Symbolic Data Analysis Of Complex Data: several directions of research

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OUTLINE

- What are Complex data?
- What are “symbolic data”?
- How “Symbolic Data” are build?
- Symbolic Data are Complex data?
- From Complex Data to Symbolic Data
- What is “Symbolic Data Analysis” (SDA)?
- Open directions of research
- Conclusion: SDA gives a framework for Complex Data Analysis (CDA)

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- SDA gives a framework for Complex Data Analysis (CDA)

What are Complex data?

Any data which cannot be considered as a standard “observations x variables” data table.

Examples

- several data tables describing different kind of observations.
- Hierarchical Data
- Textual Data in each cell of the data table
- Time series Data in each cell .

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What are “symbolic data”?

Any data taking care on the variation inside classes of standard observation.

- **each cell of the data table can contain:**
- **A number, a category, an interval, a sequence of categorical values, a sequence of weighted values , a Bar Chart, a histogram, a distribution, …**

Example of SYMBOLIC DATA

TEAM OF THE FRENCH CUP	WEIGHT	NATIONALITY	NB OF GOALS
MARSEILLES	[75 , 89]	{French}	{0.8 (0), 0.2 (1)}
LYON	[80, 95]	{Fr, Alg, Arg }	{0.1 (0), 0.3 (1), …}
PARIS-ST G.	[76, 95]	{Fr, Tun }	{0.4 (0), 0.2 (1), …}
NANTES	[70, 85]	{Fr, Engl, Arg }	{0.2 (0), 0.5 (1), …}

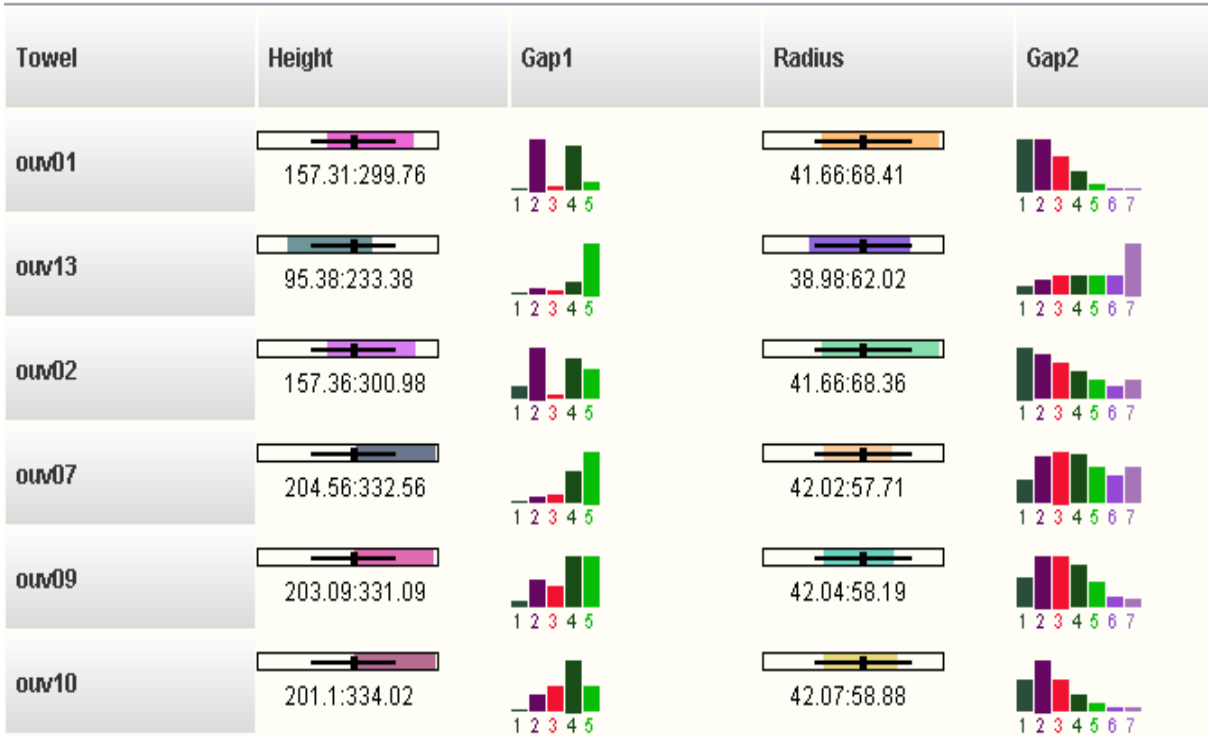
Here the variation (of weight, nationality, …) concerns the players of each team.

Therefore each cell can contain:

A number, an interval, a sequence of categorical values, a sequence of weighted values as a histogram, a distribution, …

THIS NEW KIND OF VARIABLES ARE CALLED « SYMBOLIC » BECAUSE THEY ARE NOT PURELY NUMERICAL IN ORDER TO EXPRESS THE INTERNAL VARIATION INSIDE EACH CONCEPT.

SYMBOLIC DATA TABLE SOFTWARE*



Towel	Height	Gap1	Radius	Gap2
ouv01	157.31:299.76	1 2 3 4 5	41.66:68.41	1 2 3 4 5 6 7
ouv13	95.38:233.38	1 2 3 4 5	38.98:62.02	1 2 3 4 5 6 7
ouv02	157.36:300.98	1 2 3 4 5	41.66:68.36	1 2 3 4 5 6 7
ouv07	204.56:332.56	1 2 3 4 5	42.02:57.71	1 2 3 4 5 6 7
ouv09	203.09:331.09	1 2 3 4 5	42.04:58.19	1 2 3 4 5 6 7
ouv10	201.1:334.02	1 2 3 4 5	42.07:58.88	1 2 3 4 5 6 7

Scoring rows by min, max of intervals or frequencies or barchart is possible.

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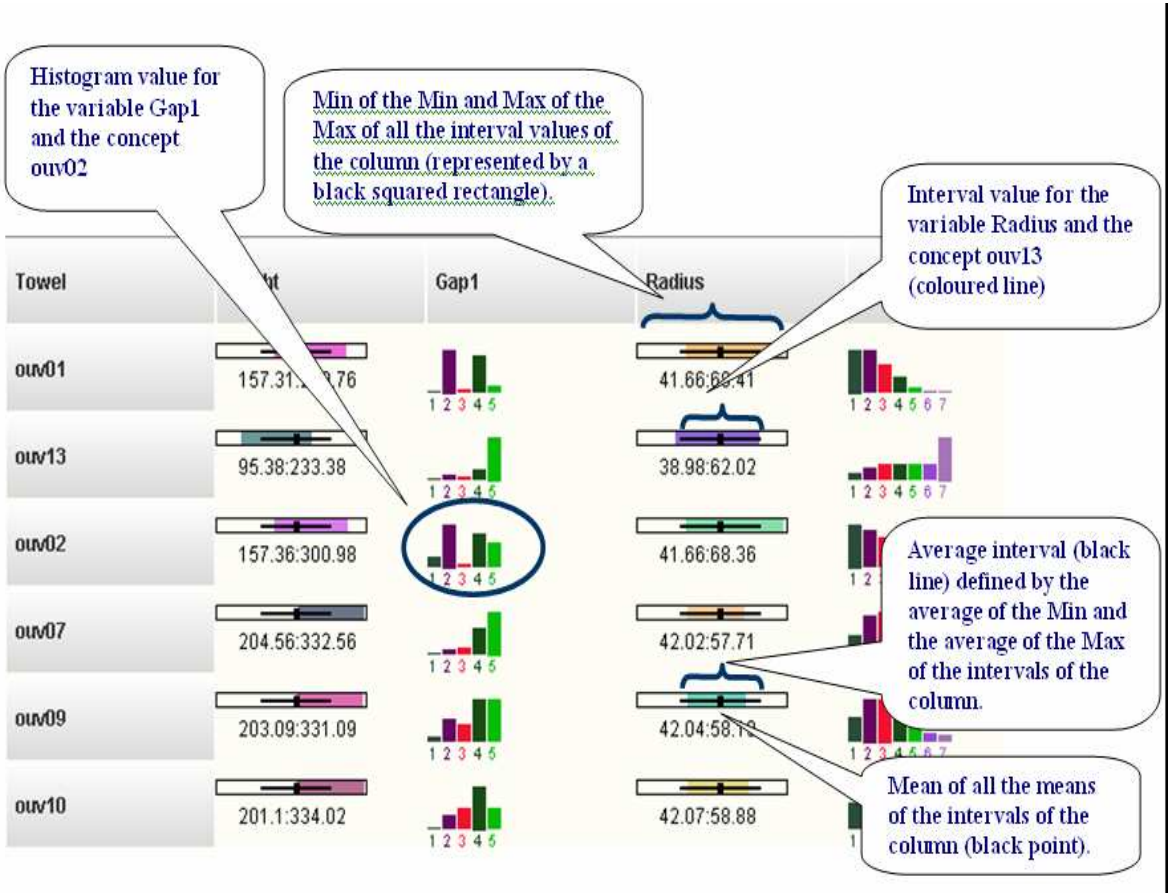
SYMBOLIC DATA TABLE SOFTWARE*



