A Case Study of Bank Branch Performance Using Linear Mixed Models

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Bank branch performance assessment

- Branches are the key contact point between customers and the central bank.
- Identify optimal branch network: number and location of branches. $\Rightarrow$ Assessment of branch performance.
- Commonly used: Data Envelope Analysis (DEA).
  - Non-parametric linear programming technique,
  - comparative ratio of inputs to outputs of each branch,
  - variables: in-branch (number of employees, operating expenses,...)
- Here: non-hierarchical linear mixed model.
  - Interaction effects,
  - variables: out-of-branch such as geographical and macroeconomic variables, in particular the influence of local socio-economic variables such as the wealth and local competition.

Linear mixed models

Definition

A linear mixed model is specified as follows:

$$Y_i = X_i\beta + Z_i u_i + \varepsilon_i$$

$$\left.\begin{array}{r} \varepsilon_i \sim \mathrm{N}(0, R_i) \\ u_i \sim \mathrm{N}(0, \Psi) \end{array}\right\} \text{independent}$$

where

$Y_i \in \mathbb{R}^{n_i}$ observations of group i

$X_i \in \mathbb{R}^{n_i \times p}$ design matrix for the fixed effects

$\beta \in \mathbb{R}^{p}$ fixed-effect coefficients

$Z_i \in \mathbb{R}^{n_i \times q}$ design matrix for the random effects

$u_i \in \mathbb{R}^{q}$ random-effect coefficients

$\varepsilon_i \in \mathbb{R}^{n_i}$ errors

$R_i \in \mathbb{R}^{n_i \times n_i}$ covariance matrix for the errors

$\Psi \in \mathbb{R}^{q \times q}$ covariance matrix for the random effects

Error structure

Often: $R_i = \sigma^2 I_{n_i}$.

Extensions:

- Let the dependent variable Y_{it} be time-dependent.

LMM with heterogeneous residual variances σ_t^2

$$\varepsilon_{it} \sim N(0, \sigma_t^2)$$

LMM with $ARMA(p, q)$-model as correlation structure

$$\varepsilon_{it} = \sum_{j=1}^{p} \phi_j \varepsilon_{i,t-j} + \sum_{j=1}^{q} \theta_j a_{t-j} + a_t$$

$\{a_t\}$ = zero mean white noise process with constant variance σ_a^2

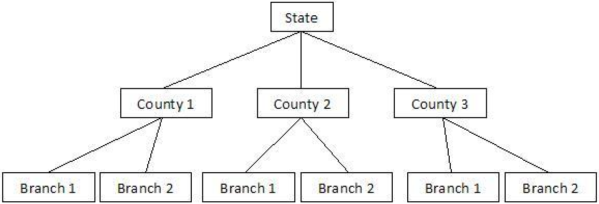
Estimation and testing

- Fixed effects:
  - Estimation usually using restricted maximum likelihood estimation (or standard maximum likelihood estimation).
  - Testing of the $H_0 : \beta_i = 0$ based on t-tests.
- Random effects:
  - Prediction using conditional expectations and estimated covariances.
- Testing with regard to covariance parameters based on likelihood ratio tests (LRT): null distribution is a mixture of χ^2 distributions.

Data

2988 branch-year records of a major US bank in the state of New York with multiple branches. 506 branches with observations over the period from 1994 to 2002.

- The data is clustered (branches within counties within state):



- The data is also longitudinal (observed over a period of 9 years).

$\Rightarrow$ Dependencies in the data.

$\Rightarrow$ Mixed model approach appropriate.

State variables

Constant over counties for each year, different outcomes for each year.

- *no.fail*: the number of branches that closed in NY during the year.
- *mshare*: the market share in NY.
- *branch.total*: the share of the number of branches in NY compared to the USA.
- *dep.total*: the share of the total deposits of the bank in NY compared to the USA.
- *av.dep*: the average deposit per branch in NY.

County variables

Constant over the branches within a county and changing for each year.

- *pop*: the population in the county (in 1000).
- *inc.pc*: the per capita income (in 1000).
- *unemp*: the unemployment rate in the county.

Branch variables

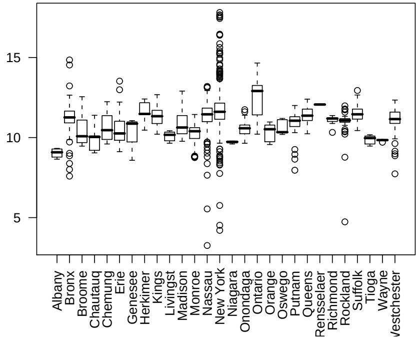
- *log.dep*: the total deposits in log form.
- *comp*: a measure of geographical competition of the branch (different for each year).
  - Scaled and standardized sum of the distances between a branch and all branches of other banks which have only one single branch or multiple branches respectively (0-100% competition).

