19th International Conference
on Computational Statistics

Paris - France, August 22-27

Empirical Mode Decomposition for Trend Extraction. Application to Electrical Data

Jean-Michel Poggi
Laboratoire de Mathématique, Université d'Orsay, France

Joint work with *Farouk Mhamdi, Mériem Jaïdane-Saïdane*
U2S, ENIT, Tunis, Tunisia

Outline

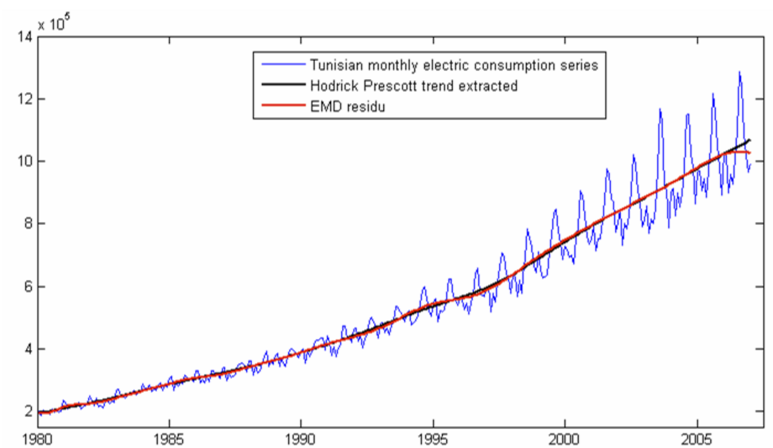
- *Motivation: Blind Trend Extraction*
- ***Empirical Mode Decomposition (EMD)***
- ***EMD trend vs. Hodrick Prescott (HP) trend***
- *Simulated seasonal series*
- ***Tunisian daily peak load 2000-2006***
- *EMD vs. HP, wavelet trends*

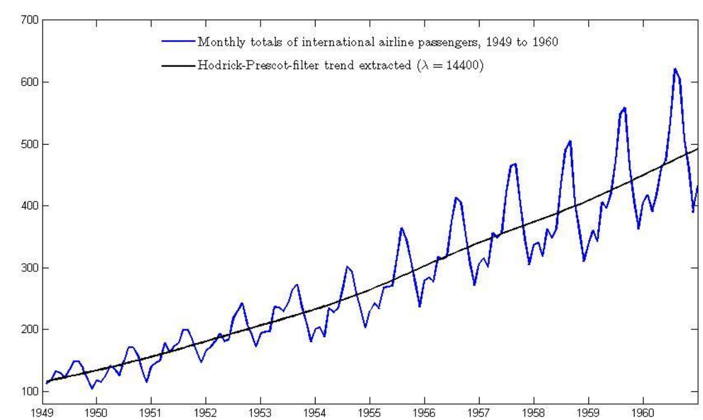
This work was supported by 2005/2009 VRR research project between ENIT and Tunisian Society of Electricity and Gas (STEG)

Trend?

- Trend = some “smooth additive component that contain information about global change” ***Alexandrov et al. (2009)***
- The problem: to ***extract trends from seasonal time series*** without strong modeling and global estimation

- Ex: long term electricity load or airline traffic forecasting





Reference : Box, G. E. P., Jenkins, G. M. and Reinsel, G. C. (1976) Time Series Analysis, Forecasting and Control. Third Edition. Holden-Da

Seasonal time series intrinsic trend

- *Additive time series model*:

  $$Y(t) = T(t) + SC(t) + e(t)$$

  i.e. ***trend*** + seasonal-cycles + error

- Several methods are used for time series components extraction including
  - local or global regressions
  - MA filtering, X11, X12
  - Hodrick Prescott filter

  see ***Alexandrov et al. (2009)*** for a recent review

- The idea: to ***directly extract trend without identifying the other components*** of the observed signal

EMD? EMD and trend?

- EMD transformation is nonlinear and suitable for non-stationary signals ***Huang et al. (2004)***
- Signal is decomposed as a superposition of local sums of oscillatory components called *Intrinsic Mode Functions (IMF)* of different time scales intrinsic to the signal

$$Y(t) = \sum IMF_k(t) + r(t)$$

with ***IMF*** = function with zero mean and having as many zero crossings as maxima or minima

and ***r*** = a monotone function

- IMFs are fully data-driven and local in time
- After the IMF extraction process (sifting) , it remains

r(t) a monotone function candidate to be an estimate of T(t) since it is a trend free of oscillatory components

EMD extraction algorithm

Residue = s(t), $I_1(t)$ = Residue

i = 1, k = 1

while **Residue not equal zero or not monotone**

while **I_i has non-negligible local mean**

U(t) = spline through **local maxima** of I_i

L(t) = spline through **local minima** of I_i

Av(t) = 1/2 (U(t) + L(t))

$I_i(t) = I_i(t) - Av(t)$, i = i + 1

end

$IMF_k(t) = I_i(t)$

Residue = Residue - IMF_k

k = k+1

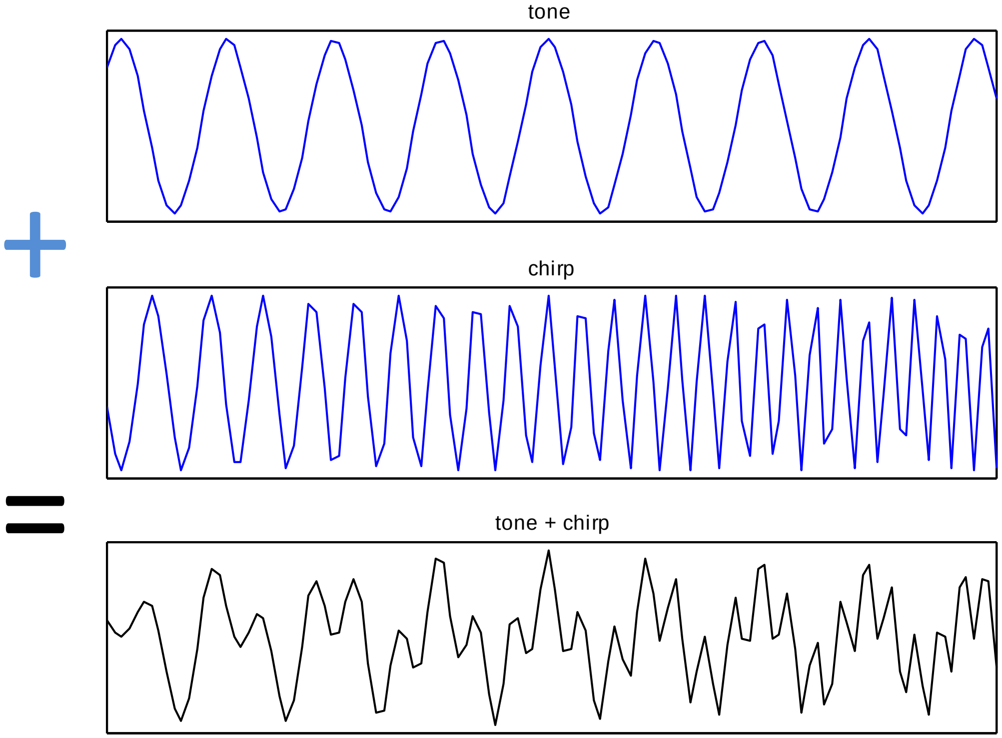
end

IMF?

Credit:
Suz Tolwinski
University of Arizona
Program in Applied Mathematics
Spring 2007 RTG

EMD acting on an example

Analyzed signal = ***tone + chirp***



Credit: Rilling and Flandrin
from http://perso.ens-lyon.fr/patrick.flandrin/emd.html

EMD acting on an example

iteration 0 - *Analyzed signal = starting point*

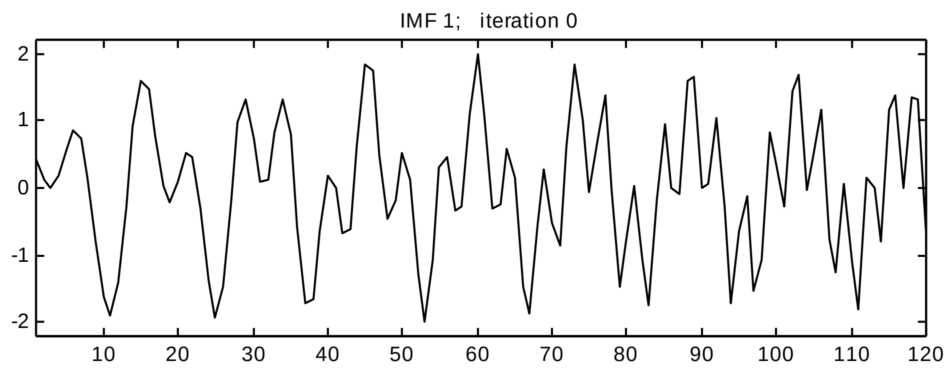
Residue = s(t)

$I_1(t)$ = Residue

i = 1

k = 1

while Residue not equal zero or not monotone
    while I_i has non-negligible local mean
        U(t) = spline through local maxima of I_i
        L(t) = spline through local minima of I_i
        Av(t) = 1/2 (U(t) + L(t))
        $I_i(t) = I_i(t) - Av(t)$
        i = i + 1
    end
    $IMF_k(t) = I_i(t)$
    Residue = Residue - IMF_k
    k = k+1
end



EMD acting on an example

Extract local maxima

Residue = s(t)

$I_1(t)$ = Residue

i = 1

k = 1

while Residue not equal zero or not monotone

while I_i has non-negligible local mean

U(t) = spline through local maxima of I_i

L(t) = spline through local minima of I_i

Av(t) = 1/2 (U(t) + L(t))

$I_i(t) = I_i(t)$ - Av(t)

i = i + 1

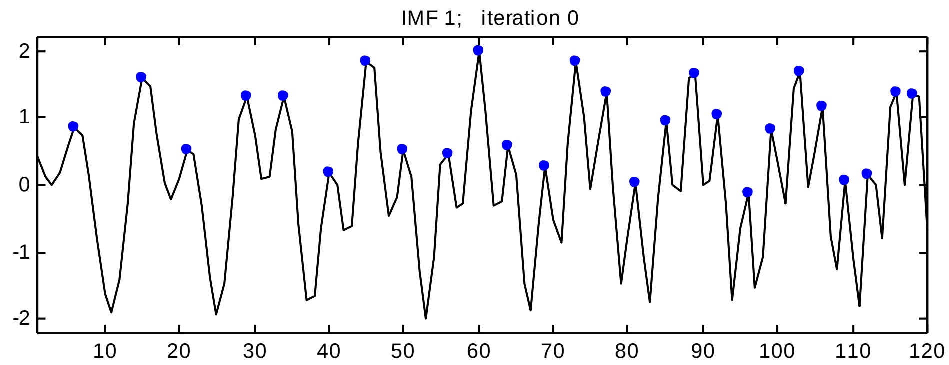
end

$IMF_k(t) = I_i(t)$

Residue = Residue - IMF_k

k = k+1

end



EMD acting on an example

Maxima envelope by interpolation

Residue = s(t)

$I_1(t)$ = Residue

i = 1

k = 1

while Residue not equal zero or not monotone

while I_i has non-negligible local mean

U(t) = spline through local maxima of I_i

L(t) = spline through local minima of I_i

Av(t) = 1/2 (U(t) + L(t))

$I_i(t) = I_i(t)$ - Av(t)

i = i + 1

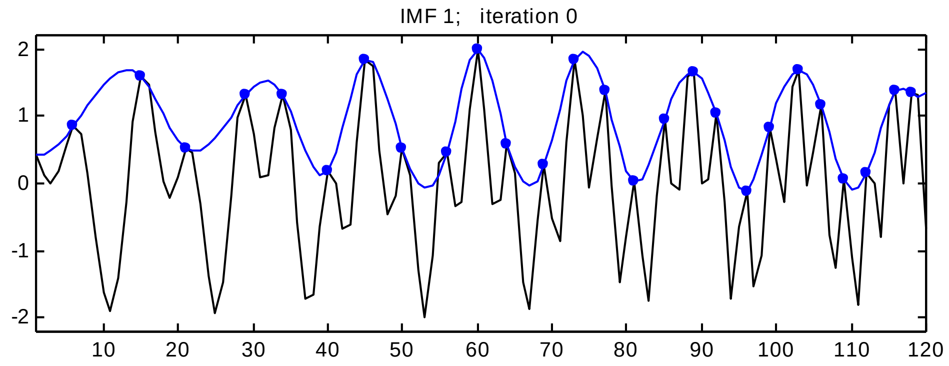
end

$IMF_k(t) = I_i(t)$

Residue = Residue - IMF_k

k = k+1

end



EMD acting on an example

Extract local minima

Residue = s(t)

$I_1(t)$ = Residue

i = 1

k = 1

while Residue not equal zero or not monotone

while I_i has non-negligible local mean

U(t) = spline through local maxima of I_i

L(t) = spline through local minima of I_i

Av(t) = 1/2 (U(t) + L(t))

