Continuous Wavelet Transform and the Annual Cycle in Temperature and the Number of Deaths

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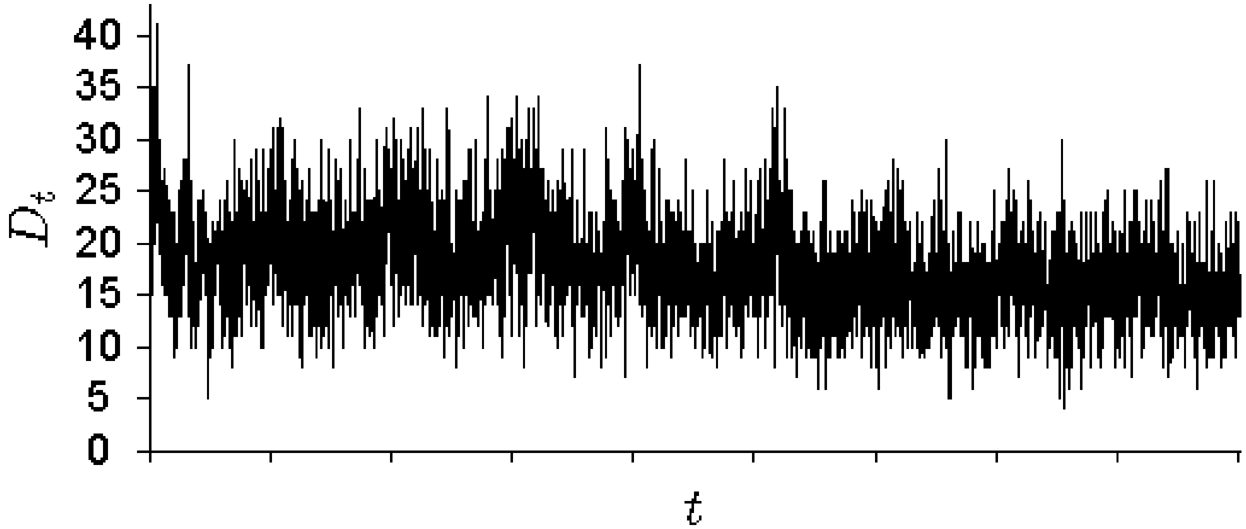
Compstat 2010

