Retrieving of the surface velocity from images of different wave bands

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Simple algorithm for the retrieving of high resolution surface currents proposed by G.K. Korotaev, E. Huot, F.-X. Le Dimet, I. Herlin, S.V. Stanichny, D.M. Solovyev and L. Wu (**Remote Sensing of Environment** Volume 112, Issue 4 15 April 2008, Pages 1464-1475), which is based on application of the variational data assimilation technique.

Basic Equation

$$\frac{\partial T}{\partial t} + u \cdot \frac{\partial T}{\partial x} + v \cdot \frac{\partial T}{\partial y} = \kappa \cdot \nabla^2 T$$

Basic Assumption

$$\frac{\partial u}{\partial t} = 0$$

$$\frac{\partial v}{\partial t} = 0$$

Cost Function

$$J = \frac{1}{2} \cdot \sum_{1}^{Q} \iint \left(T(x, y, t_q) - T^*(x, y, t_q) \right)^2 dx \cdot dy$$
$$+ \frac{1}{2} \alpha \cdot \int \iint \left((u_x)^2 + (u_y)^2 + (v_x)^2 + (v_y)^2 \right) \cdot dt \cdot dx \cdot dy$$
$$+ \frac{1}{2} \beta \cdot \int \iint (u_x + v_y)^2 \cdot dt \cdot dx \cdot dy = \min$$

Adjoint Equations

$$\frac{\partial \tilde{T}}{\partial t} + \frac{\partial\left(\overline{u} \cdot \tilde{T}\right)}{\partial x} + \frac{\partial\left(\overline{v} \cdot \tilde{T}\right)}{\partial y} + \kappa \cdot \nabla^2 \tilde{T} = \sum_{1}^{Q} \int \left(\overline{T}(x, y, t) - T^*(x, y, t)\right) \cdot \delta(t - t_q) \cdot dt$$

$$\frac{\partial \tilde{u}}{\partial t} - \tilde{T} \cdot \frac{\partial \overline{T}}{\partial x} + \alpha \cdot \nabla^2 \overline{u} + \beta \cdot \left(\frac{\partial^2 \overline{u}}{\partial x^2} + \frac{\partial^2 \overline{v}}{\partial x \partial y} \right) = 0$$

$$\frac{\partial \tilde{v}}{\partial t} - \tilde{T} \cdot \frac{\partial \overline{T}}{\partial y} + \alpha \cdot \nabla^2 \overline{v} + \beta \cdot \left(\frac{\partial^2 \overline{u}}{\partial x \partial y} + \frac{\partial^2 \overline{v}}{\partial y^2} \right) = 0$$

Simple Generalization

$$\frac{\partial T}{\partial t} + u \cdot \frac{\partial T}{\partial x} + v \cdot \frac{\partial T}{\partial y} = \kappa \cdot \nabla^2 T$$

$$\frac{\partial S}{\partial t} + u \cdot \frac{\partial S}{\partial x} + v \cdot \frac{\partial S}{\partial y} = \kappa \cdot \nabla^2 S$$

New Cost Function

$$J = \frac{1}{2} \cdot \sum_{1}^{Q} \iint \left(T(x, y, t_q) - T^*(x, y, t_q) \right)^2 dx \cdot dy + \frac{K}{2} \cdot \sum_{1}^{Q} \iint \left(S(x, y, t_q) - S^*(x, y, t_q) \right)^2 dx \cdot dy$$

$$+ \frac{1}{2} \alpha \cdot \int \iint \left((u_x)^2 + (u_y)^2 + (v_x)^2 + (v_y)^2 \right) \cdot dt \cdot dx \cdot dy$$

$$+ \frac{1}{2} \beta \cdot \int \iint \left(u_x + v_y \right)^2 \cdot dt \cdot dx \cdot dy = \min$$

