Evaluation of Deformable Image Registration Spatial Accuracy Using a Bayesian Hierarchical Model

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Outline

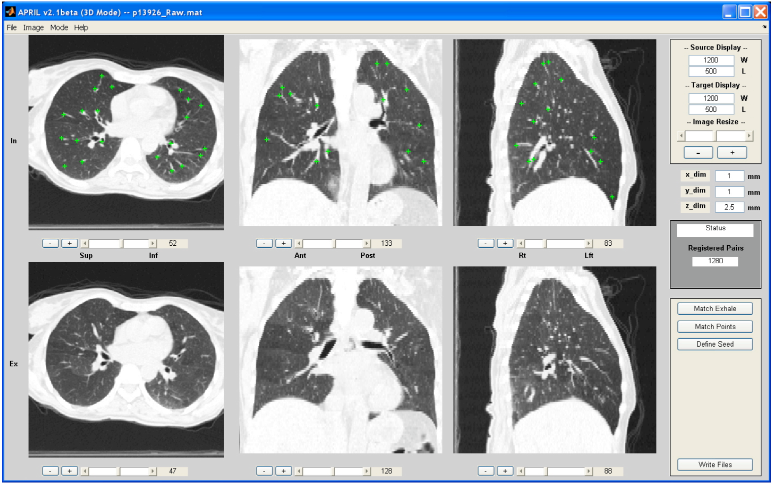
- Overview of Data and Goals
- Bayesian model for DIR/mulitple rater Data
- Application
- Summary

Deformable image registration (DIR) methods

- The use of deformable image registration (DIR) methods is standard practice in the administration and management of radiation therapy for thoracic malignancies.
- The DIR provides the spatial correspondence between the underlying anatomy in a source image and similar anatomy in one or more target images.
- No rigorous framework is available for comparing the relative performance of the various DIR algorithms.

Data description

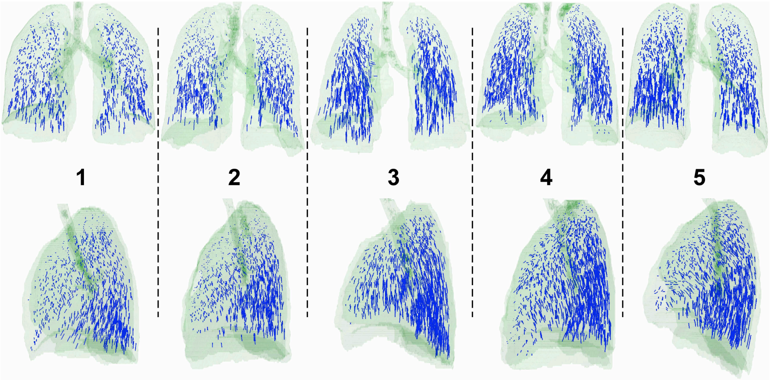
- Images collected from esophageal cancer who were free from pulmonary disease
- Extreme exhale images regarded as source images; extreme inhale images regarded as target images
- Landmarks were chosen to be vessel or bronchial bifurcations
- 1000 landmarks manually identified using APRIL software by a single expert reader in 5 pairs of thoracic 4D CT images


APRIL v2.1beta (3D Mode) -- p13926_Raw.mat
File Image Mode Help
In
Ex
52
Sup
Inf
47
133
Ant
Post
128
83
Rt
Lft
88
-- Source Display --
1200 W
500 L
-- Target Display --
1200 W
500 L
-- Image Resize --
x_dim 1 mm
y_dim 1 mm
z_dim 2.5 mm
Status
Registered Pairs
1280
Match Exhale
Match Points
Define Seed
Write Files

Data description (cont)

- Two deformable image registration (DIR) algorithms mapped landmarks in source images to target images
  - Optical flow method (OFM) (Horn and Schunck, 1981; Guerrero et al 2006)
  - Moving Least Squares (MLS) method (Schaefer et al 2006).
- 1000 target landmarks identified twice by reader 1 and once by readers 2 and 3.

