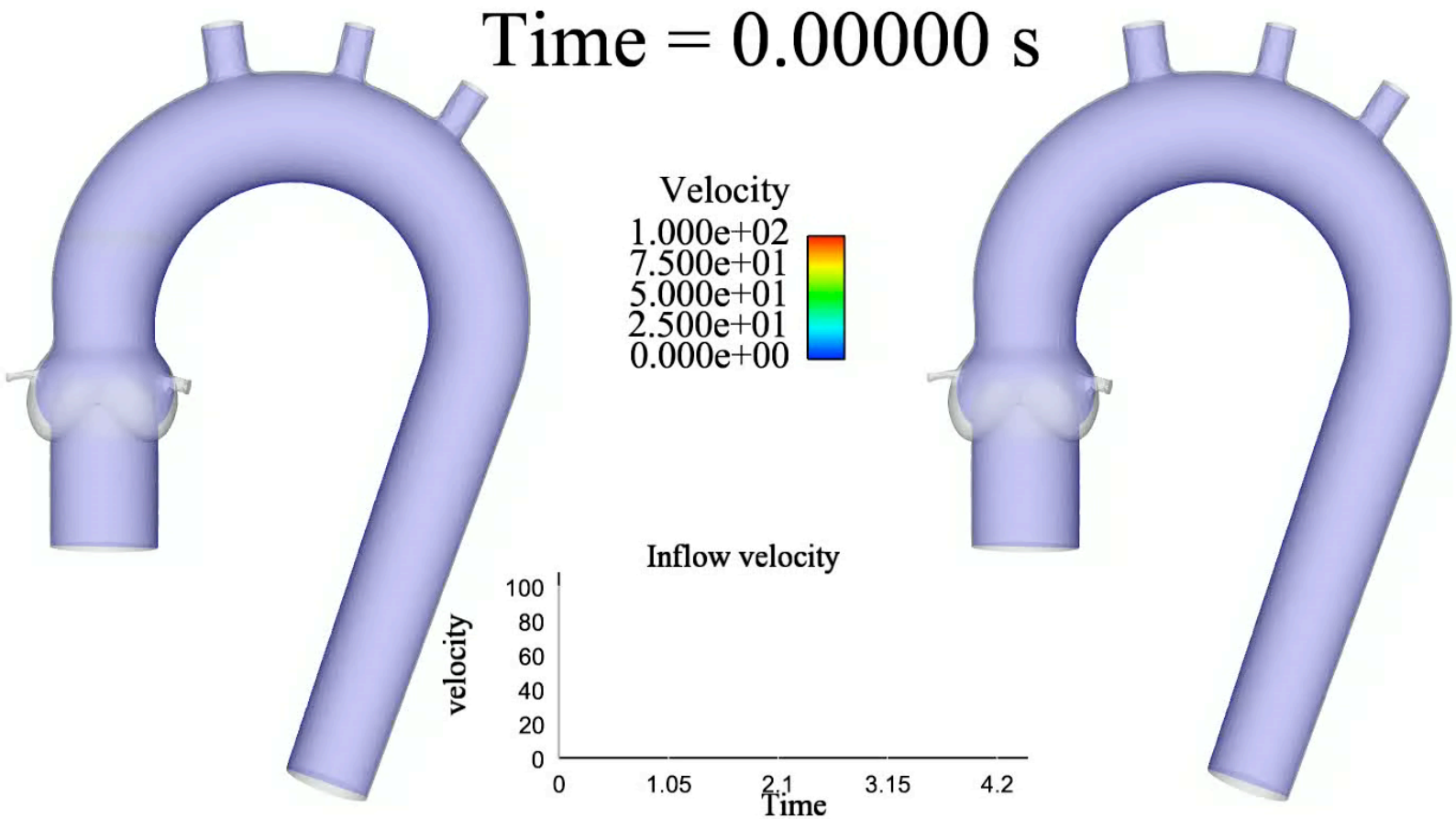


- Two Solvers : FELiScE
  Finite Element for Life Science et Engineering
  [Macs & Reo Team - INRIA Saclay & Rocquencourt].