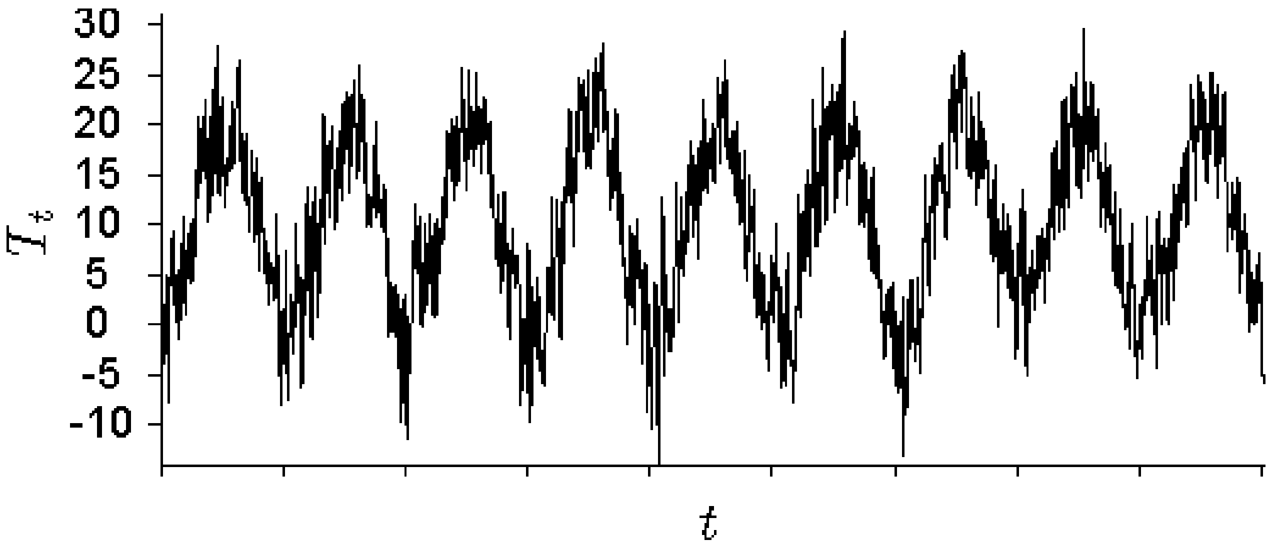
The time series I



The daily time series of the number of deaths due to cardiovascular diseases in Prague, Czech Republic, Jan 2001 – Dec 2008.

Data provided by the Czech Statistical Office.

The time series II



The daily time series of the average temperature in Prague, Czech Republic, Jan 2001 – Dec 2008.

Data provided by the Czech Hydrometeorological Institute.

Wavelets I

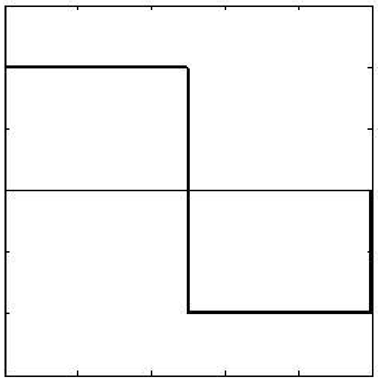
WAVELET (Wave + let = small wave):

$$\int_{-\infty}^{\infty} \psi(t)\mathrm{d}t = 0, \qquad \int_{-\infty}^{\infty} |\psi(t)|^2 \mathrm{d}t = 1$$

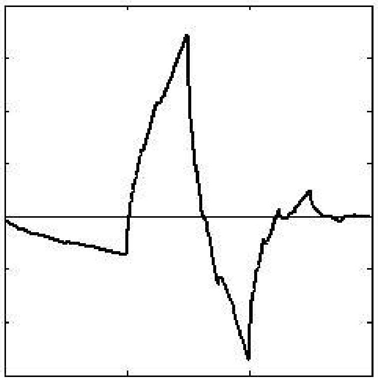
Wavelet is a small wave

Wavelet is effectively nonzero only inside a finite interval

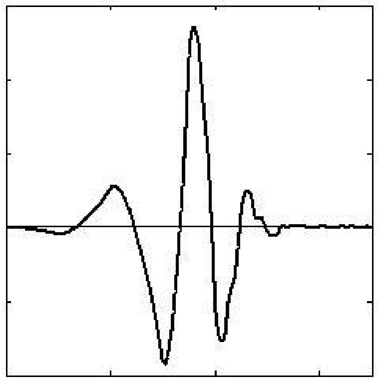
Examples of a possible choice of ψ:



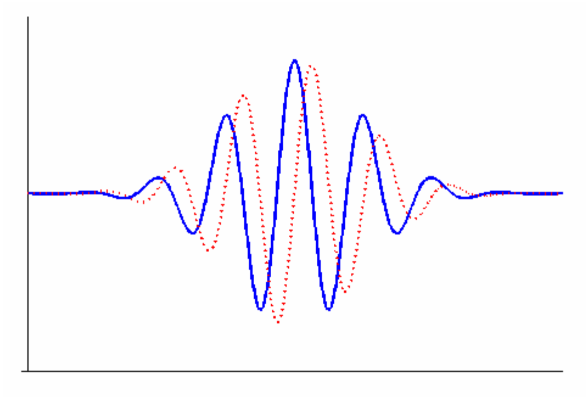
Haar



Daubechies 4



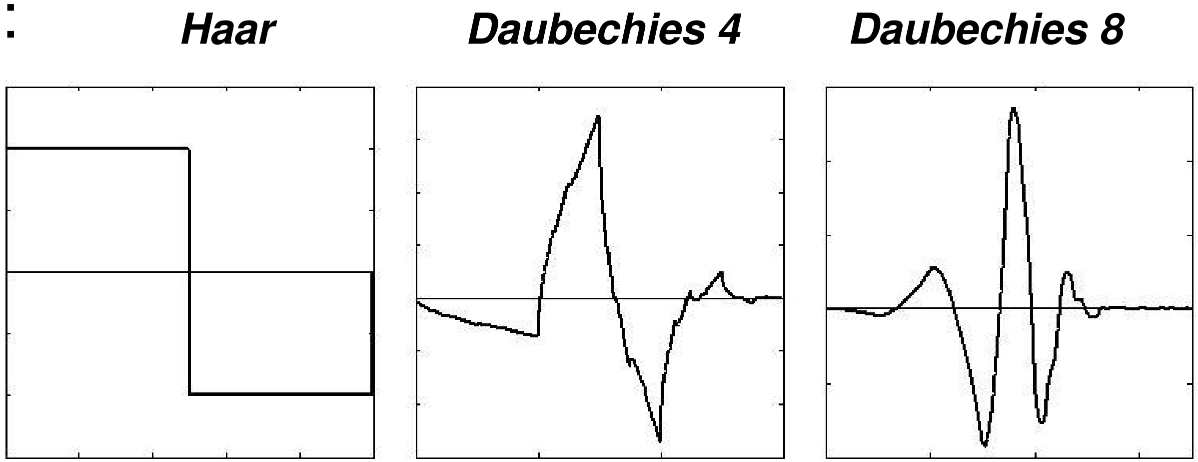
Daubechies 8



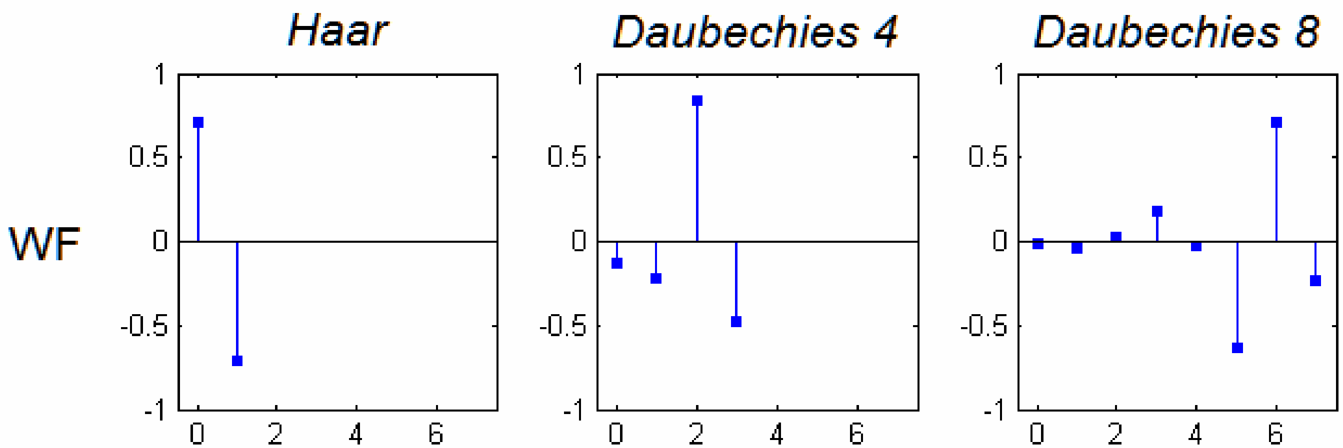
Morlet

Wavelets II

“Continuous” case:



“Discrete” case:

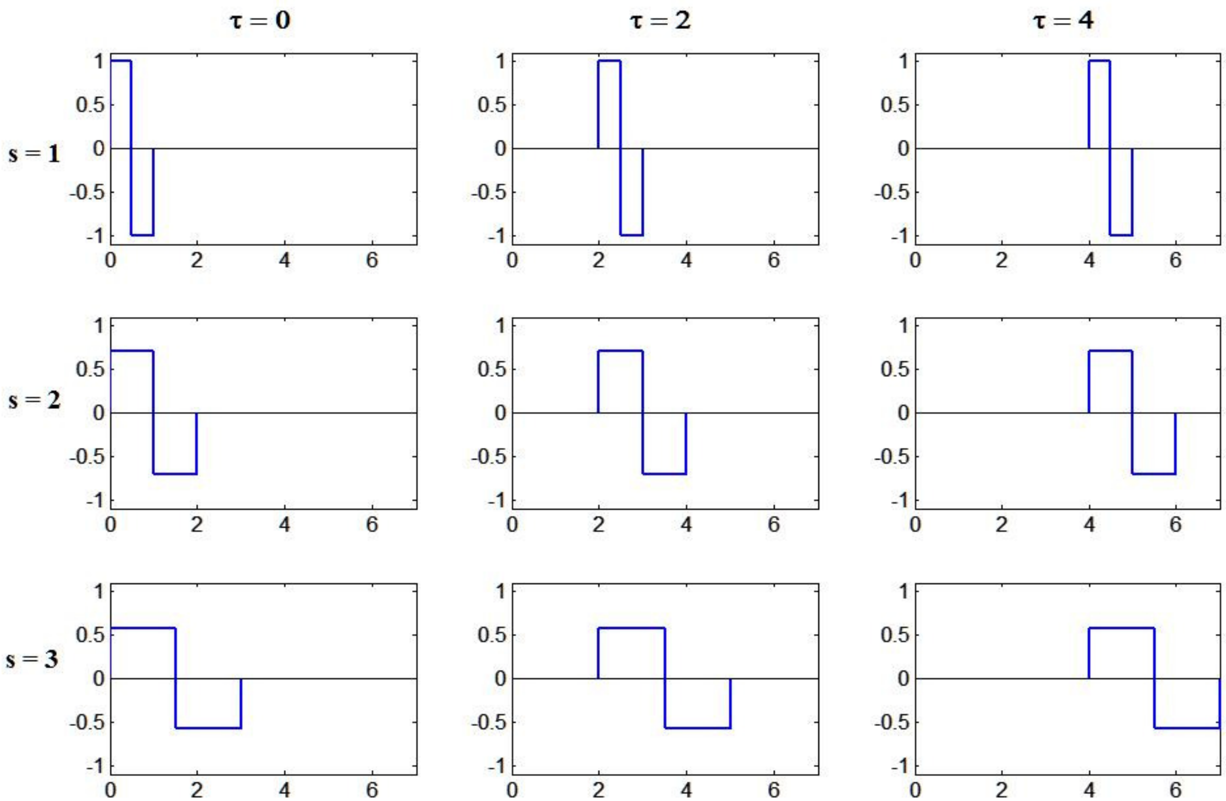


Daughter wavelets

Daughter wavelets → $$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)$$

Shift in time (→ τ)

scaling (→ s)



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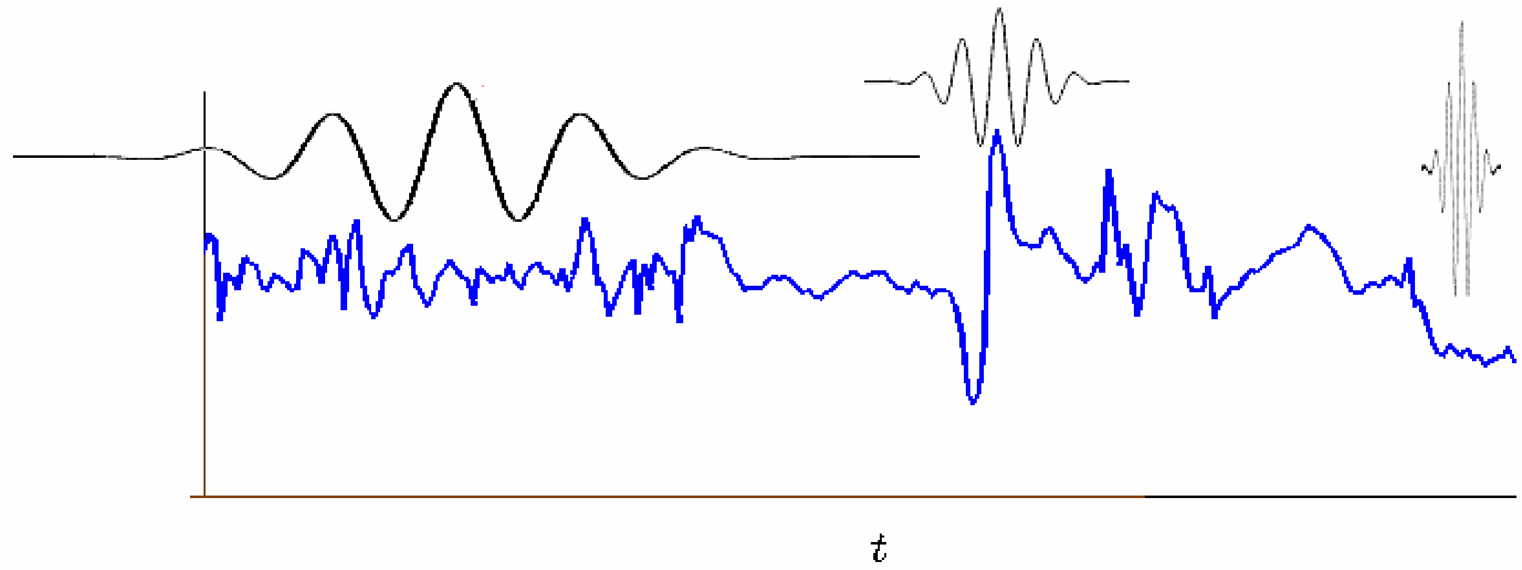
Continuous wavelet transform (CWT)

