Alma Mater Studiorum – University of Bologna

A flexible IRT Model for health questionnaire: an application to HRQoL

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Statement of the problem

Descrivi la tua salute di OGGI

Per ogni sezione, metti UNA "X" nella casella vicina alla frase che descrive meglio la tua salute OGGI

Movimento (camminare)
- Non ho nessuna difficoltà a camminare ☒
- Ho un po' di difficoltà a camminare ☐
- Ho molte difficoltà a camminare ☐

Mi prendo cura di me *(per esempio lavarmi o vestirmi da solo/a)*
- Non ho nessuna difficoltà a lavarmi o vestirmi da solo/a ☒
- Ho un po' di difficoltà a lavarmi o vestirmi da solo/a ☐
- Ho molte difficoltà a lavarmi o vestirmi da solo/a ☐

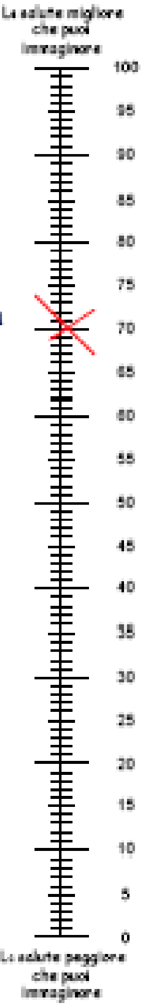
Fare le attività di tutti i giorni *(per esempio andare a scuola, fare hobby, sport, giocare, fare delle cose con la famiglia o con gli amici)*
- Non ho nessuna difficoltà a fare le cose di tutti giorni ☒
- Ho un po' di difficoltà a fare le cose di tutti i giorni ☐
- Ho molte difficoltà a fare le cose di tutti i giorni ☐

Avere dolore o fastidio
- Non ho nessun dolore o fastidio ☐
- Ho un po' di dolore o fastidio ☒
- Ho molto dolore o fastidio ☐

Sentirsi preoccupato/a, triste o infelice
- Non sono per niente preoccupato/a, triste o infelice ☐
- Sono un po' preoccupato/a, triste o infelice ☒
- Sono molto preoccupato/a, triste o infelice ☐

