Multivariate stochastic volatility model with cross leverage

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joint work with

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Time varying volatility models

Asset return

$$y_t = \log p_t - \log p_{t-1}$$

p_t: asset price at time t. $\rightarrow$ variance varies over time.

- GARCH (Generalized Autoregressive Conditional Heterogeneity) model.
- EGARCH (Exponential GARCH) model.
- Stochastic Volatility (SV) model.

Univariate SV model

- SV model

$$\begin{aligned} y_t &= \epsilon_t \exp(h_t/2), \quad \epsilon_t \sim \mathcal{N}(0,1), \\ h_{t+1} &= \mu + \phi(h_t - \mu) + \eta_t, \quad \eta_t \sim \mathcal{N}(0,\sigma_\eta^2), \end{aligned} \tag{1}$$

- Leverage effect.

$$Corr(\epsilon_t, \eta_t) = \rho < 0, \tag{2}$$

Efficient Sampler for $\{h_t\}_{t=1}^n$

Efficient sampler for SV models.

- Efficient sampler for SV model without leverage.
  - Block sampler: Shephard and Pitt (1997), Watanabe and Omori (2004), *Biometrika*. So (2006). *Statistics and Computing*.
  - 'Approximation' sampler (High efficiency).
    Kim, Shephard and Chib (1998). *Rev. Econ. Stud.*
    Chib, Nardari and Shephard (2002). *J. Econometrics.*
- Efficient sampler for SV model with leverage.
  - Block sampler: Omori and Watanabe (2008). *CSDA*.
  - 'Approximation' sampler:
    Omori, Chib, Shephard and Nakajima (2007). *J. Econometrics.*
    Nakajima and Omori (2009, 2010). *CSDA*.

MSV model with cross leverage

Multivariate Stochastic Volatility model

$$\mathbf{y}_t = \mathbf{V}_t^{1/2}\boldsymbol{\varepsilon}_t, \quad t = 1, \ldots, n, \tag{3}$$

$$\boldsymbol{\alpha}_{t+1} = \boldsymbol{\Phi}\boldsymbol{\alpha}_t + \boldsymbol{\eta}_t, \quad t = 1, \ldots, n-1, \tag{4}$$

$$\boldsymbol{\alpha}_1 \sim \mathcal{N}_p\left(\mathbf{0}, \boldsymbol{\Sigma}_0\right), \tag{5}$$

where $\boldsymbol{\Sigma}_0 = \boldsymbol{\Phi}\boldsymbol{\Sigma}_0\boldsymbol{\Phi} + \boldsymbol{\Sigma}_{\eta\eta}$, and

$$\begin{aligned} \mathbf{V}_t^{1/2} &= \mathrm{diag}\left(\exp(\alpha_{1t}/2), \ldots, \exp(\alpha_{pt}/2)\right) \\ \boldsymbol{\Phi} &= \mathrm{diag}(\phi_1, \ldots, \phi_p), \end{aligned}$$

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \Big| \boldsymbol{\alpha}_t \sim \mathcal{N}_{2p}(\mathbf{0}, \boldsymbol{\Sigma}), \quad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_{\varepsilon\varepsilon} & \boldsymbol{\Sigma}_{\varepsilon\eta} \\ \boldsymbol{\Sigma}_{\eta\varepsilon} & \boldsymbol{\Sigma}_{\eta\eta} \end{pmatrix}$$

Previous Literature

- MSV without leverage.
  - Smith and Pitts (2006). $p = 2$. (simple) block sampler for α_t's.
  - Bos and Shephard (2006). single move sampler.
  - Yu and Meyer (2006). $p = 2$. single move sampler.
  - So and Kwok (2006). QML. Long memory.
- MSV with leverage.
  - Chan, Kohn and Kirby (2006). simple block sampler. Asymmetry using (Wong, Carter and Kohn (2003))

$$\boldsymbol{\Sigma}^{-1} = \mathbf{TGT}, \quad \mathbf{T} = \mathrm{diag}\left(\sqrt{G^{11}}, \ldots, \sqrt{G^{pp}}\right),$$

  - Asai and McAleer (2006). Monte Carlo likelihood. Constrained covariance matrix (without cross leverage).
- Survey: see e.g. Chib, Omori and Asai (2009)

Efficient MCMC estimation

1. Generate $\boldsymbol{\alpha}|\boldsymbol{\phi}, \boldsymbol{\Sigma}, \mathbf{y}$. (two options A or B)

   A. *Multi-movesampler*

      (A1) Generate K stochastic knots $(k_1, \ldots, k_K)$ and set $k_0 = 0$, $k_{K+1} = n$.