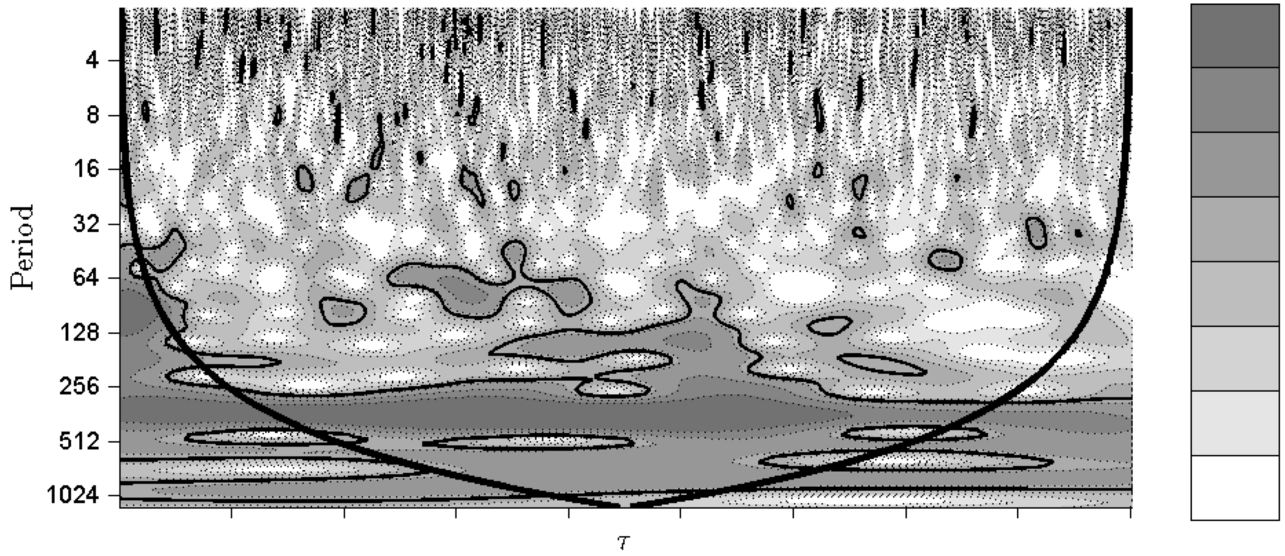
Coefficients of the continuous wavelet transform