Adjoint Equations

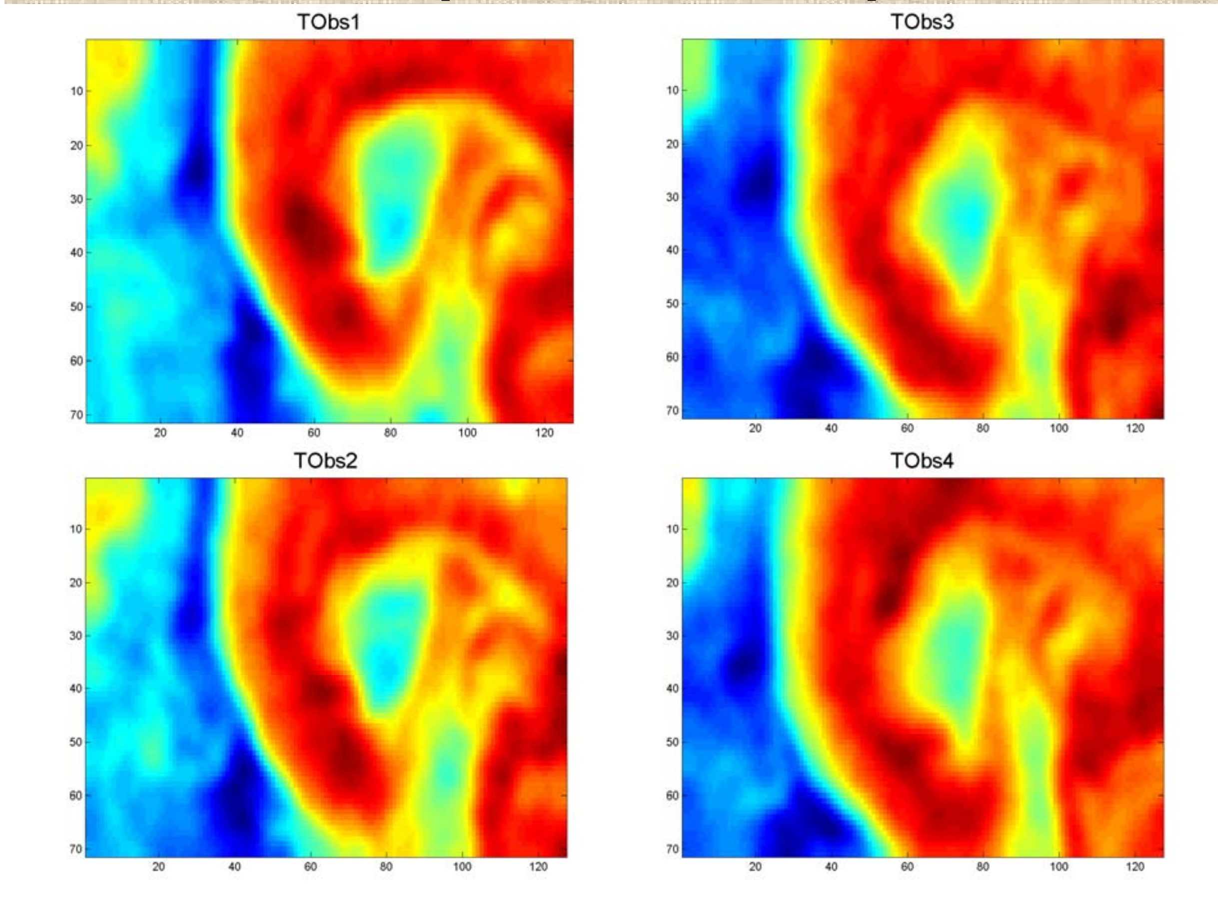
$$\frac{\partial \tilde{T}}{\partial t}+\frac{\partial\left(\overline{u}\cdot\tilde{T}\right)}{\partial x}+\frac{\partial\left(\overline{v}\cdot\tilde{T}\right)}{\partial y}+\kappa\cdot\nabla^2\tilde{T}=\sum_1^Q\int\left(\overline{T}(x,y,t)-T^*(x,y,t)\right)\cdot\delta(t-t_q)\cdot dt$$

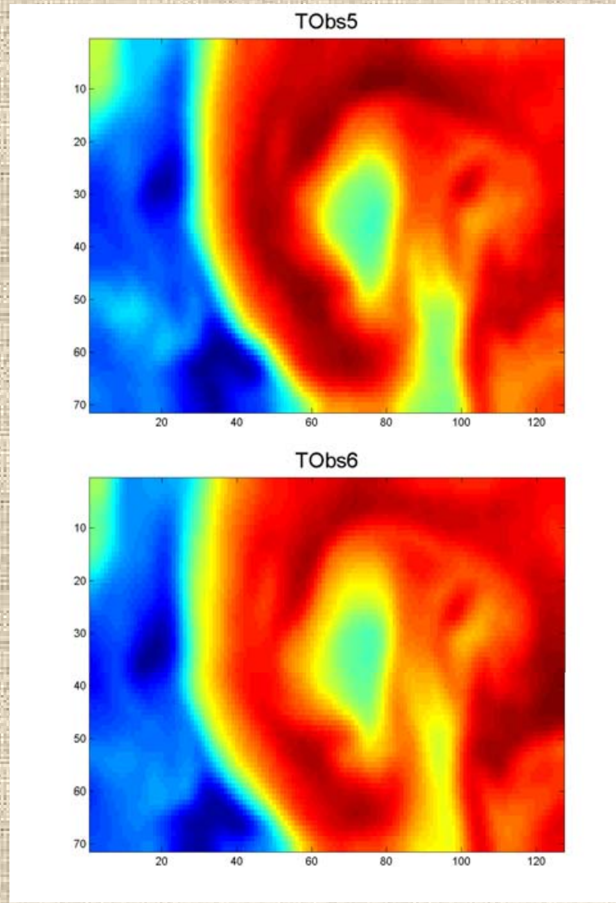
$$\frac{\partial \tilde{S}}{\partial t}+\frac{\partial\left(\overline{u}\cdot\tilde{S}\right)}{\partial x}+\frac{\partial\left(\overline{v}\cdot\tilde{S}\right)}{\partial y}+\kappa\cdot\nabla^2\tilde{S}=\sum_1^Q\int\left(\overline{S}(x,y,t)-S^*(x,y,t)\right)\cdot\delta(t-t_q)\cdot dt$$