      (A2) Generate $\{\boldsymbol{\alpha}_t\}_{t=k_{i-1}+1}^{k_i} | \{\boldsymbol{\alpha}_t | t \leq k_{i-1}, t > k_i\}, \boldsymbol{\phi}, \boldsymbol{\Sigma}, Y_n$ for $i = 1, \ldots, K+1$

   B. *Single-movesampler*

      Generate $\boldsymbol{\alpha}_t | \boldsymbol{\alpha}_{\backslash t}, \boldsymbol{\phi}, \boldsymbol{\Sigma}, Y_n$ for $t = 1, \ldots, n$

2. Generate $\boldsymbol{\Sigma}|\boldsymbol{\phi}, \{\boldsymbol{\alpha}_t\}_{t=1}^n, \mathbf{y}$.(Independent MH)
3. Generate $\boldsymbol{\phi}|\boldsymbol{\Sigma}, \{\boldsymbol{\alpha}_t\}_{t=1}^n, \mathbf{y}$.(Independent MH)

Multi-move sampler for α_t's

Approximation of the log conditional likelihood

$$\begin{aligned}\mathbf{y}_t|\boldsymbol{\alpha}_t,\boldsymbol{\alpha}_{t+1} &\sim N(\boldsymbol{\mu}_t,\boldsymbol{\Sigma}_t), \quad t=1,\ldots,n \\ \boldsymbol{\mu}_t &= \mathbf{V}_t^{1/2}\boldsymbol{\Sigma}_{\varepsilon\eta}\boldsymbol{\Sigma}_{\eta\eta}^{-1}(\boldsymbol{\alpha}_{t+1}-\Phi\boldsymbol{\alpha}_t)I(t<n), \\ \boldsymbol{\Sigma}_{\mathbf{t}} &= \mathbf{V}_t^{1/2}(\boldsymbol{\Sigma}_{\varepsilon\varepsilon}-\boldsymbol{\Sigma}_{\varepsilon\eta}\boldsymbol{\Sigma}_{\eta\eta}^{-1}\boldsymbol{\Sigma}_{\eta\varepsilon}I(t<n))\mathbf{V}_t^{1/2},\end{aligned}$$

$\rightarrow$ Approximate $L=\sum_{s=t}^{t+k} l(\boldsymbol{\alpha}_s,\boldsymbol{\alpha}_{s+1})-\frac{1}{2}\boldsymbol{\eta}_{s+k}'\boldsymbol{\Sigma}_{\eta\eta}^{-1}\boldsymbol{\eta}_{s+k}$, where $l(\boldsymbol{\alpha}_s,\boldsymbol{\alpha}_{s+1})=\log f(\mathbf{y}_s|\boldsymbol{\alpha}_s,\boldsymbol{\alpha}_{s+1})$, using Taylor expansion around the conditional mode of $\mathbf{x}_s\equiv\mathbf{R}_s^{-1}\boldsymbol{\eta}_s$ ($\boldsymbol{\Sigma}_{\eta\eta}=\mathbf{R}_s\mathbf{R}_s'$, $\boldsymbol{\Sigma}_0=\mathbf{R}_0\mathbf{R}_0'$).

Approximation of the log conditional likelihood

$$
\begin{aligned}
&\log f(\mathbf{x}_s, \ldots, \mathbf{x}_{s+k} | \boldsymbol{\alpha}_s, \boldsymbol{\alpha}_{s+k+1}, \mathbf{y}_s, \ldots, \mathbf{y}_{s+k+1}) \\
\approx\ & \text{const} - \frac{1}{2}\sum_{t=s}^{s+k} \mathbf{x}_t'\mathbf{x}_t + \hat{L} + \left.\frac{\partial L}{\partial \mathbf{x}'}\right|_{\mathbf{x}=\hat{\mathbf{x}}} (\mathbf{x} - \hat{\mathbf{x}}) \\
& + \frac{1}{2}(\mathbf{x} - \hat{\mathbf{x}})' \left. E\left(\frac{\partial L^2}{\partial \mathbf{x}\partial \mathbf{x}'}\right)\right|_{\mathbf{x}=\hat{\mathbf{x}}} (\mathbf{x} - \hat{\mathbf{x}}) \\
=\ & \text{const} + \log g(\mathbf{x}_s, \ldots, \mathbf{x}_{s+k} | \boldsymbol{\alpha}_s, \boldsymbol{\alpha}_{s+k+1}, \mathbf{y}_s, \ldots, \mathbf{y}_{s+k+1})
\end{aligned}
$$

where, $\mathbf{x}_s = \mathbf{R}_s^{-1}\boldsymbol{\eta}_s$, $\boldsymbol{\Sigma}_{\eta\eta} = \mathbf{R}_t\mathbf{R}_t'$.
$\rightarrow$ Construct the auxiliary state space model to obtain the sample from g (Omori and Watanabe (2008)).