Analyzed time series

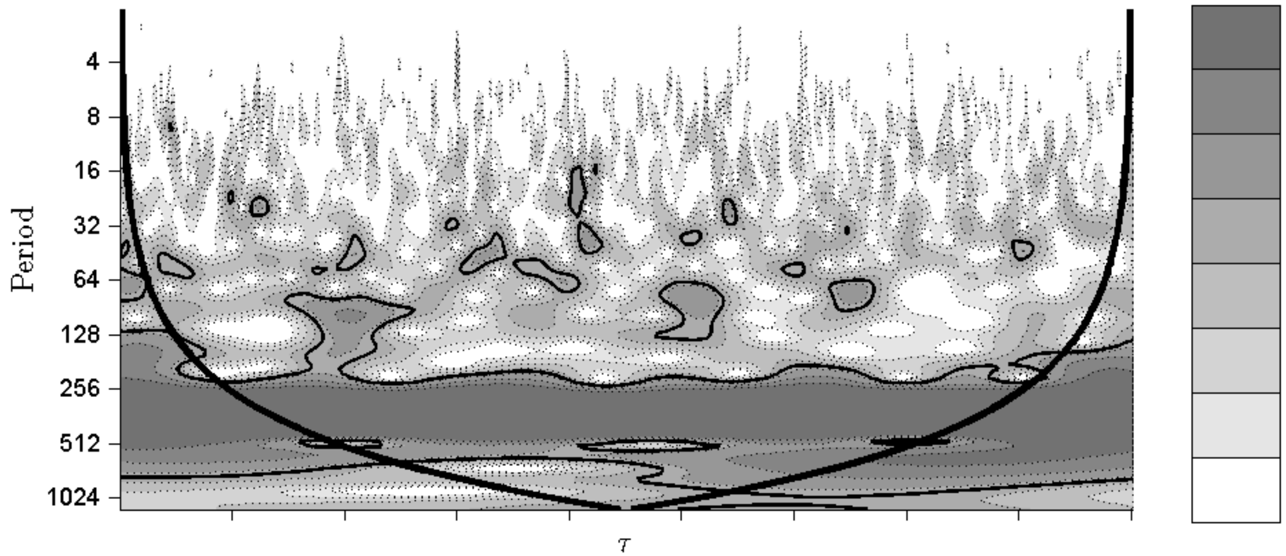
Daughter wavelets

$$W(\tau, s) = \sum_{t=0}^{N-1} x_t \psi_{\tau,s}(t)$$





$$\frac{|W_x(\tau,s)|^2}{\hat{\sigma}_x^2}$$



$$\frac{|W_y(\tau,s)|^2}{\hat{\sigma}_y^2}$$

Cross-wavelet transform

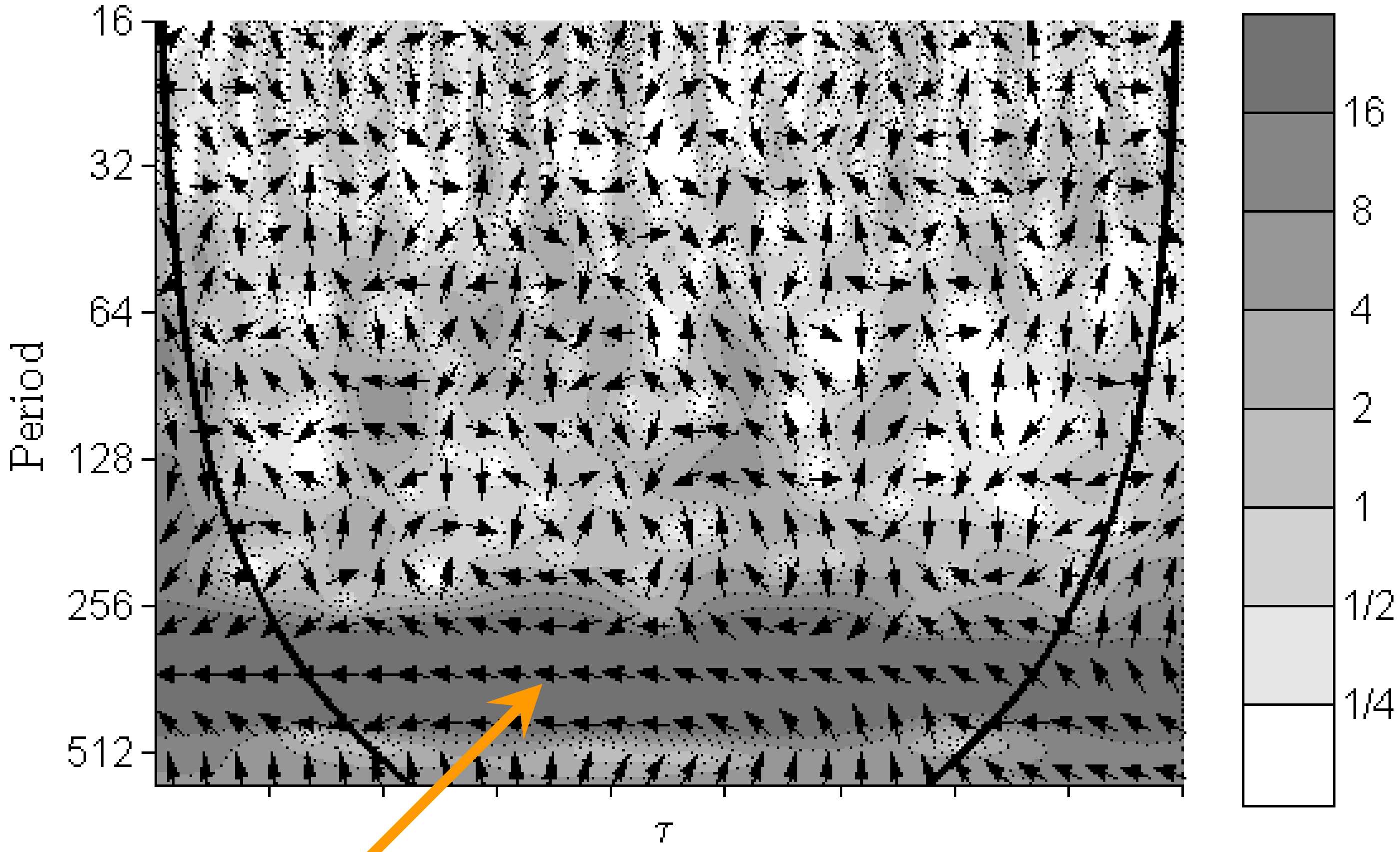
$$W_{xy}(\tau,s)=W_x(\tau,s)W_y(\tau,s)$$

$$W_{xy}(\tau,s)=\left|W_{xy}(\tau,s)\right|\arg W_{xy}(\tau,s)$$

Cross-wavelet power:

Local measure of comovement of two time series in time and scale

Local phase between the time series in time and scale



Strongest comovement between the time series occurs in the annual cycle with ANTI-PHASE relation.

