Journée BANG

Collective behaviour of cells : chemotaxis and swarming

Nicolas Vauchelet

vauchelet@ann.jussieu.fr

22 septembre 2009

Plan

1. Chemotaxis
2. Swarming

Plan

Chemotaxis

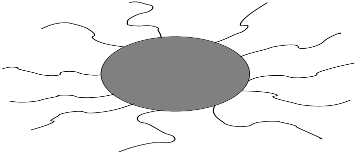
Chemotaxis : phenomenon in which cells direct their movement according to a certain chemicals in their environment.

Examples :

- movement of living systems towards food,
- secretion of cAMP by amoeba Dictyostelium discoideum,
- angiogenesis, tumors birth, $\cdots$

Consequences : aggregation, networks formation, $\cdots$

Bacteria behaviour



Some bacteria like *Escherichia Coli* have several flagella allowing them to move.

We distinguish two phases :

- *run* phases : swim in a straight line.
- *tumble* phases : reorientation of the bacterium.

The overall movement of a bacterium is the result of alternating tumble and swim phases. They are able to find favorable locations with high concentration of attractant.

Kinetic model

At a microscopic level of description, we use a kinetic model to describe the evolution of the distribution function f :

$$\partial_t f + v \cdot \nabla_x f = \int_{\mathbb{S}^2} (T(v' \to v) f(v') - T(v \to v') f(v))\, dv'.$$

where $\mathbb{S}^2$ is the sphere of $\mathbb{R}^2$, $T(v' \to v)$ represents the rate of cells reorientation : *turning kernel*.

- The left hand side represents the *run* phase.
- The right hand side describe the *tumble* phase depending on the chemoattractant concentration S :

$$\alpha S - \Delta S = \rho := \int_{\mathbb{S}^2} f(v)\, dv.$$

Tumbling Kernel

Cells have a small memory effect allowing them to compare the present chemical concentrations to previous ones and thus to respond to temporal gradients along their paths. Then they decide to change their directions or to continue, depending on the concentration profile of the chemical S.
$\Rightarrow$ The tumbling kernel is given by

$$T(v,v') = \phi(S_t + v \cdot \nabla_x S).$$

A simple model for the function ϕ is

$$\phi(x) = \begin{cases} 1 & \text{si } x < 0, \\ 1/4 & \text{si } x > 0. \end{cases}$$

References

- Y. Dolak, C. Schmeiser, J. Math. Biol. 51, 595-615 (2005).
- F.A. Chalub, P.A. Markowich, B. Perthame, C. Schmeiser, Monatsh. Math. 142, 123-141 (2004).
- F. Filbet, Ph. Laurençot, B. Perthame, J. Math. Biol. 50, 189-207 (2005).
- T. Hillen, H.G. Othmer, SIAM J. Appl. Math. 61(3), 751-775 (2000).
- N. Bournaveas, V. Calvez, S. Gutierrez, B. Perthame, CPDE 33, 79-95 (2008).

Numerical results

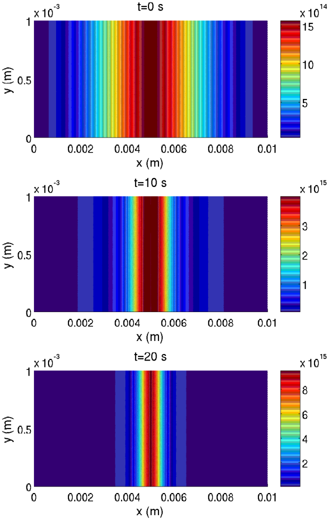


FIG.: Dynamics of the density of bacteria.

Numerical results

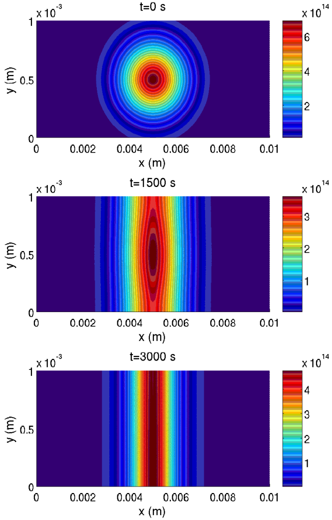


FIG.: Dynamics of the density of bacteria.

