Motion estimation using the variational data assimilation framework
First results using an *Extended Image Model*

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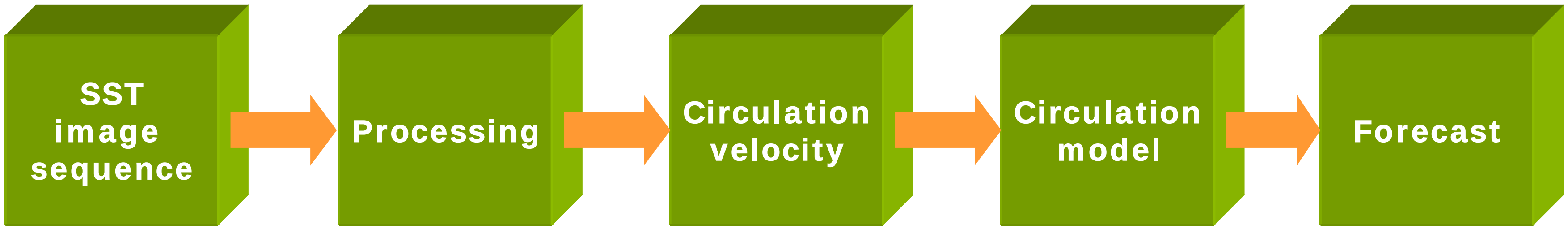
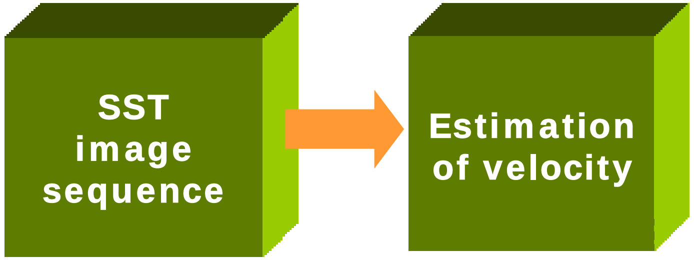
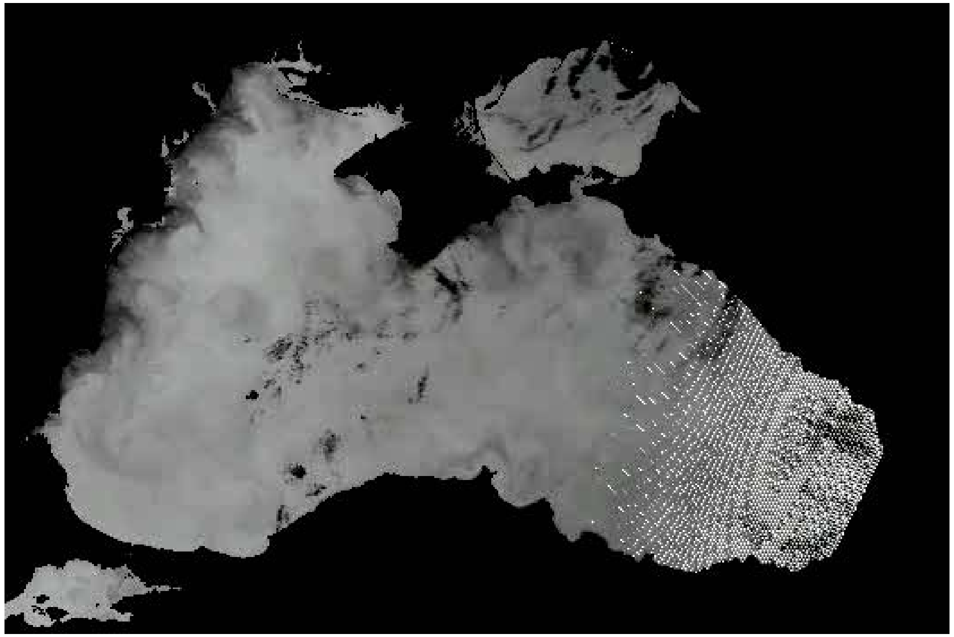
Context

1. Derive **pseudo-observations** of circulation velocity from sequence of Sea Surface Temperature (**SST**) image sequences.
2. These pseudo-observations are in turn **assimilated** in an ocean circulation model.



Objective





- Common approach:
  - Image processing techniques (correlation, optical flow, …)
  - Problem: missing data, cloud coverage

    ➔ impossible to compute derivatives.

- Data assimilation approach:
  - Image Model: expression of the transport of temperature by surface velocity
  - Assimilation of SST within the Image Model

    ➔ estimation of initial velocity field even when data are missing.

First *Image Model*

- Definition

$$\begin{cases} \dfrac{\partial T}{\partial t} + \nabla T \cdot \mathbf{v} = K_T \Delta T \\ \\ \dfrac{\partial u}{\partial t} = 0 \\ \\ \dfrac{\partial v}{\partial t} = 0 \end{cases}$$

- Assimilation system

$$\begin{cases} \dfrac{\partial X}{\partial t}(\mathbf{x};t) + F(X)(\mathbf{x};t) & = 0 \\ X(\mathbf{x};t=0) - X_0(\mathbf{x}) & = 0 \\ Y(\mathbf{x};t) - H(X)(\mathbf{x};t) & = \varepsilon_o \end{cases}$$

Variational framework

- Cost function

$$J(X_0) \;=\; \frac{1}{2}\int_{\Omega,t}(H(X)-Y)^{\mathrm{T}}\mathbf{R}^{-1}(H(X)-Y)$$

- In our application

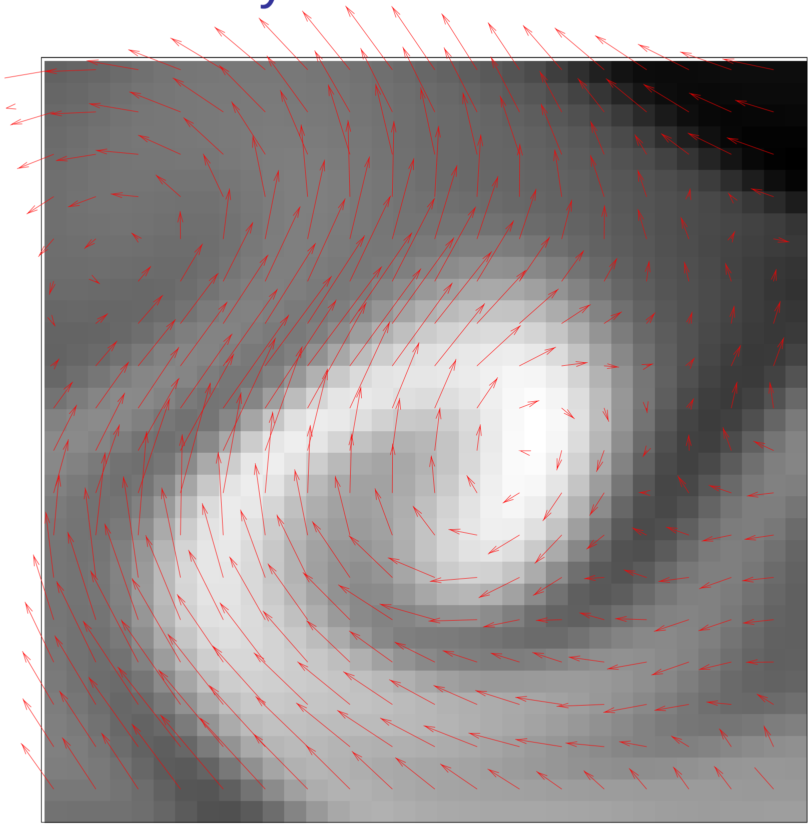
$$J \;=\; \frac{1}{2}\int_{\Omega,t}(T-T_{obs})^2$$

- In practice

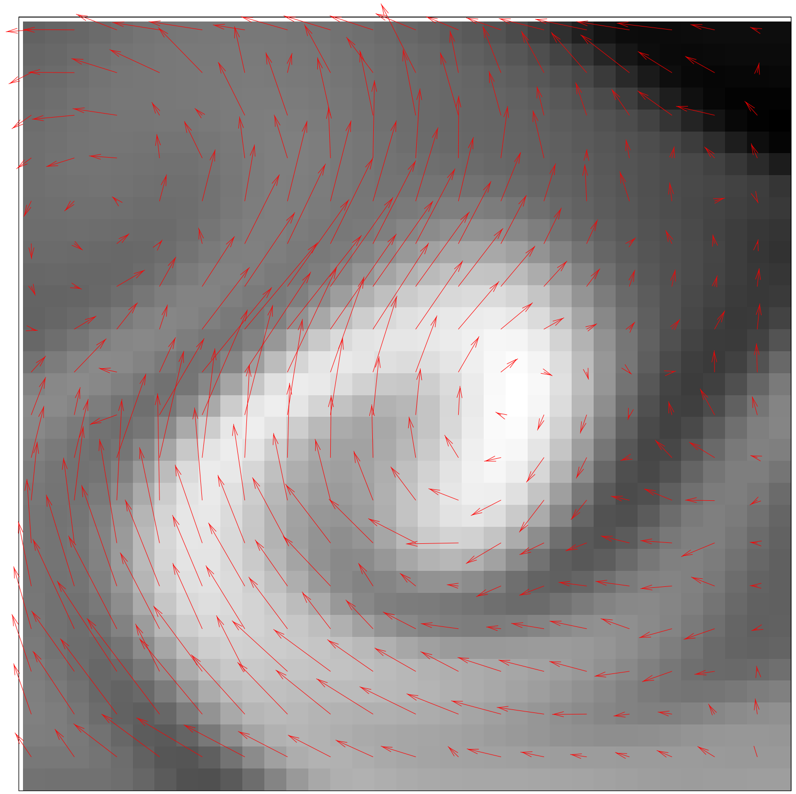
$$J \;=\; \frac{1}{2}\int_{\Omega,t}(T-T_{obs})^2+\frac{\lambda}{2}\int_{\Omega,t}(\alpha|\bigtriangledown \operatorname{div}\mathbf{v}|^2+\beta|\bigtriangledown \operatorname{curl}\mathbf{v}|^2)]$$