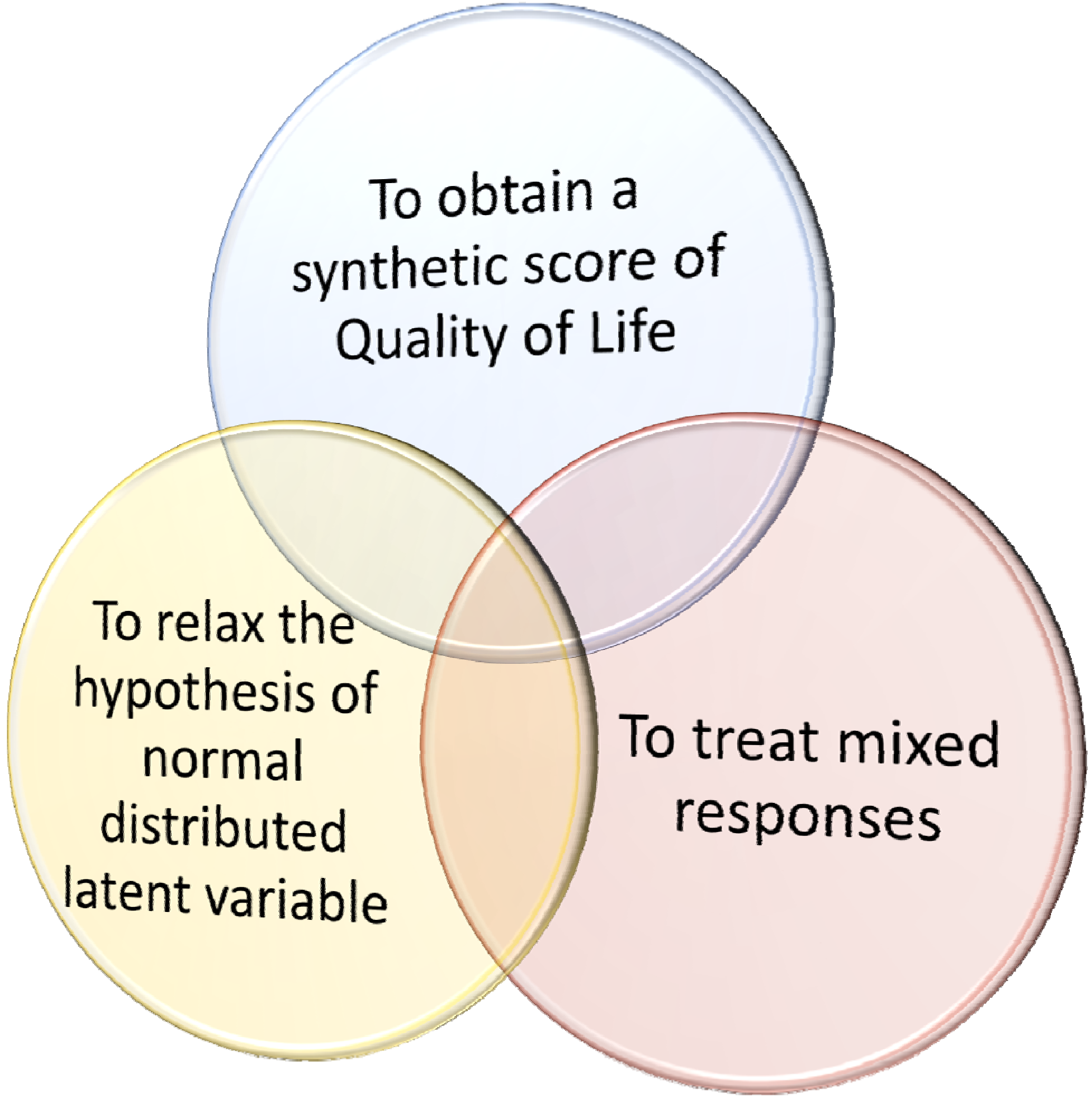
Quanto è buona la tua salute OGGI

- Per aiutarci a sapere quanto è buona o cattiva la tua salute OGGI, abbiamo disegnato questa linea numerata, come un termometro che va da 0 a 100
- Considera che:
  0 rappresenta la peggiore salute che puoi immaginare
  100 rappresenta la migliore salute che puoi immaginare
- Per favore scrivi UNA X sul punto della linea numerata che indica quanto è buona o cattiva la tua salute OGGI