Five images

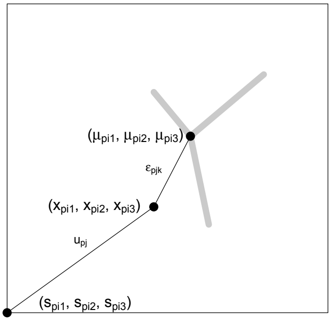


Analysis goals

- Characterize reader errors across and within image sets.
- Compare accuracy of two (or more) DIR algorithms across image sets

Expert readings

- Expert readers provide discretized spatial location of landmark in source image



A model for expert reader data

- Let μ_{pij} denote the true value of coordinate j for landmark i in source image p, x_{pijk} denote latent (unobserved) continuous version of reader's landmark identification, and s_{pijk} denote the corresponding discretized reading obtained from expert reader k.

$$\begin{aligned} x_{pijk} &\sim N(\mu_{pij}, \tau^2_{pjk}) \\ s_{pijk} &= \lfloor x_{pijk} \rfloor, \end{aligned}$$

where s_{pijk} satisfies

$$s_{pijk} \leq x_{pijk} < s_{pijk} + 1. \quad (1)$$

Likelihood

Conditionally on μ_{pij} and τ_{pjk}^2, it follows that the likelihood function for the reader landmark identifications $\boldsymbol{s} = \{s_{pijk}\}$ can be expressed as

$$\begin{aligned}\mathcal{L}(\boldsymbol{s}|\mu_{pij}, \tau_{pjk}^2) &= \prod_{p=1}^{5}\prod_{i=1}^{200}\prod_{k=1}^{4}\frac{1}{\tau_{pjk}}\exp\left\{-\frac{1}{2}\left(\frac{x_{pijk}-\mu_{pij}}{\tau_{pjk}}\right)^2\right\} \\ & I(s_{pijk} < x_{pijk} < s_{pijk}+1).\end{aligned}$$

Prior distributions

$$\begin{aligned}\tau^2_{pjk} &\sim IG(\alpha_{jk}, 1/\lambda_{jk}).\\ \pi(\alpha_{jk}, \lambda_{jk}) &\propto \sqrt{\alpha_{jk} PG(1, \alpha_{jk}) - 1}/\lambda_{jk}\\ \pi(\mu_{pij}) &\propto 1,\end{aligned}$$

where $IG(\cdot, \cdot)$ denote the inverse gamma distribution and $PG(\cdot, \cdot)$ denotes the polygamma function.

Model for DIR algorithm data

- DIR algorithms provide continuous mappings between the source image volume and the target image volume, which means that errors must be correlated within each dimension
- Let y_{pijl} denote voxel coordinate j for landmark i in source image p identified by DIR algorithm l. In image p, let $\boldsymbol{y}_{pjl} = (y_{p1jl}, \ldots, y_{p200jl})$ and $\boldsymbol{\mu}_{pj} = (\mu_{p1j}, \ldots, \mu_{p200j})$, and define the distance between two landmarks i and i' to be

$$d_{pii'} = ||\boldsymbol{\mu}_{pi} - \boldsymbol{\mu}_{pi'}|| = \sqrt{\sum_{j=1}^{3}(\mu_{pij} - \mu_{pi'j})^2}. \quad (2)$$

Gaussian process

- we assume that for each image, coordinate readings from the DIR algorithm within each dimension follow a Gaussian process (GP) of the form

$$\boldsymbol{y}_{pjl} \sim N(\boldsymbol{\mu}_{pj}, \boldsymbol{\Omega}_{pjl}).$$

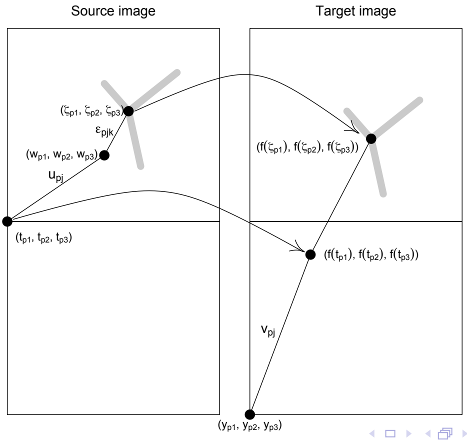
  Here, $\boldsymbol{\Omega}_{pjl}$ is an exponential covariance matrix with $(i, i^{'})$ entry given by

$$\boldsymbol{\Omega}_{pjl}(i, i^{'}) = \sigma^2_{pjl} \exp\left(-\gamma_{pjl} d_{pii'}\right),$$

  where γ_{pjl} is an unknown decay parameter controlling correlations among coordinates at different locations in an image. Small values of γ_{pjl} induce strong correlations.