$$\frac{|W_{xy}(\tau,s)|^2}{\hat{\sigma}_x \hat{\sigma}_y}$$

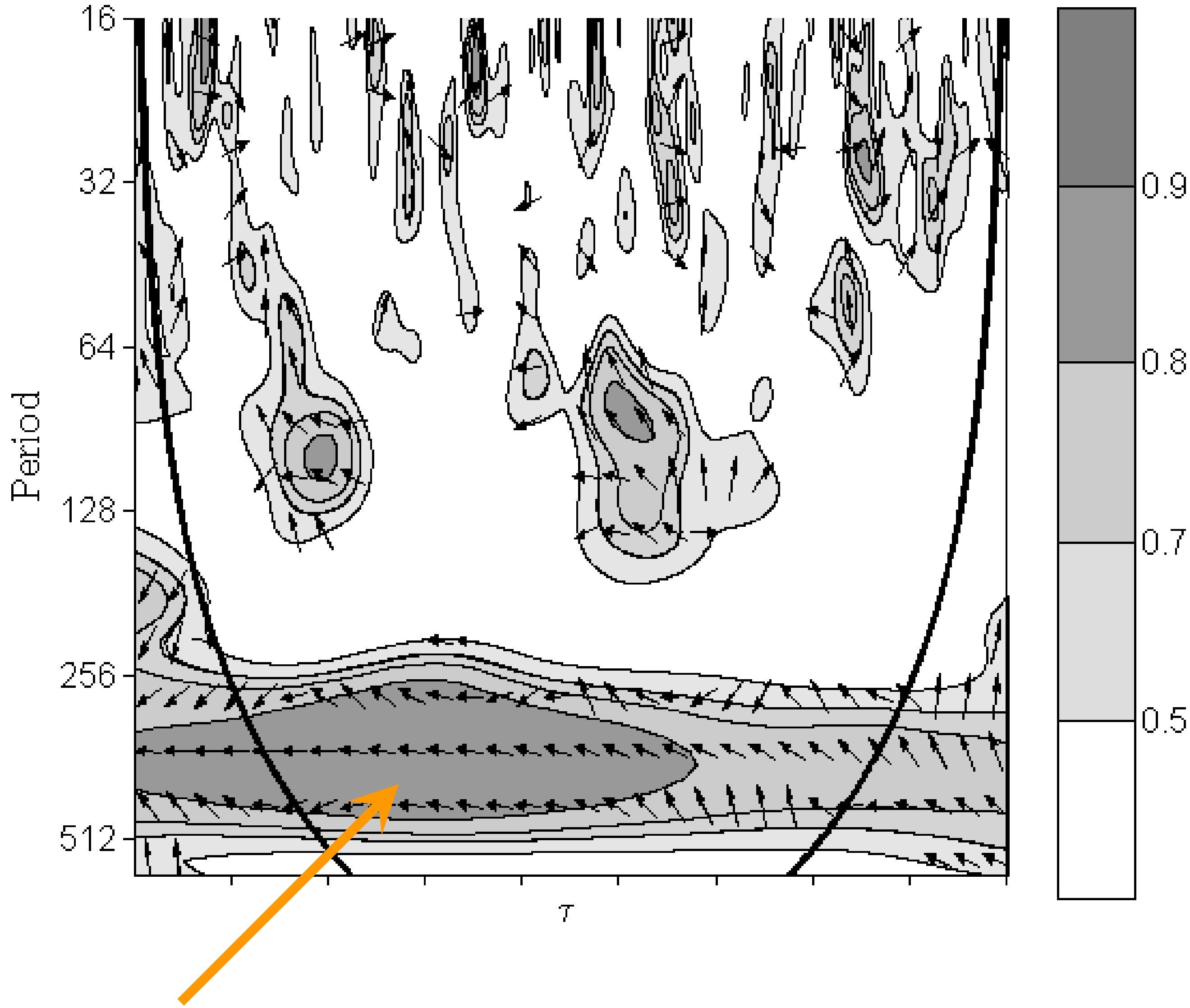
Wavelet squared coherence

Smoothing in time and scale

$$R^2(\tau,s) = \frac{\left|S(s^{-1}W_{xy}(\tau,s))\right|^2}{S(s^{-1}\left|W_x(\tau,s)\right|^2)S(s^{-1}\left|W_y(\tau,s)\right|^2)}$$

Wavelet squared coherence:

Might be informally interpreted as the square of the local correlation coefficient.



Negative correlation between the time series in the annual cycle region.

Conclusions I

- The relationship between the daily time series of the deaths due to cardiovascular diseases in Prague, Czech Republic and the daily time series of the average temperature in Prague, Czech Republic is very complex.

- The continuous wavelet transform enables to uncover this complex structure

- Both the number of deaths and temperature exhibit pronounced annual components.

Conclusions II

- These annual components are negatively correlated – high temperature in summer implies lower number of deaths, whereas low temperature in winter implies higher number of deaths.

- In other frequency regions the correlations are positive and transient in nature. For example, a positive correlation of components is present in the regions of periods between 16 and 32 days – where an increase in temperature implies an increase in the number of deaths.

- The relationship between the time series is a function of time and frequency. Further research is necessary to provide detailed explanation for such a behavior.

References

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Acknowledgements

• We acknowledge the support of the Grant Agency of the Czech Republic No. 402/09/0369, Modeling of Demographic Time Series in the Czech Republic.

• We acknowledge the use of data provided by the Czech Statistical Office and the Czech Hydrometeorological Institute.

• Crosswavelet and wavelet coherence software were provided by A. Grinsted.