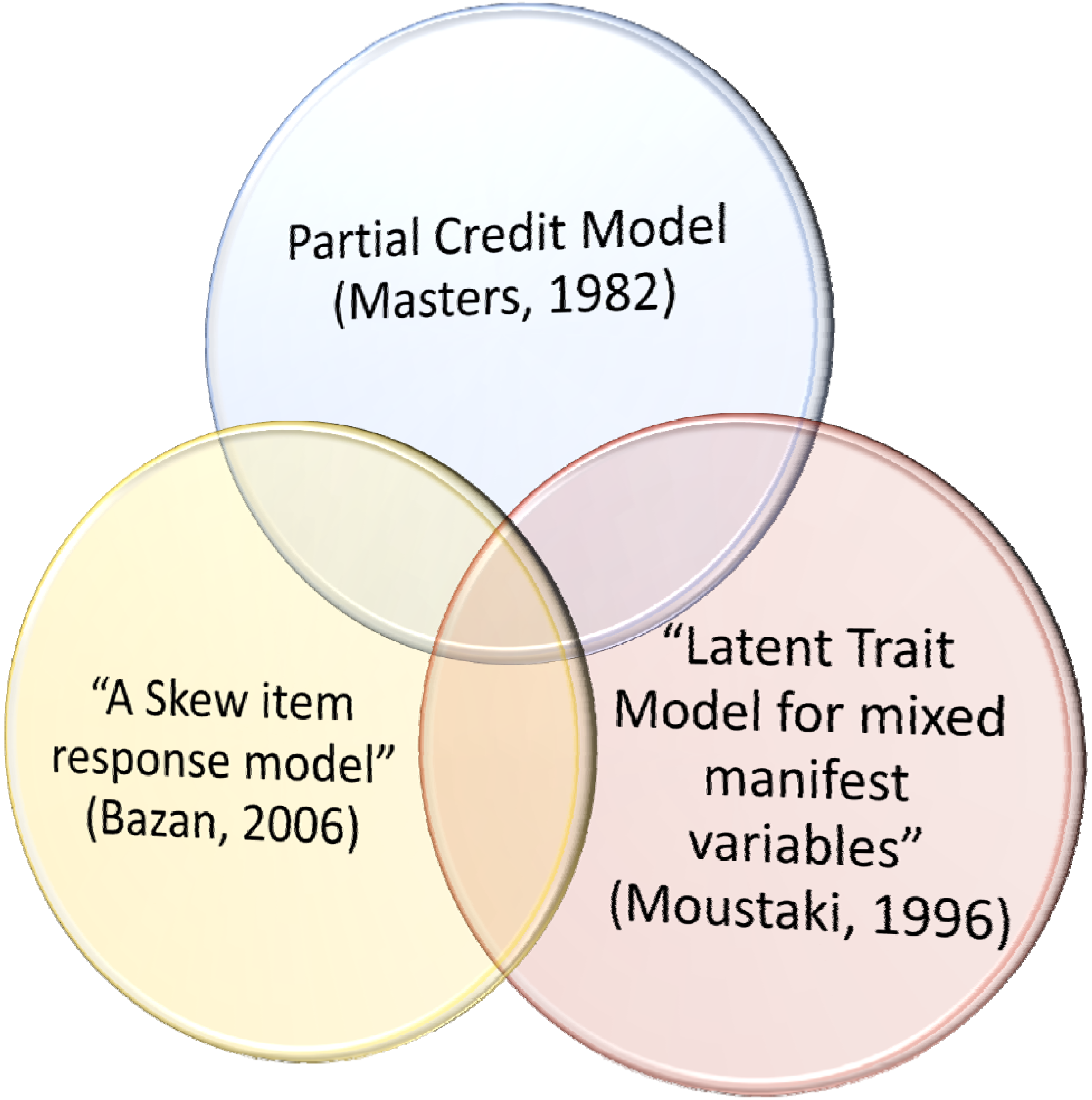


* Scalone et al, Value in Health, 2007

What you NEED



What you HAVE



A flexible IRT Model

- r ⟶ continuous items

  s ⟶ dichotomous items

  c ⟶ ordered polytomous items

  $s + r + c = q$ ⟶ total number of items

- Letting

  w_{ij} with j=1...r be the answer of subject i to the continuous item j

  v_{ij} with j=r + 1 ... r + s be the answer of subject i to the dichotomous item j

  $\mathbf{t}_{ij}$ with j= r + s + 1 ... $r + s + c$ be the answer of subject i to the ordered polytomous item j

Assumptions and constrains

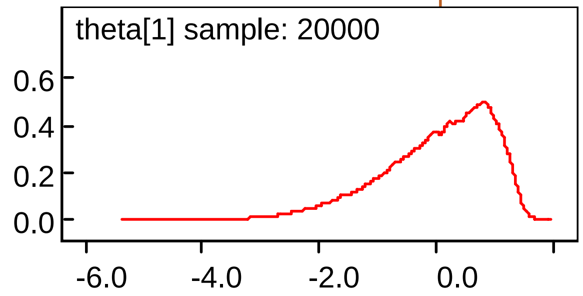
- Items are independent conditionally on θ
- $\theta_i \approx SN(\alpha, \beta, \delta) \quad i = 1 \ldots n$ (Azzalini, 1985)
- E(θ)=0 and Var(θ)=1

SKEW NORMAL CENTERED PARAMETERIZED (SN_{cp})

Given $Z \sim SN(0,1,\delta)$

$$\theta \mid Z = \frac{Z - \left(\frac{2}{\pi}\right)^{1/2} \delta}{\sqrt{\left(1 - \frac{2}{\pi}\delta^2\right)}} \sim \mathrm{SN}_{cp}(0,1,\delta)$$