Scoring variables is also possible in order to select the most discriminate variables of the rows

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- Open directions of research.

First step: From Standard Data TABLE 1 To random variables in each cell TABLE 2

Table 1

	Y_1		Y_j	
w_1				
w_i			x_{ij}	
w_n				

In each cell a number or a category

Table 2

	X_1		X_j	
C_1				
C_i			$\mathbf{X_{ij}}$	
C_k				

In each cell a Random Variable $C_i \longrightarrow D_j$

Standard data table: w_i x Y_j

(Observations) x (Random Variables)

$Y_j(w_i) = \mathbf{x}_{ij}$ = a number or a category among Dj the domain of Y_j

Random Variables data table: C_i x X_j

(Classes of Observations) x (Random variables of random variable values)

$X_j\ (C_i) = \mathbf{X_{ij}}$ **is a random variable:**

$\mathbf{X_{ij}}\ (w) = \mathbf{x}$ = a number or a category

RANDOM VARIABLES FROM STANDARD DATA

Table1 : Standard data table

PLAYERS OF THE FRENCH CUP	WEIGHT Y_1	NATIONALITY Y_2	NB OF GOALS Y_3
ZIDANE	80	FRENCH	12

Table 2 : Random Variable data table

TEAM	WEIGHT X_1	NATIONALITY X_2	NB GOALS X_3
MARSEILLES	X_{11}	X_{12}	X_{13}
LYON	X_{21}	X_{22}	X_{23}
PARIS-ST G.	X_{31}	X_{32}	X_{33}
NANTES	X_{34}	X_{42}	X_{43}

➢**Table 1**: Here the variables Y_j (weight, nationality, …) are random variables defined on the players :

$Y_j: \Omega \rightarrow D_j$: $Y_1(\text{Zidane}) = 80$ ➡ Weight(Zidane) = 80

➢**Table 2**: here the variables Xj are random variables defined on the teams with Random Variable Xij as value:

➢$X_j(C_i) = X_{ij}$ where $X_{ij}: C_i \rightarrow D_j$ is a RV: $X_{i1}(\text{Zidane}) = 80$
Weight(Zidane) = 80 if Zidan belongs to C_i

SECOND STEP: From TABLE 2 (Random variable in each cell) To TABLE 3 (Symbolic Data in each cell)

Table 2

	X_1		X_j	
C_1				
C_i			$\mathbf{X_{ij}}$	
C_n				

In each cell a Random Variable $\mathbf{X_{ij}}$: $C_i \longrightarrow D_j$

Table 3

	Y'_1		Y'_j	
C_1				
C_i				
C_k				

In each cell a symbolic data

Random Variables data table: C_i x X_j

(Classes of Observations) x (Random variables of random variable values)

X_j (Ci) = $\mathbf{X_{ij}}$ **is a random variable:**

$\mathbf{X_{ij}}$ (w) = **x** = a number or a category

Symbolic Variables : C_i x Y'_j

(Classes of Observations) x (Random variables of Symbolic values)

Y'_j (Ci) = $\mathbf{H_{ij}}$ **is a barchart value or an interval representing variation inside class C_i for the variable Y_j .**

FROM RANDOM VARIABLE DATA TABLE 2 TO SYMBOLIC DATA TABLE 3

Standard case Table 1: the variables Y_j are random variables with numerical or categorical values.

Symbolic case Table 2: the variables X_j are Random Variables with Random Variables X_{ij} as values.

Symbolic data Table 3: the random variables X_{ij} are represented by:

. Probability densities

. Distributions

. Bar charts

. Inter-quartile intervals

. Parameters (mean square, standard déviation, …)

Symbolic Representation in TABLE 3 of the Random Variables of TABLE 2

TEAM OF THE FRENCH CUP	WEIGHT X_1	NATIONALITY X_2	NB OF GOALS X_3
MARSEILLES	[75 , 89]	{French}	{0.8 (0), 0.2 (1)}
LYON	[80, 95]	{Fr, Alg, Arg }	{0.1 (0), 0.3 (1), …}
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- In Table 2: the random variable X_1 is associated to "Weight", its value for the team "Lyon" is the Random Variable X_{21} defined on the players of Lyon with value a weight inside the Domain D_1 of possible weights.
- The representation of the Random Variable X_{12} is the interquartile interval: [80, 95]
- The representation of the Random Variable X_{13} is the Bar Chart: [0.8 (0), 0.2 (1)]

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WHY SYMBOLIC DATA CANNOT BE REDUCED TO A CLASSICAL DATA TABLE?

Symbolic Data Table

Players category	Weight	Size	Nationality
Very good	[80, 95]	[1.70, 1.95]	{0.7 Eur, 0.3 Afr}