$I_i(t) = I_i(t)$ - Av(t)

i = i + 1

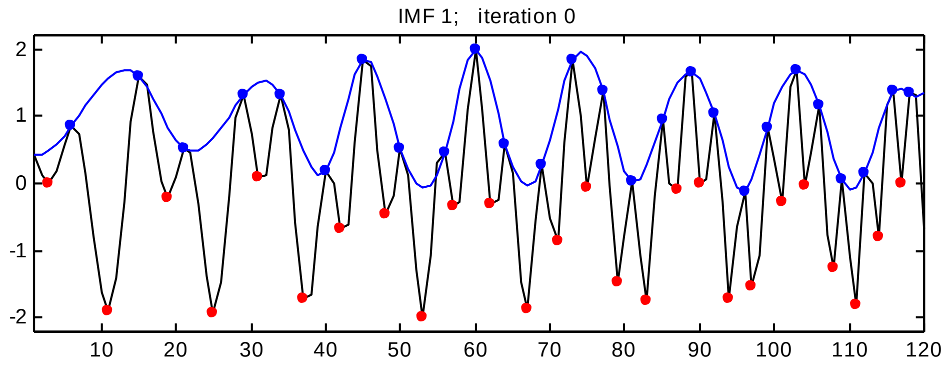
end

$IMF_k(t) = I_i(t)$

Residue = Residue - IMF_k

k = k+1

end



EMD acting on an example

Minima envelope by interpolation

Residue = s(t)

$I_1(t)$ = Residue

i = 1

k = 1

while Residue not equal zero or not monotone

while I_i has non-negligible local mean

U(t) = spline through local maxima of I_i

L(t) = spline through local minima of I_i

Av(t) = 1/2 (U(t) + L(t))

$I_i(t) = I_i(t)$ - Av(t)

i = i + 1

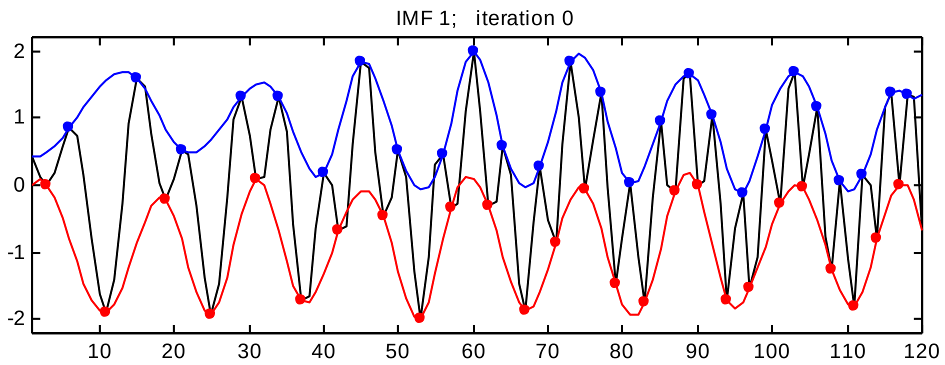
end

$IMF_k(t) = I_i(t)$

Residue = Residue - IMF_k

k = k+1

end



EMD acting on an example

Mean *of maxima and minima* ***envelopes***

Residue = s(t)

I_1(t) = Residue

i = 1

k = 1

while Residue not equal zero or not monotone

while I_i has non-negligible local mean

U(t) = spline through local maxima of I_i

L(t) = spline through local minima of I_i

Av(t) = 1/2 (U(t) + L(t))

I_i(t) = I_i(t) - Av(t)

i = i + 1

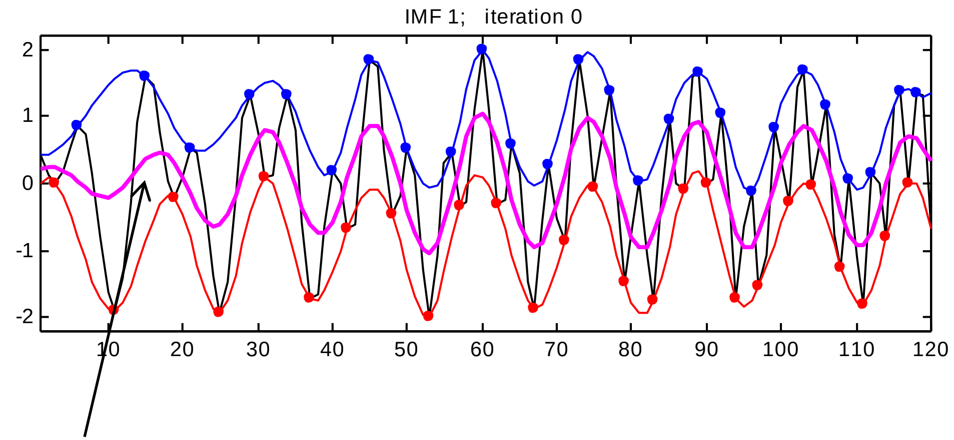
end

IMF_k(t) = I_i(t)

Residue = Residue - IMF_k

k = k+1

end



Local low frequency component

EMD acting on an example
Subtract mean envelope from signal

Residue = s(t)

I_1(t) = Residue

i = 1

k = 1

while Residue not equal zero or not monotone

while I_i has non-negligible local mean

U(t) = spline through local maxima of I_i

L(t) = spline through local minima of I_i

Av(t) = 1/2 (U(t) + L(t))

I_i(t) = I_i(t) - Av(t) (“residue”-->)

i = i + 1

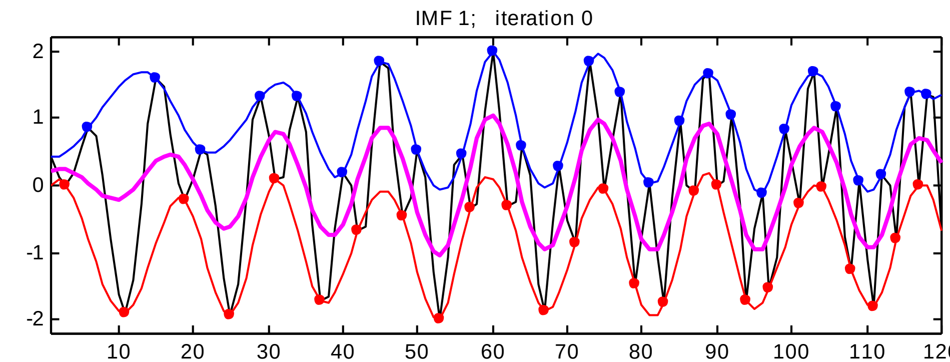
end

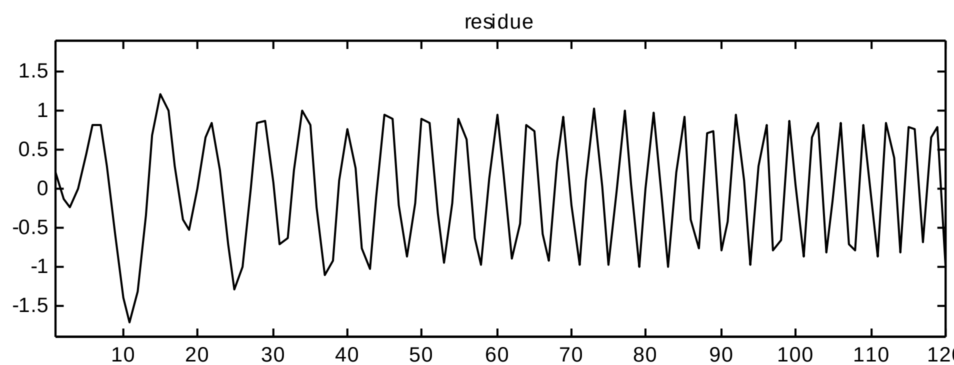
IMF_k(t) = I_i(t)

Residue = Residue - IMF_k

k = k+1

end





EMD acting on an example

Is the residue a IMF? No

Residue = s(t)

$I_1(t)$ = Residue

i = 1

k = 1

while Residue not equal zero or not monotone

while I_i has non-negligible local mean

U(t) = spline through local maxima of I_i

L(t) = spline through local minima of I_i

Av(t) = 1/2 (U(t) + L(t))

$I_i(t) = I_i(t)$ - Av(t) ("residue"-->)

i = i + 1

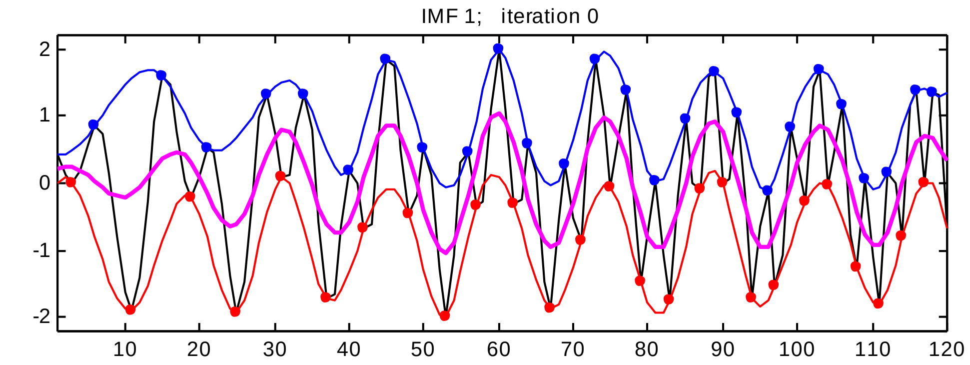
end

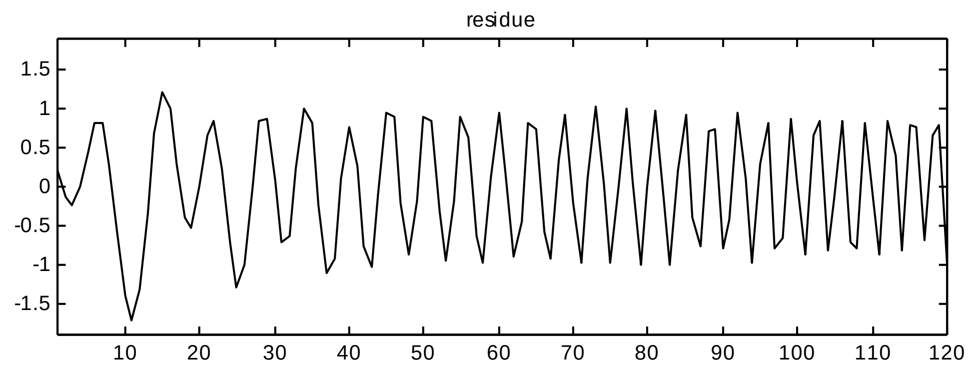
$IMF_k(t) = I_i(t)$

Residue = Residue - IMF_k

k = k+1

end





EMD acting on an example
No, so iterate the loop (sifting)

Residue = s(t)

$I_1(t)$ = Residue

i = 1

k = 1

while Residue not equal zero or not monotone

while I_i has non-negligible local mean

U(t) = spline through local maxima of I_i

L(t) = spline through local minima of I_i

Av(t) = 1/2 (U(t) + L(t))