δ = - 0,98



A flexible IRT model

- The conditional joint density function $g(\mathbf{y_i}|\theta)$ of the observed variables

$$\mathbf{y_i} = \begin{pmatrix} \mathbf{w_i} \\ \mathbf{v_i} \\ \mathbf{t_i} \end{pmatrix}$$

is

$$g(\mathbf{y}_i \mid \theta_i) = \prod_{j=1}^{q} g(y_{ij} \mid \theta_i) = \prod_{j=1}^{r} h(w_{ij} \mid \theta_i) \prod_{j=r+1}^{r+s} k(v_{ij} \mid \theta_i) \prod_{j=r+s+1}^{r+s+c} l(\mathbf{t}_{ij} \mid \theta_i)$$

where

- *h(.)* is the Normal density function of mean $\theta_i - b_j$ and variance σ_j^2
- *k(.)* is the Bernoulli probability function of parameter $\mu_{ij} = r^{-1}(\theta_i - b_j)$ and r(.)=logit link
- *l(.)* is the Multinomial probability function of parameters “PCM” and 1

Partial Credit Model (Masters, 1982)

- The probability of subject *i* scoring **x** to item *j* (item with k_j+1 levels of answer), given the latent variable θ is

$$p_{ijx}(\theta_i) = \frac{\exp \sum_{t=1}^{x} (\theta_i - b_{jt})}{1 + \sum_{k=1}^{k_j} \exp \sum_{t=1}^{k} (\theta_i - b_{jt})}, \text{for} \quad x = 1 \ldots k_j$$

$$p_{ijx}(\theta_i) = \frac{1}{1 + \sum_{k=1}^{k_j} \exp \sum_{t=1}^{k} (\theta_i - b_{jt})}, \text{for} \quad x = 0$$

A flexible IRT model

- The log-likelihood for a random sample of *n* individuals can be expressed as

$$\log L = \sum_{i=1}^{n} \log f(\mathbf{y_i}) = \sum_{i=1}^{n} \log \int_{-\infty}^{+\infty} g(\mathbf{y_i} \mid \theta) h(\theta) d\theta$$

where *h(θ)* is now the Skew Normal distribution function of mean 0 and variance 1.

Bayesian estimation of the parameters

- Joint Posterior distribution of the parameters of the model

$$p(b_j,\theta_i \mid \mathbf{y_i}) \propto \prod_{j=1}^{r} g(w_{ij} \mid b_j,\theta_i)h(\sigma_j)\prod_{j=r+1}^{r+s} g(v_{ij} \mid b_j,\theta_i)\prod_{j=r+s+1}^{r+s+c} g(\mathbf{t}_{ij} \mid b_j,\theta_i)\prod_{j=1}^{q} h(b_j)h(\theta_i \mid \delta)r(\delta)$$

where

$\theta_i \sim SN_{cp}(0,1,\delta)$

$b_j \sim N(0,100)$

$\sigma_j \sim \text{invgamma}(10,10)\ j=1\ldots r$

$\delta \sim U(-1,1)$

Bayesian estimation of the parameters

- Bayesian parameter estimates were obtained using Gibbs sampling algorithms as implemented in the computer program WinBUGS 1.4 (Spiegelhalter, Thomas, Best, & Lunn, 2003).

- The value taken as the MCMC estimate is the mean over iterations sampled starting with the first iteration following burn-in.

- The R-Package CODA (Best, Cowles, & Vines, 1995) was used to compute convergence Geweke's diagnostic.

