Holt's exponential smoothing model for interval-valued time series

André Luis Santiago Maia[1], Francisco de A.T. de Carvalho[1]

Centro de Informática, Universidade Federal de Pernambuco, Av. Prof. Luiz Freire - s/n - Cidade Universitária - CEP 50740-540 - Recife (PE) - Brazil

Abstract Interval-valued time series are interval-valued data that are collected in a chronological sequence through time. This paper adapts an approach to forecasting interval valued-time series based on Holt's exponential smoothing method. In the adapted Holt's method for interval-valued time series, the smoothing parameters are estimated by using techniques for non-linear optimization problems with bound constraints. The practicality of the method is demonstrated by simulation studies and applications using real interval-valued stock market time series.

Keywords: Symbolic Data Analysis, Time Series Forecasting, Interval-Valued Data, Exponential Smoothing.

1 Introduction

Exponential smoothing (ES) methods (e.g., Gardner [5]) have become very popular because of their (relative) simplicity compared to their good overall performance. The Holt's smoothing method (originally presented in Holt [6], reprinted 2004 and Winters [8]), also referred to as double exponential smoothing, is an extension of ES designed for trended time series.

This paper addresses the forecasting of time series with interval-valued data, i.e., when interval-valued variables are collected in an ordered sequence over time, we say that we have an *interval-valued time series* (see Maia et al. [7]). Thus, an ITS is as

$$\mathbf{I}_1, \mathbf{I}_2, \ldots, \mathbf{I}_n \qquad \text{or} \qquad \begin{bmatrix} X_1^U \\ X_1^L \end{bmatrix}, \begin{bmatrix} X_2^U \\ X_2^L \end{bmatrix}, \ldots, \begin{bmatrix} X_n^U \\ X_n^L \end{bmatrix},$$

where n denotes the number of intervals of the time series (sample size). Interval-valued data arise quite naturally in many situations where such data represent uncertainty (for instance, confidence intervals), variability (minimum and maximum of daily temperature), etc. Interval-valued data have been considered in the field of *Symbolic Data Analysis* (SDA) [2], [1], [4]. Concerning ITS, Maia et al. [7] have introduced approaches to modelling and forecasting based on the ARIMA models, based on an artificial neural networks (ANN) model and based on a hybrid methodology that combines both ARIMA and ANN models.

The interval Holt's exponential smoothing method (Holt$^{\mathrm{I}}$) follows similar representation for usual quantitative data and has the following form:

$$\widehat{\mathbf{L}}_t^{\mathrm{I}} = \mathcal{A}\mathbf{I}_t + (\mathbf{I} - \mathcal{A})(\widehat{\mathbf{L}}_{t-1}^{\mathrm{I}} + \widehat{\mathbf{T}}_{t-1}^{\mathrm{I}}), \tag{1}$$

$$\widehat{\mathbf{T}}_t^{\mathrm{I}} = \mathcal{B}(\widehat{\mathbf{L}}_t^{\mathrm{I}} - \widehat{\mathbf{L}}_{t-1}^{\mathrm{I}}) + (\mathbf{I} - \mathcal{B})\widehat{\mathbf{T}}_{t-1}^{\mathrm{I}}, \tag{2}$$

where $\mathcal{A}$ and $\mathcal{B}$ denote the (2×2) smoothing parameters matrices and $\mathbf{I}$ is an (2×2) identity matrix. The estimation of the optimum $\mathcal{A}$ and $\mathcal{B}$ matrices is obtained by the L-BFGS-B (*limited memory algorithm for bound constrained optimization*), method developed by Byrd et al. [3].

The advantage of the use of the model introduced in order to forecast ITS is illustrated through a comparison between the performance of the Holt^{I} (fitting simultaneously the interval boundaries) and the performanc of the usual Holt's exponential smoothing method (fitted independently, on the lower and upper boundaries). The results of the simulation and the applications demonstrated that the usual Holt model performs more poorly than the Holt^{I} model introduced in this paper, i.e., fitting simultaneously the interval boundaries is itself an advantage.

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