On calculation of Blaker's binomial confidence limits

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Summary

- Blaker (2000) proposed
  - a new type of binomial confidence limts
  - a numerical algorithm for their calculation
- Theoretical work good, but numerics a bit careless
- My contribution: A better algorithm

Outline

- Blaker's confidence interval – what is it (recap)
- Original Blaker's algortithm . . . and what is wrong with it
- Remedy – new algorithm
- Concluding remarks

Blaker's confidence interval

- Task: Exact (conservative) two-sided confidence interval (CI) for binomial parameter p

- Clopper-Pearson (1934):

  Both probabilities

  $$P(\text{whole CI below true } p) \leq \alpha/2,$$

  $$P(\text{whole CI above true } p) \leq \alpha/2$$

  controlled

- Les conservative exact alternatives:

  $$P(\ldots \text{below} \ldots) + P(\ldots \text{above} \ldots) \leq \alpha$$

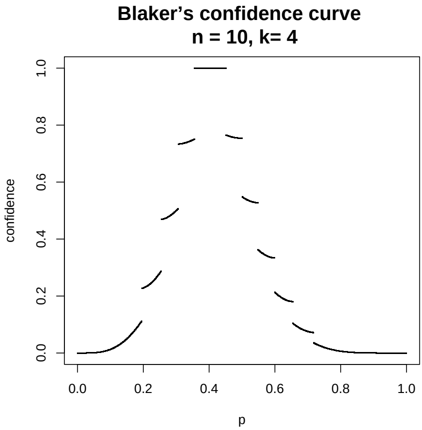
  only controlled

Blaker's confidence interval

- Proposals of alternatives:
  - Sterne, Crow (1954, 1956)
  - Blyth, Still, Casella (1983, 1986)
  - Blaker (2000)
- Virtues of Blaker's CI:
  - Blaker's CI $\subseteq$ Clopper-Pearson CI
  - Monotonicity w.r.t. α
  - Easy calculation - short R program in Blaker (2000)

Blaker's confidence interval

- Based on confidence function $\beta(\cdot)$ defined in terms of binomial tail probabilities (details: Blaker (2000))



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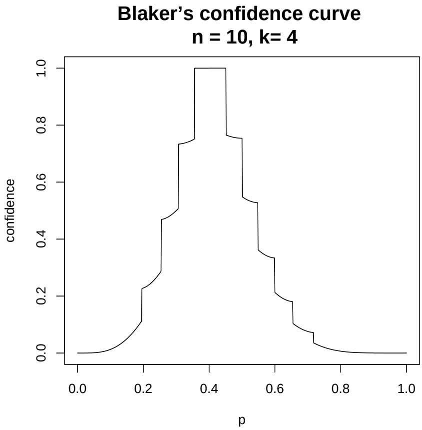
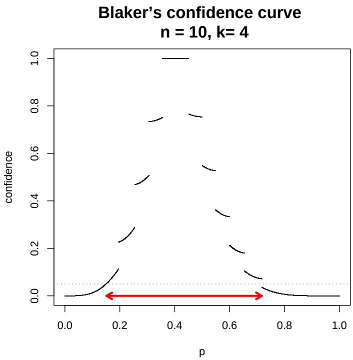


Blaker's confidence interval

- Roughly: $(1-\alpha)$ confidence interval is where $\beta(p) \geq \alpha$



- Calculation of confidence limits:
  Numerical search for points where $\beta(\cdot)$ crosses α level

Blaker's confidence interval

- More precisely:
- $C = \{p; \beta(p) \geq \alpha\}$ often not an interval
- Blaker's interval defined as $conv(C)$ (convex hull)
- Calculation of confidence limits:
  Numerical search for the leftmost and rightmost points where $\beta(\cdot)$ crosses α level

