Random Sampling over Data Streams for Sequential Pattern Mining

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Plan

1. Motivations
2. Preliminary Concepts
3. Related Work
   - Sequential Pattern Mining
   - Synopsis Construction
4. Sampling in static databases
5. Extending to Data Streams
6. Experimental Results
7. Conclusion and Summary
Motivations

- A new problem: data modeled as a potentially infinite flow of transactions
- Many recent real-world applications:
  1. Network traffic monitoring
  2. Trend analysis
  3. Sensor network data analysis

Classical mining approaches are inefficient for this new problem.
In many cases, it may be acceptable to generate approximate solutions: synopsis structures?

Definitions

Let $D$ be a database of customer transactions where each transaction $T$ consists of:
1. A customer-id, denoted by $C$
2. A transaction time, denoted by $T$
3. A set of items (called itemset) involved in the transaction, denoted by $I$
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- Related Work: Sequential Pattern Mining
- Construction
- Sampling in static databases
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**Sequence**
- Let \( I = \{ i_1, i_2, \ldots, i_m \} \) be a set of literals called items
- A sequence \( S \) is an ordered list of itemsets
- Sequence inclusions

**Exemple**

- \( I = \{ a, b, c, d \} \)
- \( it_1 = (bcd), it_2 = (ab) \)
- \( S = \langle (bcd)(ab) \rangle \) (5-sequence)
- \( \langle (ab)(a) \rangle \) \( \mathcal{C}_1 \)
- \( \langle (a)(b) \rangle \) \( \mathcal{C}_2 \)
- \( \langle (b)(ab) \rangle \) \( \mathcal{C}_3 \)

**Definition (Support)**

The support of a sequence \( S \) is defined as:

\[
\text{Support}(S, D) = \frac{|\{C \in D | S \preceq C \text{trans} \}|}{|\{C \in D \}|}
\]

**Sequential Pattern mining**

Extract all frequent sequences \( S \), i.e verifying:

\[
\text{Support}(S, D) \geq \sigma
\]

with \( 0 \leq \sigma \leq 1 \)
### Classical and incremental approaches

#### Classical approaches

1. Levelwise generate-and-prune:
   - SPADE: inverted database representation
   - SPAM: binary representation

2. Pattern-Growth:
   - PrefixSPAN: multiple database projection

#### Incremental approaches

Taking into account the dynamic evolution of a customer database. ISE, ISM and IncSPAN (no deletion)

### Remarks

1. Generation: Joint operations are known to be blocking operations [Babcock et al., 2002]
2. There is more than 1 pass over $D$ for all these algorithms however stream mining requires one-pass algorithms

### Data streams approaches

- SMDS
- SPEED
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Synopsis Construction

Requirements
- Broad Applicability
- One Pass Constraint
- Time and Space Efficiency
- Robustness
- Evolution sensitive

Techniques
1. Sampling Methods like Reservoir Sampling
2. Histograms
3. Wavelets
4. Sketches

Reservoir Sampling (Vitter 1985)

Main idea
An unbiased reservoir is maintained by probabilistic insertions and deletions

- Initialization: the first \( n \) points are directly added to the reservoir.
- When the \((t+1)\)th point from the reservoir is received, it is added with a probability \( \frac{n}{t+1} \) and replaces a random point in the reservoir.
Observations

- Insertion probabilities reduce with stream progression
- Unbiased reservoir maintained

Disadvantages

- The reservoir may not represent data stream evolutions
- Applications focusing on recent events from the data streams may get inaccurate results
- Smaller and smaller portions of the sample remain relevant with time

Biased Reservoir Sampling (Aggarwal 2006)

Main idea

- Use a temporal bias function to regulate the stream sample.
- This ensures that recent points from the data streams have higher probability to get inserted into the reservoir.

- Helps obtaining a biased and unbiased sample
- The bias is useful for applications focusing on representing the recent behavior of the data streams
Observations

- An easy to use memory-less bias functions class is the exponential bias functions defined as:

  \[ f(r, t) = e^{-\lambda(t-r)} \]

  The parameter \( \lambda \in [0,1] \) defines the bias rate.

- The bias function is proportional to \( p(r, t) \).

- \( p(r, t) \) is the probability that a point inserted at the instant \( r \) is still belonging to the reservoir when a point arrives at instant \( t \).

- In the special case of exponential bias functions the maximum reservoir requirement is bounded by \( \frac{1}{\lambda} \) for small \( \lambda \) values.