Estimation of surface velocity

Result on synthetic data:



Ground truth



Estimated result

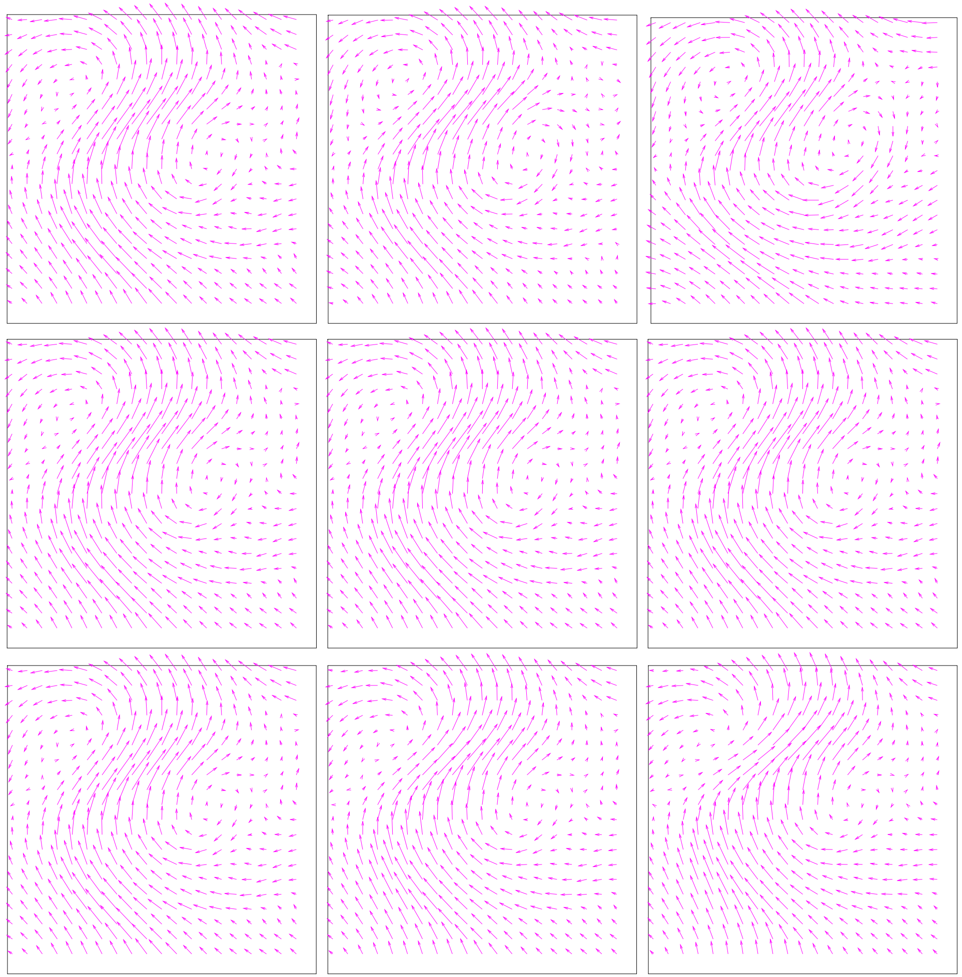
Extended Image Model

- Why introducing a new model?
  - ➔ To take into account the physical knowledge of the ocean surface dynamic

- Idea
  - ➔ The velocity evolution equations come from the shallow water model
  - ➔ The transport equation of temperature is the same as in the 1st *IM*

$$\begin{cases} \dfrac{\partial T}{\partial t} + \nabla T \cdot \mathbf{v} = K_T \Delta T \\[2em] \dfrac{du}{dt} - fv = g' \dfrac{\partial h}{\partial x} + K_v \Delta u \\[2em] \dfrac{dv}{dt} + fu = g' \dfrac{\partial h}{\partial y} + K_v \Delta v \\[2em] \dfrac{\partial h}{\partial t} + \dfrac{\partial (uh)}{\partial x} + \dfrac{\partial (vh)}{\partial y} = 0 \end{cases}$$

Model comparison: velocity simulations



→ 3D model

→ 1st *Image Model*

→ *Extended Image Model*

Model comparison: SST simulations

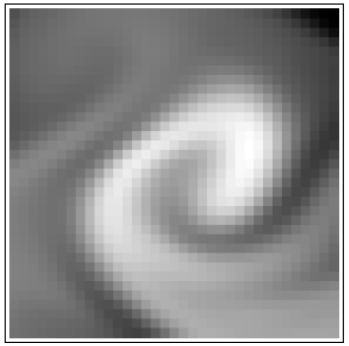
→ 3D model

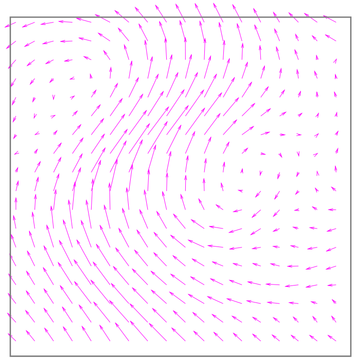
→ 1st *Image Model*

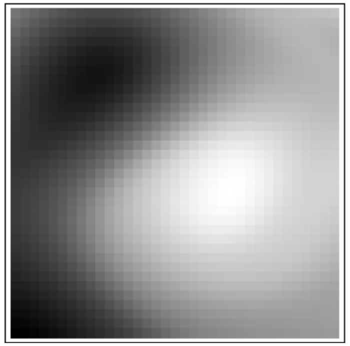
→ *Extended Image Model*

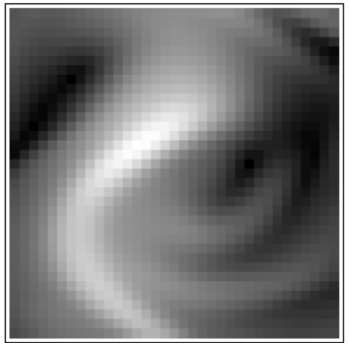
Twin Experiments

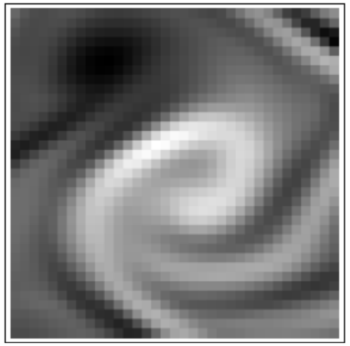
- We use the output of a 3D model (OPA) as initial conditions:


temperature


velocity


thickness


magnitude of velocity

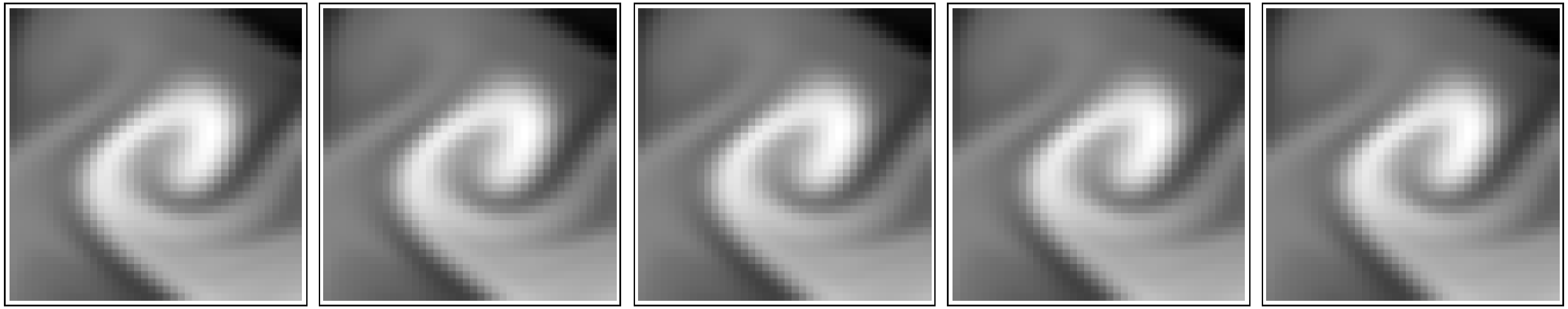

vorticity of velocity

- The cost function doesn't include any regularization term:

$$J(X_0) = \frac{1}{2}\int_{\Omega,t}(T - T_{obs})^{\mathrm{T}}\mathbf{R}^{-1}(T - T_{obs})$$

Twin Experiments: small displacements

- Generation of temperature images every hour with the *EIM:*



- Initial conditions for the assimilation process:
  - the temperature field T_0 is taken equal to the first observation:

$$\forall(i,j)\ T_0(i,j) = T_1(i,j)$$

  - the velocity field:

$$\forall(i,j)\ \mathbf{v}_0(i,j) = (0,0)^T$$

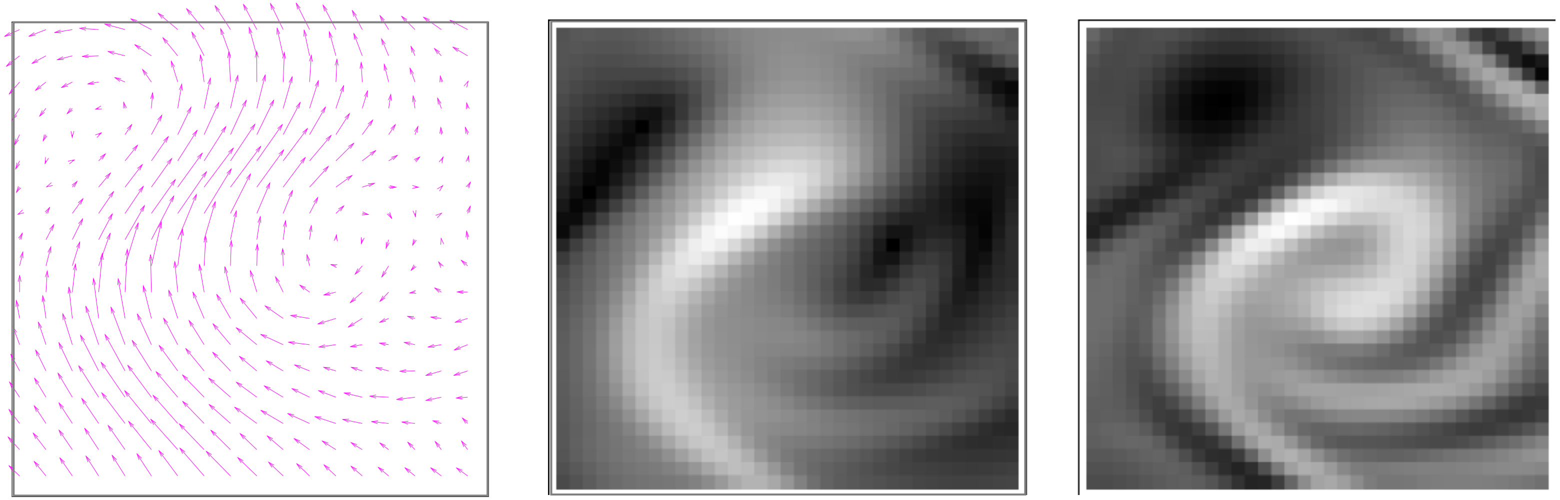
  - the thickness field is constant

$$\forall(i,j)\ h_0(i,j) = h_m$$

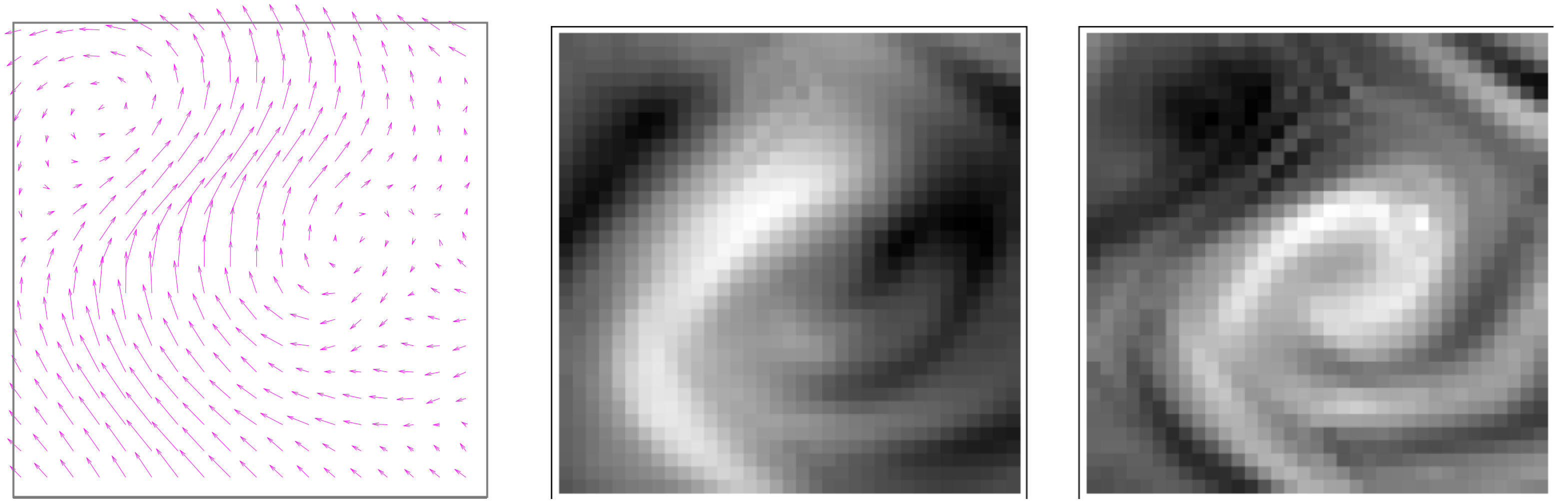
  where h_m is the average thickness of the ocean layer.

Twin Experiments: small displacements results

Ground truth
given by the
3D model



Velocity
estimation



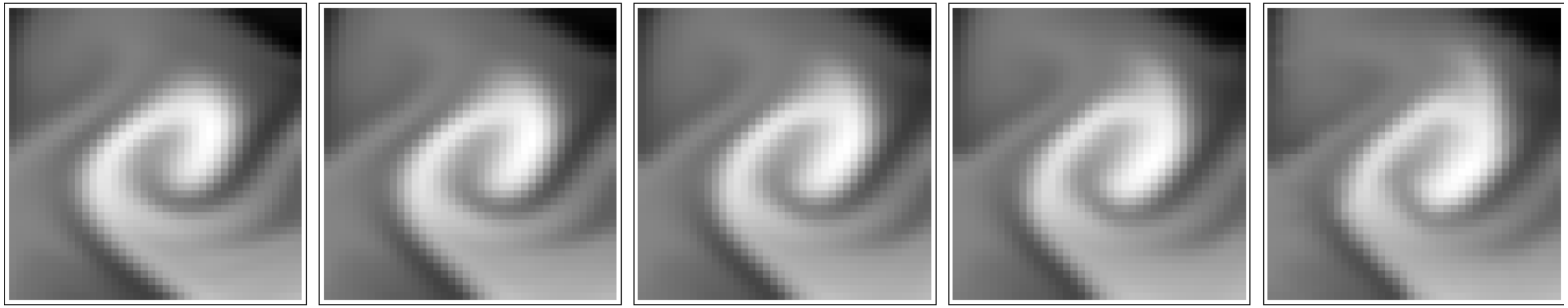
Twin Experiments: small displacements results

- Quantitative evaluation of the results

	Min	Avg	Max	Unit	%
Angular error	0.030	0.991		o	
Euclidian error	0.001	0.018	0.058	m/s	0.006
Vorticity error	0.000	0.009	0.040		5.867

Twin Experiments: large displacements

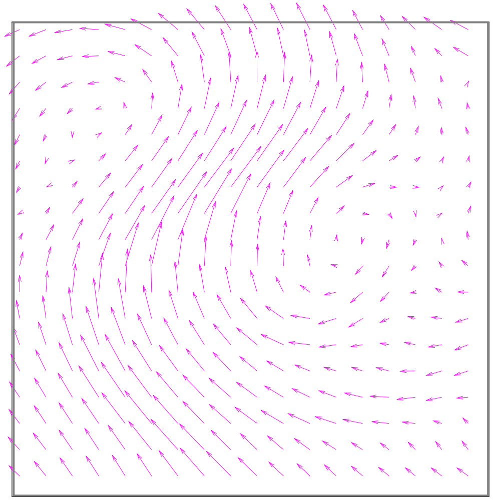
- Generation of temperature images every day with the *EIM:*

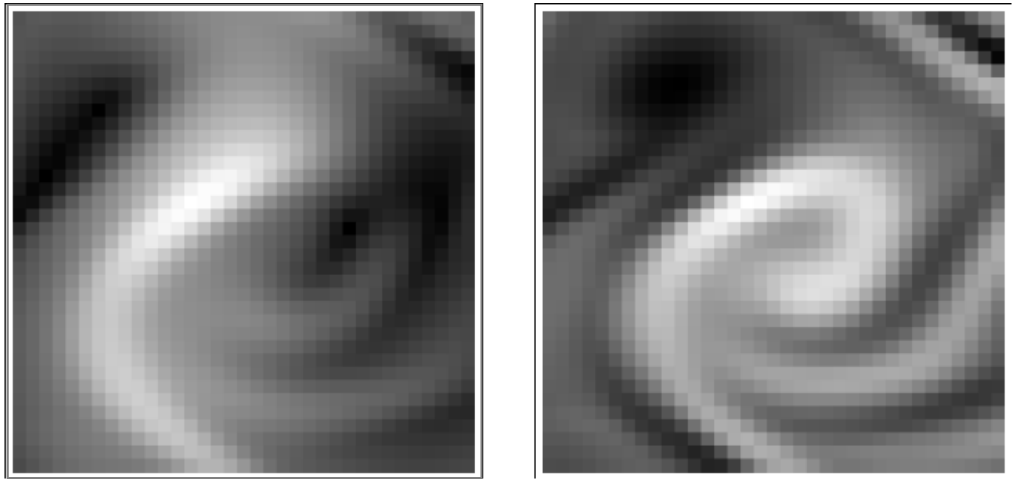


- Initial conditions for the assimilation process:
  - the temperature field T_0 is taken equal to the first observation:
  $$\forall(i,j)\ T_0(i,j) = T_1(i,j)$$
  - the velocity field:
  $$\forall(i,j)\ \mathbf{v}_0(i,j) = (0,0)^T$$
  - the thickness field is constant
  $$\forall(i,j)\ h_0(i,j) = h_m$$
  where h_m is the average thickness of the ocean layer.