Approximation of the log conditional likelihood

$$
\begin{aligned}
\mathbf{d} &= \partial L/\partial \boldsymbol{\alpha}|_{\mathbf{x}=\hat{\mathbf{x}}} \\
\mathbf{Q} &= -E\left[\frac{\partial^2 L}{\partial \mathbf{x}\partial \mathbf{x}'}\right]_{\mathbf{x}=\hat{\mathbf{x}}} \\
&= \begin{pmatrix}
\mathbf{A}_{s+1} & \mathbf{B}'_{s+2} & \mathbf{O} & \cdots & \mathbf{O} \\
\mathbf{B}_{s+2} & \mathbf{A}_{s+2} & \mathbf{B}'_{s+3} & \cdots & \mathbf{O} \\
\mathbf{O} & \mathbf{B}_{s+3} & \mathbf{A}_{s+3} & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \mathbf{B}'_{s+m} \\
\mathbf{O} & \cdots & \mathbf{O} & \mathbf{B}_{s+m} & \mathbf{A}_{s+m}
\end{pmatrix} \\
\mathbf{A}_t &= -\,E\left[\frac{\partial^2 L}{\partial \boldsymbol{\alpha}_t \partial \boldsymbol{\alpha}'_t}\right]\Bigg|_{\mathbf{x}=\hat{\mathbf{x}}}, \quad \mathbf{B}_t = -\,E\left[\frac{\partial^2 L}{\partial \boldsymbol{\alpha}_t \partial \boldsymbol{\alpha}'_{t-1}}\right]\Bigg|_{\mathbf{x}=\hat{\mathbf{x}}}
\end{aligned}
$$

Approximation of the log conditional likelihood

Calculate

- $\mathbf{D}_{s+1} = \hat{\mathbf{A}}_{s+1}$
- $\mathbf{D}_t = \hat{\mathbf{A}}_t - \hat{\mathbf{B}}_t \mathbf{D}_{t-1}^{-1} \hat{\mathbf{B}}_t'$, $t = s+2, \ldots, s+m$
- $\mathbf{K}_t$: Choleski decomposition of $\mathbf{D}_t$ ($\mathbf{D}_t = \mathbf{K}_t \mathbf{K}_t'$).

Define auxiliary variable $\hat{\mathbf{y}}_t = \hat{\gamma}_t + \mathbf{D}_t^{-1} \mathbf{b}_t$ $(s+1 \leq t \leq s+m)$ where

$$
\begin{aligned}
\hat{\gamma}_t &= \hat{\alpha}_t + \mathbf{D}_t^{-1} \hat{\mathbf{B}}_t' \hat{\alpha}_{t+1} \quad (\hat{\gamma}_{s+m} = \hat{\alpha}_{s+m}) \\
\mathbf{b}_t &= \mathbf{d}_t - \hat{\mathbf{B}}_t \mathbf{D}_{t-1}^{-1} \mathbf{b}_{t-1} \quad (\mathbf{b}_{s+1} = \mathbf{d}_{s+1})
\end{aligned}
$$