Dependent variable

- Performance measure of total deposits of a branch.
  - One of the main business drivers of banks.
  - Easily collected and amenable for statistical analysis.

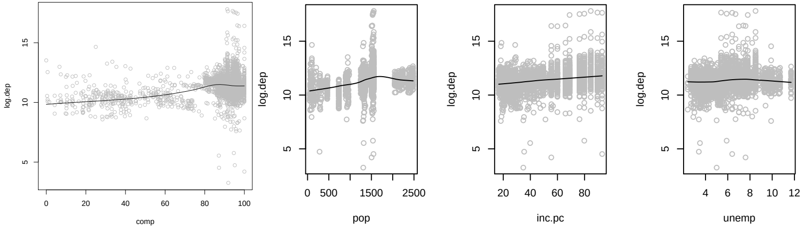
Aim

A model describing the (log-)deposits of branch i in county j in year t: $log.dep_{ijt}$.



Explorative data analysis

- Overall influence of *comp* is weakly positive.
- Weak positive influences of *pop* and *inc.pc*, no influence of *unemp*.



- No clear influence of any of the state variables.

$\Rightarrow$ Very variable effects.

Model formulation and fit: fixed and random effects

- Fixed effects for all branch, county and state variables and their interactions.
  - Select significant effects with t-tests at the 5% level.
- Random intercepts and slopes on the branch level b_{ij0}, b_{ij1}, as well as random intercepts on the county level b_j,
  - Distributions:

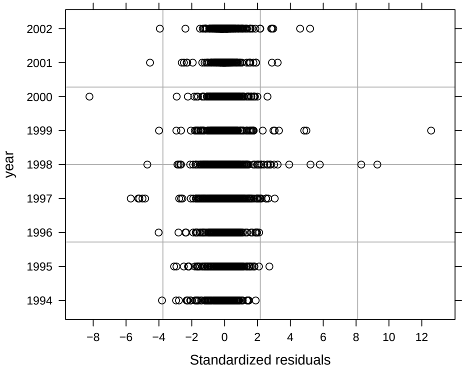
$$u_j \sim N(0, \psi_{00}^2)$$

$$u_{ij} = (u_{ij0}, u_{ij1})^T \sim N_2(0, \Psi) \text{ where } \Psi = \begin{pmatrix} \psi_0^2 & \psi_{01} \\ \psi_{01} & \psi_1^2 \end{pmatrix}.$$

  - Select significant effects with likelihood ratio tests at the 5% level.
- Non-hierarchical model, since random effects b_{ij0}, b_{ij1} are crossed with fixed effects of the county variables, e.g. pop_{jt}.

Variance structure of the errors

Since the number of observations and the values of *log.dep* in each year are varying, the within-group errors might be varying for each year, too.

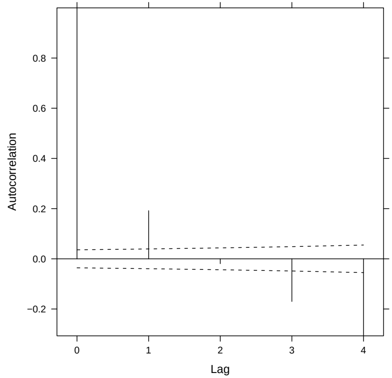


$H_0 : \sigma_t^2 = \sigma^2 \; \forall t$ rejected at the 5% level.

$\Rightarrow$ Include heterogeneous residual variances σ_t^2 for each year t.

Autocorrelation of the within-group errors

Since the observations are taken longitudinally on the same subjects, the within-group errors are probably autocorrelated.



$H_0 : \phi_1 = \theta_1 = 0$ rejected at the 5% level.
$\Rightarrow$ Include $ARMA(1,1)$-model as correlation structure.

Final model

- Fixed effects:

Variable	Estimate	Std. Error	p-value
Intercept	1.12 E+1	4.41 E−1	0.0000
pop	5.77 E−4	8.83 E−5	0.0000
inc.pc	−7.32 E−4	1.44 E−3	0.6121
unemp	−2.69 E−1	6.91 E−2	0.0001
no.fail	5.50 E−4	2.95 E−4	0.0624
mshare	−6.23 E+0	2.23 E+0	0.0054
branch.t	−3.90 E+0	1.80 E+0	0.0303
dep.t	3.44 E+0	1.68 E+0	0.0410
av.dep	1.70 E−6	2.87 E−7	0.0000

