Nonlinear regression model of copper bromide laser

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INTRODUCTION

Subject of study

Low-temperature impulse copper bromide (CuBr) laser from the group of metal vapor lasers:

- **wavelengths 510.6 nm and 578.2 nm**
- **the most efficient and produces the highest output power in the visible region, up to 100-150 W**
- **with wide application in medicine, chemistry, in investigation of the atmosphere, aerial and submarine location, in modern micro and nano laser technologies.**

This laser is developed in the Laboratory of Metal vapor lasers, Institute of solid state physics, Bulgarian Academy of Sciences, Sofia.

Schematic experimental design

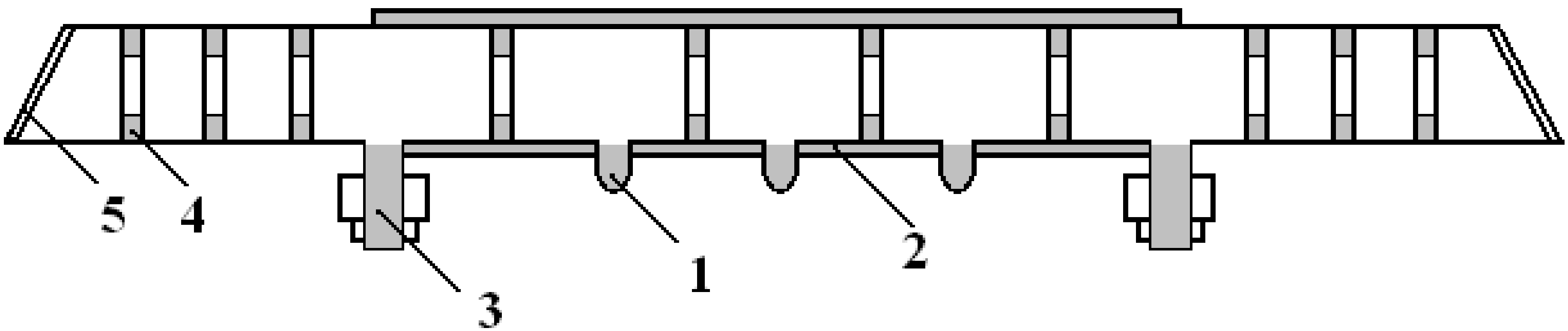

Fig. 1. Laser tube of a CuBr laser: 1- reservoirs with the copper bromide, 2-insulation in the active zone, 3- external copper electrodes, 4-quartz diaphragms, 5-mirrors

AIMS OF THE STUDY

- **To treat available experimental data for CuBr laser**
- **To obtain nonlinear regression models, describing the dependence of output laser power *Pout* on basic input parameters**
- **To investigate the predictive ability of the nonlinear model**

Applied statistical software: SPSS, Mathematica

DESCRIPTION OF THE DATA

The initial data of more than 300 experiments

- 10 input laser variables (predictors):

- **D (mm) – inside diameter of the laser tube**
- **dr (mm) – inside diameter of the rings (diaphragms)**
- **L (cm) – length of the active area (electrode separation)**
- **Pin (kW) – input electrical power**
- **PL (kWm-1) – input electrical power per unit length**
- **PH2 (Torr) – hydrogen gas pressure**
- **Prf (kHz) – pulse repetition rate**

- **PNe (Torr) - neon gas pressure**
- **C (pF) – equivalent capacity of the capacitor bank**
- **Tr (^{0}C) – temperature of the CuBr reservoirs**

Dependant variable: *Pout* - the output laser power (W)

Initial sample:

a random sample of 109 experiments, partially stratified.

PREVIOUS RESULTS

It was established that:

- **only the first 6 of 10 variables show statistically significant influence on the output power *Pout*. These are: D, dr, L, Pin, PL and PH2.**
- **They show a strong multicolinearity.**
- **There was carried out factor analysis via PCA with varimax rotation: 3 factors were obtained.**
- **There were constructed:**
  - **multiple linear regression models (MLR) and**
  - **multivariate adaptive regression splines (MARS) models**

Previous publications

[1]. Iliev I. P., Gocheva-Ilieva S. G., Denev N. P. and Sabotinov N. V., “Statistical study of the copper bromide laser efficiency”, Sixth Intern. Conf. of the Balkan Physical Union 2006, Istanbul – Turkey, Proc. AIP CP899, p. 680 (2007).

[2]. Iliev I. P. and Gocheva-Ilieva S. G., “Statistical techniques for examining copper bromide laser parameters”, Int. Conf. of Numer. Analysis and Appl. Math., ICNAAM 2007, Corfu - Greece, Proc. AIP CP936, 267-270 (2007).