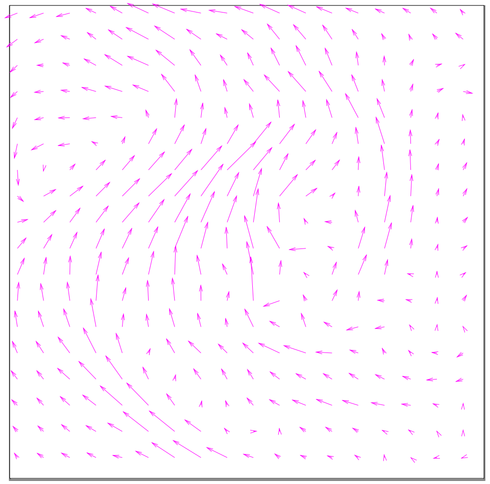
Twin Experiments: large displacements results

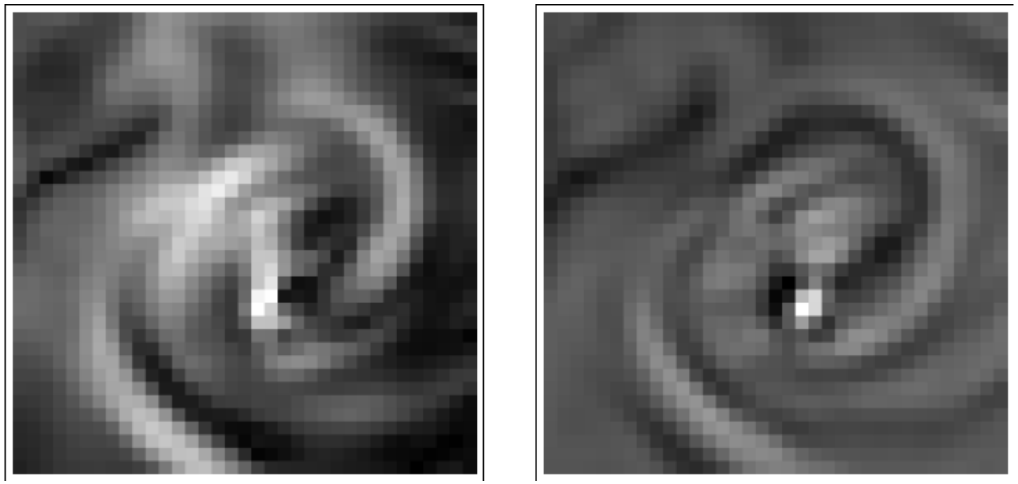
Ground truth given by the 3D model





Velocity estimation





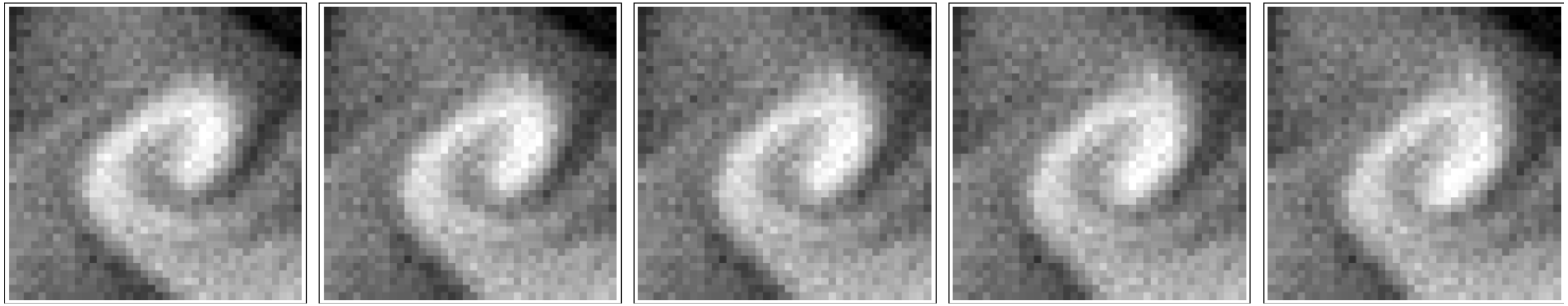
Twin Experiments: large displacements results

- Quantitative evaluation of the results

	Min	Avg	Max	Unit	%
Angular error	0.041	6.460	14.891	o	
Euclidian error	0.001	0.115	0.267	m/s	0.047
Vorticity error	0.000	0.040	0.172		21.659

Twin Experiments: noisy observations

- Generation of temperature images every day with the *EIM* + Gaussian noise

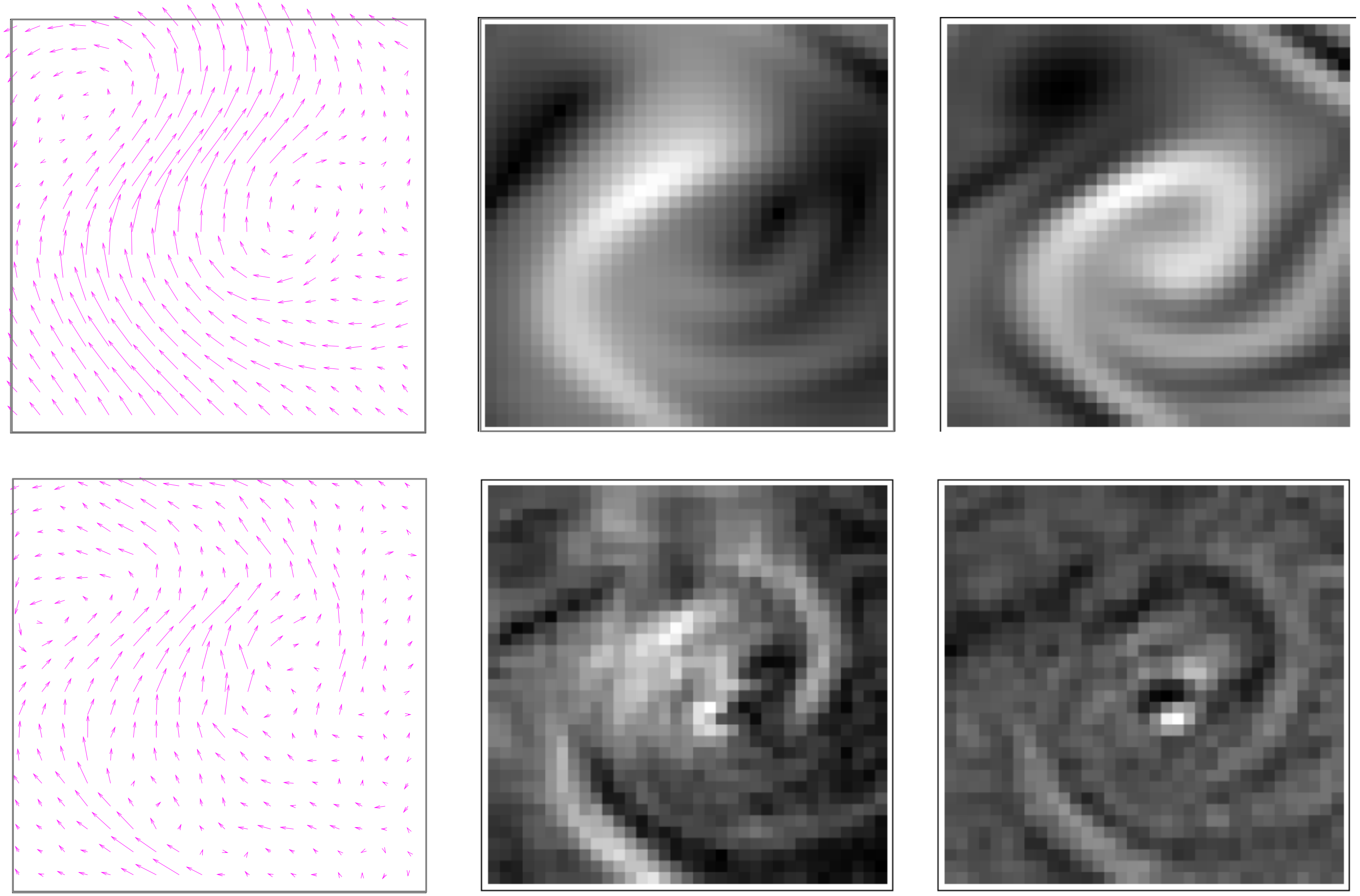


- Initial conditions for the assimilation process:
  - the temperature field T_0 is taken equal to the first observation:
    $$\forall (i,j)\ T_0(i,j) = T_1(i,j)$$
  - the velocity field:
    $$\forall (i,j)\ \mathbf{v}_0(i,j) = (0,0)^T$$
  - the thickness field is constant
    $$\forall (i,j)\ h_0(i,j) = h_m$$
    where h_m is the average thickness of the ocean layer.

Twin Experiments: noisy observations results

Ground truth given by the 3D model

Velocity estimation



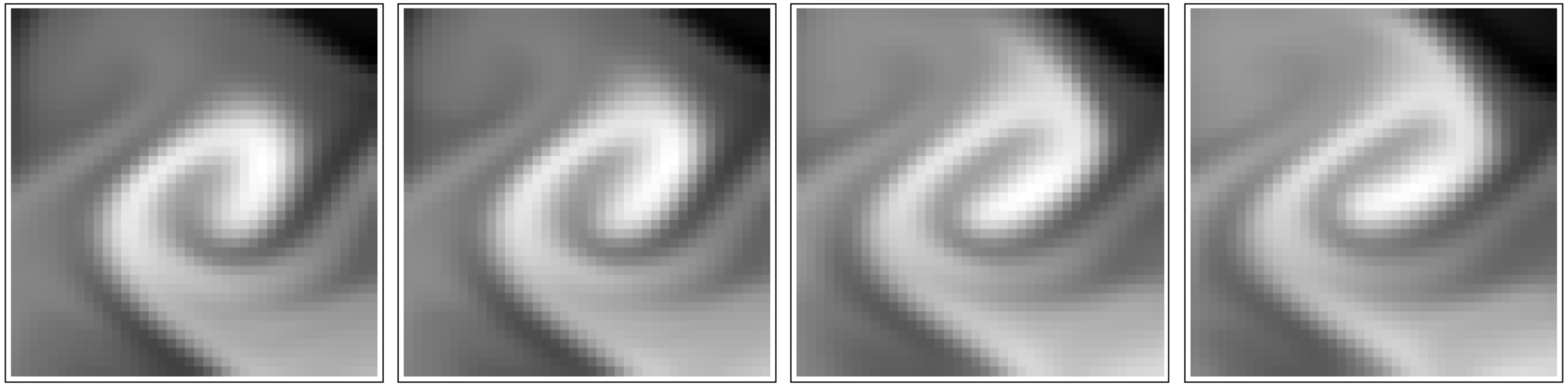
Twin Experiments: noisy observations results

- Quantitative evaluation of the results

	Min	Avg	Max	Unit	%
Angular error	**0.038**	**6.703**	**15.843**	**o**	
Euclidian error	**0.001**	**0.120**	**0.284**	**m/s**	**0.050**
Vorticity error	**0.000**	**0.038**	**0.160**		**20.373**