Original Blaker's algorithm

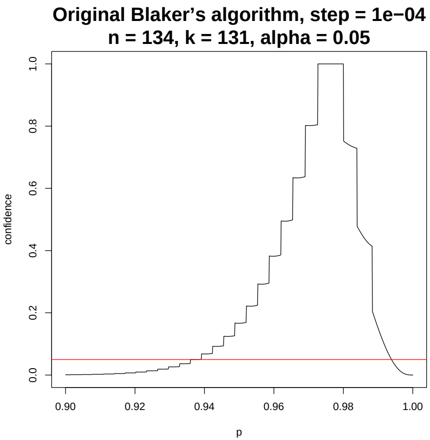
- Short R program in Blaker (2000), correction Blaker (2001) (to be found at A. Agresti's web, too)
- Calculation of p_L (lower confidence limit):
  1. Start in Clopper-Pearson limit p_L^{CP}
  2. Iterate $p := p + \Delta p$
     (fixed step $\Delta p > 0$, default 10^{-4})
     while $\beta(p) < \alpha$
  3. Once $\beta(p) \geq \alpha$, set $p_L := p - \Delta p$, finish
- Calculation of p_U (upper confidence limit) analogous

Original Blaker's algorithm

- What is wrong?
- Constant step $\longrightarrow$ drastic slowdown when higher accuracy required
- Algorithm may skip short intervals and fail

Original Blaker's algorithm

- Example of failure:



Original Blaker's algorithm

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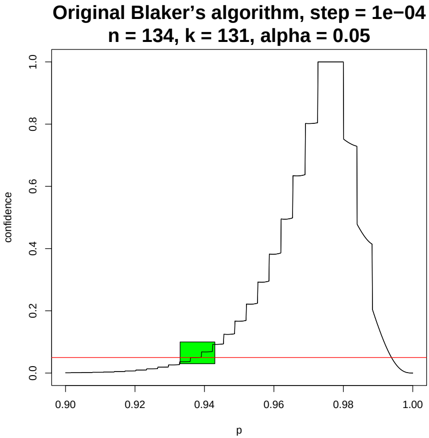

Original Blaker's algorithm

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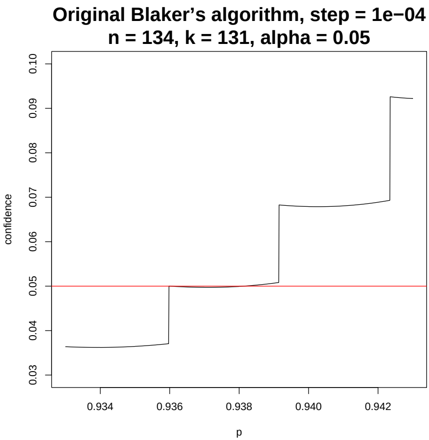

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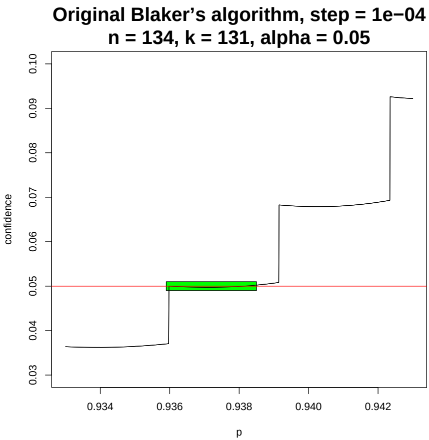

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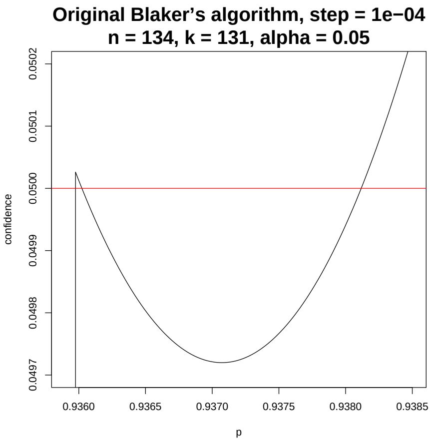

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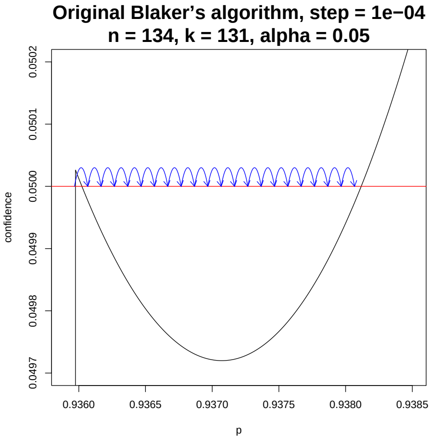