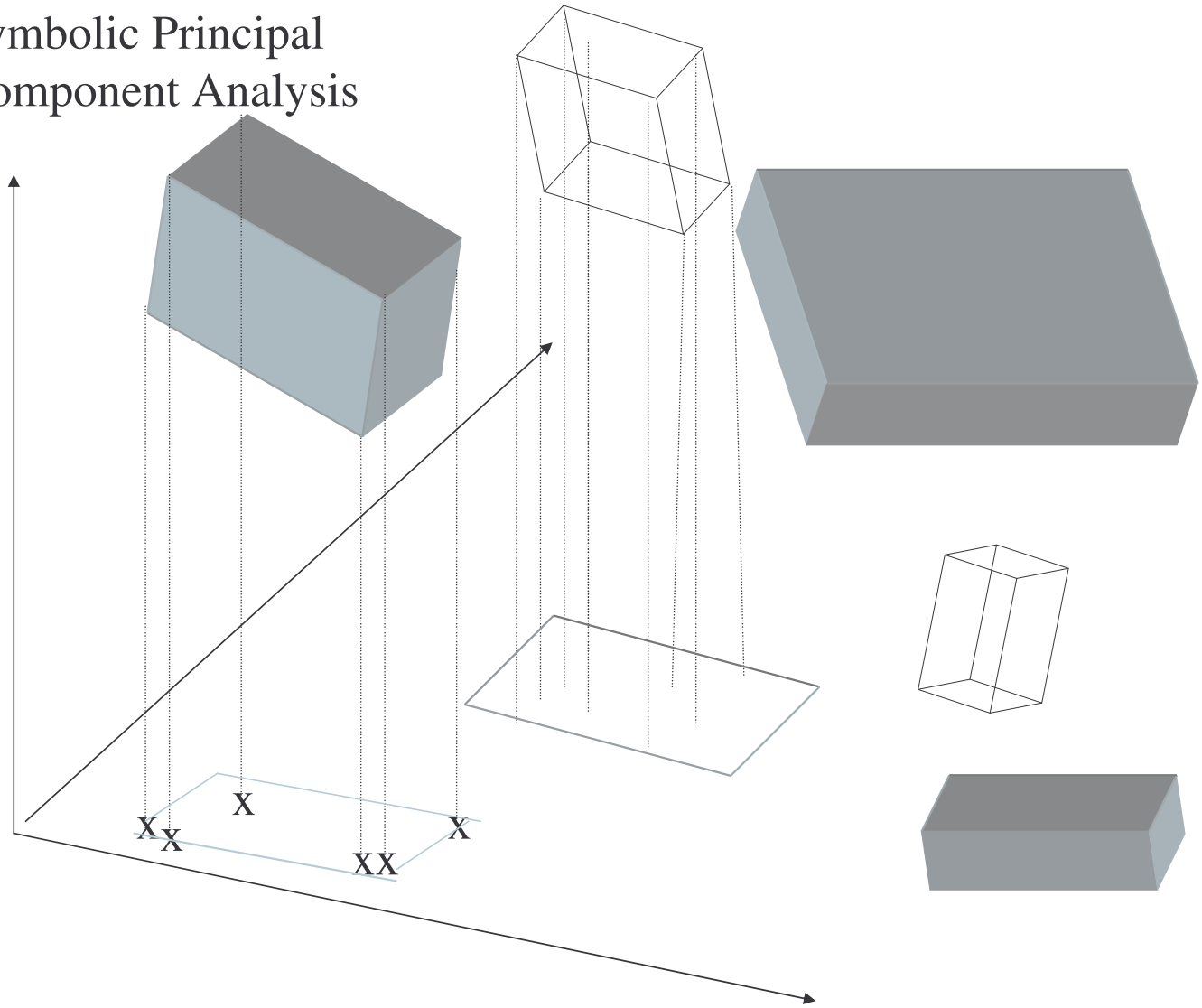
Transformation in classical data

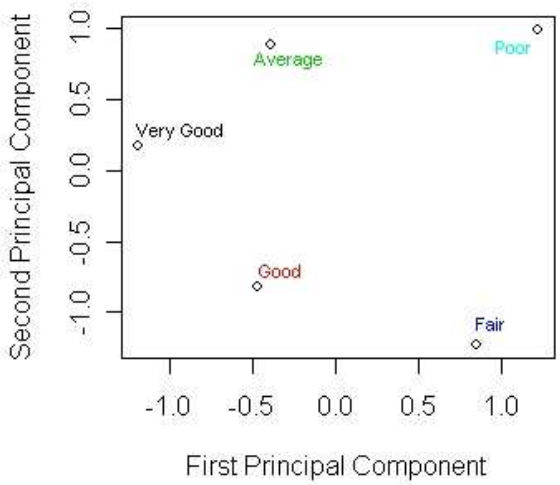
Players category	Weight Min	Weigh t Max	Size Min	Size Max	Eur	Afr
Very good	80	95	1.70	1.95	0. 7	0.3

Concern:

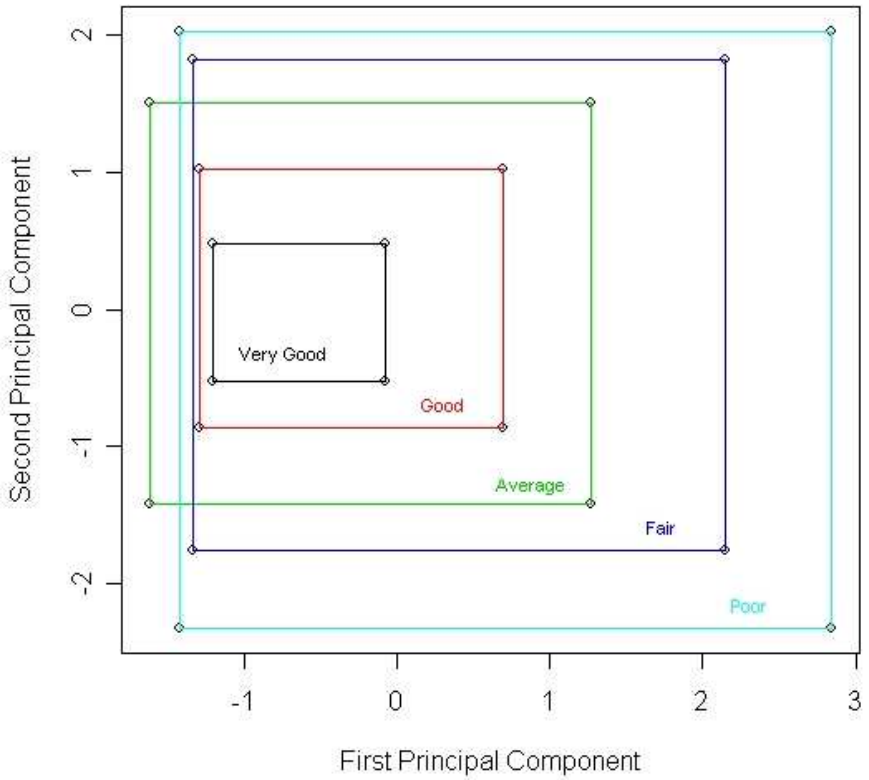
The initial variables are lost and the variation is lost!