Realistic simulations

- Temperature images come from the 3D model OPA, one image every day:



- Initial conditions for the assimilation process **no *a priori***:
  - the temperature field T_0 is taken equal to the first observation:

$$\forall(i,j)\ T_0(i,j) = T_1(i,j)$$

  - the velocity field:

$$\forall(i,j)\ \mathbf{v}_0(i,j) = (0,0)^T$$

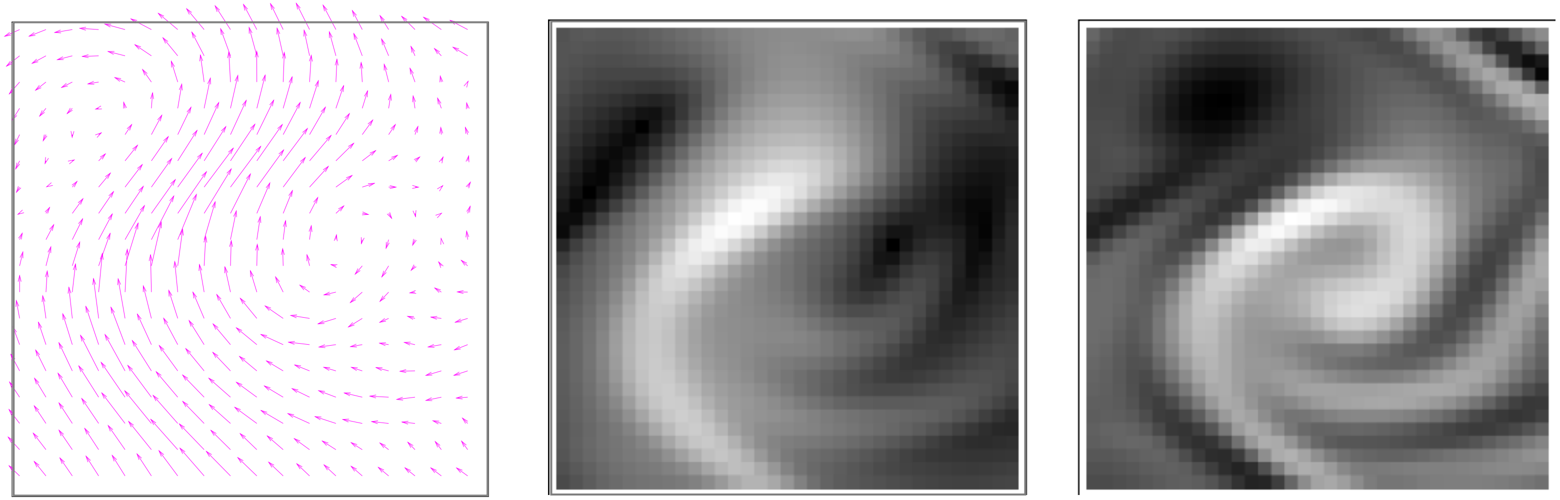
  - the thickness field is constant

$$\forall(i,j)\ h_0(i,j) = h_m$$

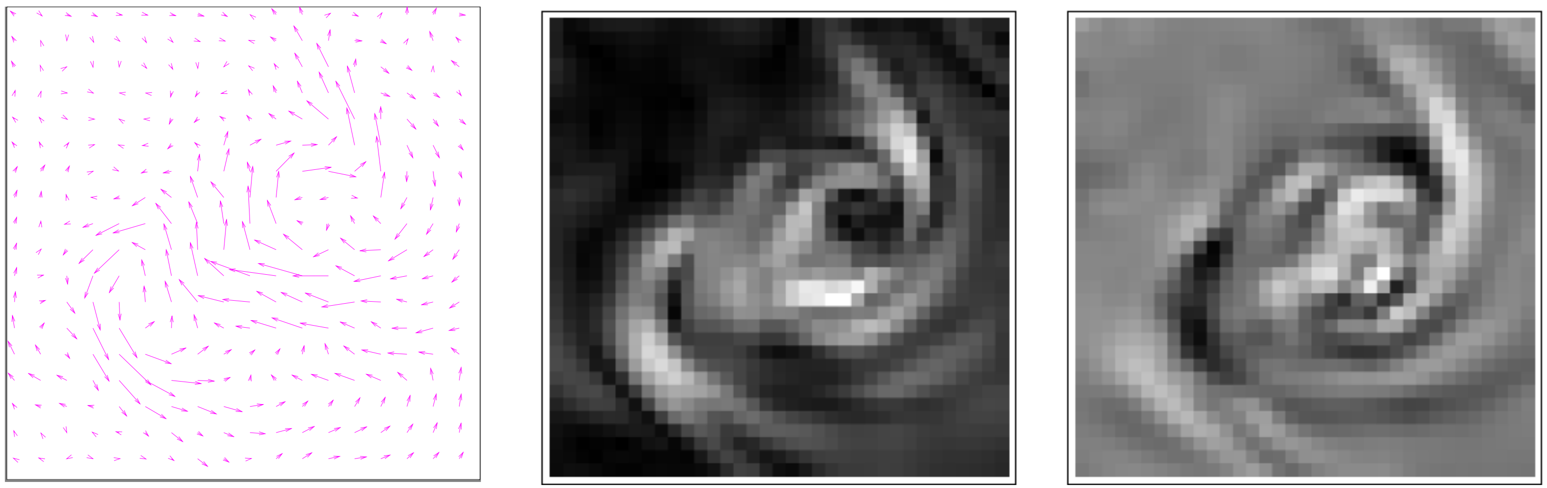
where h_m is the average thickness of the ocean layer.

Realistic simulations, no *a priori*: results

Ground truth given by the 3D model



Velocity estimation



Realistic simulations, no *a priori*: results

- Quantitative evaluation of the results

	Min	Avg	Max	Unit	%
Angular error	**0.171**	**10.406**	**136.61**	**o**	
Euclidian error	**0.003**	**0.185**	**0.662**	**m/s**	**0.053**
Vorticity error	**0.000**	**0.080**	**0.286**		**40.986**

NOT GOOD !

How to enhance the estimation quality?

Two strategies:

1. **Addition of a regularization term in the cost function**

   It is important to notice that the addition of a regularization term does not have the same signification as in *Optical Flow* approaches. In these approaches, the regularization term is mandatory to constraint the solution of the ill-posed problem commonly named *aperture problem*. In our approach, this problem is actually resolved by the use of the evolution model (*EIM*) itself.

2. **Initialization of the minimization process in the neighborhood of the global solution**

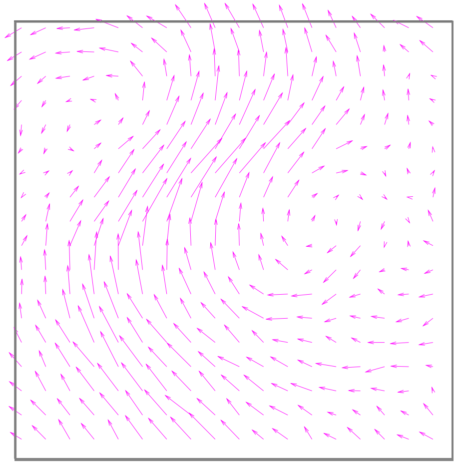
   It is important to not mixed up with the use of a background information commonly used in the data assimilation framework:

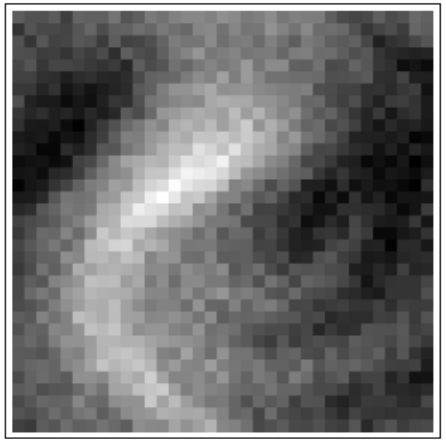
$$
\begin{aligned}
J(X_0) &= \frac{1}{2}\int_{\Omega,t}(H(X)-Y)^{\mathrm{T}}\mathbf{R}^{-1}(H(X)-Y) \\
&\quad +\frac{1}{2}\int_{\Omega,t}(X-X_b)^{\mathrm{T}}\mathbf{B}^{-1}(X-X_b)
\end{aligned}
$$

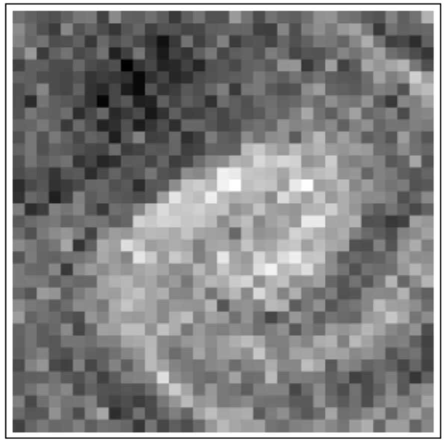
   The *background term* is a regularization term. It constrains the solution to be in the neighborhood of the X_b. In our approach, we cannot trust enough the initialization to use this background term in the cost function. We only use the initialization to start the minimization process in the neighborhood of an acceptable solution.