$I_i(t) = I_i(t) - Av(t)$

i = i + 1

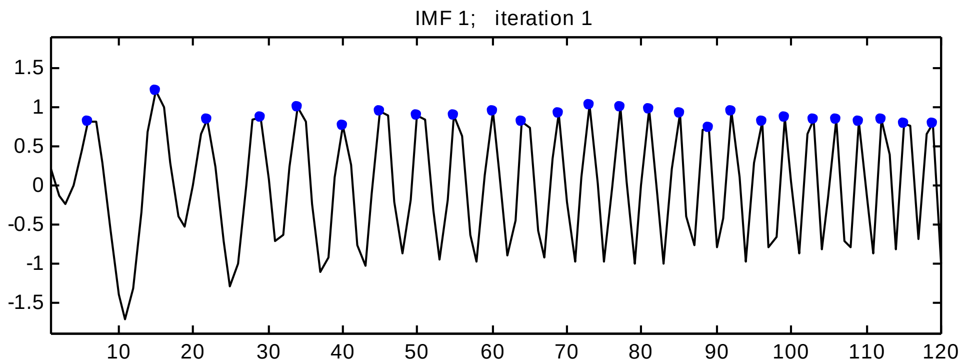
end

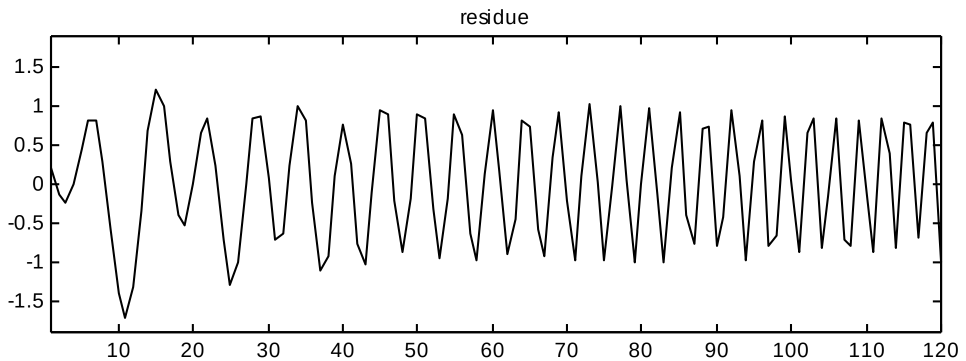
$IMF_k(t) = I_i(t)$

Residue = Residue - IMF_k

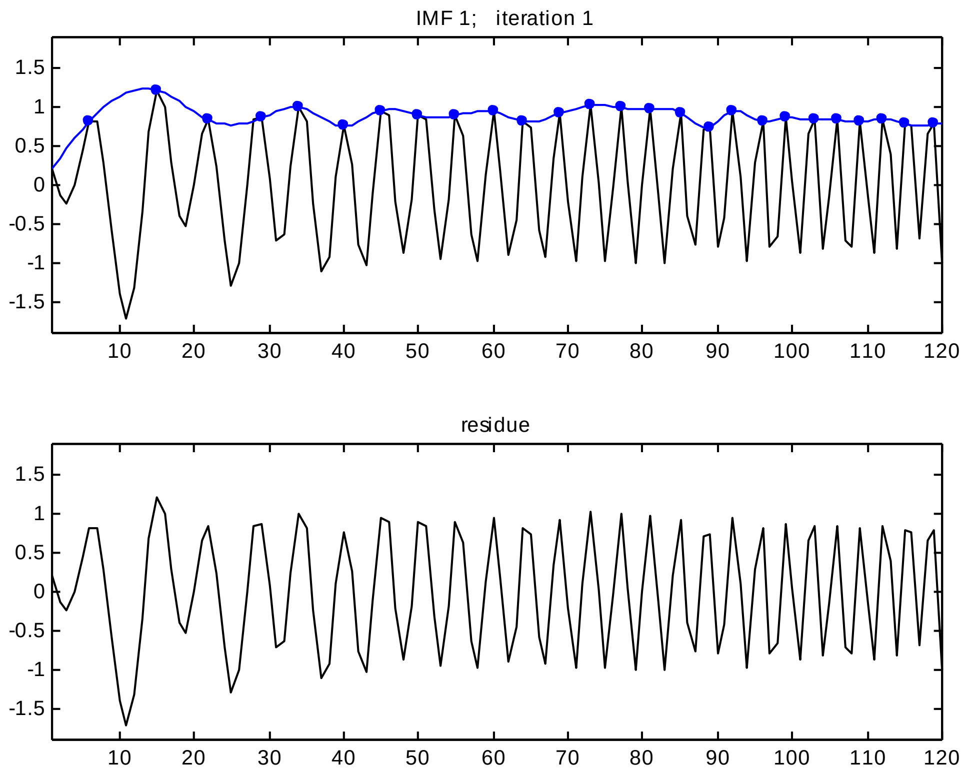
k = k+1

end

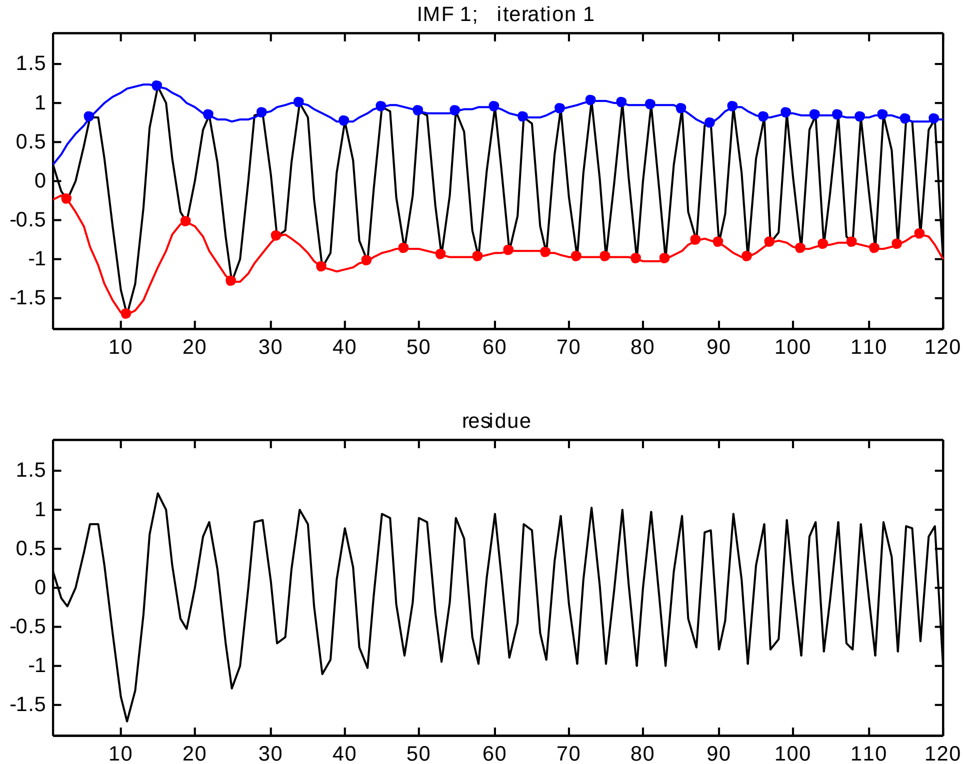




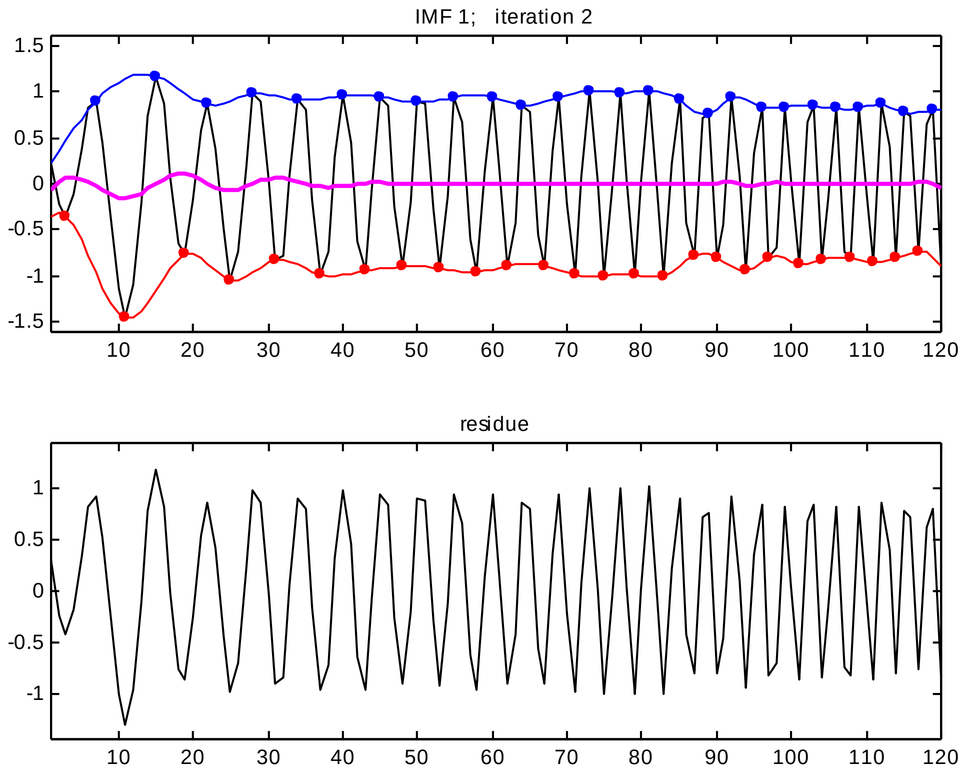
EMD acting on an example IMF1 - iteration 1 - maxima



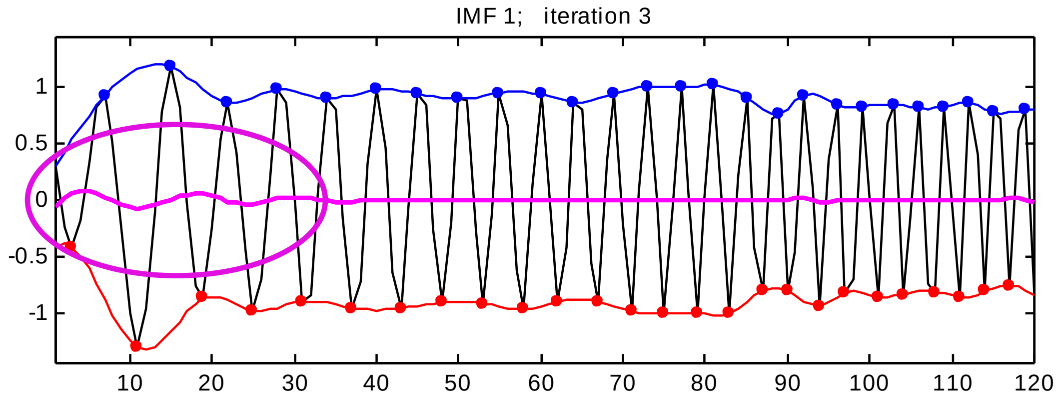
EMD acting on an example
IMF1 - iteration 1 - minima