Results

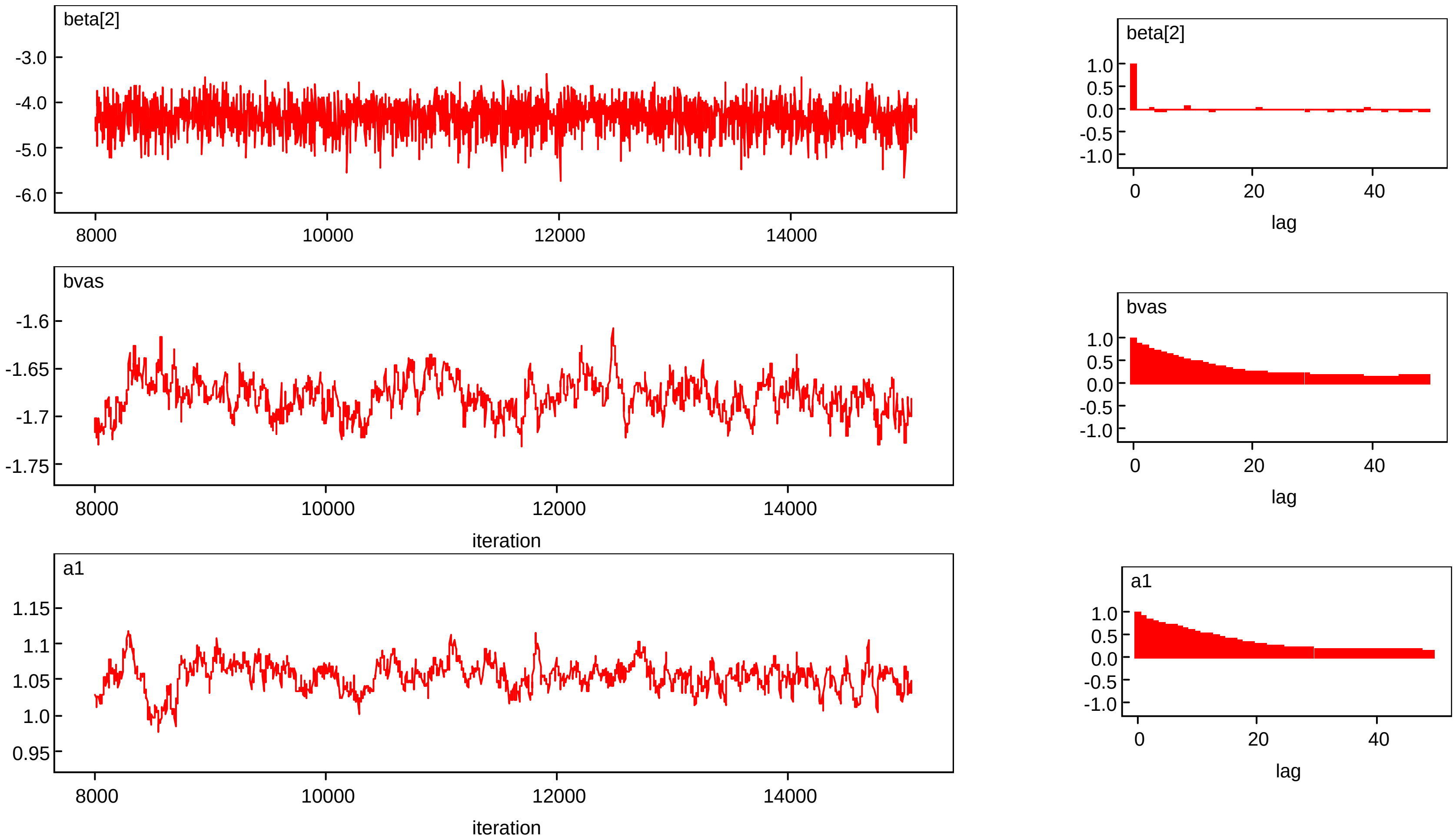
- **Model 1**: Partial Credit Model
  - 10,000 iterations with the first 3,000 as burn-in
- **Model 2**: IRT model for mixed responses
  - 25,000 iterations with the first 10,000 as burn-in
- **Model 3**: IRT for mixed responses and skew latent variable
  - 15,000 iterations with the first 5,000 as burn-in

Results

	Model 1				Model 2				Model 3			
	Posterior mean	SD	MC error	Median	Posterior mean	SD	MC error	Median	Posterior mean	SD	MC error	Median
HRQol												
[43] 11111 VAS=100	1.162	1.347	0.028	1.005	1.094	1.244	0.033	0.926	0.615	0.614	0.021	0.674
[1] 11111 VAS=85	1.147	1.370	0.027	1.004	0.949	1.195	0.027	0.783	0.577	0.636	0.022	0.661
[6] 11111 VAS=50	1.164	1.369	0.025	1.031	0.654	1.271	0.037	0.508	0.348	0.752	0.034	0.450
[29] 31122 VAS=65	-2.752	1.176	0.041	-2.702	-2.455	1.219	0.056	-2.397	-2.142	0.814	0.039	-2.156