Symbolic Principal Component Analysis



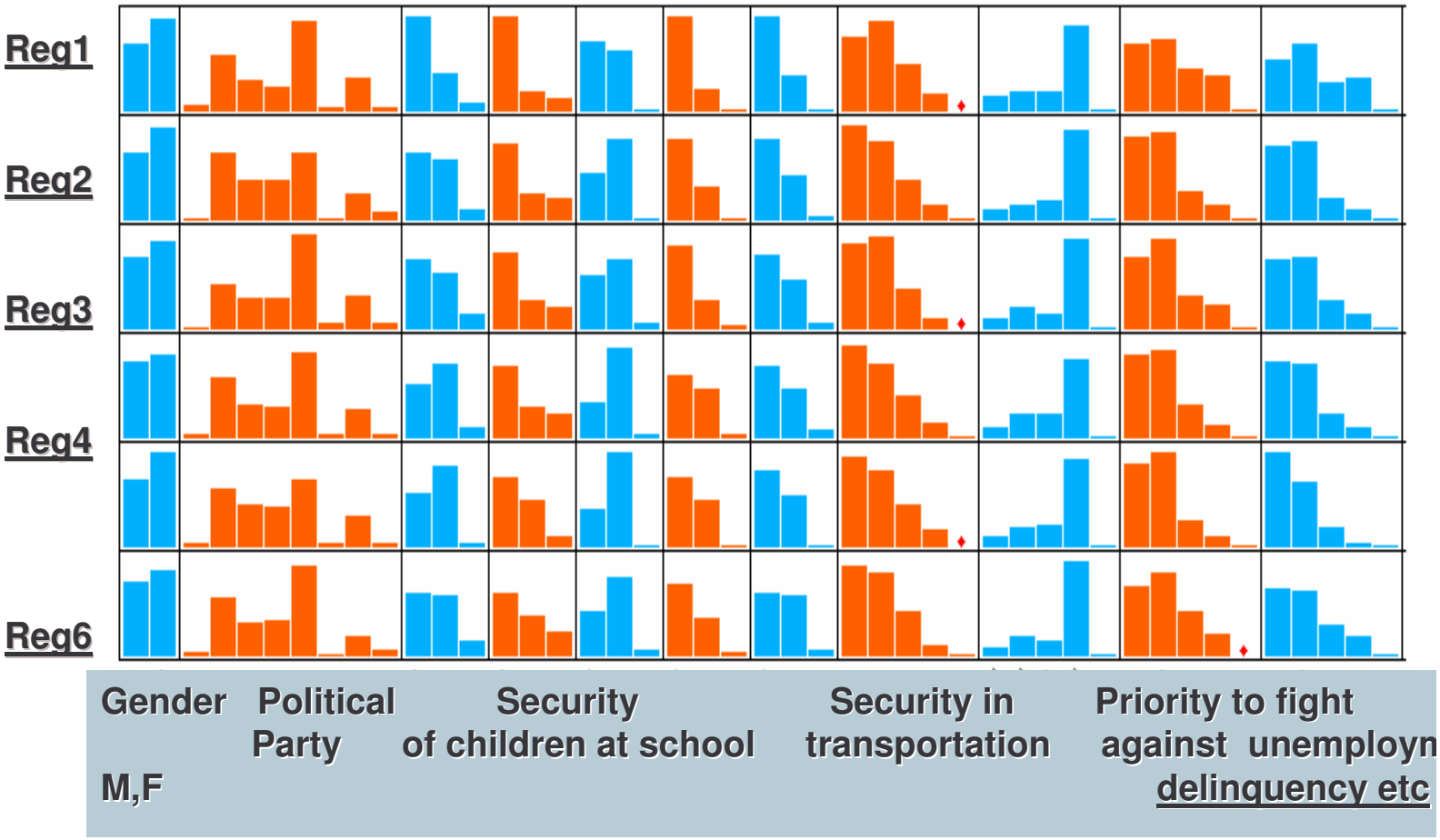


Classical Analysis
Loose variation

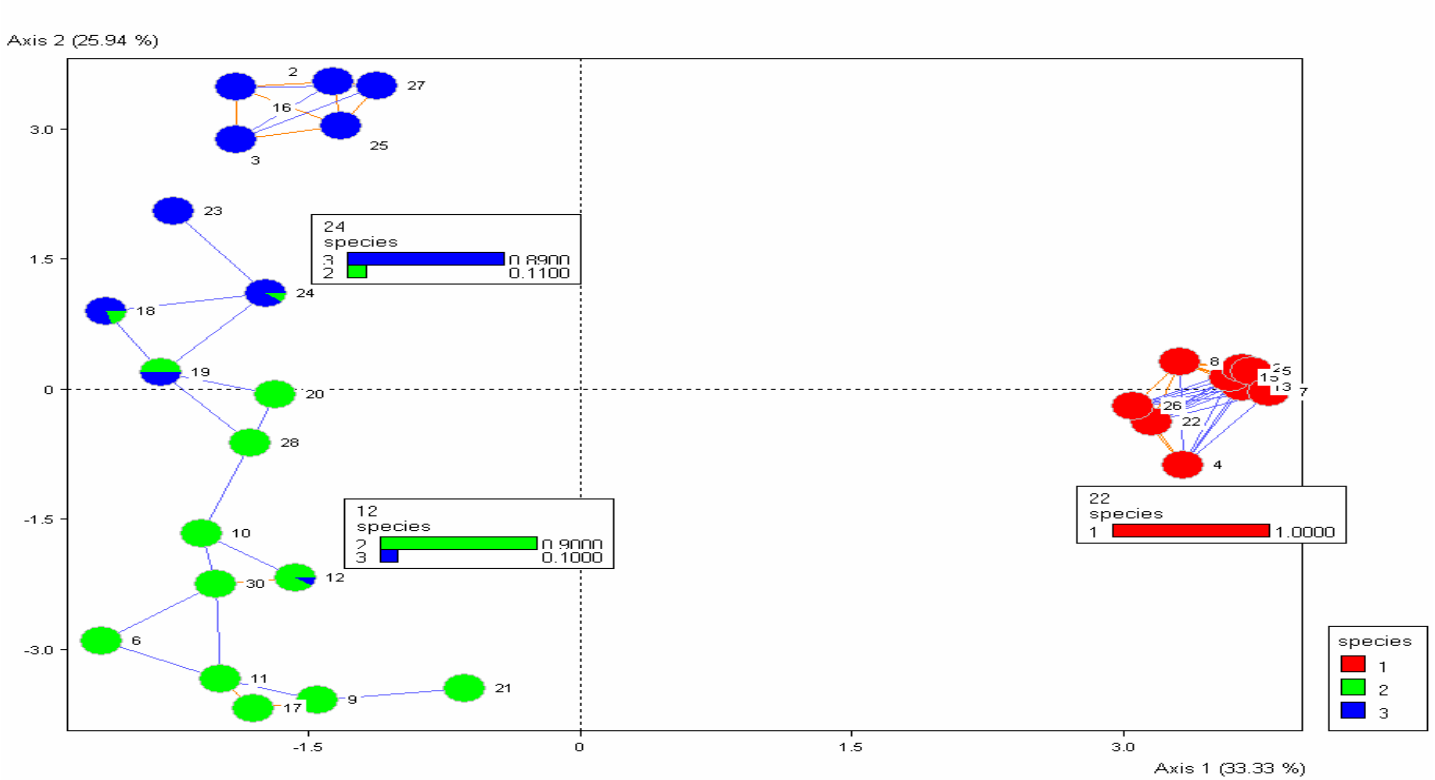


Symbolic Analysis
Take care of
variation

Tackle security problems in regions



PCA and NETWORK OF BAR CHART DATA of 30 Iris Fisher Data Clusters*



Any symbolic variable can be projected. Here the species variable.

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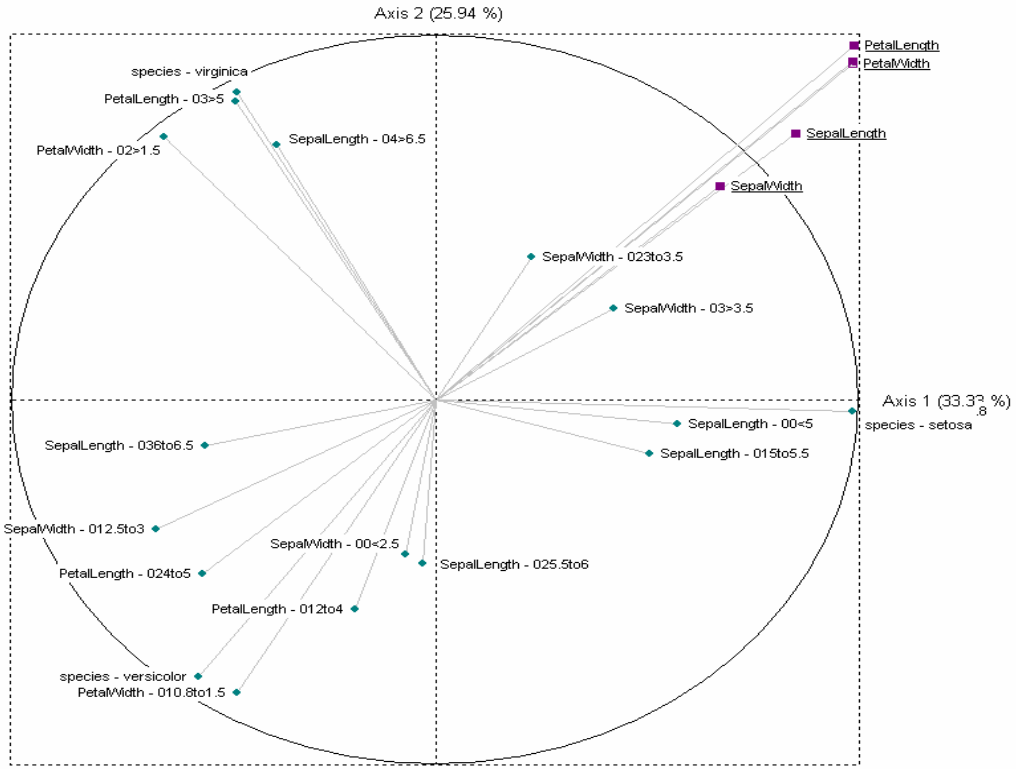
The Symbolic Variables contributions are inside the smallest hyper cube containing the correlation circle of the bins



Conclusion:

Symbolic data are complex data as they cannot be reduced to standard data without loosing much information.