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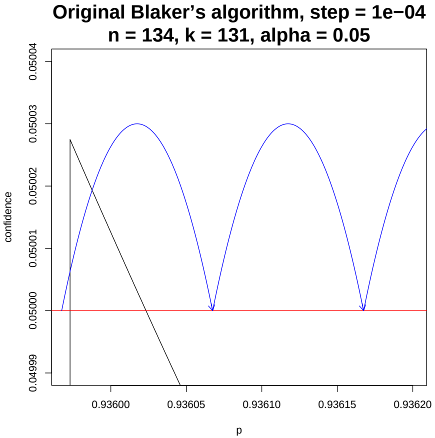
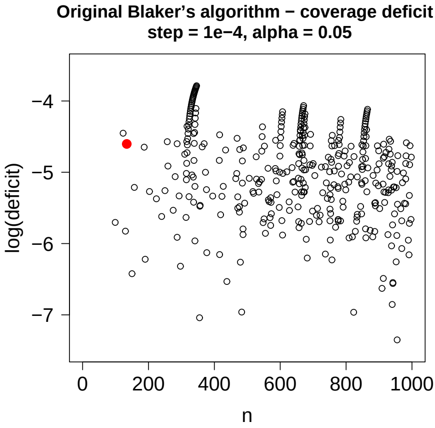


Original Blaker's algorithm

- Statistics of coverage deficits – $n = 1, \ldots, 1000$:



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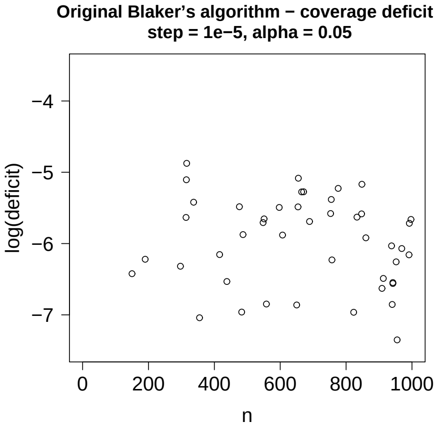

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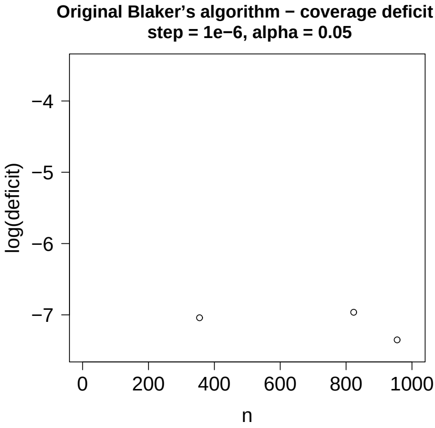

New algorithm – lemmas

- p_L^{CP} … Clopper-Pearson lower limit
  p^* … first discontinuity point of $\beta(\cdot)$ to the right
- Lemma 1: $p_L^{CP} \leq p_L \leq p^*$
- Lemma 2: $\beta(\cdot)$ crosses α level only once on $[p_L^{CP}, p^*]$
- Analogously for p_U

New algorithm – description

- Search for the lower confidence limit p_L:
  (For p_U analogously)

1. Modify $\beta(\cdot)$:

$$\beta^*(p) = \begin{cases} \beta(p) & p < p^* \\ +\infty & p \geq p^* \end{cases}$$

   Remark: Modification is computationally easy

2. Apply interval halving to $\beta^*(\cdot)$ between p_L^{CP} and 1

   Remark: With unmodified $\beta(\cdot)$, halving safe only on $[p_L^{CP}, p^*]$, naive "global" use fails

Concluding remarks

- New algorithm is fast and accurate
- Implementation: $\sim$ 50 lines of R code
- R package to come (hopefully) soon

- Slightly extended version:
  `www.cs.cas.cz/~klaschka/c10/417_ext.pdf`

Thank you for your attention