Approximation of the log conditional likelihood

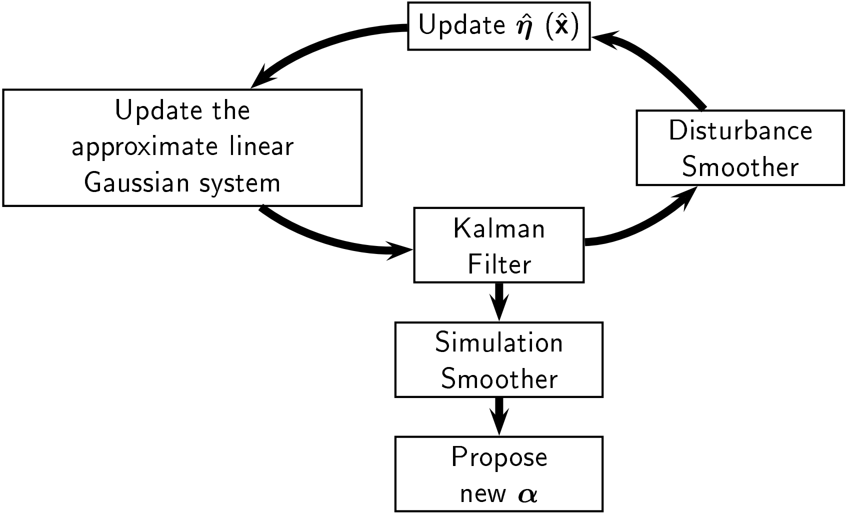
Approximate linear Gaussian system (← proposal)

$$\begin{aligned}\hat{\mathbf{y}}_t &= \mathbf{Z}_t \boldsymbol{\alpha}_t + \mathbf{G}_t \mathbf{u}_t \\ \boldsymbol{\alpha}_{t+1} &= \mathbf{T}_t \boldsymbol{\alpha}_t + \mathbf{H}_t \mathbf{u}_t \\ \mathbf{u}_t &= (\mathbf{u}_{1t}', \mathbf{x}_t')' \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_{2p})\end{aligned}$$

where

$$\begin{aligned}\mathbf{Z}_t &= \mathbf{I} + \mathbf{D}_t^{-1} \hat{\mathbf{B}}_t' \mathbf{T}_t \\ \mathbf{G}_t &= [\mathbf{K}_t'^{-1}, \mathbf{D}_t^{-1} \hat{\mathbf{B}}_t' \mathbf{R}_t] \\ \mathbf{H}_t &= [\mathbf{O}, \mathbf{R}_t]\end{aligned}$$

Multi-move sampler



MCMC estimation (Single move sampler for $\{\boldsymbol{\alpha}_t\}$)

Generate $\boldsymbol{\alpha}_t|\{\boldsymbol{\alpha}_s\}_{s\neq t}, \boldsymbol{\phi}, \boldsymbol{\Sigma}, Y_n$: using MH algorithm. Propose a candidate $\boldsymbol{\alpha}_t^\dagger$ from $\boldsymbol{\alpha}_t^\dagger \sim \mathrm{N}(\mathbf{m}_{\alpha_t}, \boldsymbol{\Sigma}_{\alpha_t})$ and accept it with probability

$$\min\left\{\exp\{g(\boldsymbol{\alpha}_t^\dagger) - g(\boldsymbol{\alpha}_t)\}, 1\right\}$$

for $t = 1, \ldots, n$ where $g(\boldsymbol{\alpha}_t) = -\frac{1}{2}\mathbf{y}_t'\boldsymbol{\Sigma}_t^{-1}\mathbf{y}_t + \mathbf{y}_t'\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\mu}_t$, and $\boldsymbol{\mu}_t = \mathbf{V}_t^{1/2}\mathbf{m}_t$, $\boldsymbol{\Sigma}_t = \mathbf{V}_t^{1/2}\mathbf{S}_t\mathbf{V}_t^{1/2}$ and

$$\mathbf{m}_t = \begin{cases} \boldsymbol{\Sigma}_{\epsilon\eta}\boldsymbol{\Sigma}_{\eta\eta}^{-1}(\boldsymbol{\alpha}_{t+1} - \Phi\boldsymbol{\alpha}_t), & t < n, \\ \mathbf{0} & t = n, \end{cases}$$

$$\mathbf{S}_t = \begin{cases} \boldsymbol{\Sigma}_{\epsilon\epsilon} - \boldsymbol{\Sigma}_{\epsilon\eta}\boldsymbol{\Sigma}_{\eta\eta}^{-1}\boldsymbol{\Sigma}_{\eta\epsilon}, & t < n, \\ \boldsymbol{\Sigma}_{\epsilon\epsilon} & t = n. \end{cases}$$

(Definitions of $\mathbf{m}_{\alpha_t}, \boldsymbol{\Sigma}_{\alpha_t}$ are omitted)

Simulation data

Number of observations : $n = 4000$.
Number of assets : $p = 5$.