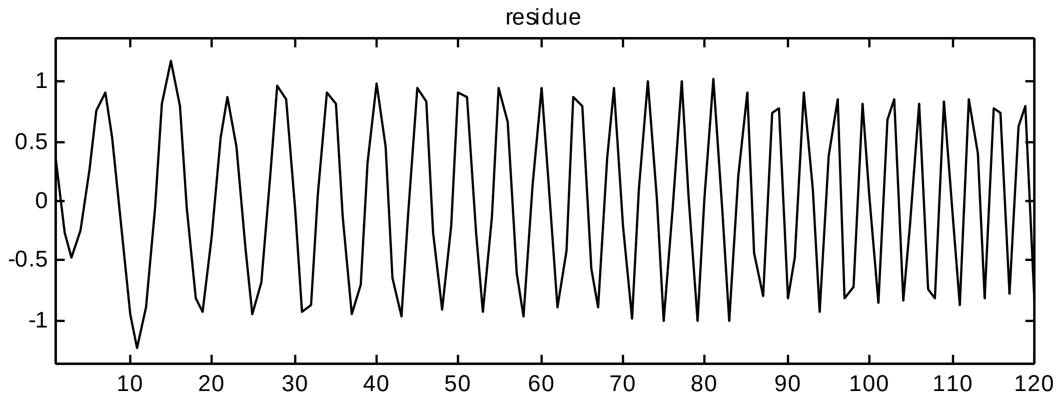


EMD acting on an example
IMF1 - iteration 2 – mean

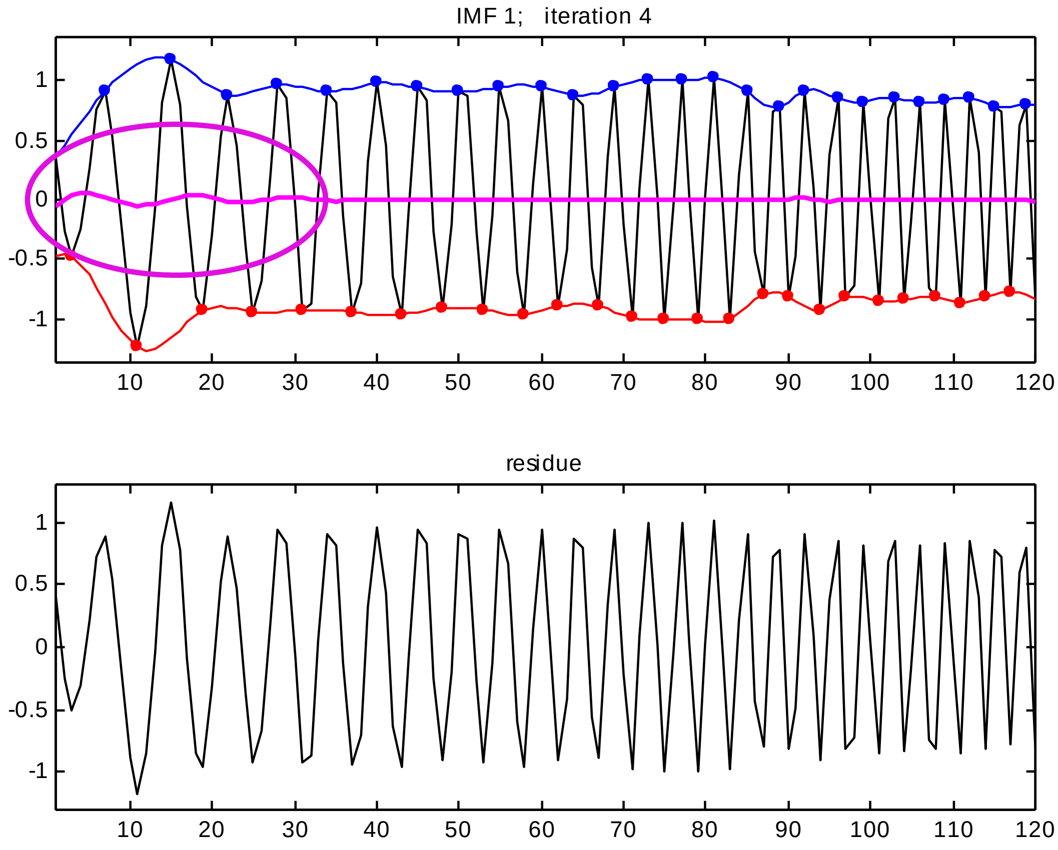


EMD acting on an example
IMF1 - iteration 3

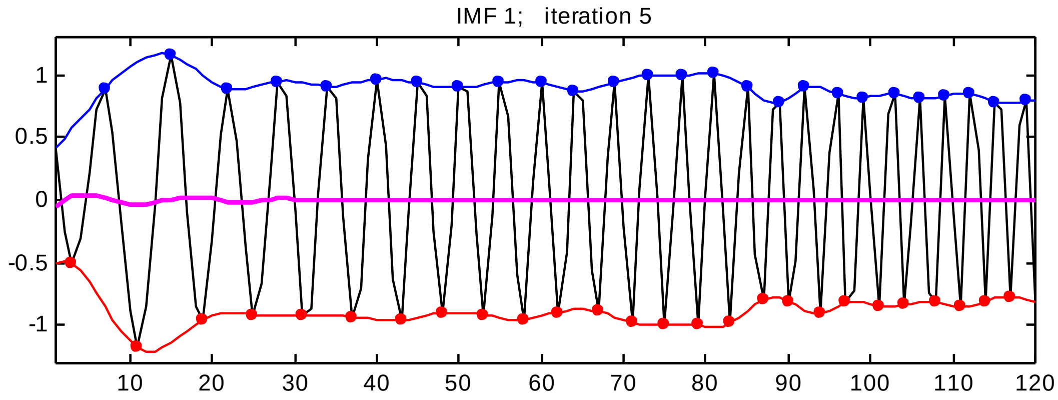


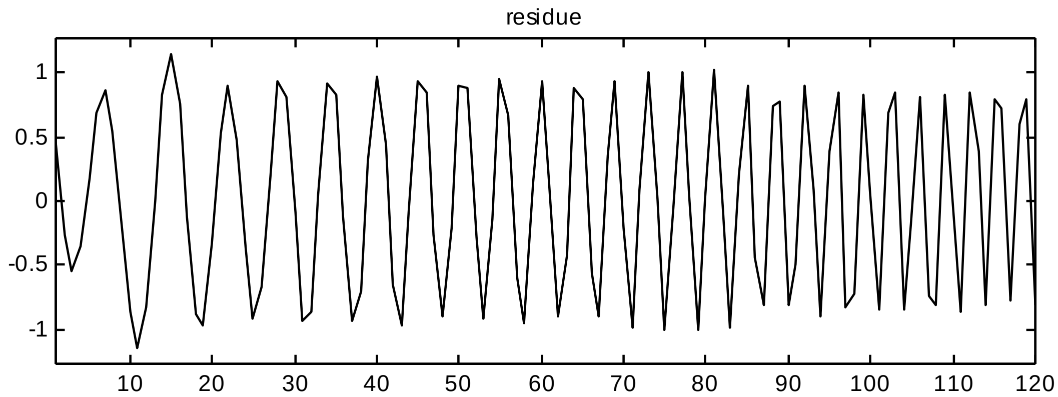


EMD acting on an example
IMF1 - iteration 4

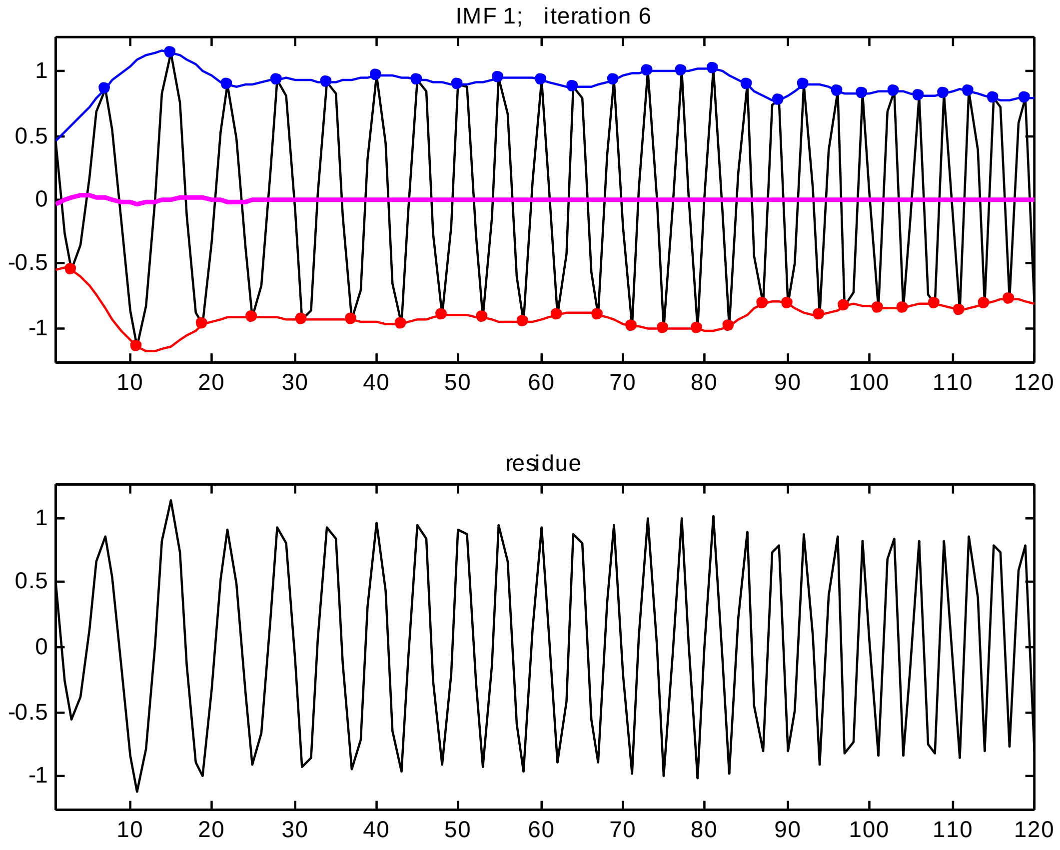


EMD acting on an example
IMF1 - iteration 5

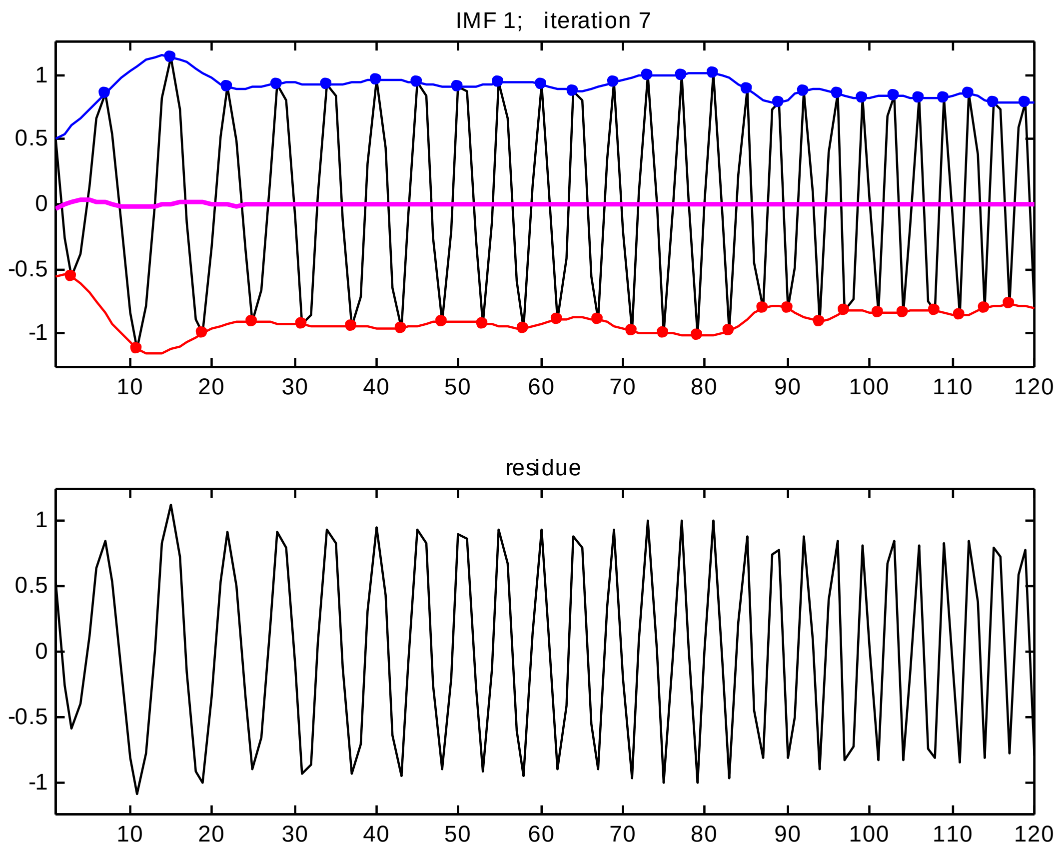




EMD acting on an example
IMF1 - iteration 6



EMD acting on an example
IMF1 - iteration 7



EMD acting on an example
IMF1 - iteration 8

Residue = s(t)

I_1(t) = Residue

i = 1

k = 1

while Residue not equal zero or not monotone

while I_i has non-negligible local mean

U(t) = spline through local maxima of I_i

L(t) = spline through local minima of I_i

Av(t) = 1/2 (U(t) + L(t))

I_i(t) = I_i(t) - Av(t)

i = i + 1

end

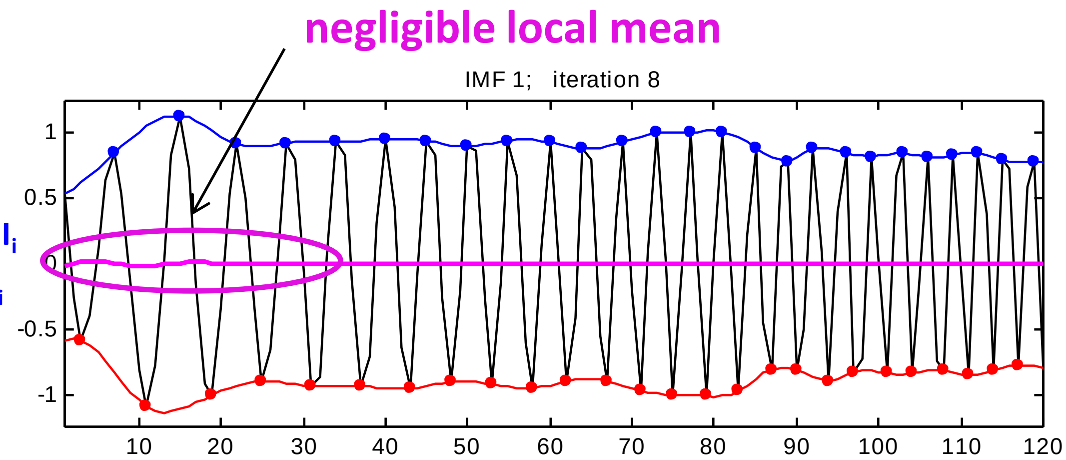
IMF_k(t) = I_i(t)

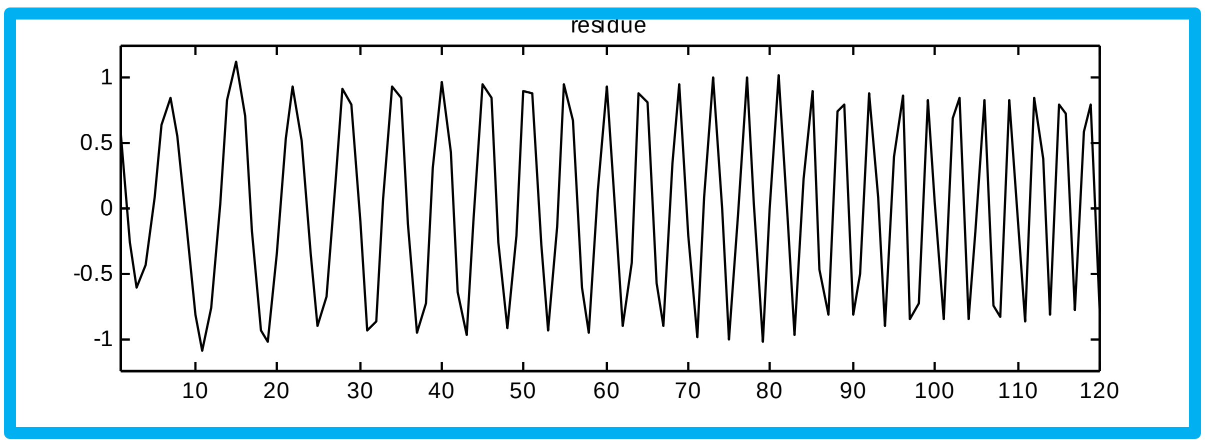
Residue = Residue - IMF_k

k = k+1

end

IMF1





EMD acting on an example
IMF2 - iteration 0

Residue = s(t)