Hydrodynamic limit

After rescaling, one derive a class of hydrodynamical model :

$$\partial_t \rho + \text{ div } (\chi(S_t, |\nabla_x S|) \nabla_x S\, \rho) = 0,$$
$$\alpha S - \Delta S = \rho.$$

where $\chi(S_t, |\nabla_x S|)$ is the chemotactic sensibility.

Hydrodynamic limit

After rescaling, one derive a class of hydrodynamical model :

$$\partial_t \rho + \text{ div } (\chi(S_t, |\nabla_x S|) \nabla_x S \, \rho) = 0,$$
$$\alpha S - \Delta S = \rho.$$

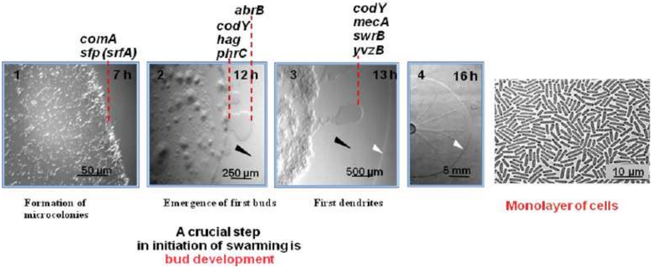
where $\chi(S_t, |\nabla_x S|)$ is the chemotactic sensibility.

Blow up profile are usually observed for this kind of system.
Rigorous mathematical study of such behaviour is challenging.

Plan

Introduction

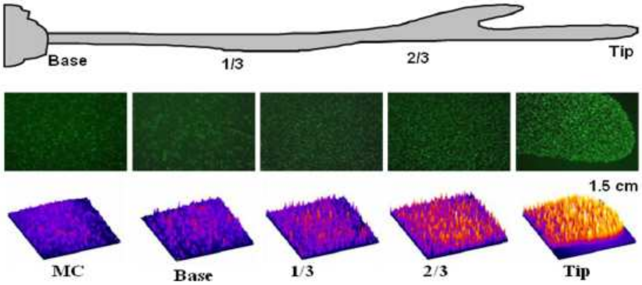
- *Swarm* describes a behaviour of an aggregate of animals of similar size and body orientation, generally moving *en mass* in the same direction.
- Since several years, swarming phenomena has been studied at *Institut de Génétique et Microbiologie, Orsay* on bacteria *Bacillus subtilis*.
  $\Rightarrow$ In a synthetic medium, formation of dendrites which are monolayer with minimal branching.



Swarmers and Followers

The gene expression along the dendrite is heterogeneous. It implies the presence of different cells type :

- highly flagellated and hypermotile cells in the tip : *swarmers*.
- immobile and less flagellated *followers*.



Relevant facts

- Swarming depends on surfactin secretion and flagella. There is at least two distinguishable cell types : hyper-motile *swarmers* (24 flagella) and largely immobile *followers* (12 flagella).
- Swarms proceed with a constant migration speed.
- Dendrites maintain a constant population density which increases sharply in the tip.
- The nutrients appears to be not limiting during a swarm process.

Mathematical model

$n(t,x)$: swarmers density,
$f(t,x)$: followers density,
$c(t,x)$: chemical factor concentration,
$S(t,x)$: surfactin concentration,
$D_m(t,x)$: diffusion coefficient of the followers,
$m_{col}(x)$: position of the mother colony.

$$\begin{cases} \partial_t n + \operatorname{div}(n(1-n)\nabla c - n\nabla S) = 0, \\ -D_c \Delta c + c = \alpha_c n, \\ \partial_t D_m = dtrace \times n, \\ \partial_t f - \operatorname{div}(D_m \nabla f) = B_f f(1-f) + B_n n, \\ \partial_t S - D_s \Delta S + \tau S = \alpha_S (f + m_{col}). \end{cases}$$

Questions

- What should be the birth rates to maintain a constant speed migration ? Is the speed migration really constant ?
- How to obtain and explain the branching phenomenon ?
- Does a numerical simulation of the model correspond to what is observed in experiment ?