AR(1) parameters: $\phi_i = 0.97$,

Variances:

$$\sigma_{i,\varepsilon\varepsilon} = \sqrt{Var(\varepsilon_{it})} = 1.2, \qquad \sigma_{i,\eta\eta} = \sqrt{Var(\eta_i)} = 0.2,$$

Correlations:

$$\rho_{ij,\varepsilon\varepsilon} = Corr(\varepsilon_i, \varepsilon_j) = 0.6, \quad \rho_{ij,\eta\eta} = Corr(\eta_i, \eta_j) = 0.7,$$

Leverages:

$$\rho_{ii,\varepsilon\eta} = Corr(\varepsilon_i, \eta_i) = -0.4, \; \rho_{ij,\varepsilon\eta} = Corr(\varepsilon_i, \eta_j) = -0.3, \; i \neq j$$

Priors:

$$(\phi_i + 1)/2 \sim \mathcal{B}(20, 1.5), \quad \mathbf{\Sigma} \sim \mathcal{IW}(10, (10 \times true\ \mathbf{\Sigma})^{-1})$$

Single-move sampler vs Multi-move sampler

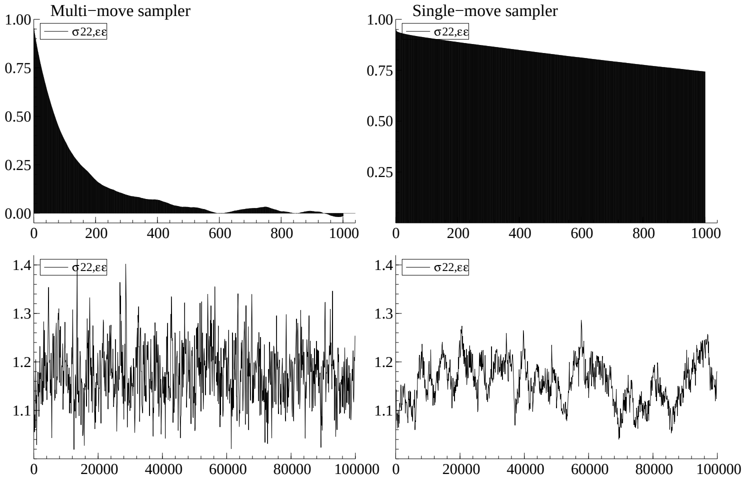
Table: Maxima of inefficiency factors for $K = 50, 100, 140, 200$, and 270

K	$\frac{n}{(K+1)}$	ϕ_j	$\sigma_{i,\varepsilon\varepsilon}$	$\sigma_{i,\eta\eta}$	$\rho_{i,\varepsilon\eta}$	$\rho_{ij,\varepsilon\varepsilon}$	$\rho_{ij,\eta\eta}$	$\rho_{ij,\varepsilon\eta}$	ν
50	79	221	407	388	168	8	323	163	152
100	40	153	160	256	84	4	219	101	66
140	29	151	242	214	86	4	185	100	29
200	20	111	236	181	81	3	160	90	48
270	15	92	257	154	69	3	155	73	46
SM	1	786	6732	901	297	27	937	509	151

Red : maximum Blue : minimum.

$\rightarrow K = 200$ is selected.

Sample ACF and sample path of $\sigma_{i,\varepsilon\varepsilon}$



Data

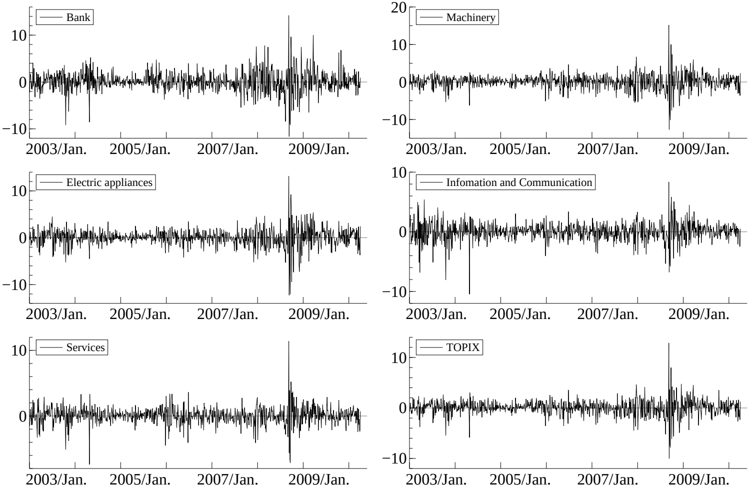
Tokyo Stock Exchange Indices.

- Five stock price indices ($p = 5$) from 33 industry sectors.