[3]. Iliev I. P., Gocheva-Ilieva S. G. and Sabotinov N. V., “Statistical approach in planning experiments with a copper bromide vapor laser”, Quantum Electron. 38(5), 436-440 (2008).

[4]. Iliev I. P., Gocheva-Ilieva S. G., Astadjov D. N., Denev N. P. and Sabotinov N. V., “Statistical analysis of the CuBr laser efficiency improvement”, Opt. Laser Technol. 40(4), 641-646 (2008).

[5]. Gocheva-Ilieva S. G. and Iliev I. P., Parametric and nonparametric empirical regression models of copper bromide laser generation, Math. Probl. Eng., Theory, Methods and Applications, Hindawi Publishing Corporation, New York, NY, Volume 2010, Article ID 697687, 15 pages (2010).

RESULTS FROM FACTOR ANALYSIS

Orthogonal factors and corresponding loadings of its grouping variables:

F1: Pin(0.913), dr(0.887), D(0.807), L(0.769);
F2: PL (-0.914)
F3: PH2 (0.929)

The generated factor scores (factor variables) have values between (-3,3).

NONLINEAR REGRESSION MODEL

Yeo-Johnson transformation (generalization of Box-Cox transformation for non-positive predictors)

$$\psi_{Y-J}(\lambda, x) = \begin{cases} \{(x+1)^{\lambda} - 1\}/\lambda, & x \geq 0, \lambda \neq 0 \\ \log(x+1), & x \geq 0, \lambda = 0 \\ -\{(-x+1)^{2-\lambda} - 1\}/(2-\lambda), & x < 0, \lambda \neq 2 \\ -\log(-x+1), & x < 0, \lambda = 2 \end{cases}$$

Model estimation of Pout in the form

$$\widehat{Pout}(\theta,\lambda) = \theta_0 + \theta_1\psi_{Y-J}(\lambda_1, F_1) + \theta_2\psi_{Y-J}(\lambda_2, F_2) + \theta_3\psi_{Y-J}(\lambda_3, F_3), \quad (1)$$

We have compiled the *Mathematica* compact code shown in Fig. 2.

The resulting parameters for the seven-dimensional model (1) are:

$$\theta_0 = 39.735372, \theta_1 = 27.167573, \theta_2 = 4.456846, \theta_3 = 11.777153, \qquad (2)$$

$$\lambda_1 = 1.290534, \lambda_2 = 0.381756, \lambda_3 = 0.767572.$$

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ψ[λ_, y_] := If[y ≥ 0 && λ ≠ 0, ((y + 1)^λ - 1)/λ,
  If[y < 0 && λ ≠ 2, -((1 - y)^(2-λ) - 1)/(2 - λ),
   If[y ≥ 0 && λ == 0, Log[y + 1],
    If[y < 0 && λ == 2, -Log[1 - y]]]]]

n = 109;
f1 = ReadList["f1-109.txt", Number] ;
f2 = ReadList["f2-109.txt", Number] ;
f3 = ReadList["f3-109.txt", Number] ;
pout = ReadList["pout-109.txt", Number] ;
data = Table[{f1[[i]], f2[[i]], f3[[i]], pout[[i]]}, {i, 1, n}];