Blood velocity profile

Blood-Blood coupling

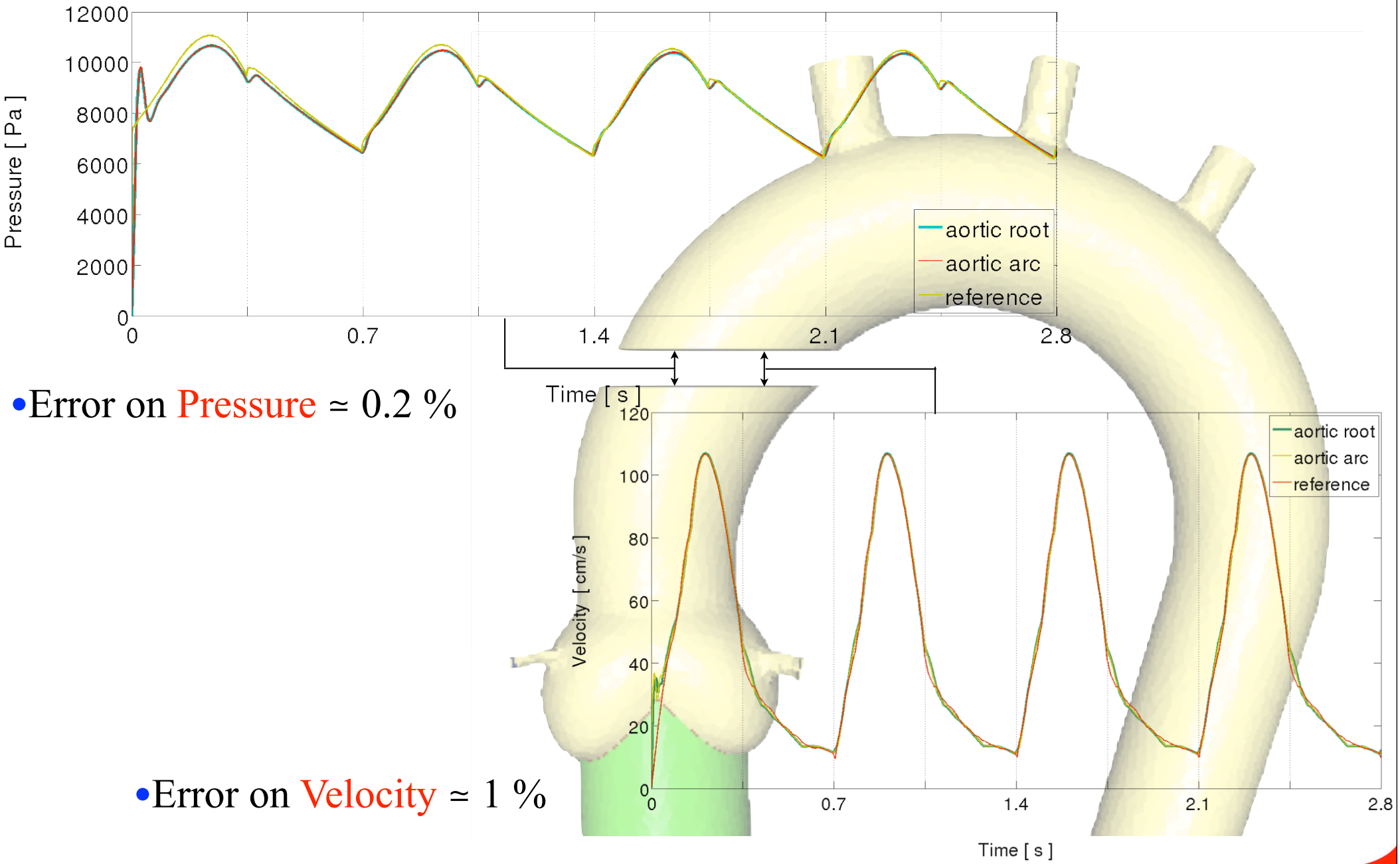
Reference solution



[Smaldone, Fernández, Gerbeau, '12]

- Time step: t = 1e-3;
- Inflow conditions: Waveform of cardiac output 9 liters/minute
  hear rate 85 beats/minute.