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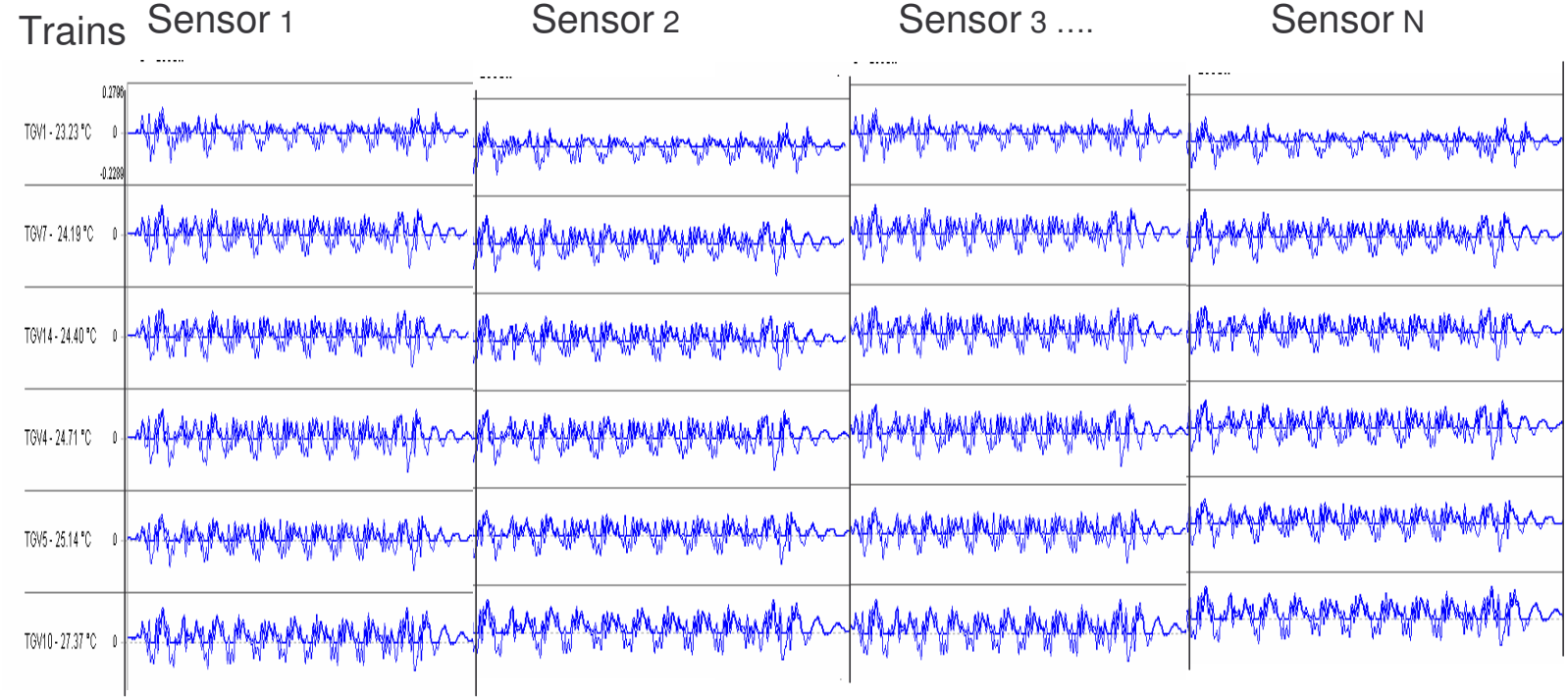
Complex data are Symbolic Data ?

- Time series data table
- Multisource data tables
- Hierarchical data
- Textual Data
- Etc.

CAN BE TRANSFORMED IN SYMBOLIC DATA

Time series data table: Anomaly detection on a bridge
LCPC (Laboratoire Central Des Ponts et Chaussées and SNCF Data



Each row represents a train going on the bridge at a given temperature,

each cell contains until 800.000 values.

Each cell is transformed in HISTOGRAM from a PROJECTION or from WAVELETS

The symbolic aggregation approach

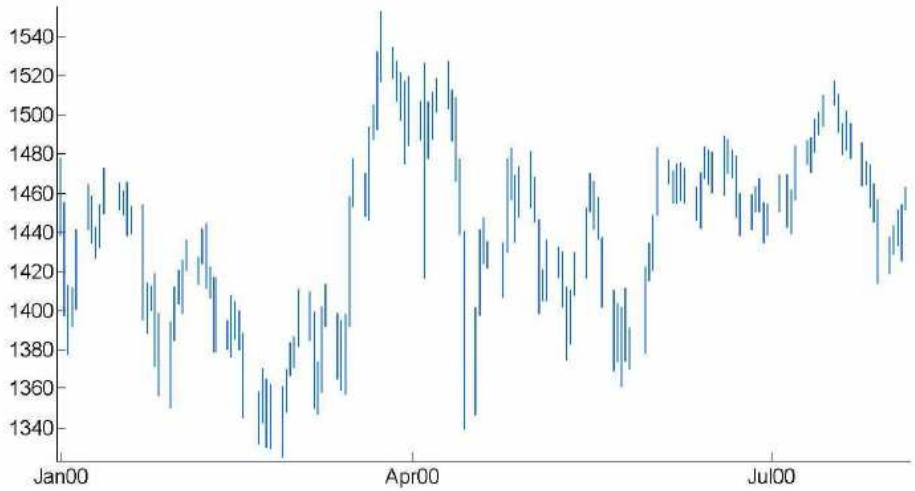


Figure 2: Interval time series of the high and low prices of the SP500 stock index.

Multisource data tables

FRANCE IS DIVIDED INTO 50 097 COUNTIES CALLED IRIS

IRIS are the level to study, initial data are confidential and multisource

Classical Data table

Household	IRIS	Size	Car Mark	SPC
Dupont	IRIS 55	2	Renault	3
Durand	IRIS 602	5	Renault	1
Boule	IRIS 498	3	Peugeot	2

Symbolic description of households in IRIS 1

IRIS	Size	Car Mark	SPC
IRIS 1	[0, 5]	Renault(43%), Citroën (21%)….	

Classical Data table

School	IRIS	TYPE
Condorcet	IRIS 605	Private
Laplace	IRIS 75	Public
Voltaire	IRIS 855	Public

Symbolic description of shools in IRIS 1

IRIS	TYPE	Spécialisation	
IRIS 1	{(private, 37%);(public, 63%)}	{(yes,17%); (no, 83%)}	

Concatenation

IRIS n = [Symb. Description of households] $\wedge$ [Symb. Description of School

NEW DATA: in one SYMBOLIC DATA TABLE describing each IRIS.