$$\frac{\partial \tilde{u}}{\partial t}-\tilde{T}\cdot\frac{\partial\overline{T}}{\partial x}-\tilde{S}\cdot\frac{\partial\overline{S}}{\partial x}+\alpha\cdot\nabla^2\overline{u}+\beta\cdot\left(\frac{\partial^2\overline{u}}{\partial x^2}+\frac{\partial^2\overline{v}}{\partial x\partial y}\right)=0$$

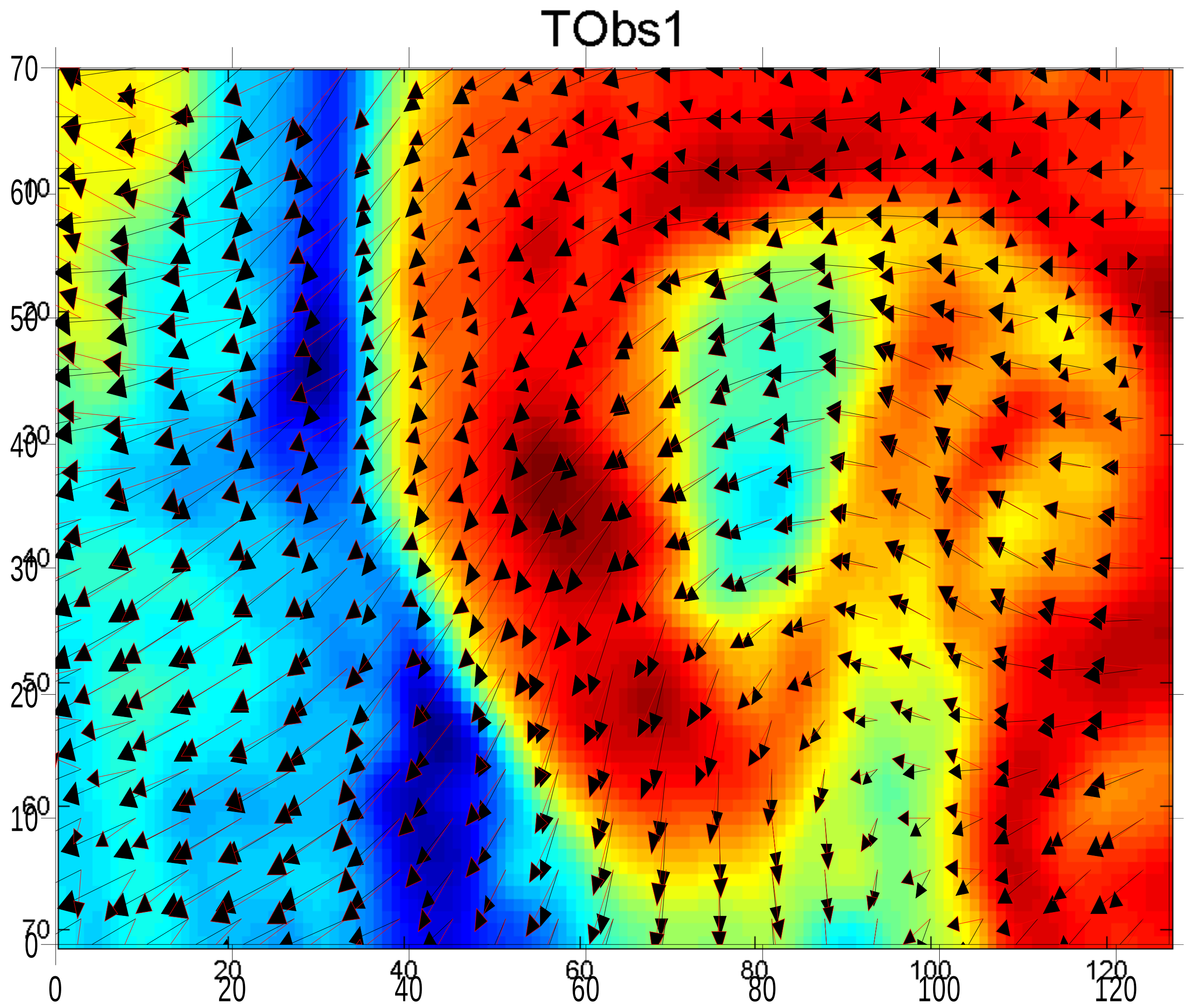
$$\frac{\partial \tilde{v}}{\partial t}-\tilde{T}\cdot\frac{\partial\overline{T}}{\partial y}-\tilde{S}\cdot\frac{\partial\overline{S}}{\partial y}+\alpha\cdot\nabla^2\overline{v}+\beta\cdot\left(\frac{\partial^2\overline{u}}{\partial x\partial y}+\frac{\partial^2\overline{v}}{\partial y^2}\right)=0$$