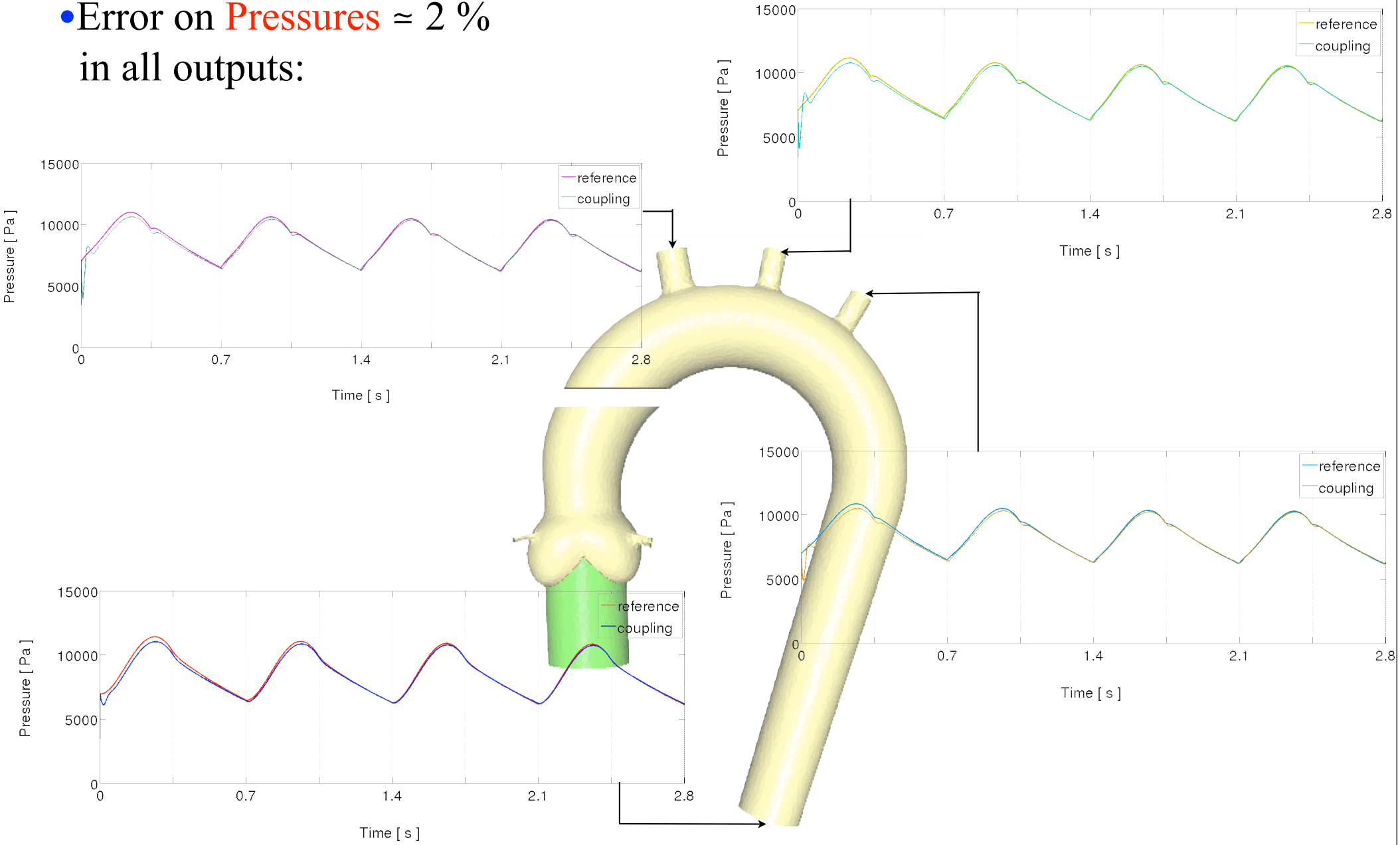
Interface Velocity and Pressure



- Error on Pressure ≃ 0.2 %
- Error on Velocity ≃ 1 %

Outputs Pressures

- Error on Pressures ≃ 2 % in all outputs:



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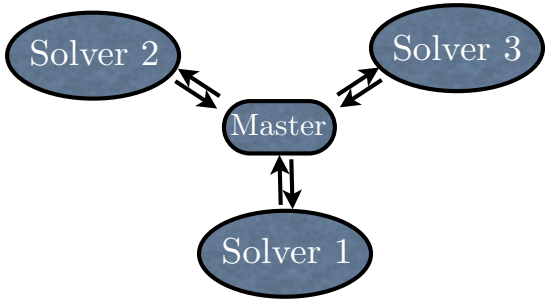
Conclusions

- Current works :



☑ Coding Software for coupling solvers;

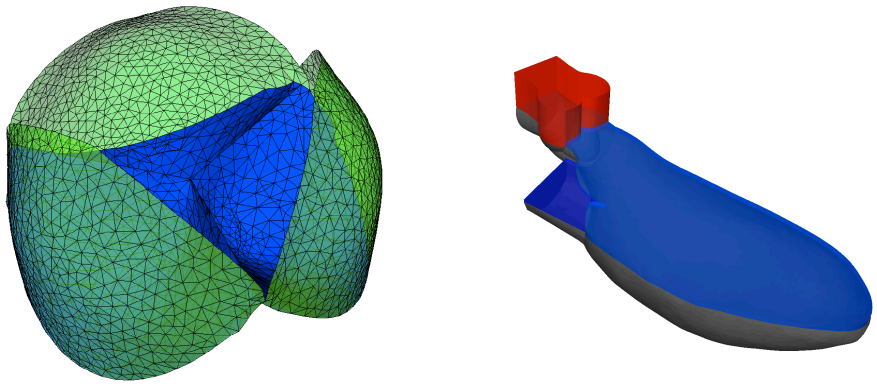
☑ Coding Solver for fluid & cardiovascular models;

☑ Efficient coupled Blood-Blood model;

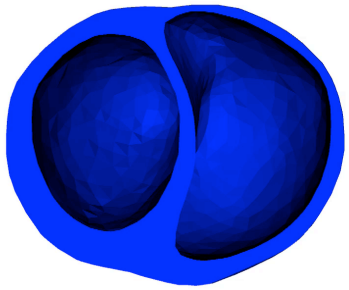
Conclusions

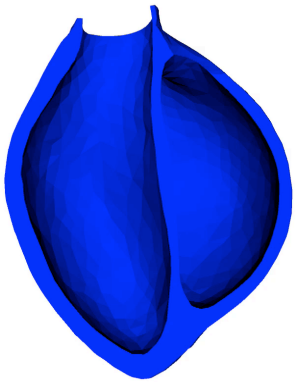
- Forthcoming works:

- Testing with Strongly coupled schemes
- Including Heart model;
- Including a semplified Aortic valve model;



[M. Astorino, '09]





[Macs & Reo Team -
INRIA Saclay & Rocquencourt]

Thanks for the attention