I_1(t) = Residue

i = 1

k = 1

while Residue not equal zero or not monotone

while I_i has non-negligible local mean

U(t) = spline through local maxima of I_i

L(t) = spline through local minima of I_i

Av(t) = 1/2 (U(t) + L(t))

I_i(t) = I_i(t) - Av(t)

i = i + 1

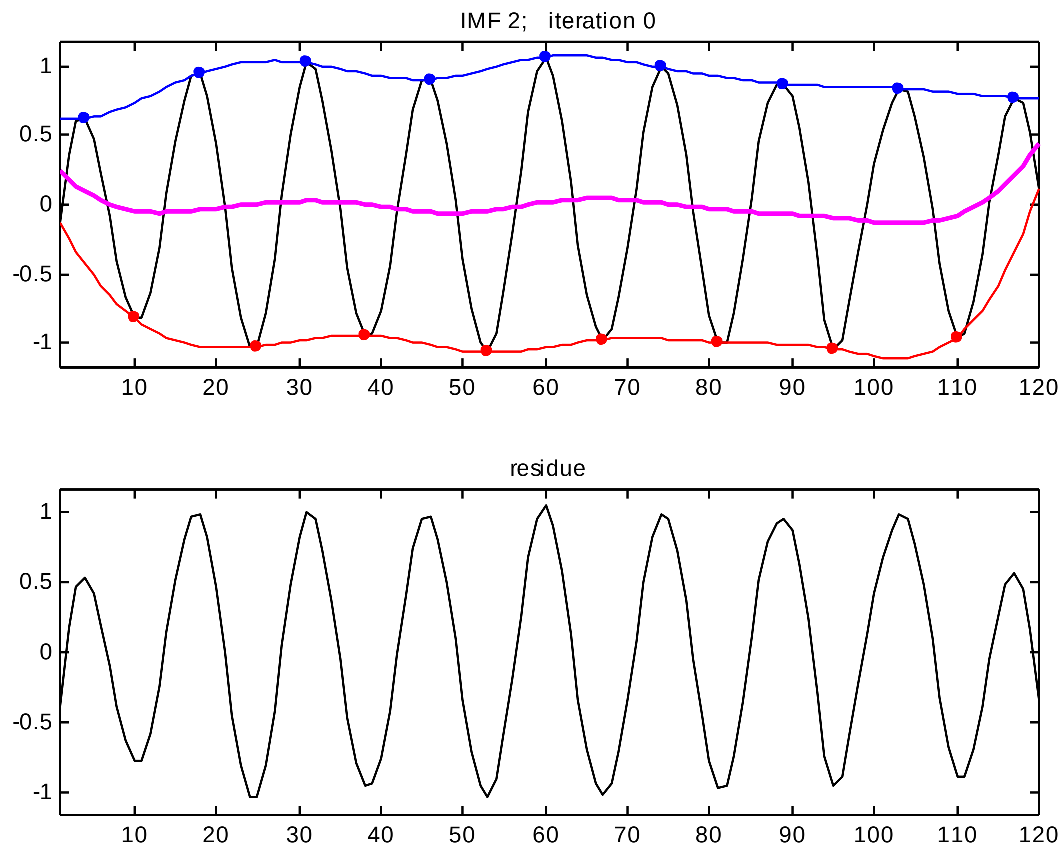
end

IMF_k(t) = I_i(t)

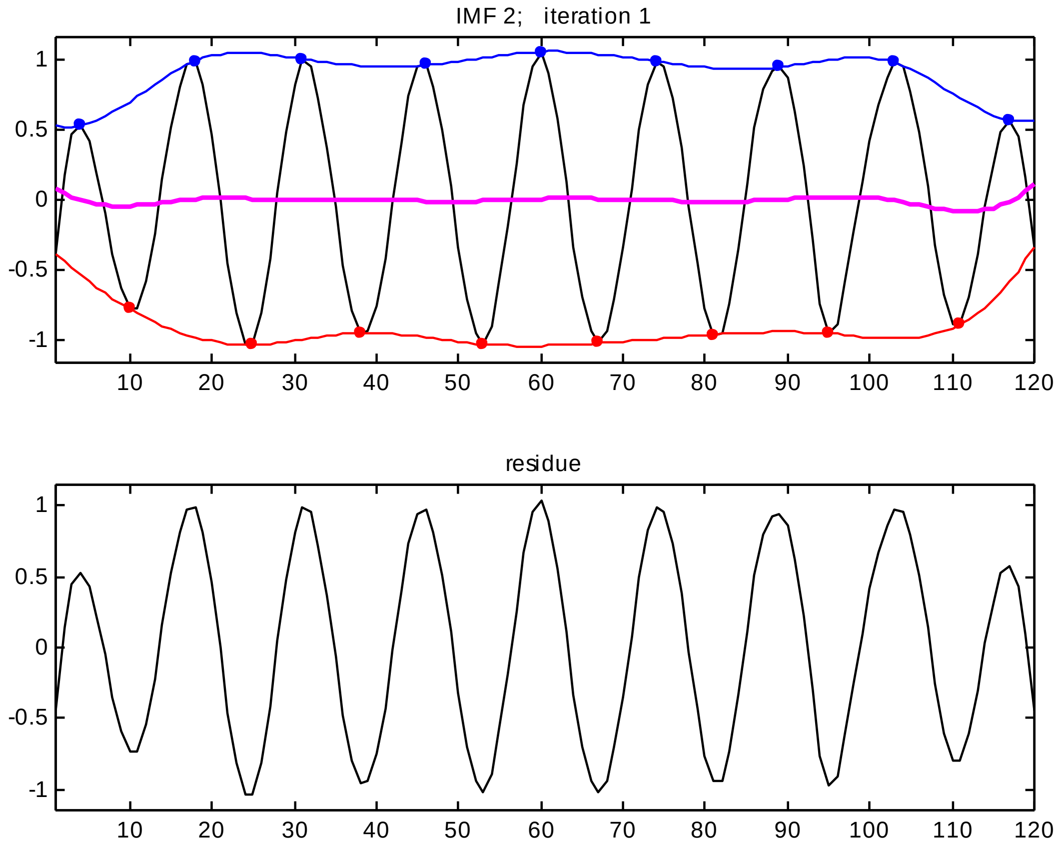
Residue = Residue - IMF_k

k = k+1

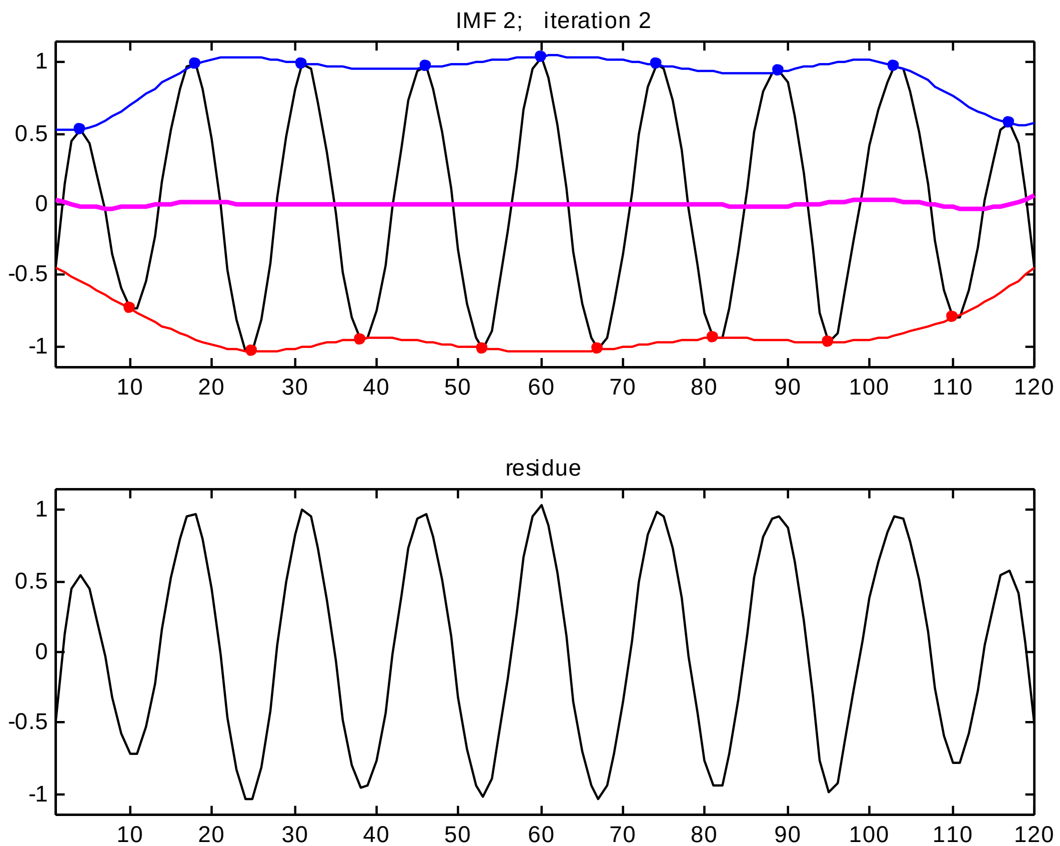
end



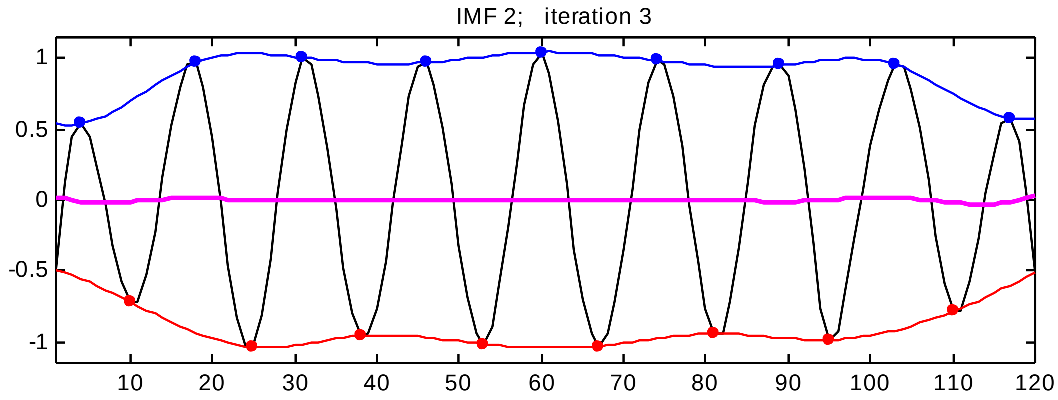
EMD acting on an example
IMF2 – iteration 1