Challenges

- All classical mining algorithms have a strong hypothesis stating that a database can be loaded into main memory.

- What about real-world databases containing gigabytes of transactions?

- Nowadays we can afford approximate solutions but can we assure bounds on the size of the samples given a desired accuracy?
Sample size

- Error:
  \[ e(s, S_D) = |\text{Support}(s, S_D) - \text{Support}(s, D)| \]

  \[ X_i \] a random variable for the \( i \)th customer with:
  - \( \Pr[X_i = 1] = p_i \) if \( i \)th customer supports the sequence \( s \)
  - \( \Pr[X_i = 0] = 1 - p_i \), if not.

**Note**

We are in presence of Poisson trials as the number \( t \) of trials in which the probability of success \( p_i \) varies from trial to trial.

The number of customers in the sample that supports the sequence \( s \):

\[ X(s, S_D) = \sum X_i = \text{Support}(s, S_D) \times |S_D| \]

The expected number of customers that support the sequence \( s \) in the sample is:

\[ E[X(s, S_D)] = \text{Support}(s, D) \times |S_D| \]

**Theorem**

Given a sequence \( s \) then \( \Pr[e(s, S_D) > \epsilon] \leq \delta \) if the reservoir size is:

\[ |S_D| \geq \ln \left( \frac{2}{\delta} \right) \frac{1}{2\epsilon^2} \]
Proof sketch

1. Start from $\Pr[\text{Support}(s, S_D) - \text{Support}(s, D) > \epsilon]$
2. Introduce $X(s, S_D)$ and $E[X(s, S_D)]$
3. Use Chernoff bounds to get the previous result

Observations

- We easily get an $(\epsilon, \delta)$-approximation
- Chernoff bound is not always very tight, but in this case it is acceptable
- We get samples of reasonable size with tolerable error:

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>$\delta$</th>
<th>$S_M$</th>
</tr>
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<tr>
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Extending to Data Streams: the challenges

- We would like to approximate sequences support by maintaining a dynamic sample
- We would like to have both biased and unbiased sample (user-defined granularity)
- Use biased reservoir approach but with respect to our \((\epsilon, \delta)\)-approximation

Analysis

We are working on biased reservoir samples, the following corollary gives an upper bound on the bias rate:

**Corollary**

Given an error bound \(\epsilon\) and a maximum probability \(\delta\) that \(\epsilon(s, S_D) > \epsilon\) we get an upper bound on the bias rate:

\[
\lambda \leq \frac{2\epsilon^2}{\ln(2/\delta)}
\]

- Proof sketch
  - \(|S_D| \leq \frac{1}{\epsilon^2}
  - \(|S_T| \leq \frac{1}{\lambda}
  - replace in the theorem
Observations

- The bias rate depends on the accuracy we want.
- The accuracy of our mining results is optimal when the reservoir is full.
- The reservoir maintained is very small in terms of space requirements.

<table>
<thead>
<tr>
<th>ϵ</th>
<th>δ</th>
<th>λ</th>
<th>S_D</th>
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Algorithm

1. Check if customer \( C_i \) is present in the reservoir.
2. If no, throw a coin.
   - If Success \( (\frac{\epsilon}{\delta}) \) add the customer to the reservoir.
   - Else replace with a random position in the reservoir.
3. If present in the reservoir then add \( C_i \) itemset.
Observations

We have to show that the replacement policy in the algorithm respects the exponential bias behaviour with $\lambda = \frac{1}{q}$.

**Proof sketch**

- Probability that a customer is in the reservoir $\frac{1}{q}$
- Probability to throw a customer is $\left(1 - \frac{1}{q}\right)^n \frac{1}{q^n}$
- $\frac{d - 1}{qn}$

- If the customer is inserted at the time $r$ and is still in the reservoir at time $t$, then it did not get ejected in $t - r$ iterations: $(1 - \frac{1}{q})^{t-r}$
- $(1 - \frac{1}{q})^{t-r} = \left(1 - \frac{1}{qn}\right)^n (\frac{1}{q^n})^{t-r}$
- For large value of $n$, $(1 - \frac{1}{q^n})^{n}$ is approximately equal to $\frac{1}{2}$
Summary

- No sampling techniques for sequential patterns mining
- We introduced approximate approaches that work for mining on static databases and we extended it to data streams
- We get a biased sample, quality does not degrade with stream progression
- Extremely easy to implement and easy to maintain (small space requirements depending on bias rate defined by the user)

Thank you for your attention