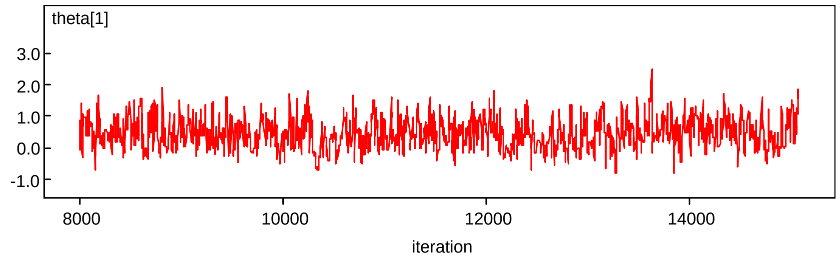
Model		**DIC**
1	PCM	181.1
2	PCM + VAS	133.3
3	PCM + VAS + skewed normal latent variable a priori	129.8

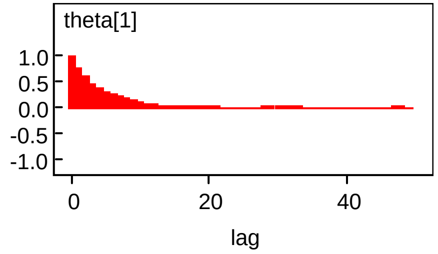
Results

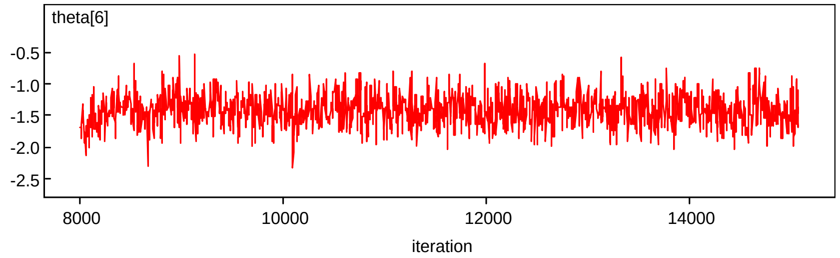


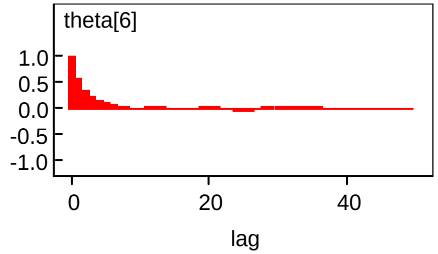
The procedure had a run length of 15,000 iterations with a burn-in period of 8,000 iterations. Every three states of the chain were included in the posterior estimates, to avoid autocorrelation.