Multisource data tables

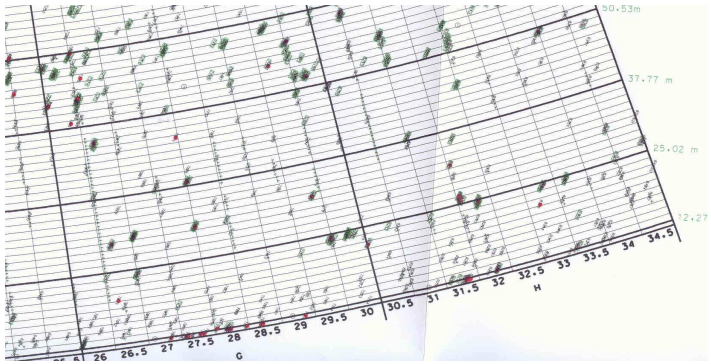
NUCLEAR POWER PLANT Nuclear thermal power station

Inspection :

Inspection machine

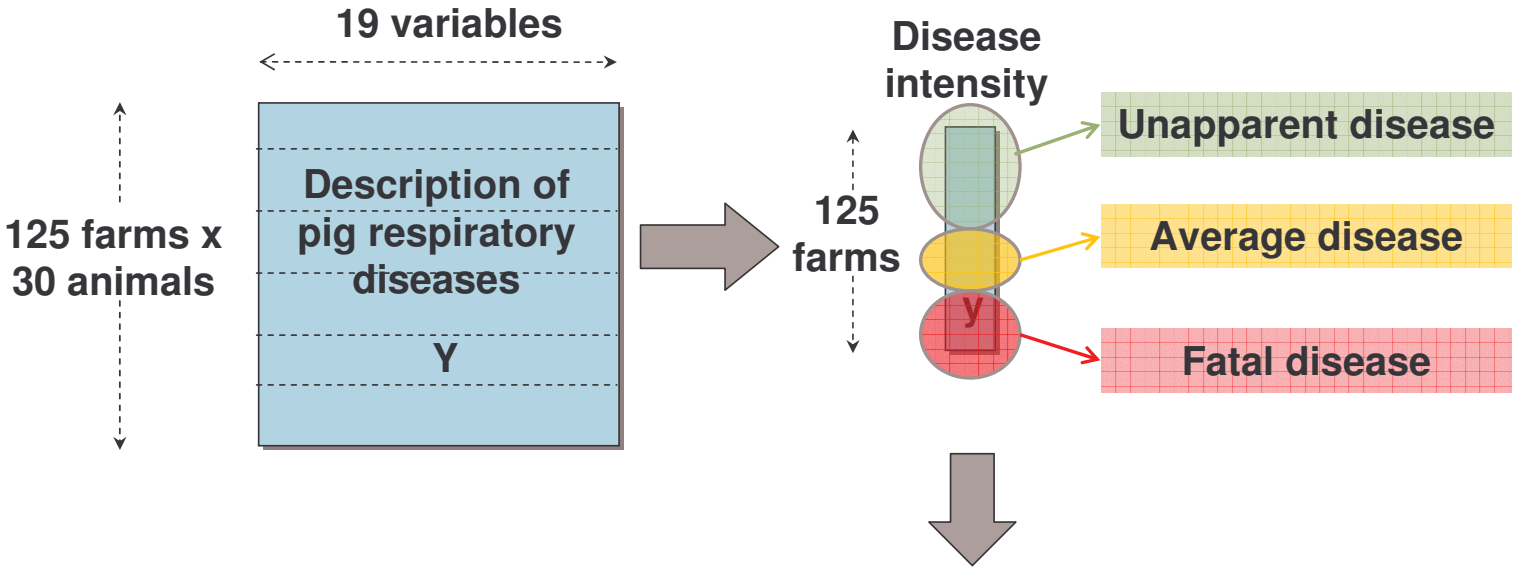
Craks

Cartography of the towel by a grid



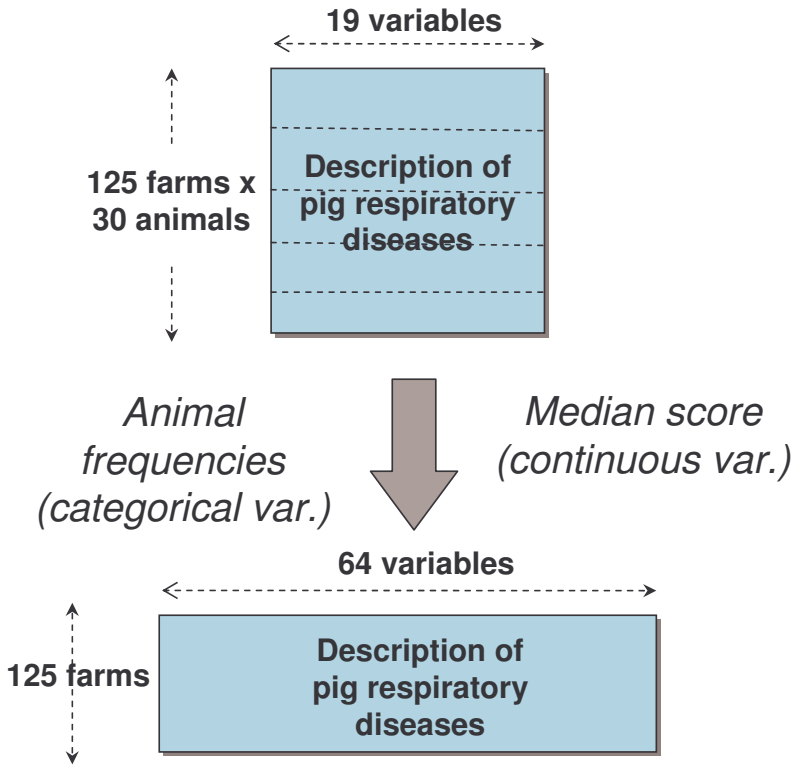
PB: FIND CORRELATIONS BETWEEN 3 CLASSICAL DATA TABLES OF DIFFERENT UNITS AND VARIABLES:
Table 1) Cracks description.
Table 2) Gap deviation of vertices of a grid at different periods compared to the initial model position.
Table 3) Gap depression from the ground.
ARE Transformed in ONE Symbolic Data Table where the concepts are interval of height

Hierarchical-Structured Data.



AFSSA: Study of pig respiratory diseases*

*C. Fablet, S. Bougeard (AFSSA)



Symbolic procedure

From numerical description of pigs to symbolic description of Farms

- Numerical variables

and

- Categorical variables

  are transformed in Bar Chart of the frequencies based on 30 animals,

Or in interval value variables

Step 1: Symbolic Description of Farms*

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- **Conclusion**

  In many cases COMPLEX DATA can be transformed in SYMBOLIC DATA.

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- **The Aim of SYMBOLIC DATA ANALYSIS?**

TO

EXTEND STATISTICS AND DATA MINING TO SYMBOLIC DATA TABLES DESCRIBING HIGHER LEVEL OBSERVATIONS (called "concepts") NEEDING VARIATION IN THEIR DESCRIPTION.

THE 4 MAIN TABLES IN SDA

1. **Standard data table T1:** observations x variables

Ex: players x variables each cell is numerical or categorical: Weight (Zidane) = 80

2. From T1 to **Random data table T2**: teams x variables

each cell is a Random Var. Ex: Weight (Lyon) = X_{21}

3. From T2 to **Symbolic data table T3** : teams x symb variables Ex: Weight (Lyon) = [80,95]

4. From T3 to **Random data table T4**: teams x variables

Example: an interval [80,95] which induces a random variable uniformly distributed on this interval.

RANDOM VARIABLES (T4) from SYMBOLIC DATA (T3)

•When the data are natively symbolic, which means that T1 and T2 are not known (ie only T3 is known) then the random variables X_{ij} of table T4 are defined from the Symbolic Data Table T3.

Examples:

Description of species of trees, mushroom, etc

Description of INSEE IRIS.