  Series 1: 'Bank'
  Series 2: 'Machinery'
  Series 3: 'Electric appliances'
  Series 4: 'Information and Communication'
  Series 5: 'Services'

- January 6, 2003 – May 28, 2010. $n = 1814$ obs.

Five stock price indices



Models and Priors

1. MSVLn: MSV model with cross leverage, multivariate normal error.
2. MSVLt: MSV model with cross leverage, multivariate t error, $(\mathbf{y}_t = \lambda_t^{-1/2}\mathbf{V}_t\boldsymbol{\varepsilon}_t)$.
3. SVLn: univariate SV model with leverage, normal error.
4. SVLt: univariate SV model with leverage, t error.

Priors are

$$\begin{aligned} \frac{\phi_i + 1}{2} &\sim \mathcal{B}(20, 1.5), \quad i = 1, \ldots, 5, \\ \boldsymbol{\Sigma} &\sim \mathcal{IW}(10, (10\boldsymbol{\Sigma}^*)^{-1}), \quad \nu \sim \mathcal{G}(0.01, 0.01), \end{aligned}$$

where

$$\boldsymbol{\Sigma}^* = \begin{pmatrix} 1.2^2(0.3\mathbf{I}_5 + 0.7\mathbf{1}_5\mathbf{1}_5') & 1.2 \times 0.2 \times (-0.1\mathbf{I}_5 - 0.1\mathbf{1}_5\mathbf{1}_5') \\ & 0.2^2(0.2\mathbf{I}_5 + 0.8\mathbf{1}_5\mathbf{1}_5') \end{pmatrix},$$

MSVLt:

	i	Mean	Stdev	95% interval	IF
	1	0.976	0.005	[0.965, 0.985]	96
	2	0.971	0.005	[0.960, 0.980]	106
ϕ_i	3	0.980	0.004	[0.971, 0.987]	101
	4	0.977	0.005	[0.967, 0.986]	89
	5	0.967	0.006	[0.954, 0.979]	101

Univariate SVLt:

	i	Mean	Stdev	95% interval	IF
	1	0.984	0.005	[0.973, 0.993]	76
	2	0.972	0.006	[0.959, 0.983]	113
ϕ	3	0.985	0.004	[0.976, 0.993]	58
	4	0.979	0.007	[0.964, 0.991]	169
	5	0.960	0.011	[0.936, 0.979]	127

The estimated means of MSVLt model are smaller.

MSVLt:

	i	Mean	Stdev	95% interval	IF
$\sigma_{i\varepsilon\varepsilon}$	1	1.630	0.140	[1.356, 1.909]	280
	2	1.501	0.085	[1.345, 1.680]	190
	3	1.435	0.104	[1.246, 1.657]	292
	4	1.247	0.091	[1.077, 1.440]	254
	5	1.075	0.055	[0.973, 1.193]	199

Univariate SVLt:

	i	Mean	Stdev	95% interval	IF
$\sigma_{i\varepsilon\varepsilon}$	1	1.697	0.184	[1.371, 2.096]	245
	2	1.612	0.104	[1.426, 1.833]	147
	3	1.504	0.157	[1.240, 1.865]	172
	4	1.299	0.123	[1.100, 1.600]	221
	5	1.088	0.065	[0.976, 1.236]	79

The estimates and their stdevs of variance parameters of MSVLt model are smaller.

MSVLt:

	i	Mean	Stdev	95% interval	IF
$\sigma_{i\eta\eta}$	1	0.184	0.018	[0.151, 0.221]	154
	2	0.167	0.016	[0.138, 0.199]	151
	3	0.150	0.014	[0.125, 0.179]	134
	4	0.153	0.015	[0.126, 0.183]	137
	5	0.160	0.015	[0.133, 0.191]	166

Univariate SVLt:

	i	Mean	Stdev	95% interval	IF
$\sigma_{i\eta\eta}$	1	0.177	0.020	[0.140, 0.219]	186
	2	0.188	0.020	[0.151, 0.230]	189
	3	0.157	0.017	[0.127, 0.193]	154
	4	0.156	0.020	[0.119, 0.200]	314
	5	0.199	0.026	[0.153, 0.255]	192

MSVLt:

	i	Mean	Stdev	95% interval	IF
$\rho_{ii,\varepsilon\eta}$	1	-0.233	0.073	[-0.373,-0.086]	26
	2	-0.392	0.065	[-0.515,-0.260]	45
	3	-0.297	0.075	[-0.440,-0.146]	51
	4	-0.191	0.088	[-0.354,-0.011]	55
	5	-0.330	0.072	[-0.465,-0.182]	57

Univariate SVLt:

	i	Mean	Stdev	95% interval	IF
$\rho_{ii,\varepsilon\eta}$	1	-0.394	0.075	[-0.533, -0.235]	17
	2	-0.611	0.058	[-0.715, -0.489]	77
	3	-0.497	0.075	[-0.632, -0.339]	43
	4	-0.316	0.095	[-0.493, -0.120]	77
	5	-0.454	0.072	[-0.587, -0.301]	44

The estimated leverage effects in MSVLt model are larger.