Realistic simulations

- Initial conditions for the assimilation process:
  - the temperature field T_0 is taken equal to the first observation:
    $$\forall(i,j)\ T_0(i,j) = T_1(i,j)$$
  - the thickness field is constant:
    $$\forall(i,j)\ h_0(i,j) = h_m$$
    where h_m is the average thickness of the ocean layer.
  - the velocity field is initialized with a ***rather good estimation***:
    $$\forall(i,j)\ \mathbf{v}_0(i,j) = (u_{opa_0} + \varepsilon, v_{opa_0} + \varepsilon)^T$$





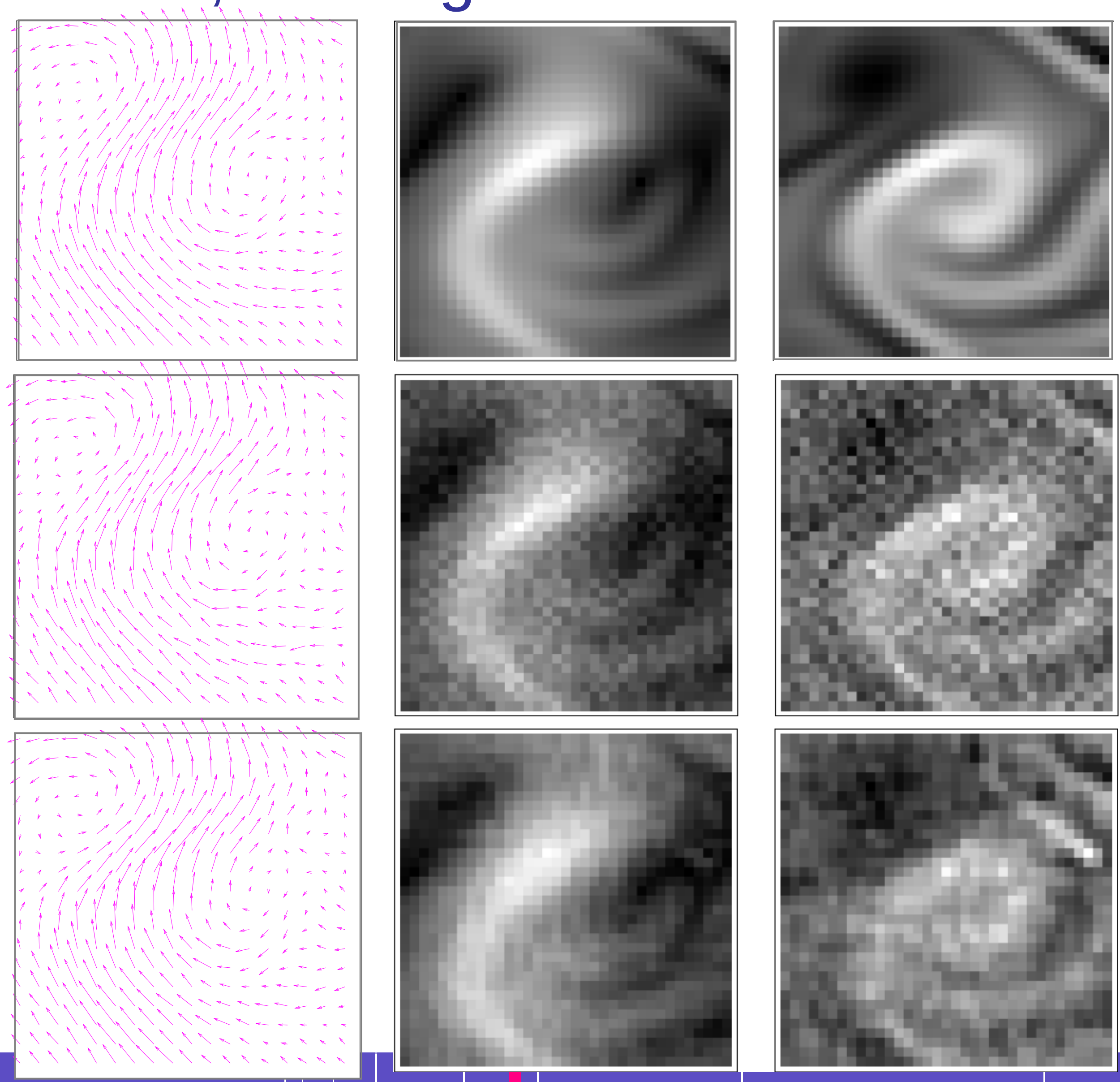


Realistic simulations, rather good initialization: results

Ground truth given by the 3D model

Velocity initialization

Velocity estimation



Realistic simulations, rather good initialization: results

- Quantitative evaluation of the differences between the reference velocity field and the initialization field

	Min	Avg	Max	Unit	%
Angular error	0.369	1.500	2.451	o	
Euclidian error	0.007	0.027	0.043	m/s	0.009
Vorticity error	0.000	0.021	0.056		11.31

- Quantitative evaluation of the differences between the reference velocity field and the estimated field

	Min	Avg	Max	Unit	%
Angular error	0.041	1.366	6.927	o	
Euclidian error	0.001	0.024	0.121	m/s	0.008
Vorticity error	0.000	0.018	0.124		18.69

Realistic simulations

- Initial conditions for the assimilation process:
  - the temperature field T_0 is taken equal to the first observation:

$$\forall(i,j)\ T_0(i,j) = T_1(i,j)$$

  - the thickness field is constant:

$$\forall(i,j)\ h_0(i,j) = h_m$$

    where h_m is the average thickness of the ocean layer.
  - the velocity field is initialized with as the result of an Optical Flow approach.

    We have used an implementation based on multiscale spline vectors using the luminance conservation hypothesis and a second order div-curl regularization [Isambert *et al.* in ECCV'2008]:

$$\begin{cases} \dfrac{\partial T}{\partial t} + \nabla T \cdot \mathbf{v}(r_i) = 0 & i \in \{1 \dots N\} \\ \min \displaystyle\int_{image} \alpha \|\nabla \mathrm{div}\, \mathbf{v}\|^2 + \beta \|\nabla \mathrm{curl}\, \mathbf{v}\|^2 & \end{cases}$$

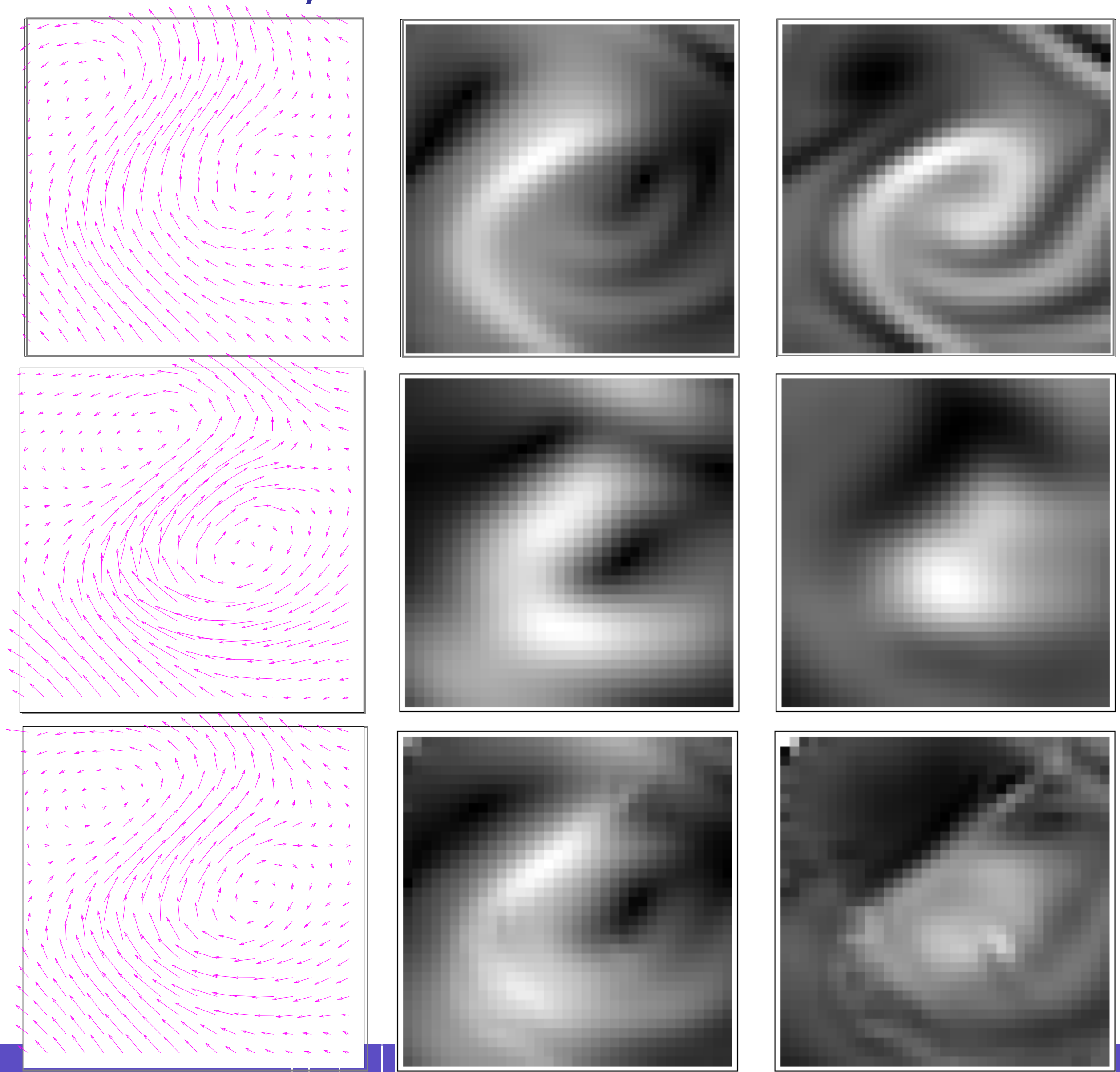
The vector field $\mathbf{v}$ must respect the conservation equation on several control points r_i and respect the div-curl constraint whole over the image.