FROM SYMBOLIC DATA TABLE (T3) TO RANDOM VARIABLE DATA TABLE (T4)

Symbolic Data Table T3

	Y'_1		Y'_j	
C_1				
C_i				
C_k				

In each cell a symbolic data

Random Variable Data Table T4

	X_1		X_j	
C_1				
C_i			$\mathbf{X_{ij}}$	
C_k				

In each cell a Random Variable $C_i \longrightarrow D_j$

Symbolic Variables : C_i x Y'_j

(Classes of Observations) x (Random variables of Symbolic values)

$Y'j$ (Ci) = **Hij is a barchart value or an interval representing variation inside class C_i for the variable Y_j .**

Random Variables data table: C_i x X_j

(Classes of Observations) x (Random variables of random variable values)

Xj (Ci) = **Xij is a random variable:**

Xij (w) = **x** = a number or a category

From TABLE T4 with random variables in each cell To TABLE T5 with parameters in each cell

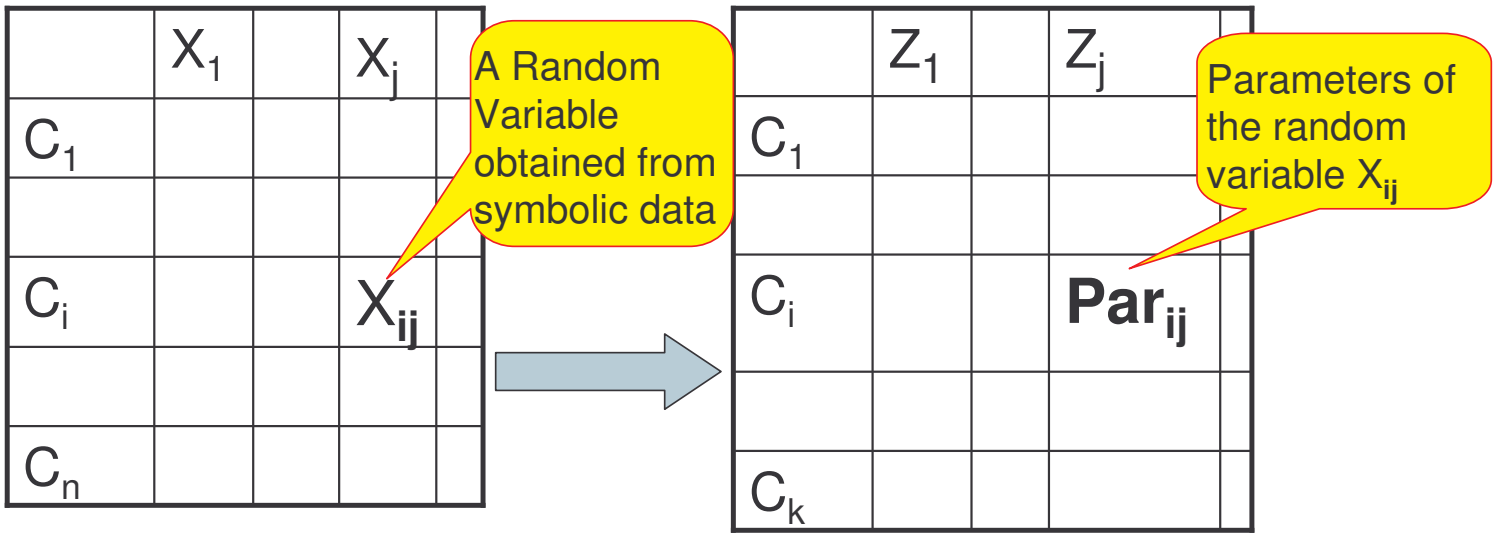


Table T4

Random data table T 4: w_i x X_j

(Classes of Observations) x (Rand Variables)
Xj (Ci) = **Xij is a random variable:**
Xij (w) = **x** = a number or a category

Table 5

Parametric data table 5: C_i x Z_j

(Classes of Observations) x (Random variables of parameter vector value)

Example: $\mathbf{Par_{ij}} = (\mu_{ij}, \sigma_{ij})$

From a random variable X_{ij} of T4 obtained from a Symbolic Data to its parameters of T5

Example: the case of interval symbolic data.

- The interval $[a_{ij} , b_{ij}]$ transformed in the random variable X_{ij} under the uniformity assumption has the following parameters:

➢ $\mu_{ij} = (a_{ij} + b_{ij})/2$

➢ $\sigma_{ij} = (a_{ij} - b_{ij})^{*2} /12$

(more details in Bock Diday (2000 Springer) in Bertrand Goupil chapter)

FOUR OPEN DIRECTION OF RESEARCH IN SDA

1. Non parametric:

Input: Symbolic data table 3

Output: Extending classical data mining to symbolic data table 3.

2. Semi parametric:

Input: Symbolic data table 3

Output: Copulas models from empirical distributions

3. Parametric:

Input: Table 4 of random variables induced from the symbolic data table 3 or Table 2 when the data are not natively symbolic.

Output: Table 5 of the parameters of the random variables of Table 2 or 4 .

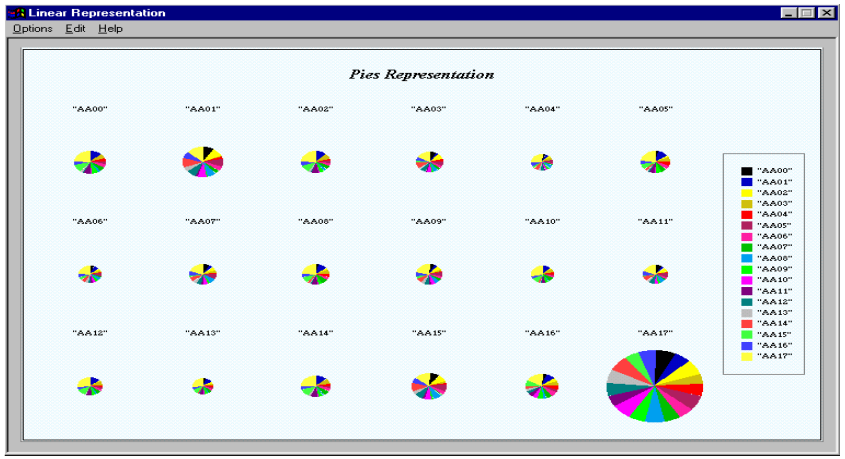
Descriptive statistic of Table 5 under models assumption (Gaussian, Dirichlet, Multinomial, etc.). Mixture decomposition.

4. Stochastic random data tables:

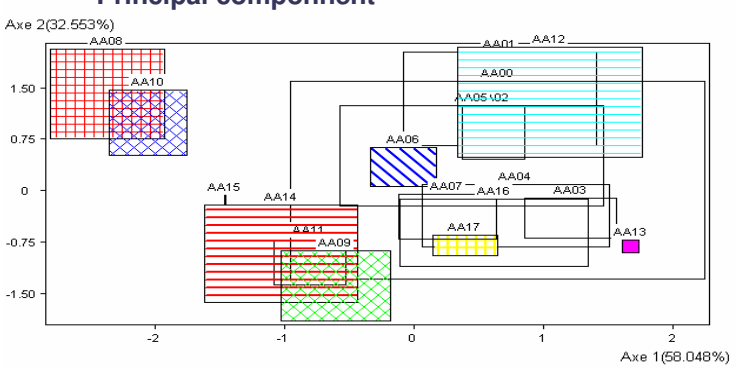
$X_{ij}^{(n)} \longrightarrow X_{ij}$ **for hudge data sets (data streams, cloud computers…) when the number of observations of the Data Table 1 increases does the classification structure (partition, hierarchy, pyramid, Galois Lattice,…) with their symbolic description converges?. (Only started: Stochastic Galois lattices with capacities).**