Discretization uncertainties

Localizations based on DIR algorithms suffer from discretization uncertainties in both the source and target image.



Nugget effect

- Hence, we add a “nugget” variance component of $1/6 + \tau^2_{pj1}$ to the diagonal elements of $\mathbf{\Omega}_{pjl}$. This leads to a GP covariance matrix $\mathbf{\Sigma}_{pjl}$ with elements

$$\mathbf{\Sigma}_{pjl}(i, i') = \begin{cases} \sigma^2_{pjl}\exp(-\gamma_{pjl} d_{pii'}) & i \neq i' \\ \sigma^2_{pjl} + 1/6 + \tau^2_{pj1} & i = i'. \end{cases} \quad (3)$$

Prior distributions

$$\sigma_{pjl}^2 \sim IG(\omega_{jl}, 1/\beta_{jl})$$
$$\pi(\omega_{jl}, \beta_{jl}) \propto \omega_{jl}\sqrt{\omega_{jl} PG(1, \omega_{jl}) - 1/\beta_{jl}}$$
$$\pi(\gamma_{pjl}) \propto \sqrt{n_p \mathrm{tr}(\mathbf{U}^2) - (\mathrm{tr}(\mathbf{U}))^2},$$

where $\mathbf{U} = (\mathbf{D}_p * \boldsymbol{\Sigma}_{pjl})\boldsymbol{\Sigma}_{pjl}^{-1}$ and $\mathrm{tr}(\mathbf{U})$ is the trace of the matrix $\mathbf{U}$ with $\mathbf{D}_p$ denoting the distance matrix between landmarks with elements defined in, and $\mathbf{D}_p * \boldsymbol{\Sigma}_{pjl}$ denoting the element-wise product of $\mathbf{D}_p$ and $\boldsymbol{\Sigma}_{pjl}$.

Inference

- Fit the model using hybrid Gibbs/Metropolis-Hastings method
- To summarize the performance of expert readers and DIR algorithms in the three-dimensional space, we define the expected registration error for the *k*th expert reader to be

$$e_{pk} = E\left[\sqrt{(x_{pi1k} - \mu_{pi1})^2 + (x_{pi2k} - \mu_{pi2})^2 + (x_{pi3k} - \mu_{pi3})^2}\right]$$

Results

Table: Posterior mean and standard error of registration errors for three expert reader and the MLS and OFM algorithms. The posterior standard error is shown in parentheses.

	Reader			DIR	
Image	1	2	3	MLS	OFM
1	0.51 (0.22)	0.46 (0.20)	0.62 (0.27)	1.93 (0.96)	9.64 (6.10)
2	0.44 (0.19)	0.37 (0.16)	0.60 (0.26)	1.99 (1.03)	8.70 (6.02)
3	0.57 (0.26)	0.47 (0.21)	0.83 (0.38)	2.07 (1.01)	12.62 (8.74)
4	0.43 (0.19)	0.38 (0.17)	0.52 (0.22)	1.97 (0.96)	6.76 (3.88)
5	0.58 (0.26)	0.48 (0.23)	0.87 (0.44)	2.72 (1.69)	5.71 (3.93)

Conclusions

- We have proposed a Bayesian hierarchical model to evaluate the spatial accuracy for deformable image registration algorithms based on landmarks identified by human experts.
- Our model explicitly accounts for the variation among multiple experts and the discretization process of readings.
- When evaluating the spatial accuracy of the DIR algorithm, our model accounts for the random errors associated with the experts' registration errors and utilizes a hierarchical model to borrow information across multiple images.
- A Gibbs sampling algorithm was developed to fit the data efficiently.

Thank you !