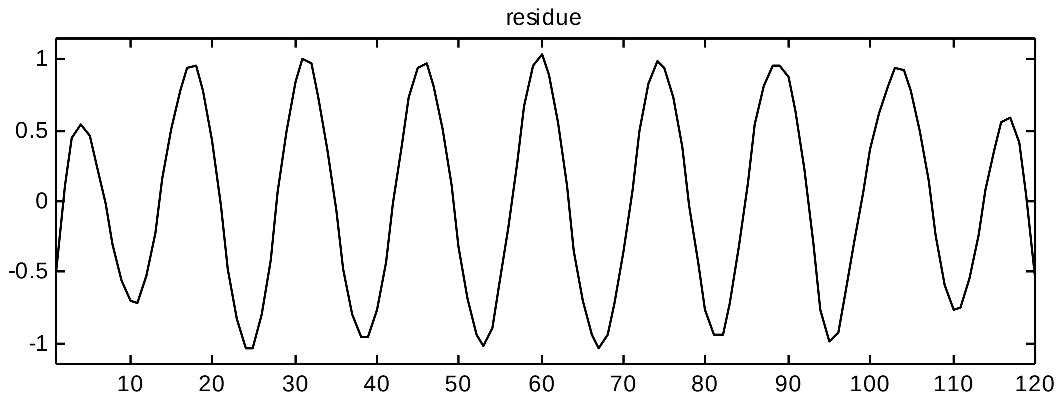


EMD acting on an example
IMF2 – iteration 2

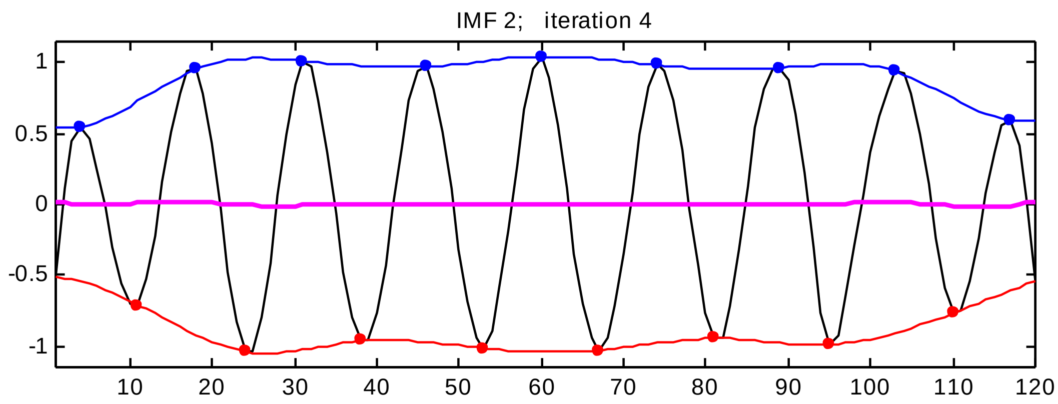


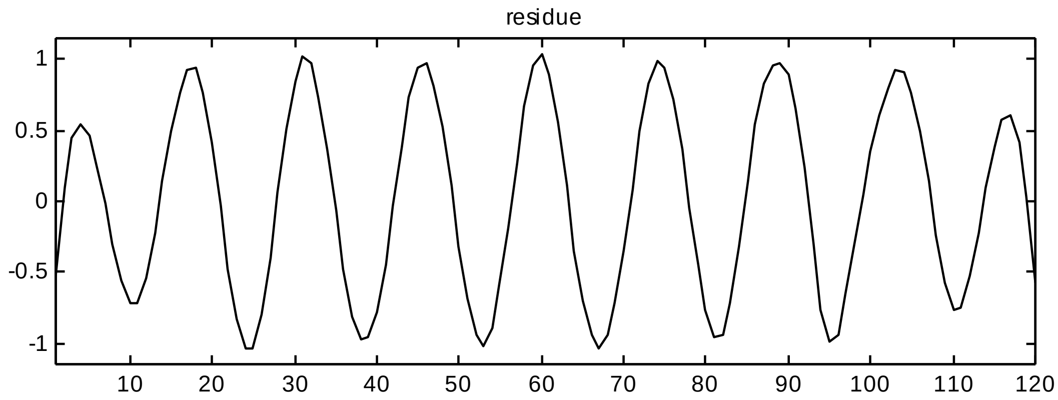
EMD acting on an example
IMF2 – iteration 3



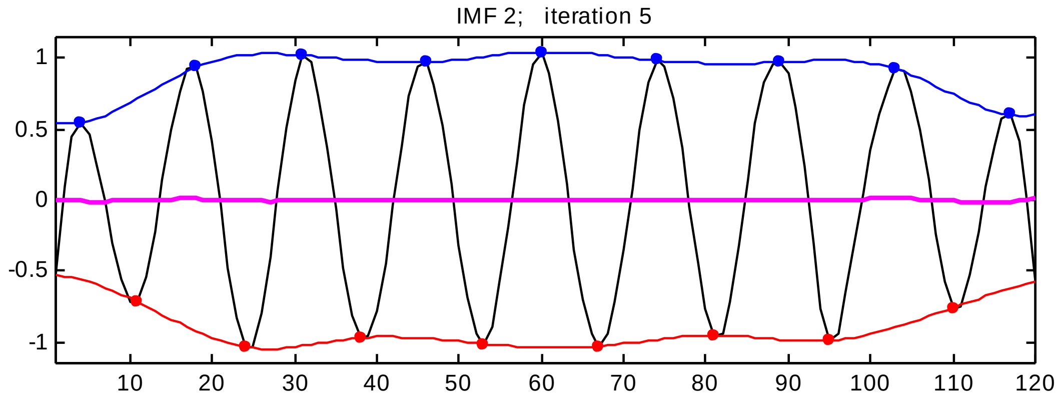


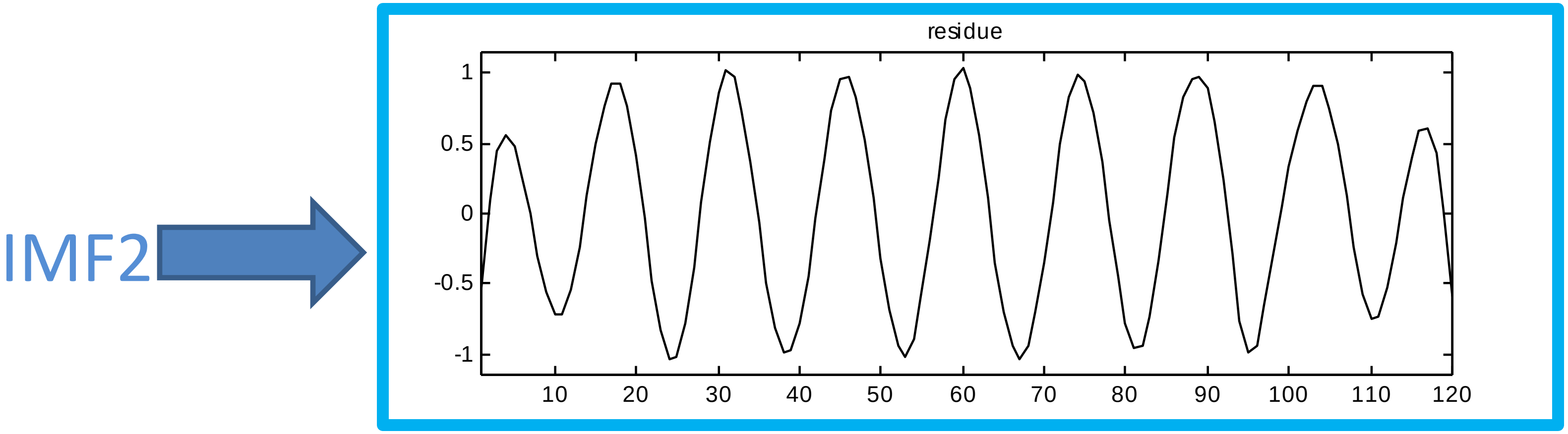
EMD acting on an example
IMF2 – iteration 4





EMD acting on an example
IMF2 – iteration 5





Trend definitions and EMD

- No consensus about what is a trend, various definitions related to data peculiarities and fields of application
- EMD already used to extract trends :
  - *Suling et al. 2009*, local trends for time scales
  - *Flandrin et al. 2004*, sum of nonzero mean IMFs
  - *Zhou et al. 2008*, power-system oscillation data
- *This paper*: long-term trend for seasonal time series and comparison with *Hodrick Prescott* (HP) and a remark about *wavelets*
- Three weeks ago:

  Moghtaderi, Borgnat, Flandrin (april 2010) in "Trend Filtering: Empirical Mode Decompositions Versus l1 and Hodrick–Prescott" introduce the energy-ratio approach to select IMFs

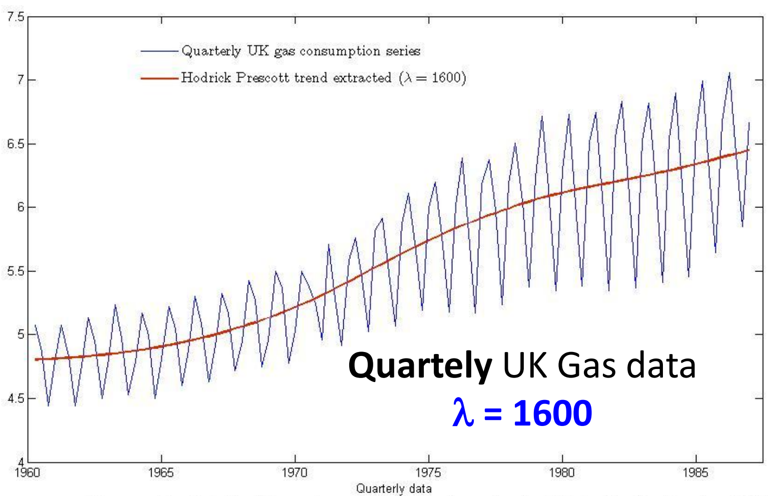
  The idea: select low frequency IMFs from coarse to fine and when it does not differ from some noise reference

Hodrick-Prescott filtering

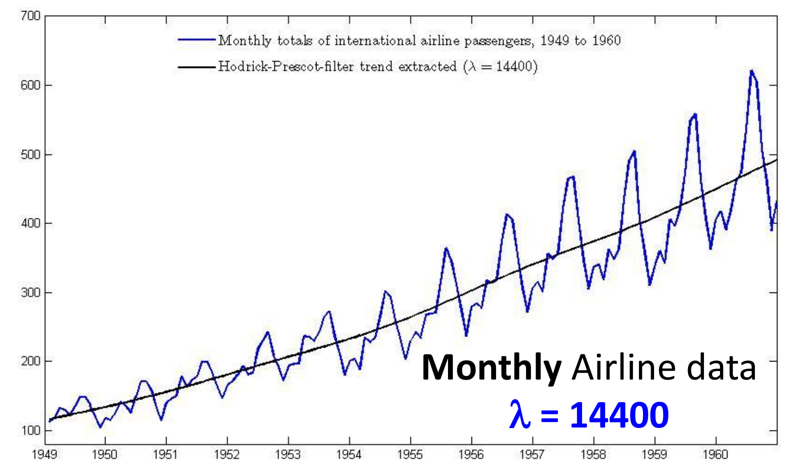
- Comparison with the nonparametric trend extraction method based on HP filtering
- Widely used by economists , *Pollock (2003)*

  Trend = argmin { (y-T)² + λ ∇²(T) }

- Penalized least squares estimation
- Usual values for λ in economic time series:



Reference : Box & Jenkins, Time series analysis: forecasting and control, Holden Day, San Francisco (1978)

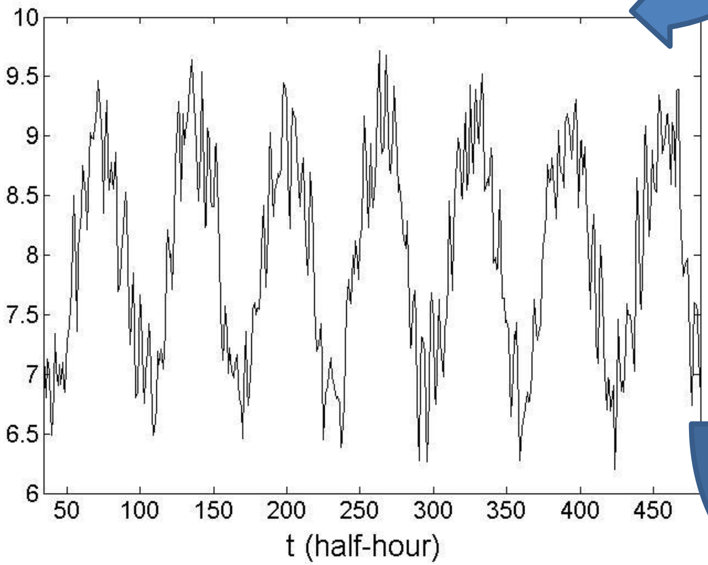


Reference : Box, G. E. P., Jenkins, G. M. and Reinsel, G. C. (1976) Time Series Analysis, Forecasting and Control. Third Edition. Holden-Da

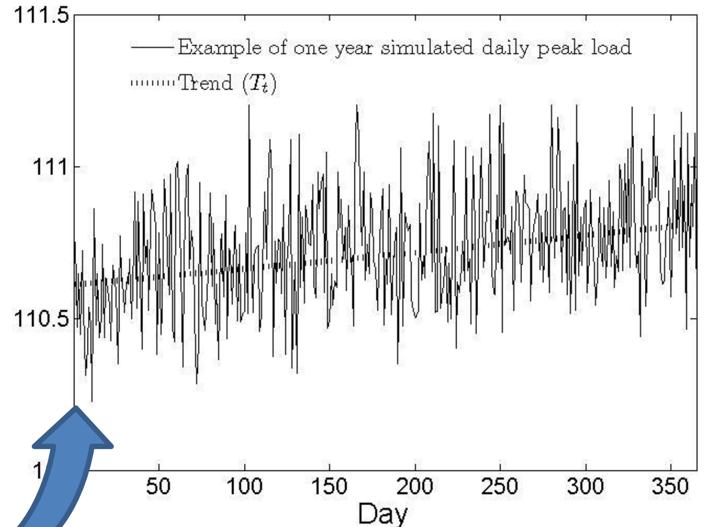
HP filter / simulated seasonal time series

- To select λ automatically, see ***Schlicht (2005)***
- Here, we prefer to use Empirical tuning based on ***simulated load curve*** for $\lambda \in [10^2, 10^{15}]$, a bootstrap-like scheme

- one week simulated **daily load**



- one year simulated **daily peak load**
- and additive trend (linear or exp.)