MSVLt:

	i	Mean	Stdev	95% interval	IF
	12	0.726	0.012	[0.702, 0.749]	2
	13	0.709	0.013	[0.683, 0.733]	3
	14	0.634	0.015	[0.604, 0.663]	3
	15	0.699	0.013	[0.673, 0.723]	4
	23	0.858	0.007	[0.845, 0.871]	2
$\rho_{ij,\varepsilon\varepsilon}$	24	0.648	0.014	[0.620, 0.676]	2
	25	0.753	0.011	[0.731, 0.774]	3
	34	0.703	0.013	[0.677, 0.727]	2
	35	0.742	0.011	[0.719, 0.764]	3
	45	0.732	0.012	[0.709, 0.754]	3

Correlations of returns are all positive ($0.6 \sim 0.8$).

MSVLt:

	i	Mean	Stdev	95% interval	IF
	12	0.763	0.055	[0.641, 0.853]	120
	13	0.727	0.063	[0.589, 0.837]	132
	14	0.639	0.079	[0.470, 0.778]	108
	15	0.674	0.074	[0.512, 0.798]	145
	23	0.831	0.037	[0.748, 0.892]	89
$\rho_{ij,\eta\eta}$	24	0.673	0.068	[0.523, 0.788]	83
	25	0.778	0.050	[0.670, 0.862]	79
	34	0.792	0.048	[0.687, 0.872]	83
	35	0.807	0.045	[0.706, 0.883]	134
	45	0.814	0.044	[0.715, 0.886]	115

Correlations of volatilities are all positive and high.

MSVLt: Cross leverages between Series 2 and others.

	i	Mean	Stdev	95% interval	IF
$\rho_{2i,\varepsilon\eta}$ $2 \to i$	1	-0.183	0.077	[-0.330,-0.031]	53
	3	-0.251	0.073	[-0.391,-0.105]	40
	4	-0.143	0.082	[-0.302, 0.019]	56
	5	-0.276	0.075	[-0.419,-0.125]	64
$\rho_{i2,\varepsilon\eta}$ $i \to 2$	1	-0.356	0.068	[-0.483,-0.215]	32
	3	-0.369	0.070	[-0.498,-0.226]	46
	4	-0.309	0.080	[-0.461,-0.146]	49
	5	-0.367	0.070	[-0.498,-0.225]	44

Leverage effects: impacts from return of Series 2 ("Machinery" industries) to the volatilities of other industries are small, while the inverse impacts are large.

MSVLt: Degrees of freedom

	i	Mean	Stdev	95% interval	IF
ν (MSVLt)		34.3	10.9	[20.8, 60.6]	167
	1	57.8	54.8	[16.0,219.1]	699
	2	117.6	97.1	[24.4,395.0]	1325
ν_i (SVLt)	3	82.9	57.9	[19.8,240.1]	683
	4	50.7	42.6	[14.7,169.8]	986
	5	85.4	86.0	[18.2,354.3]	861

Model comparison based on DIC

Compute the DIC defined by

$$DIC \equiv E_{\theta|Y_n}[D(\theta)] + p_D,$$

where

$$p_D = E_{\theta|Y_n}[D(\theta)] - D(E_{\theta|Y_n}[\theta]), \quad D(\theta) \equiv -2\log f(Y_n|\boldsymbol{\theta}).$$

- Use an auxiliary particle filter to evaluate the log likelihood $\log f(\mathbf{y}|\theta)$. The number of particles $I = 10,000$.
- Calculate via $E_{\theta|Y_n}[D(\theta)] \approx \frac{1}{M}\sum_{m=1}^{M} D(\theta^{(m)}),\ M = 100$
- $\theta^{(m)}$'s are resampled posterior samples.
- Repeat ten times for numerical standard errors.

Factor MSV model (Competing model)

Mean factor model (factor & error: symmetric SV).