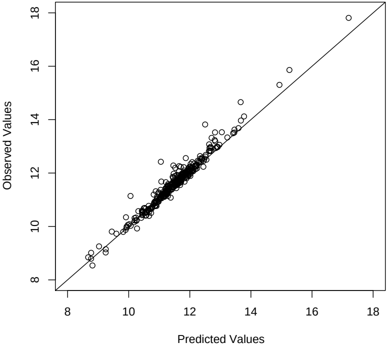
Interact.	Estimate	Std. Error	p-value
unemp× *no.fail*	−1.04 E−4	4.39 E−5	0.0184
unemp× *mshare*	1.37 E+0	3.67 E−1	0.0002
unemp× *branch.t*	1.05 E+0	2.82 E−1	0.0002
unemp× *dep.t*	−9.43 E−1	2.66 E−1	0.0004
inc.pc× *av.dep*	1.05 E−8	3.94 E−9	0.0078

- Random intercepts on the county level are not significant.
- Model diagnostics:
  - ✓ Residuals scatter around 0: 93.9% of all observations in [-2,2]-interval.
  - ✓ Assumption of normality appropriate for most years.
  - ✓ Zero mean and normality assumptions for random effects are plausible.

Check of the predictive capability

Approach:

- The final model is estimated with the data of 1994 to 2001.
- The values of 2002 are predicted using this restricted model.

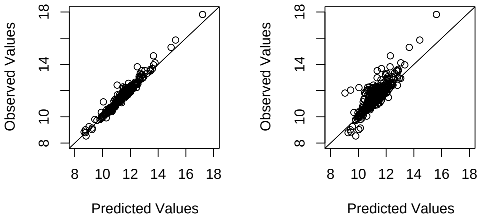


Points approximately lie on the line $y = x$. $\Rightarrow$ Quite good prediction.

Comparison to alternative models

Is the mixed model an improvement in the model fit compared to a linear model, to generalized least squares (GLS) model with heteroscedastic and correlated within-group errors (but no random effects) and to a hierarchical mixed model with the same error structure?

- The *AIC* of the linear mixed model is much smaller (531 vs. 7937 of the linear model and 3675 of the GLS model).
- The predictive capability of the hierarchical model is inferior to that of the non-hierarchical model.



⇒ Inclusion of random effects and flexible non-hierarchical structure significantly improve the fit.

Macroeconomic effects

- Positive effects of the market share, the average deposit per bank and the share of the number of branches in NY compared to the USA.
    1. ✓
    2. ✓
    3. No obvious explanation.
- Negative effects of the number of branches that closed during the year and the share of the total deposits in NY compared to the USA.
    1. Closures of branches $\leftrightarrow$ bad economic environment.
    2. No obvious explanation.

Geographical effects

- Positive effects of the county's population and the per capita income.
  1. population $\uparrow \Rightarrow$ deposits $\uparrow$
  2. income $\uparrow \Rightarrow$ deposits $\uparrow$
- Unclear effects of the unemployment rate and the local competition.
  1. Possible explanations:
     - Unemployed people have less cash flow (negative effect).
     - Unemployed people and people threatened by unemployment save more money because of the financial insecurity (positive effect).
  2. $\rightarrow$ next slide

Local competition

- No uniform influence of the geographical competition: *opposing trends*, if the competition increases.
    - Competition stimulates business.
    - More branches, same population $\Rightarrow$ less deposits.

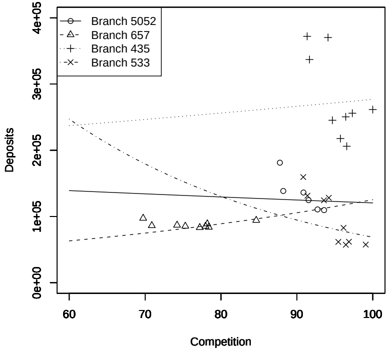
$\Rightarrow$ Classify branches as being in a "rural"/"urban", "rich"/"poor" area with low/high unemployment and compare average effect of competition.

- Stronger effect in rural areas: more likely to be influenced by marketing activities.
- Stronger effect in rich areas: choose banks more deliberately.
- Stronger effect in areas with high unemployment: more worried about money, susceptible to competing offers.

Branch-specific effects

- Some branches have more deposits than others if all other influences are disregarded: e.g. long-term customer loyalty or a particular good location in an area.
- Varying influence of the geographical competition on the deposits: see previous slide.

Influence of these branch-specific effects for four randomly chosen branches from Rockland (533), Suffolk (657), Nassau (5052) and New York (435):



Conclusion and outlook

- Regression analysis ...
  - summarizes information about *all* branches in the sample,
  - directly indicates causes of low performance, and
  - can be used to forecast deposits of new branches.

$\Rightarrow$ Easy evaluation of a single existing branch and of the potential of a new location.

- Outlook:
  - Use other performance variables such as fee income or the number of new deposit and/or lending accounts.
  - Include in-branch variables or competitive factors.

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