HP vs EMD and suitable λ

- ***Calibration of HP parameter*** λ for different kinds of artificial trends for daily data

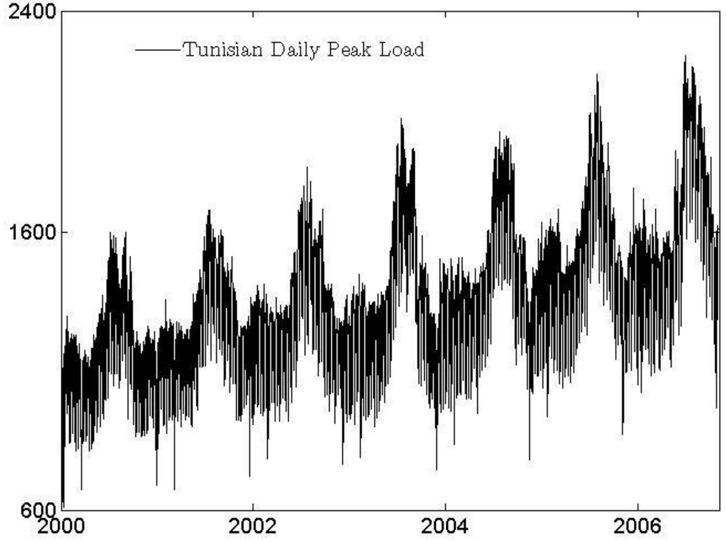
- ***Comparison*** of HP and EMD trends for different ***linear, quasi linear*** and ***exponential*** **trends**

- **EMD-trend very close to the optimal HP one**
  $\Rightarrow$ **EMD is an effective alternative for trend extraction**

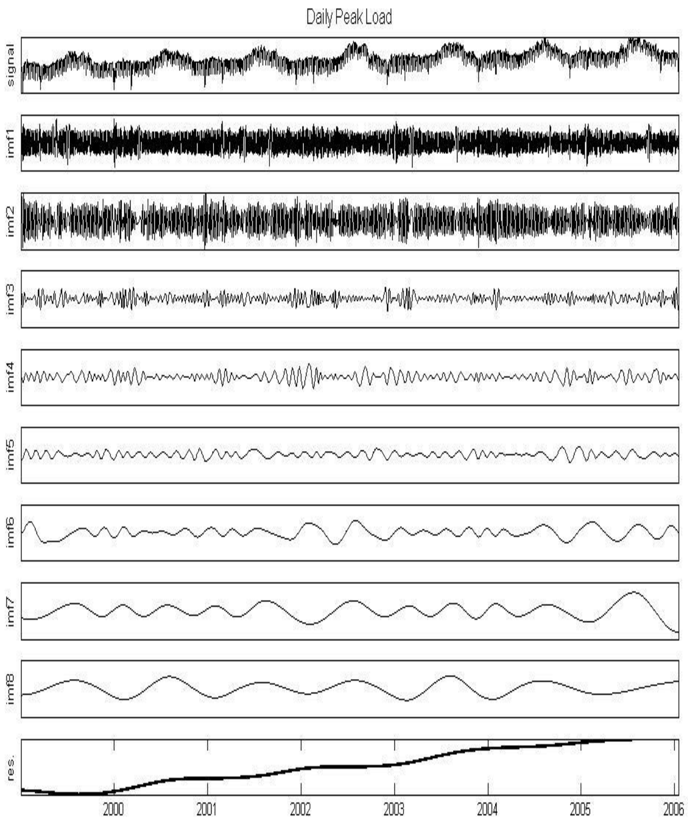
- Then we can use these values to compare *EMD-trend* and "optimal" *HP one* on real electrical daily data