- Jacquier, Polson and Rossi (1999). error: constant covariance matrix. single move.
- Pitt and Shephard (1999). mixture sampler.
- Chib, Nardari and Shephard (2006). Jumps and heavy-tailed errors. mixture sampler.
- Yu and Meyer (2006). Bivariate model.
- Lopes and Carvalho (2006). AR(1) for factor coefs. Markov switching in volatility level.
- Han (2006). AR(1) for factors.
- So and Choi (2009). Threshold model.

Factor MSV model with leverage

$$
\begin{aligned}
\mathbf{y}_t &= \mathbf{B}\mathbf{f}_t + \lambda_t^{-1/2}\mathbf{V}_{1t}^{1/2}\boldsymbol{\varepsilon}_{1t}, && (6)\\
\mathbf{f}_t &= \mathbf{V}_{2t}^{1/2}\boldsymbol{\varepsilon}_{2t}, && (7)\\
\boldsymbol{\alpha}_{t+1} &= \boldsymbol{\Phi}\boldsymbol{\alpha}_t + \boldsymbol{\eta}_t, && (8)
\end{aligned}
$$

$$
\begin{pmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\eta}_t \end{pmatrix} \sim \mathcal{N}_{2(p+q)}\left(\mathbf{0}, \begin{pmatrix} \boldsymbol{\Sigma}_{\varepsilon\varepsilon} & \boldsymbol{\Sigma}_{\varepsilon\eta} \\ \boldsymbol{\Sigma}_{\varepsilon\eta} & \boldsymbol{\Sigma}_{\eta\eta} \end{pmatrix}\right),
$$

$$
\lambda_t \sim \mathcal{G}(\nu/2, \nu/2).
$$

where $\boldsymbol{\varepsilon}_t = (\boldsymbol{\varepsilon}_{1t}', \boldsymbol{\varepsilon}_{2t}')'$ and

$$
\begin{aligned}
\mathbf{V}_t &= \mathrm{diag}(\mathbf{V}_{1t}, \mathbf{V}_{2t}) = \mathrm{diag}(\exp(\alpha_{1t}), \ldots, \exp(\alpha_{p+q,t})), && (9)\\
\boldsymbol{\Phi} &= \mathrm{diag}(\phi_1, \ldots, \phi_{p+q}). && (10)
\end{aligned}
$$

Factor MSV model with leverage

$$\mathbf{\Sigma}_{\varepsilon\varepsilon} = \mathrm{diag}(\sigma_{1,\varepsilon\varepsilon}, ..., \sigma_{p+q,\varepsilon\varepsilon}) \tag{11}$$

$$\mathbf{\Sigma}_{\eta\eta} = \mathrm{diag}(\sigma_{1,\eta\eta}, ..., \sigma_{p+q,\eta\eta}) \tag{12}$$

$$\mathbf{\Sigma}_{\varepsilon\eta} = \mathrm{diag}\left(\rho_1\sigma_{1,\varepsilon\varepsilon}^{1/2}\sigma_{1,\eta\eta}^{1/2}, ..., \rho_{p+q}\sigma_{p+q,\varepsilon\varepsilon}^{1/2}\sigma_{p+q,\eta\eta}^{1/2}\right), \tag{13}$$

and ($b_{ij} = 0$ for ($i < j$, $i \leq q$) and $b_{ii} = 1$ ($i \leq q$)).

1. FMSVLn-q: q factor MSV model with leverage, normal error.
2. FMSVLt-q: q factor MSV model with leverage, multivariate t error.

Model	DIC	(s.e.)	DIC_{max}	DIC_{min}	Ranking
MSVLt	24997.4	(1.8)	25009.1	24986.5	1
MSVLn	25006.0	(0.9)	25011.5	25001.2	2
FSVLt-1	25617.2	(3.0)	25639.8	25600.6	7
FSVLn-1	25676.3	(1.8)	25688.8	25666.4	8
FSVLt-2	25173.2	(2.2)	25184.8	25160.5	5
FSVLn-2	25200.3	(1.4)	25214.8	25194.2	6
FSVLt-3	25067.8	(2.4)	25084.1	25054.4	3
FSVLn-3	25082.7	(1.5)	25093.2	25072.6	4

Summary

- Efficient multi-move sampler for MSV models with cross leverage.
- Comparison with single-move samplers.
- Illustrative examples and empirical study.
- Future work: Evaluation of the portfolio selections, Value at Risk. Dynamic correlation models.

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