Differentiation tests for the mean shape and its variance

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Outline

1. Statistical shape analysis

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2. Renal tumors in early childhood

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3. Experimental results

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Landmarks

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Every object o_i in a space V of dimension m is thus represented in a space of dimension $k \cdot m$ by a set of landmarks:

$$\forall i = 1 \dots n,\ o_i = \{l_1 \dots l_k\}, l_j \in \mathbb{R}^m. \tag{1}$$

Removing the scale

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1. For every $i, i = 1, ..., n$, the size of each object is determined as the euclidian norm of their landmarks.

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2. The landmarks are standardized by dividing them by the size of their object:

$$\tilde{l}_j^i = \frac{l_j^i}{\|o_i\|}. \tag{3}$$

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2. We center all the landmarks by subtracting this mean:

$$\bar{l}_j^i = l_j^i - z^i \tag{5}$$

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We do not need any further procrustes analysis nor any complicated stochastic geometry. It is easy to show that the partial procrustean distance is equivalent to the euclidean distance in our case.

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If X demotes a random variable defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with values in a metric space (Ξ, d), an element $m \in \Xi$ is called a mean of $x_1, x_2, \ldots, x_k \in \Xi$ if

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That means that the mean shape is defined as the shape with the smallest variance of all shapes in a group of objects.

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The stopping rule is $\tilde{m} = T(\tilde{m})$.

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Renal tumors in early childhood

- Wilms-tumors (nephroblastoma) growing next to the kidney.

  Genetic cause. There are four types of tissue (a, b, c, d) and three stages of development (I, II, III).

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- Neuroblastoma growing next to nerve tissue.

  Quite frequent (80 per year).
- Clear cell carcinoma growing next to bones.

  Rare (12 per year).

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The data

Research sample:

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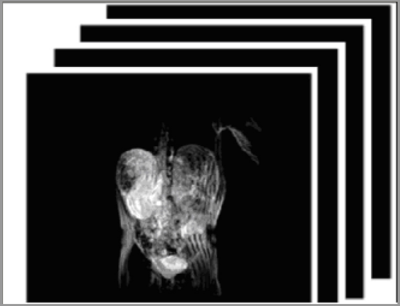
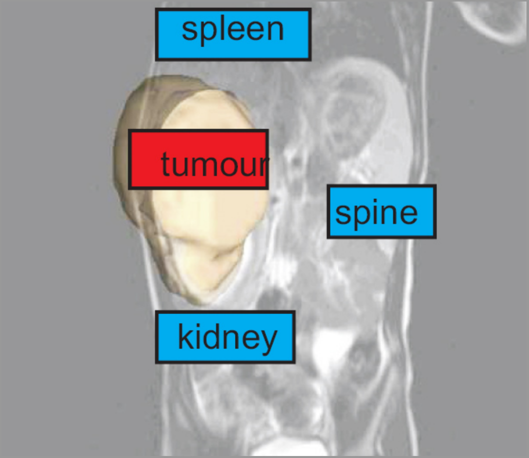


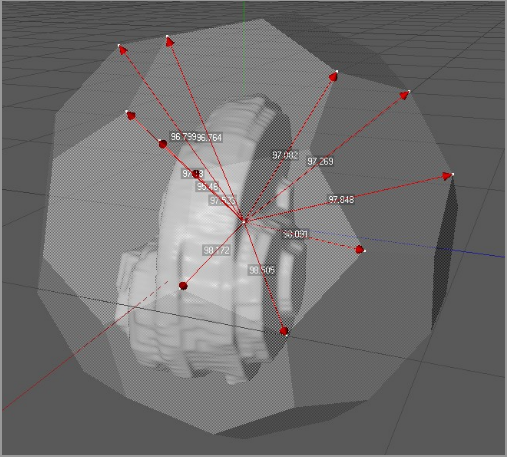
MRI image of a renal tumor in frontal view.

The three-dimensional object



Three-dimensional model of a tumor.

The platonic body C60



For every object, we consider the platonic body C60 whose center lies in the center of the object. This platonic body has 60 edges which give us 60 three-dimensional landmarks for every object.