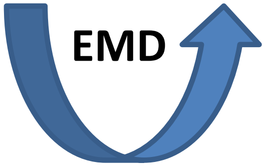
Tunisian daily peak load 2000-2006

Daily peak load



IMFs + final trend

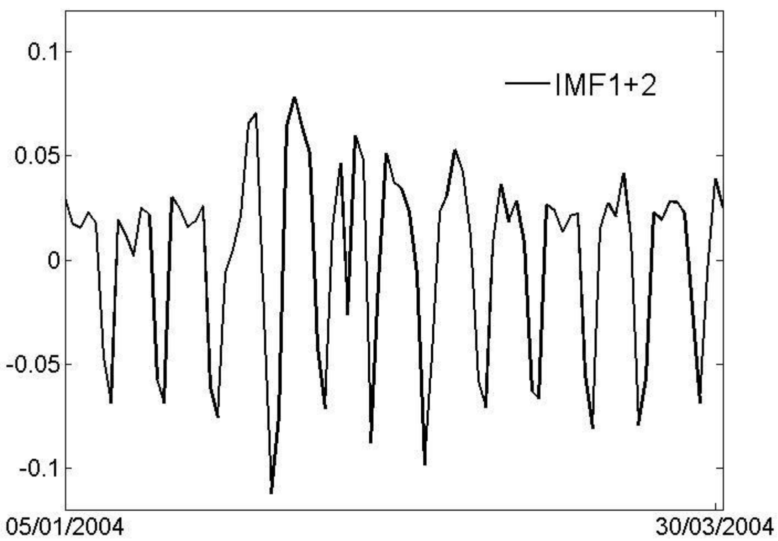




IMFs / seasonal load components

Sum of IMFs 1-2

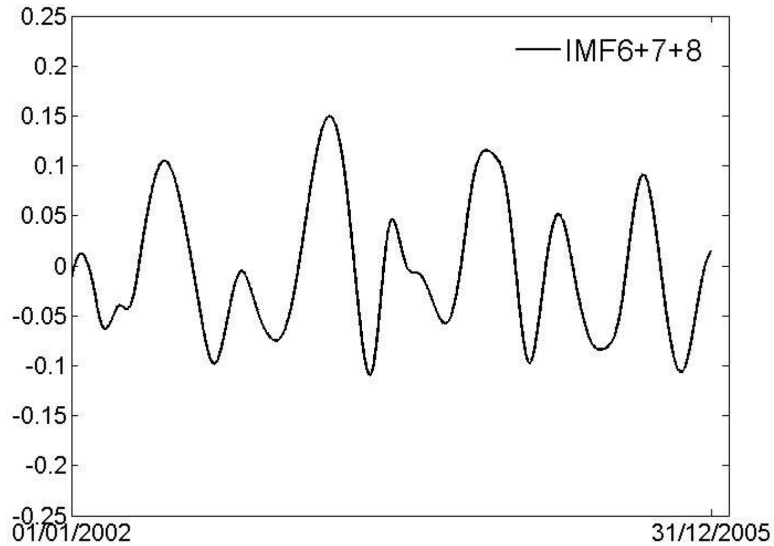
Short term week component



12 weeks

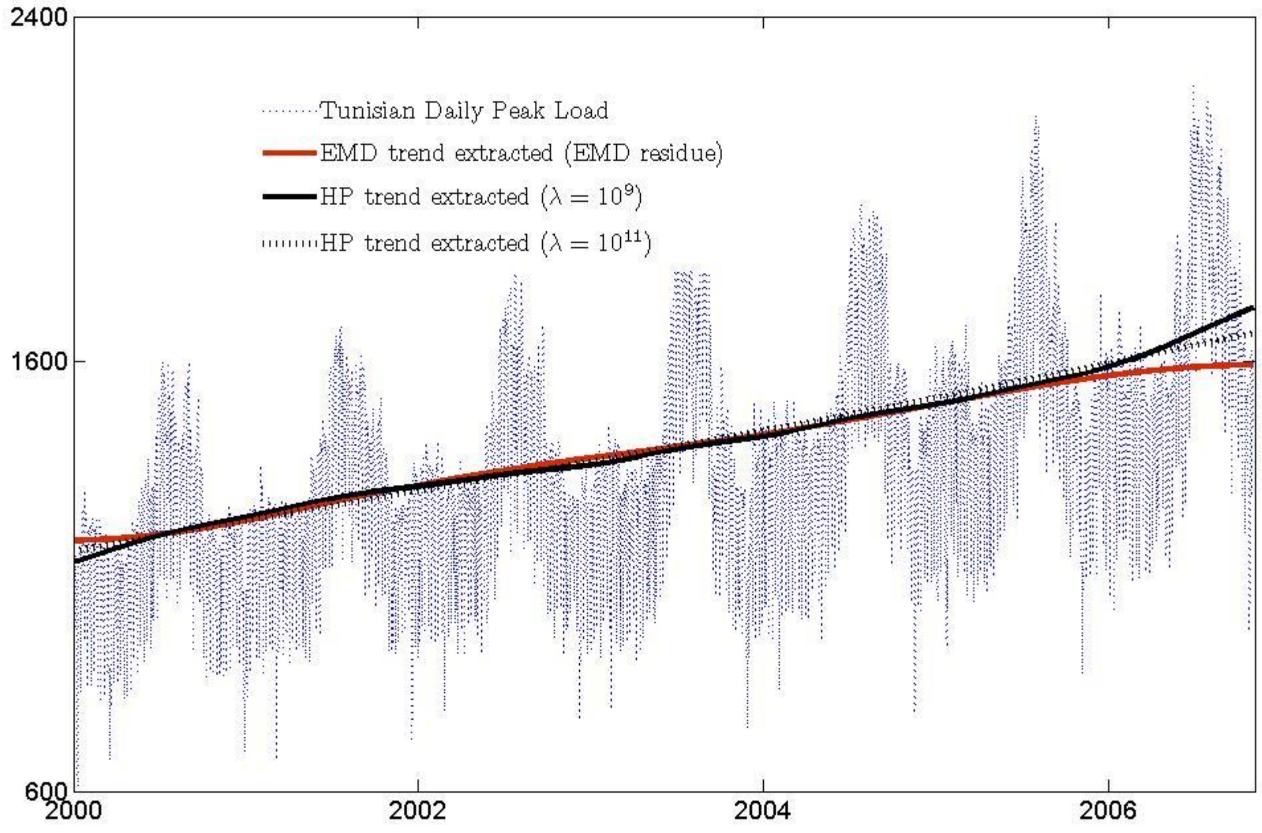
Sum of IMFs 6-7-8

Long term annual component

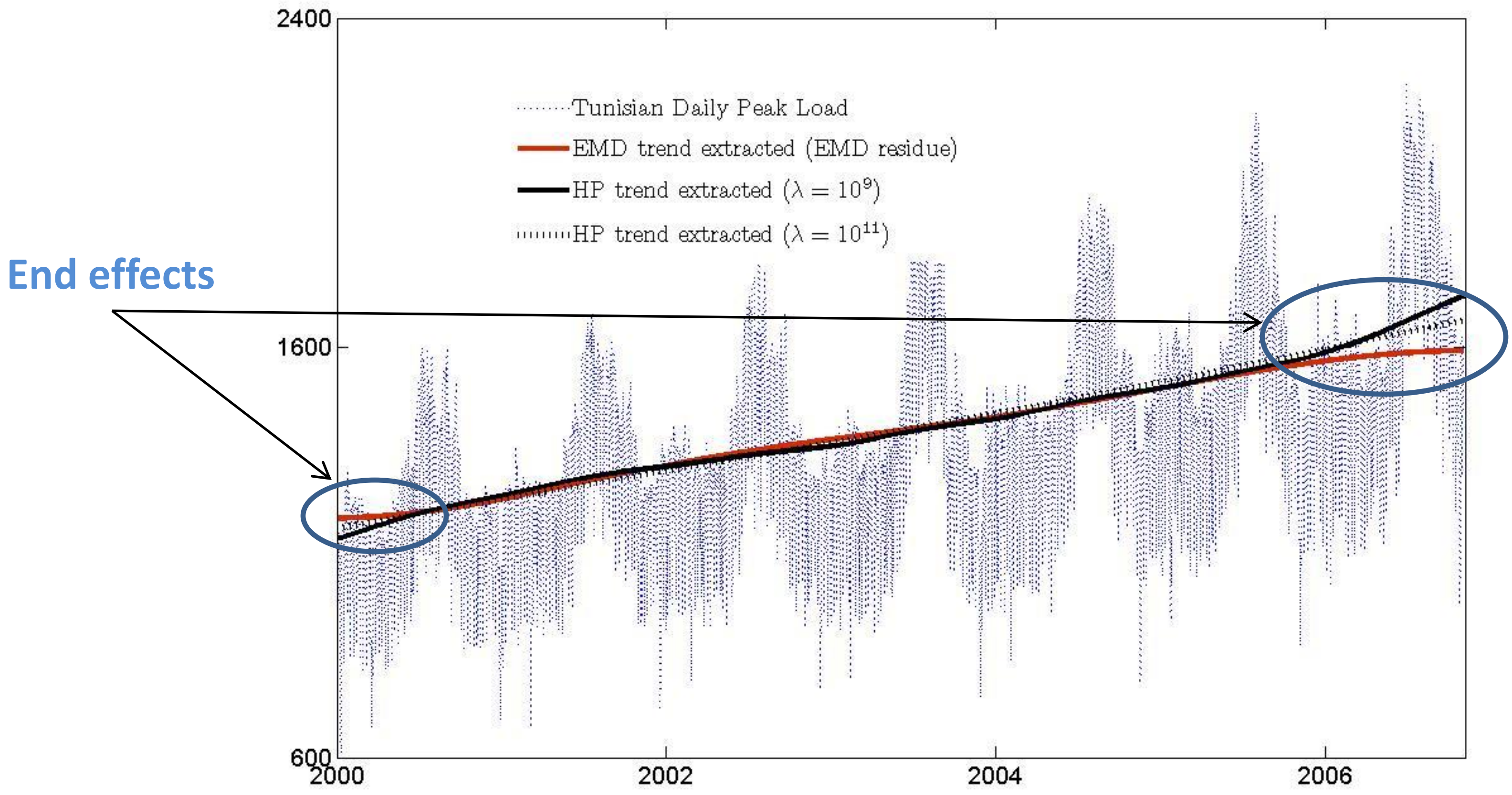


4 years

Tunisian daily peak load
HP and EMD long-term trends



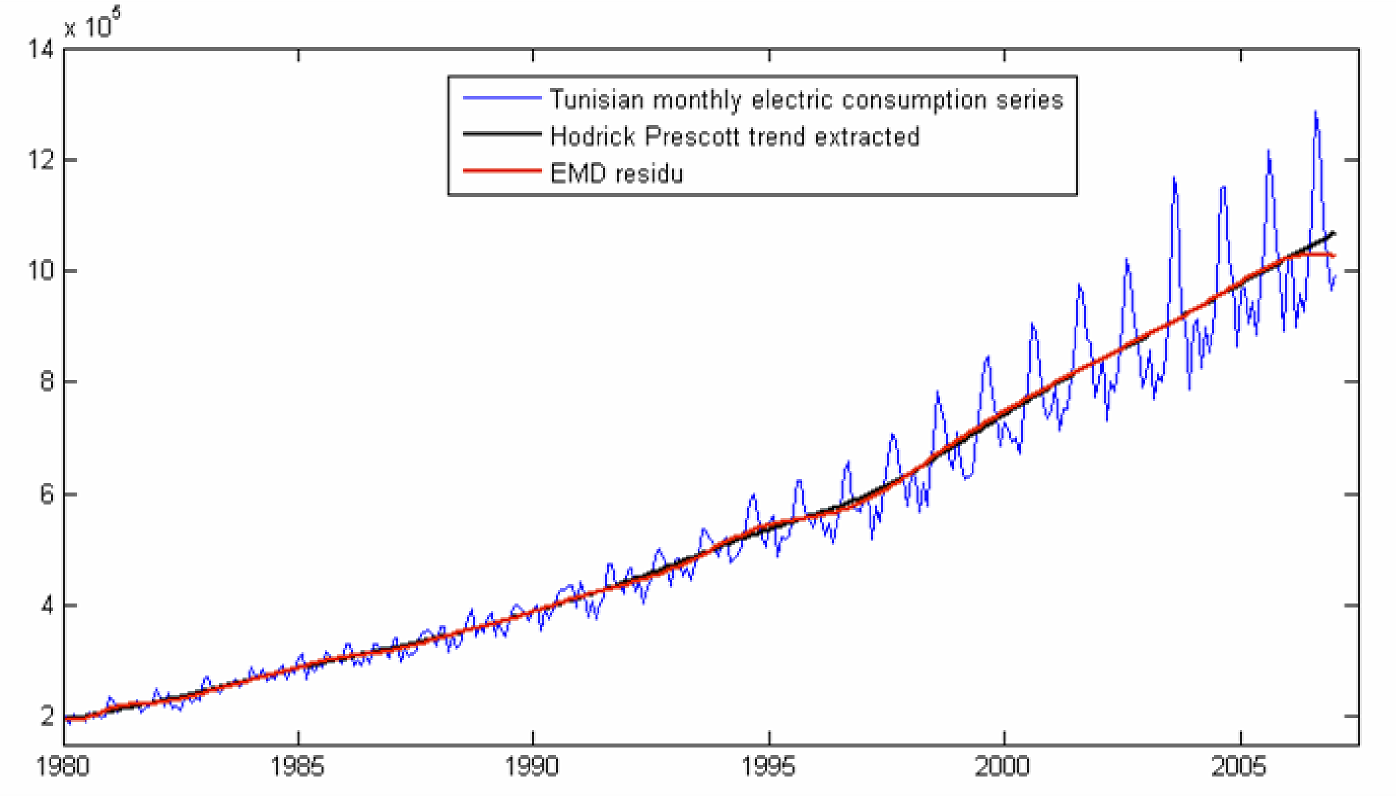
Tunisian daily peak load
HP and EMD trends: end effects



Dealing with EMD end effects:

- **Windowing the signal**, *Ren et al. (2006)*
- **Extrapolate** end maxima and minima, *Zhaohua et al. (2009)*

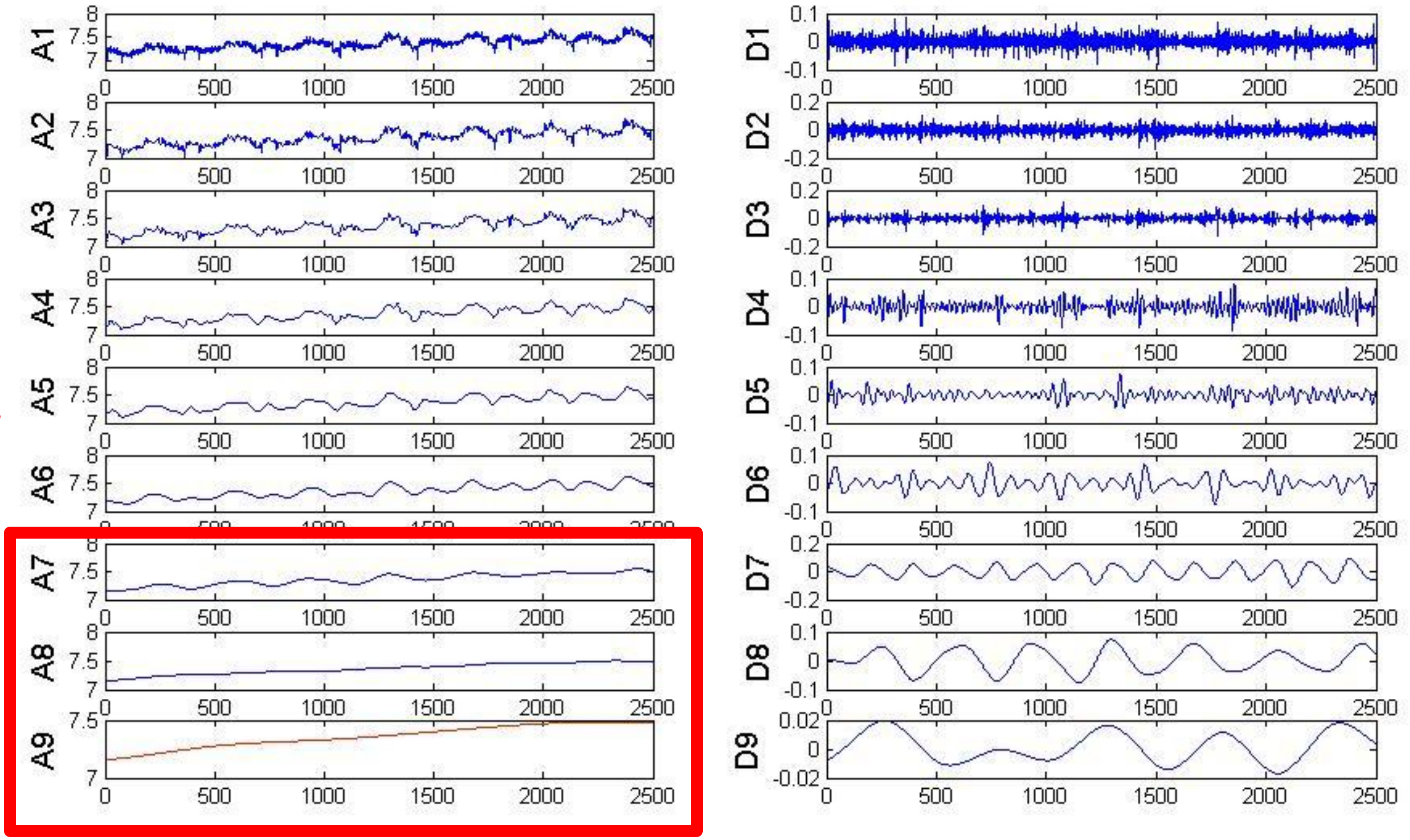
Tunisian monthly electrical data
HP and EMD trends (1980-2006)



Wavelet trends: *A7, A8, A9?*

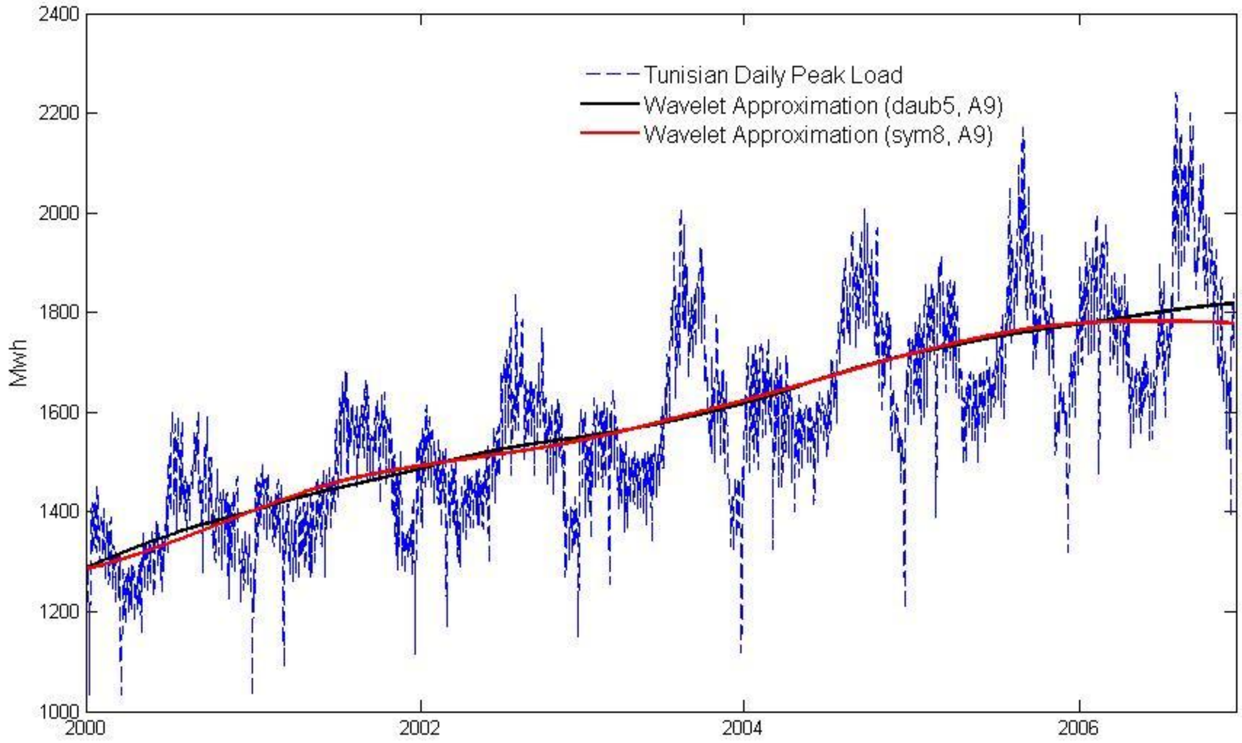
Approximations (/trends?) **Details** (/IMFs?)

Choosing Good Level?



Wavelet trends: *daub5*, *sym8*?

Choosing Good Wavelet?



Conclusion

- EMD eligible method for *trend extraction from seasonal time series*

- Comparison with
  - HP Filter: widely used method in economics (λ?)
  - Wavelets: another time-scale/frequency method (wavelet?)

- EMD trend extraction does not require any tuning parameter thanks to its *adaptive nature*

- *Perspective: extensions to other trends and comparisons with proposals from Moghtaderi et al. (2010)*

References

- **Alexandrov, Bianconcini, Bee Dagum, Maass, Mcelroy (2009)** *U.S. Census Bureau*
- **Flandrin, Goncalves, Rilling (2004)** *EUSIPCO*
- **Huang, Shen, Long, Wu, Shih, Zheng, Yen, Tung, Liu (1998)** *Royal Society London*
- **Ould Mohamed Mahmoud, Mhamdi, Jaidane-Saidane (2009)** *IEEE PowerTech*
- **Moghtaderi, Borgnat, Flandrin (april 2010)** *Submitted CSDA and AADA*
- **Pollock (2003)** *Journal of Statistical Planning and Inference*
- **Ren, Yang, Wu, Yan (2006)** *International Technology and Innovation Conf.*
- **Schlicht (2005)** *Journal of Japan Statistic Society*
- **Suling, Yanqin, Qiang, Jian (2009)** *World Congress on Engineering and Computer Science*
- **Wu, Huang, Long, Peng (2007)** *PNAS*
- **Zhou, Trudnowski, Pierre, Sarawgi, Bhatt (2008)** *IEEE Power Elec. Syst.*