Realistic simulations, OF initialization: results

Ground truth
given by the
3D model

Velocity
initialization

Velocity
estimation



Realistic simulations, OF initialization: results

- Quantitative evaluation of the differences between the reference velocity field and the initialization field

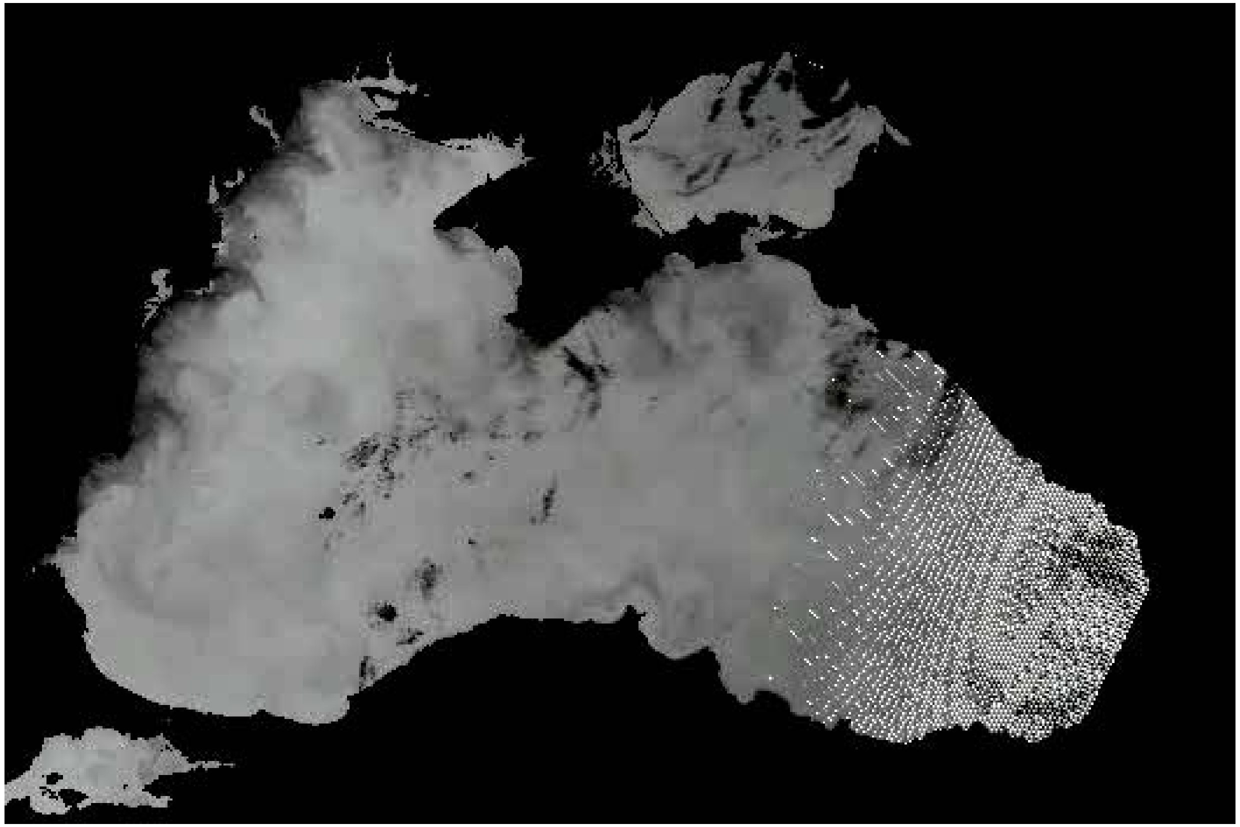
	Min	Avg	Max	Unit	%
Angular error	0.129	6.338	16.023	o	
Euclidian error	0.002	3.492	9.685	m/s	0.044
Vorticity error	0.000	0.038	0.104		91.41

- Quantitative evaluation of the differences between the reference velocity field and the estimated field

	Min	Avg	Max	Unit	%
Angular error	0.046	3.303	8.171	o	
Euclidian error	0.001	0.060	0.149	m/s	0.022
Vorticity error	0.000	0.021	0.062		13.56

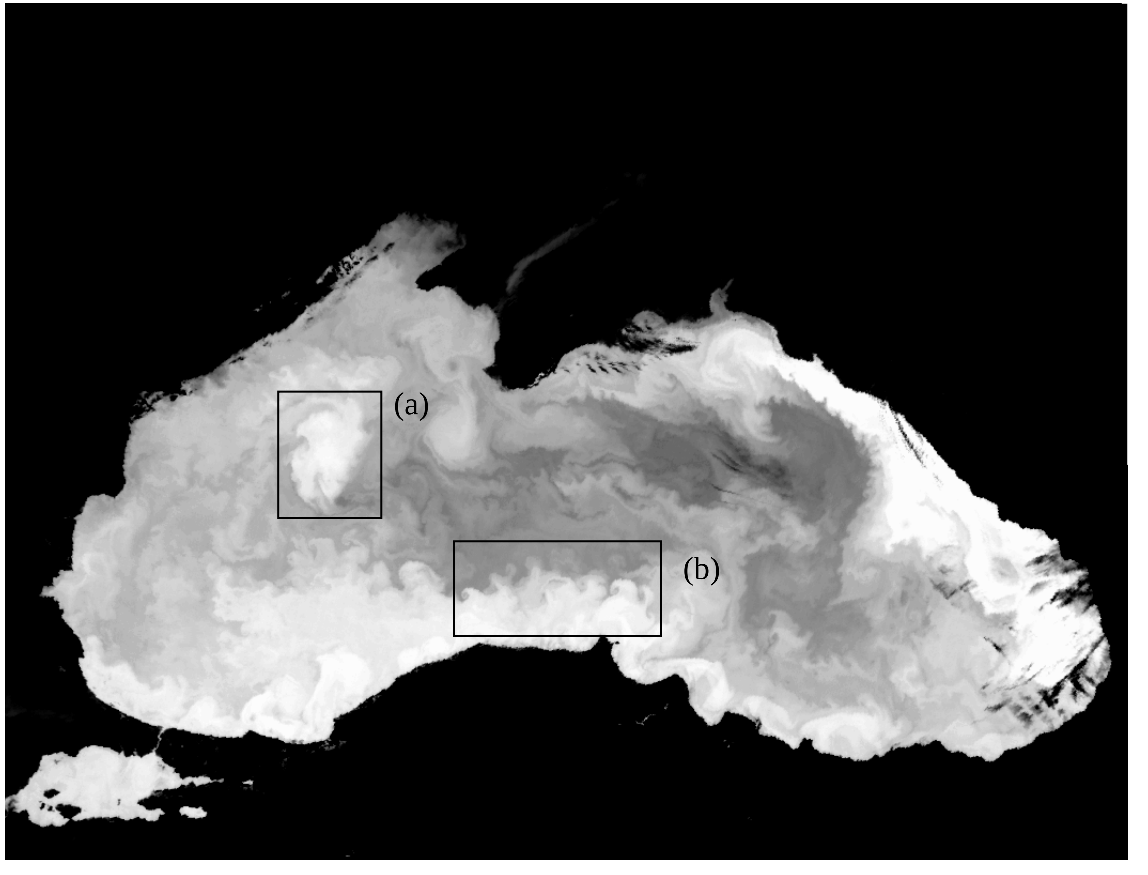
First results on Black Sea images

- NOAA/AVHRR SST sequence captured in December 1999

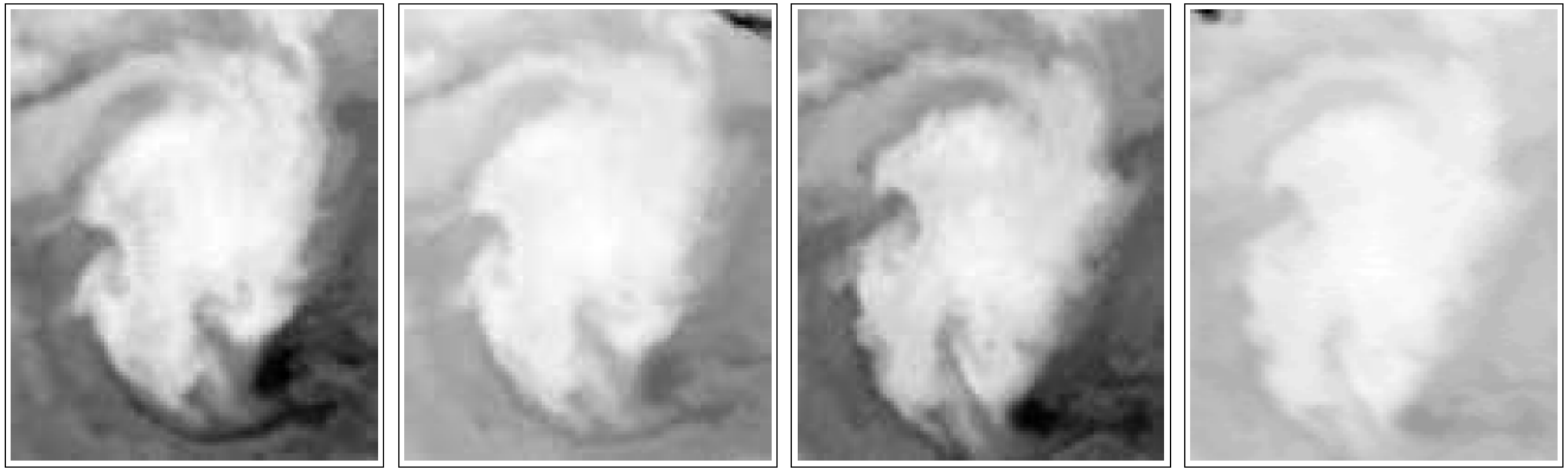


First results on Black Sea images

- NOAA/AVHRR SST sequence captured in December 1999



Mesoscale eddy: frame box (a)

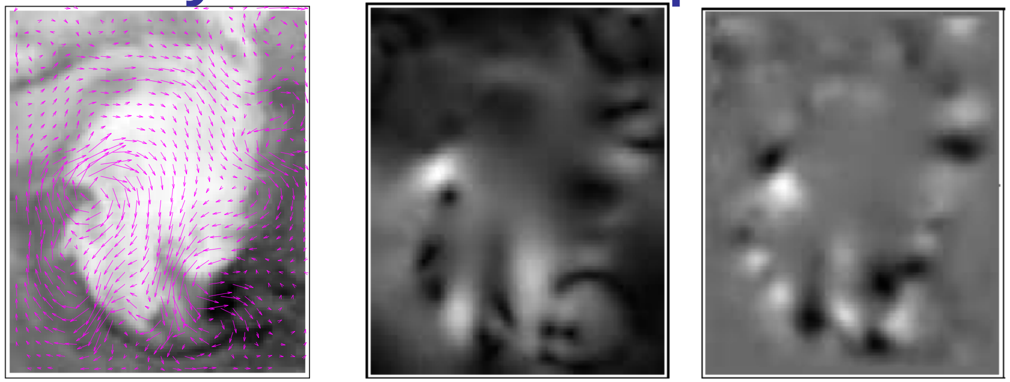


- We can make a comparison between estimation results obtain with:

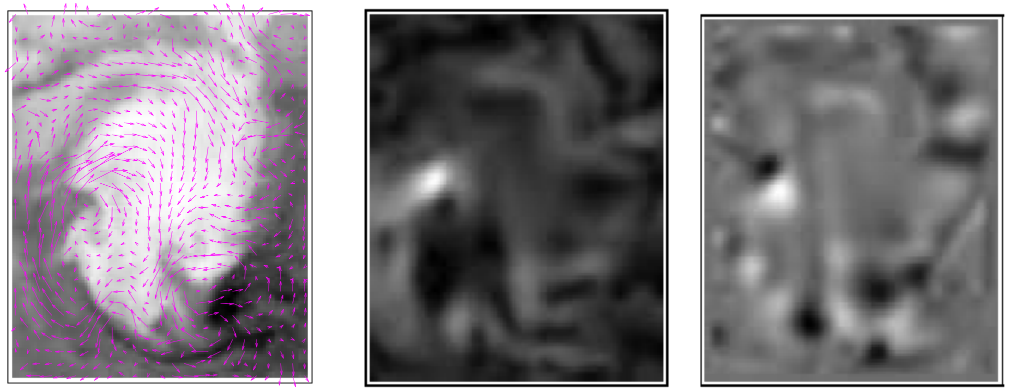
  1. Assimilation of SST images in the first *Image Model* (without initialization for the velocity but an additional regularization term in the cost function)
  2. The Optical Flow approach (multiscale spline vector implementation with second order div-curl regularization)
  3. Assimilation of SST images in the *Extended Image Model* (with an initialization of the velocity field corresponding to the Optical Flow estimation but no regularization)

Mesoscale eddy: result comparison

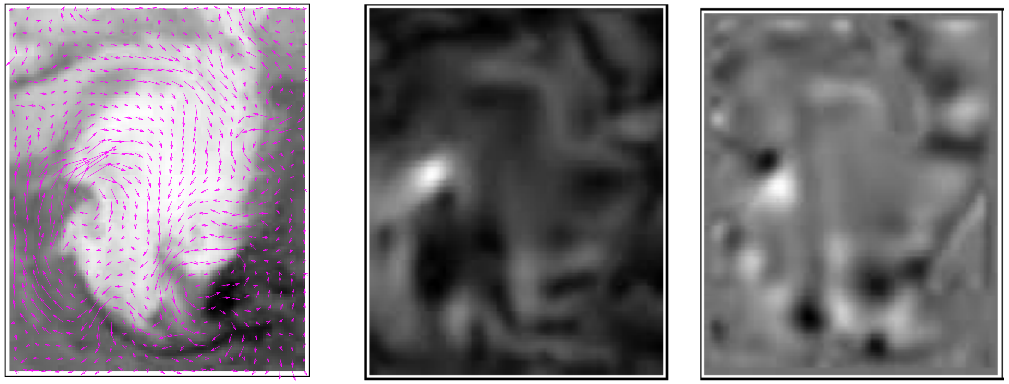
Estimation
with the
1^{st} *IM*



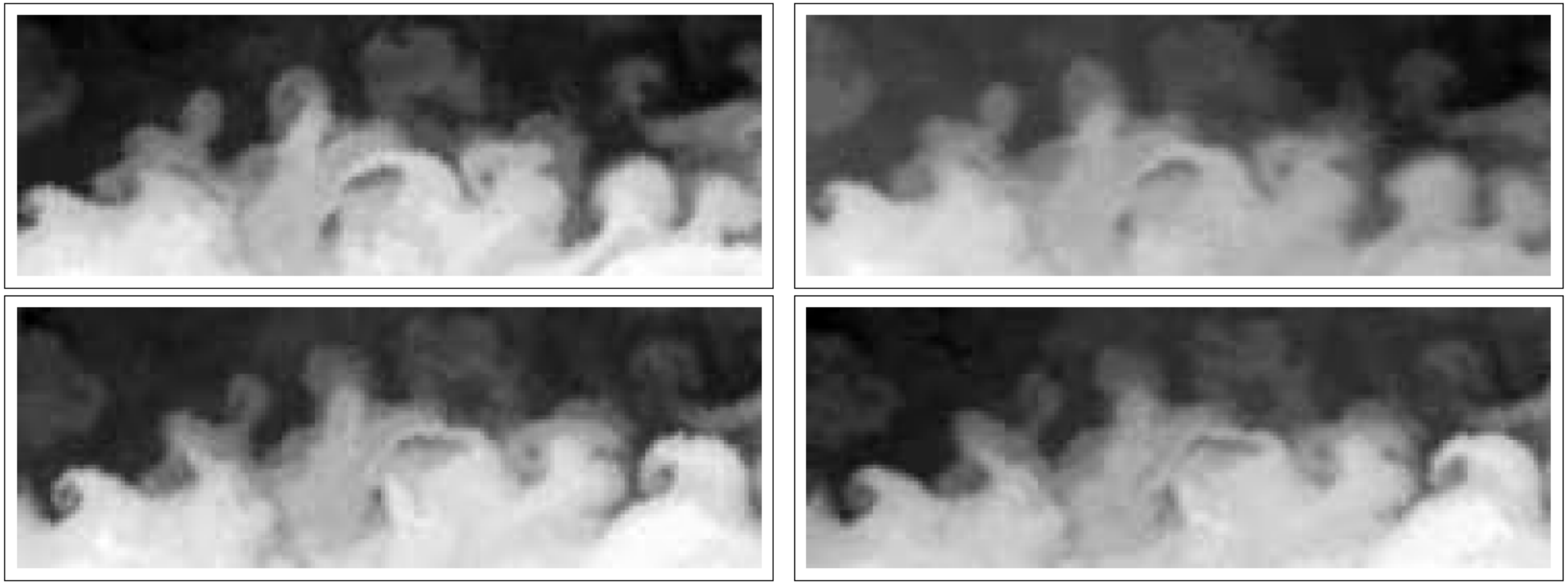
Estimation
with OF



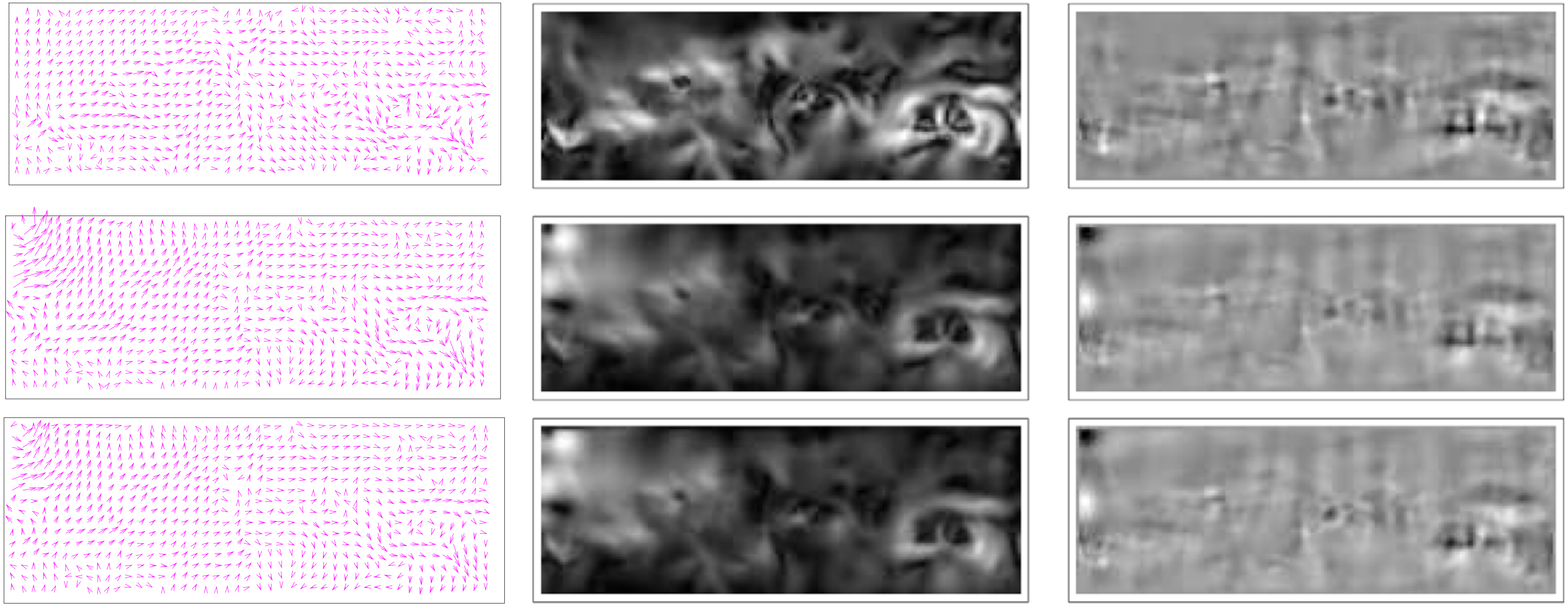
Estimation
with the
EIM



Rim currents: frame box (b)



Rim currents: result comparison



Conclusion and perspectives

- Advantages
  - We have proposed an *Extended Image Model* closer to the physic behavior of the sea surface dynamic than the first *Image Model* was.
  - Assimilation in these models provide the estimation of surface motion even when data are missing.
- But
  - The minimization process fall very often in local minima.
  - The quality of estimation is too dependant from the initialization.
  - ➔ (unfortunately) we need to add a regularization term.
- How to evaluate the quality of the estimation for real images?
  - Compare with operational model outputs
  - Compare to drifters trajectory