Calibration of hitting probabilities via multilevel splitting

Ioannis Phinikettos Axel Gandy

Department of Mathematics
Imperial College London

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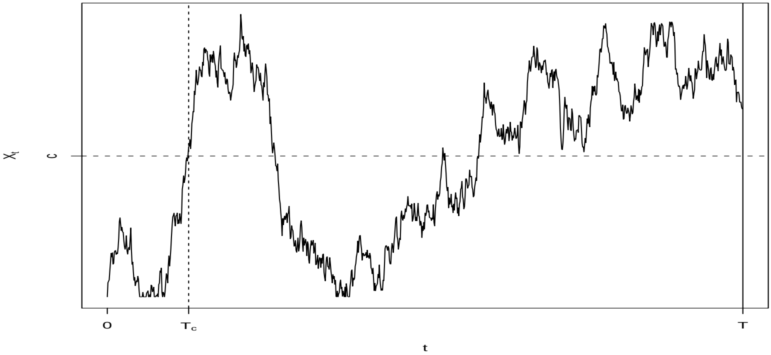
Calibration of hitting thresholds via multilevel splitting

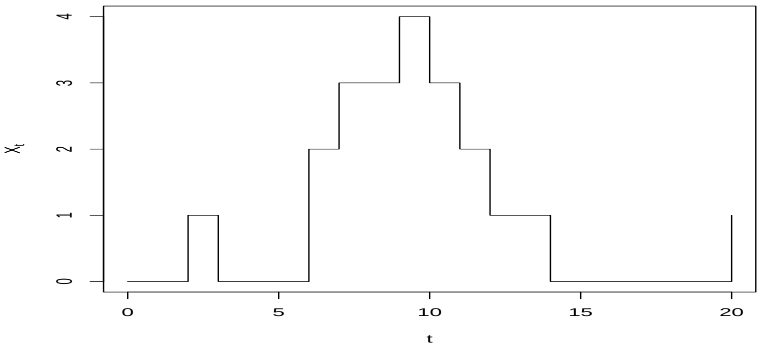
Description

- $X = \{X_t \in [0, \infty), t \geq 0\}$
  - $X_0 = 0$
  - $T_k = \inf\{t \geq 0 : X_t \geq k\}$
  - $c = \inf\{k : P(T_k \leq T) \geq \alpha\}, \quad \alpha \in (0, 1)$



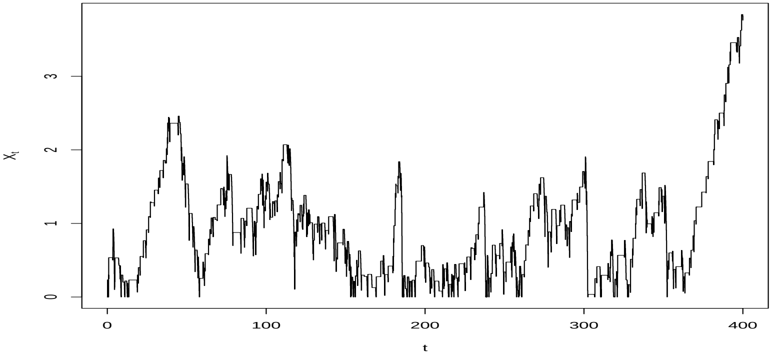
Examples

- Poisson CUSUM chart
  - $X_t = \max(X_{t-1} + P_t - 1, 0), \quad t \in \mathbb{N}$
  - $P_t \sim \text{Poisson}(\lambda), \quad \lambda > 0$



Examples

- Survival analysis CUSUM chart (Gandy et al., 2010)
  - $T_i \sim F \quad i \leq \eta$
    $T_i \sim G \quad i > \eta$
  - Partial likelihood - $L_j(t) \quad j = 0, 1$
  - Log-likelihood ratio statistic - K_t
  - $X_t = K_t - \min_{s \leq t} K_s$

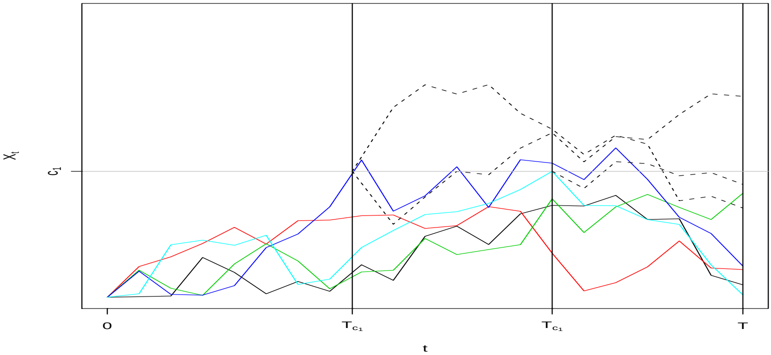


Methods

- Monte Carlo
  - $X_i \sim X (1 \leq i \leq N)$
  - $Y_i = supX_i$
  - $\hat{c} = Y_{([N\beta])}, \quad \beta = 1 - \alpha$

Methods

- Multilevel splitting (Glasserman et al., 1999)
  - $\alpha_1 > \alpha_2 > \cdots > \alpha_m = \alpha$
  - $c_1 < c_2 < \cdots < c_m = c$
  - $c_i = \inf\{k : P(T_k \leq T) \geq \alpha_i\}$
  - $c_i = \inf\{k \geq c_{i-1} : P(T_k \leq T | T_{c_{i-1}} \leq T) \geq \frac{\alpha_i}{\alpha_{i-1}}\}$



Results - Poisson CUSUM chart

- $\alpha_i = \alpha^{\frac{i}{m}}$ (Lagnoux, 2005)
- $c_i = \inf\{k \geq c_{i-1} : P(T_k \leq T | T_{c_{i-1}} \leq T) \geq \alpha^{\frac{1}{m}}\}$
- $N = [10^4/m]$
- $T = 100, \lambda = 1$

α c	0.01 28	0.001 35
$m = 2$	0.367	0.534
$m = 3$	0.322	0.489
$m = 4$	0.284	0.515
$m = 5$	0.268	0.594
$m = 7$	0.558	0.813
$m = 10$	0.828	1.273
MC(m=1)	0.418	1.038

Table: Root mean square error of the estimates.

Results - Survival analysis CUSUM chart

- $\alpha_i = \alpha^{\frac{i}{m}}$
- $c_i = \inf\{k \geq c_{i-1} : P(T_k \leq T | T_{c_{i-1}} \leq T) \geq \alpha^{\frac{1}{m}}\}$
- $N = [10^4/m]$
- $T = 400$

α	0.01	0.001
c	7.11	9.25
$m = 2$	0.0621	0.1044
$m = 3$	0.0561	0.0897
$m = 4$	0.0594	0.0812
$m = 5$	0.0638	0.0877
$m = 7$	0.0694	0.0973
$m = 10$	0.0822	0.1081
MC(m=1)	0.0975	0.2852

Table: Root mean square error of the estimates.

Conclusion - Further work

- Multilevel Splitting
  - Better than crude MC
  - Rare events
- Algorithm
  - Work on theoretical underpinning

References

- Gandy, A.; Kvaloy, J.; Bottle, A. Zhou, F., Risk-adjusted monitoring of time to event, Biometrika, Biometrika Trust, 2010
- Glasserman, P.; Heidelberger, P.; Shahabuddin, P. Zajic,T., Multilevel splitting for estimating rare event probabilities, Operations Research, Operations Research Society of America, 1999, 585-600
- Lagnoux, A., Rare event simulation, Probability in the Engineering and Informational Sciences, Cambridge Univ Press, 2005, 20, 45-66