<< Statistics`NonlinearFit`
NonlinearRegress[data, theta0 + theta1 * ψ[λ1, x1] +
  theta2 * ψ[λ2, x2] + theta3 * ψ[λ3, x3], {x1, x2, x3 },
 {theta0, theta1 , theta2, theta3, λ1, λ2, λ3}]
```

Fig. 2. *Mathematica* code for calculating the nonlinear model (1)-(2).

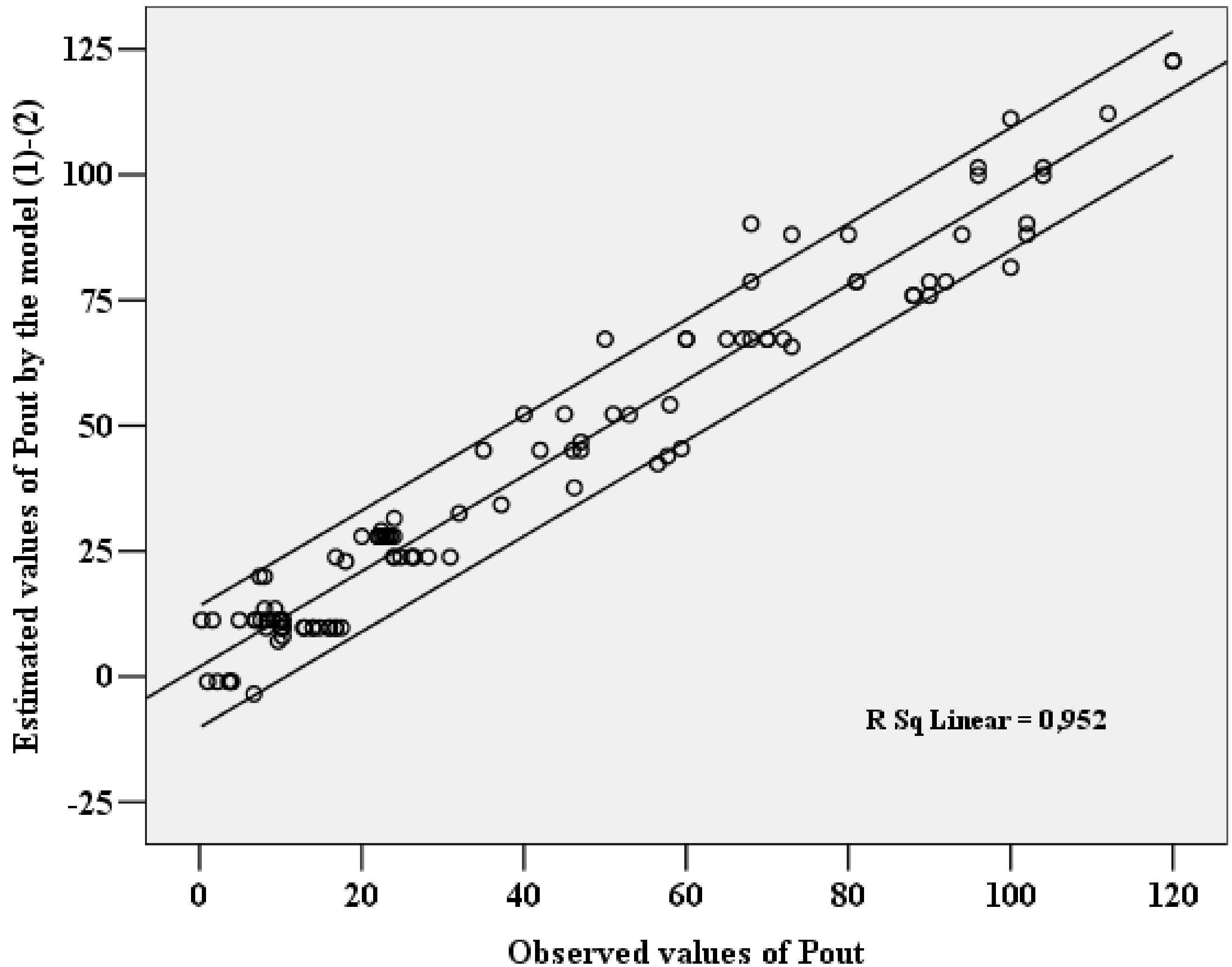

Fig. 3. The observed vs estimated values of laser generation Pout.

ASSESMENT OF THE MODEL PREDICTIVE ABILITY

Cross-validation technique:

The sample was randomly divided in 2 subsets ("teaching" and "evaluation" data subset), with 70:30 percents of data, respectively.

The obtained parameters of the nonlinear model for the 70% teaching subset are:

$$\theta_0 = 40.063269, \theta_1 = 26.973166, \theta_2 = 4.283957, \theta_3 = 11.859342, \qquad (3)$$

$$\lambda_1 = 1.257025, \lambda_2 = 0.389093, \lambda_3 = 0.767572.$$

The predicted values for the 30% evaluation subset versus experimental data are shown in Fig.4.

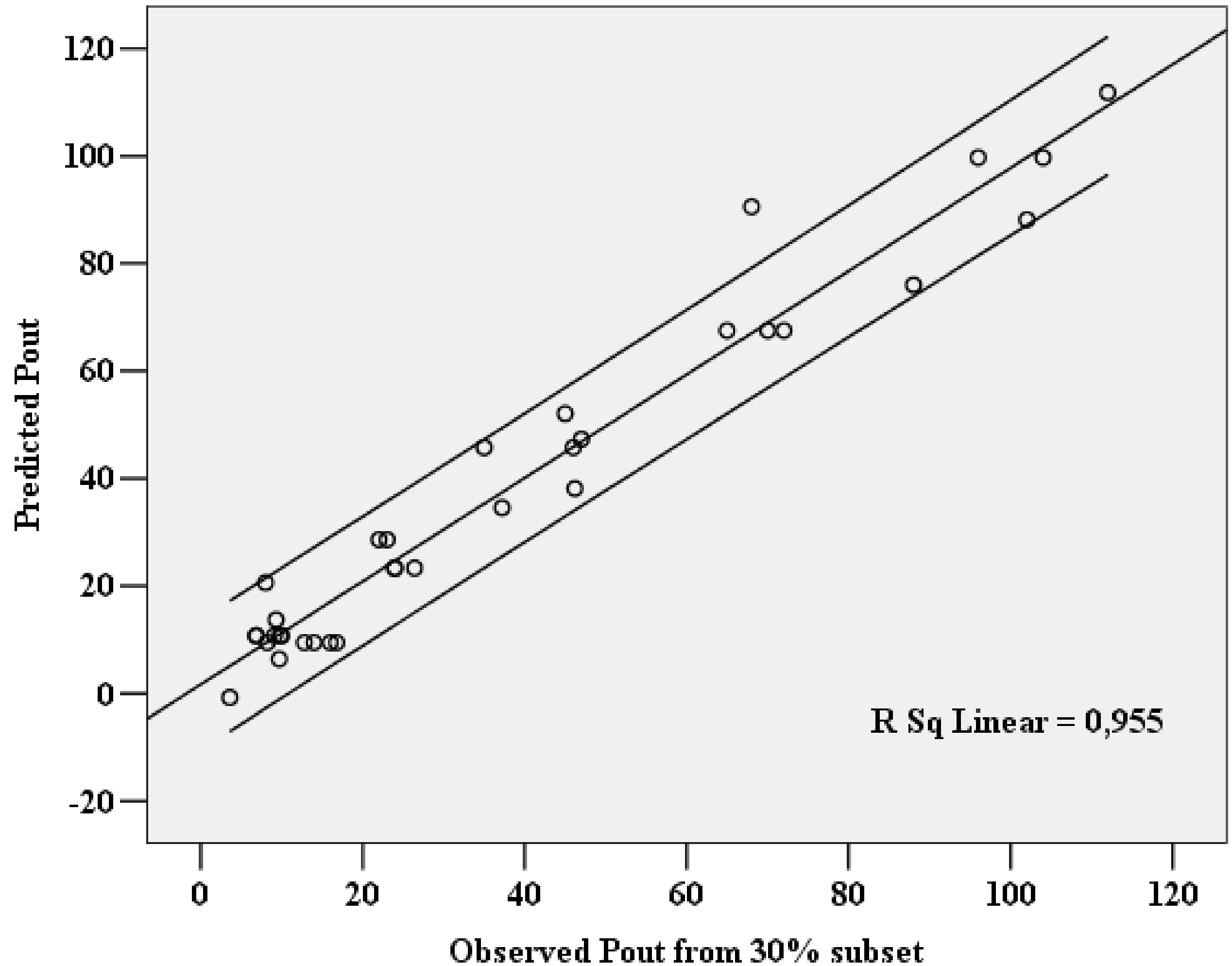

Fig. 4. Predicted values for Pout compared to the initial observed values for a 30% evaluation data set.

DISCUSSION AND CONCLUSION

Model	R^2 of the estimates	R^2 adj.	Std. Err.
Model (1), (2)	0.952	0.950	7.58979
Model (1), (3) for a 70% subset	0.950	0.948	7.94853
Model (1), (3) for a 30% subset	0.955	0.954	6.96404

Table 1. Basic statistics of the basic nonlinear regression model (1) and cross evaluation of 70% and 30% randomly extracted sets.

From the results given in Table 1 it is seen that the nonlinear model (1), (2) fits the data very well. Also, the indexes of model (1), (3) are relatively good and fall only a little behind those of (1), (2). The substituted in (1), (3) values from the 30% evaluation data set, which are not included in the extraction of parameters (3) confirm the good quality of the constructed models.

We can conclude that nonlinear models of the suggested type are stable and fit the data well. The indexes of these estimates exceed the analogical statistics, obtained for the same data set using multivariate linear regression. They are almost equal of the statistics from the second degree polynomial regression and fall behind the accuracy of the polynomial regression of the third degree and the MARS models based on linear regression splines and splines with first and second order interactions (see Gocheva-Ilieva and Iliev (2010)).

One can conclude that the obtained nonlinear regression model is fully applicable for estimation and prediction of the output laser power of CuBr lasers.

Thank you for your attention!