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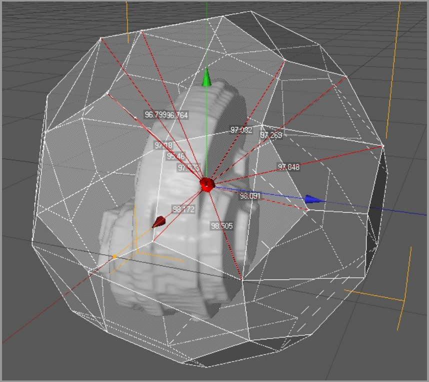

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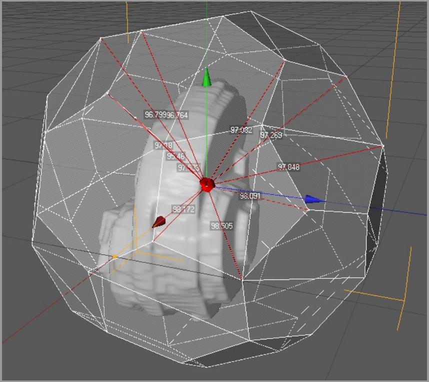

Only real measured points on the border of the tumor are taken, the approximated part of the three-dimensional object is not used.

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The test hypotheses are:

$$\begin{array}{ll} \textit{Hypothesis:} & H_0 : P = Q \\ \textit{Alternative:} & H_1 : P \neq Q \end{array}$$

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4. Comparing the u_0-value to all possible u-values. Computing the rank (small u-value mean a small rank).

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4. Comparing the u_0-value to all possible u-values. Computing the rank (small u-value mean a small rank).
5. Calculate the p-value for H_0. $p_{r=i} = \frac{1}{\binom{N}{n}}$ for $i = 1, \ldots, \binom{N}{n}$, where r is the rank for which we assume a uniform distribution.

Wilms tumors against non Wilms tumors

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- **Comparing the non Wilms tumors to the mean shape of the Wilms tumors.**

  $u = 257 \qquad \text{rank} = 1 - 2$

  Random sample: $n = 1000 \qquad p = 0,002.$

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- **Comparing the Neuroblastoma to the mean shape of the Wilms tumors.**

  $u = 257 \qquad \text{rank} = 15 - 40$

  Random sample: $n = 1000 \qquad p = 0.040.$

Mean variance of a set of shapes

We define the mean variance in the sense of Fréchet of a set of objects as the average of the distances to the mean shape.
If X denotes a random variable defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with values in a metric space (Ξ, d) and $m \in \Xi$ is the mean of $x_1, x_2, ..., x_k \in \Xi$, σ^2 is the variance of $x_1, x_2, ..., x_k \in \Xi$ if

$$\sum_{j=1}^{k} d(d(x_j, m)^2, \sigma^2)^2 = \inf_{\alpha \in \Xi} \sum_{j=1}^{k} d(d(x_j, m)^2, \alpha)^2. \qquad (10)$$

That means that the variance is defined as the mean of the distances between the "mean shape" and the objects.

The variance test

In this section we propose a test to compare the mean variance of two groups of objects.

step 1: Definition of the set of objects

There is a set M that can be divided into a subset A, realisation of a distribution P with variance σ_1^2 and a subset B, realisation of a distribution Q with variance σ_2^2.

Hypothesis: $H_0 : \sigma_1^2 = \sigma_2^2$
Alternative: $H_1 : \sigma_1^2 \neq \sigma_2^2$

The variance test

step 2: Computation of the variance

The variance is calculated by means of a straightforward generalisation of the algorithm of Ziezold (1994).

step 3: Computation of the F-value

$$F = \frac{|\hat{\sigma}_1^2|}{|\hat{\sigma}_2^2|}.$$

step 4: Determination of all the possibilities of dividing the set into two subsets with given sizes

step 5: Comparison of the F-value to all possible F-values. Computation of the rank (small F-value mean a small rank).

The variance test

step 6: Computation of the p-value for H_0

$p_{r=i} = 1 - \frac{1}{\binom{N}{n}}$ for $i = 1, \ldots, \binom{N}{n}$, where r is the rank for which we assume a rectangular distribution on the right side and $p_{r=i} = \frac{1}{\binom{N}{n}}$ on the left side.

Results of variance test

For the renal tumours, the F-value for the differentiation of the variance of the group of nephroblastomas to the group of neuroblastomas is 1.28128 and the rank is 315.
So the corresponding p-value is $1 - 0.315 = 0.685$ and we have to accept the null hyptothesis that the variance is similiar in both groups.
Both kind of tumours seem to have more or less the same dispersion and a possible difference in the dispersion can be excluded as cause for difficulties in distinguishing the two kinds of tumours.

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- Wilms tumors can be clearly differentiated from neuroblastomas.
- It is possible to differentiate the whole set of non-Wilms tumors from the mean shape of Wilms tumors.
- But we cannot use statistical shape analysis to say if a given general tumor is not a Wilms tumor.
- The variance of the set of Wilms tumors is not significantly different from the variance of the set of the other tumors.