1. Non parametric: Extending Data Mining to Symbolic Data

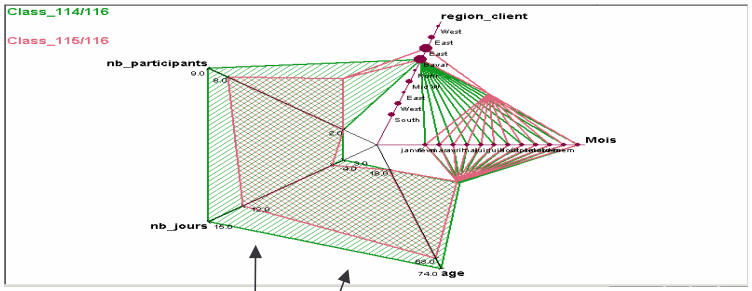
Kohonen map



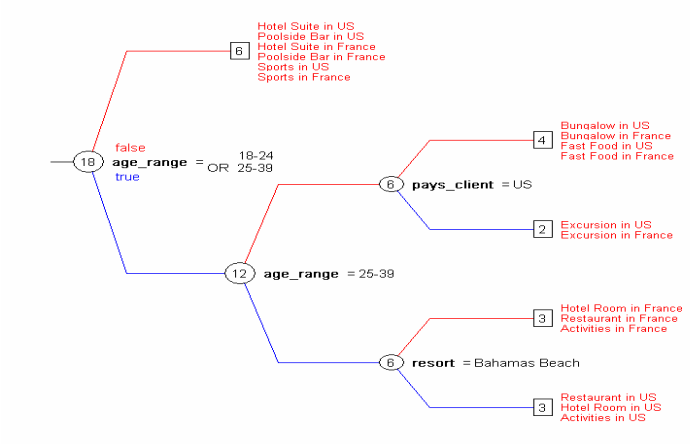
Principal componnent



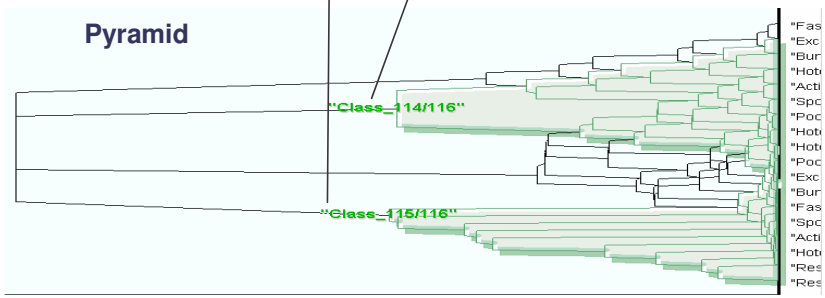
Zoom stars overlapping



Top down clustering tree or decision tree



Pyramid



1 Extending Data Mining to Symbolic Data: SOME RECENT ADVANCES

- PCA of bar chart data

- Symbolic Decision Trees, Regression

- Symbolic Text Mining

- Symbolic Time series

- Mixture Decomposition of symbolic Data by copulas

- Spatial Symbolic Clustering

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2. Semi parametric:

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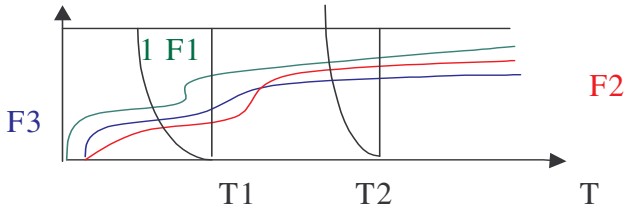
2. Semi parametric:

Input: Symbolic data table 3 where each cell contains a distribution.

Output: Copulas models from empirical distributions

DEFINITION OF A "POINT-DISTRIBUTION OF DISTRIBUTIONS"

$$G_{T_n}(x) = Pr(\{F_i \in F / F_i(T_n) \leq x\}) = card(\{F_i \in F / F_i(T_n) \leq x\})/N$$



DEFINITION OF A "K-POINT JOINT DISTRIBUTION OF DISTRIBUTIONS"

$$H_{T1,\ldots,T_k}(x_1,\ldots,x_k) = Pr(\{F_i \in F / F_i(T_1) \leq x_1\} \wedge \ldots \wedge \{F_i \in F / F_i(T_k) \leq x_k\}).$$

PROPOSITION 1

. G_{T_n} IS A DISTRIBUTION.

. $H_{T1,\ldots,T_k}$ IS A K-DIMENSIONAL JOINT DISTRIBUTION FUNCTION WITH MARGIN

$G_{T1},\ldots, G_{Tk}$

WHAT LINK BETWEEN

. THE JOINT H

. THE MARGINAL G_{Ti} **?**

$H_{T1,\ldots,Tk}(X_1,\ldots,X_k) = \mathbf{C}(G_{T1}(X_1),\ldots. G_{Tk}(X_k))$

C IS A K-COPULAS

. DEFINITION OF C ?

. EXISTENCE ?

. PROPERTIES ? (UNICITY,…)

(Vrac, Cuvelier Dissertation…)

FOUR OPEN DIRECTION OF RESEARCH IN SDA

1. Non parametric:

Input: Symbolic data table 3

Output: Extending classical data mining to symbolic data table 3.

2. Semi parametric:

Input: Symbolic data table 3

Output: Copulas models from empirical distributions

3. Parametric:

Input: Table 4 of random variables induced from the symbolic data table 3 or from Table 2 when the data are not natively symbolic.

Output: Table 5 of the parameters of the random variables of Table 2 or 4 .

Descriptive statistic of Table 5 under models assumption (Gaussian, Dirichlet, Multinomial, etc.). Mixture decomposition, Biclassification, Partitioning, Galois Lattice, Pyramid , hierarchy

4. Stochastic random data tables:

$X_{ij}^{(n)} \longrightarrow X_{ij}$ **for hudge data sets (data streams, cloud computers…) when the number of observations of the Data Table 1 increases does the classification structure (partition, hierarchy, pyramid, Galois Lattice,…) with their symbolic description converges?. (Only started: Stochastic Galois lattices with capacities).**

3. Parametric: Statistical description of Table 5

RECALL:

Table 1 Players x standard Random Variables (RV), Table 2 Teams x (RV of RV),

Table 3 Teams x Symbolic Data (SD) Example: Lyon Weight [80,95],

Table 4 Teams x RV Example: Uniform RV associated to the interval [80,95],

Table 5 Each cell contains the parameters of the T4 Random Variables,

Example: mean = (80 + 95)/ 2 , mean square = $(95 - 80)^2 / 12$

EXAMPLE of descriptive statistic of Table 5:

Sample Mean, Sample Variance (Bertrand and Goupil (2000))

For an interval-valued random variable Y, the symbolic sample mean is given by

$$\bar{Y} = \frac{1}{2m} \sum_{u \in E} (b_u + a_u)$$

and the symbolic sample variance is given by

$$S^2 = \frac{1}{3m} \sum_{u \in E} (b_u^2 + b_u a_u + a_u^2) - \frac{1}{4m^2} [\sum_{u \in E} (b_u + a_u)]^2,$$

for observations $Y_u = [a_u, b_u), u = 1, \ldots, m$

INFERENCE ON THE PARAMETRIC DATA TABLE 5

Sample mean, variance, correlation of parametric data table 5 follow which model?

- Infering Models on mean, mean square, correlation ,
- Finding models on classes:
  - Mixture decomposition,
  - Biclassification…
  - Lattices, Pyramids, hierarchies...

CONCLUSION

- Symbolic Data (SD) are complex data as they cannot be transformed in standard data.
- Complex data can be transformed in SD.
- Therefore SDA is a tool for extracting knowledge from classes of standard data or from complex data.
- Much open research for non parametric or parametric SDA from the symbolic data or from their induced random variable distributions.
- Much to be done for stochastic symbolic data for cloud computers

THREE SDA Books

WILEY, 2008
"Symbolic Data Analysis and the SODAS software." 457 pages
E. Diday, M. Noirhomme , (www.wiley.com)

WILEY, 2006
L. Billard , E. Diday "Symbolic Data Analysis, conceptual statistic and Data Mining".www.wiley.com

SPRINGER, 2000 :
"Analysis of Symbolic Data"
H.H., Bock, E. Diday, Editors . 450 pages.

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