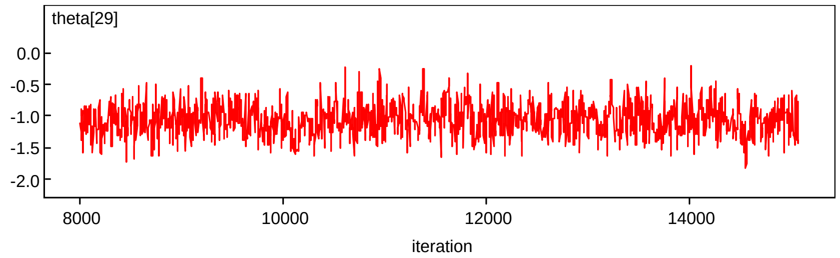
Results

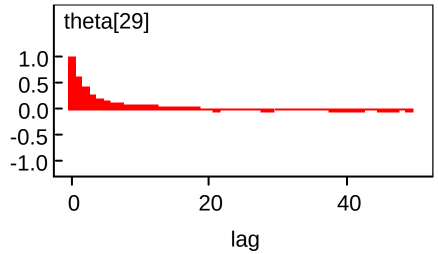

theta[1]
3.0
2.0
1.0
0.0
-1.0
8000
10000
12000
14000
iteration

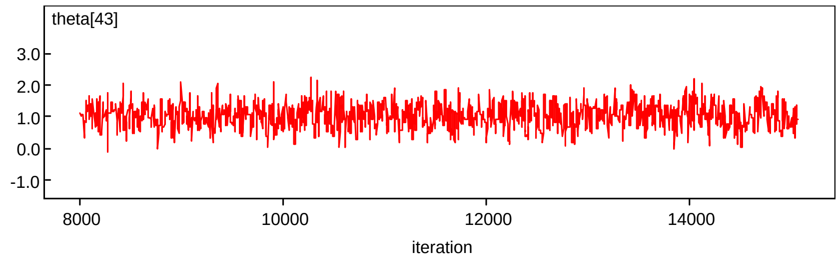

theta[1]
1.0
0.5
0.0
-0.5
-1.0
0
20
40
lag

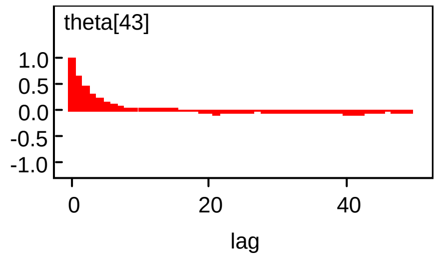

theta[6]
-0.5
-1.0
-1.5
-2.0
-2.5
8000
10000
12000
14000
iteration


theta[6]
1.0
0.5
0.0
-0.5
-1.0
0
20
40
lag

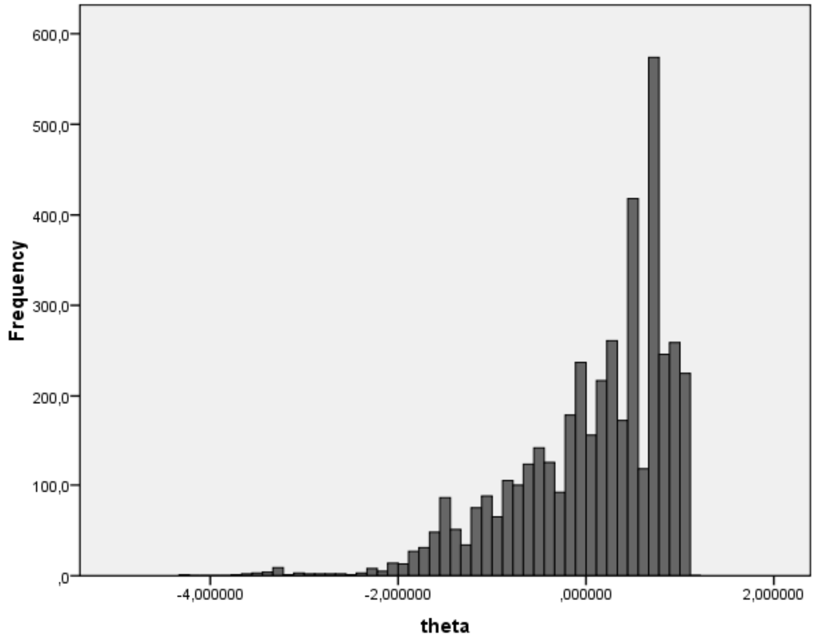

theta[29]
0.0
-0.5
-1.0
-1.5
-2.0
8000
10000
12000
14000
iteration


theta[29]
1.0
0.5
0.0
-0.5
-1.0
0
20
40
lag


theta[43]
3.0
2.0
1.0
0.0
-1.0
8000
10000
12000
14000
iteration


theta[43]
1.0
0.5
0.0
-0.5
-1.0
0
20
40
lag

Results



- The HRQoL mean value is 0.06 (s.d. 0.80)
- The maximum value is 1.065 and the minimum is -4.25
- The right-skewed shape of the histogram is expected, as well as the mean centered on 0.

Some limits

- Long computational times
- Not user-friendly software

Further developments

- Generalized Partial Credit model
- Covariates

References

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