Example 1. Processing of four IR images of 10.8 mcm (channel 4) and two images of 3.7 mcm (channel 2)





Simulated velocities overlapped on the channel 4 images


TObs1
70
60
50
40
30
20
10
0
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10
20
30
40
50
60
70
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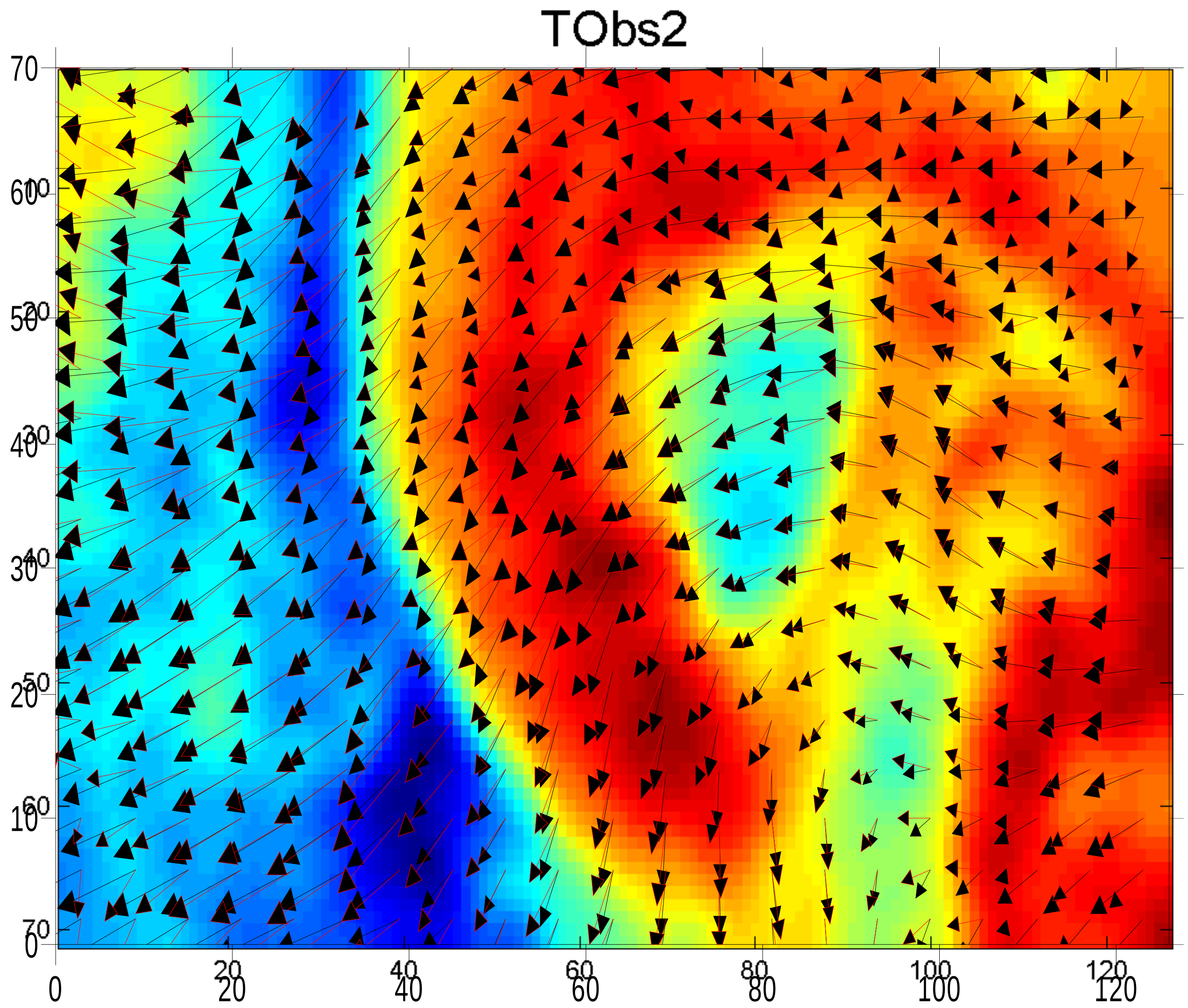
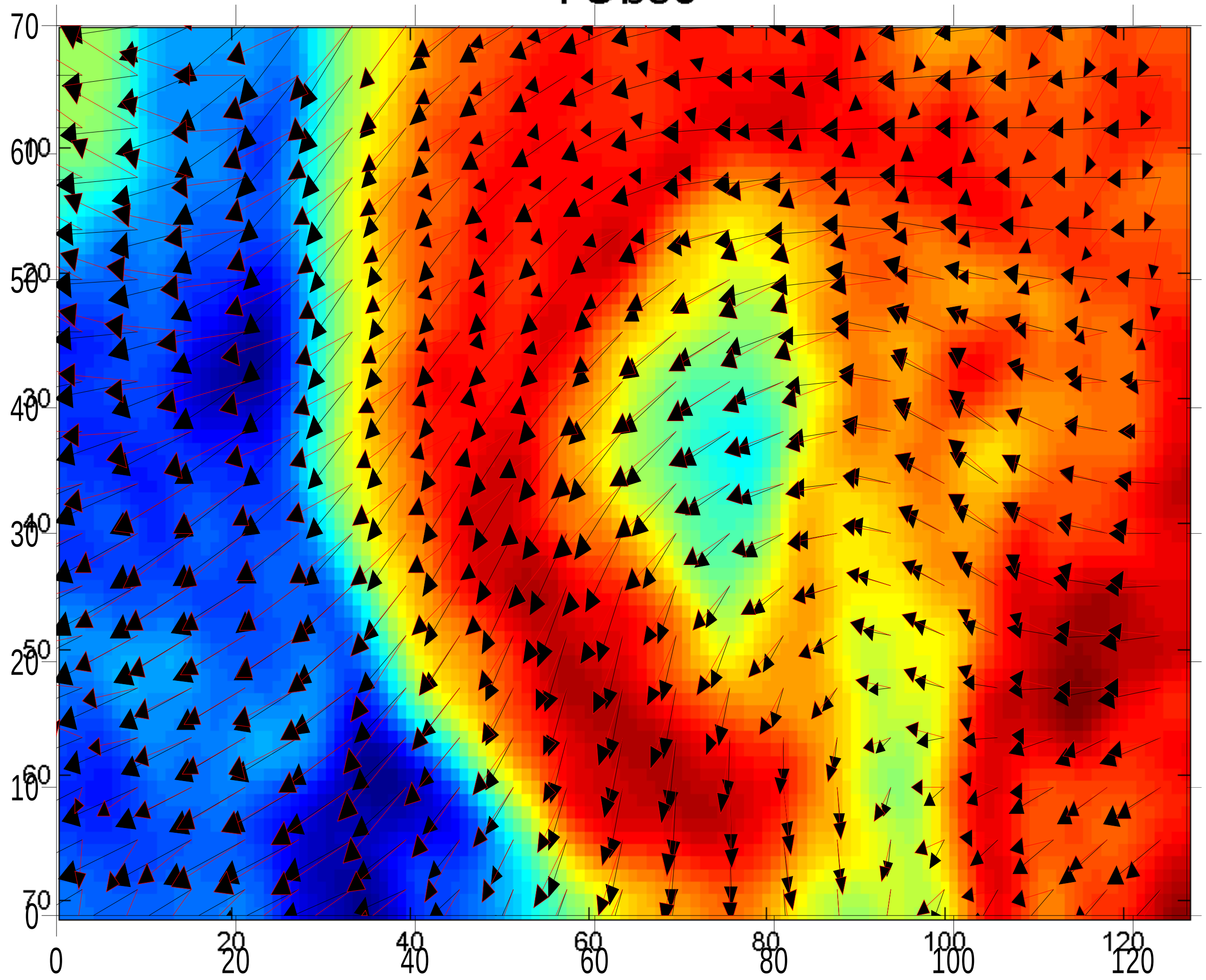
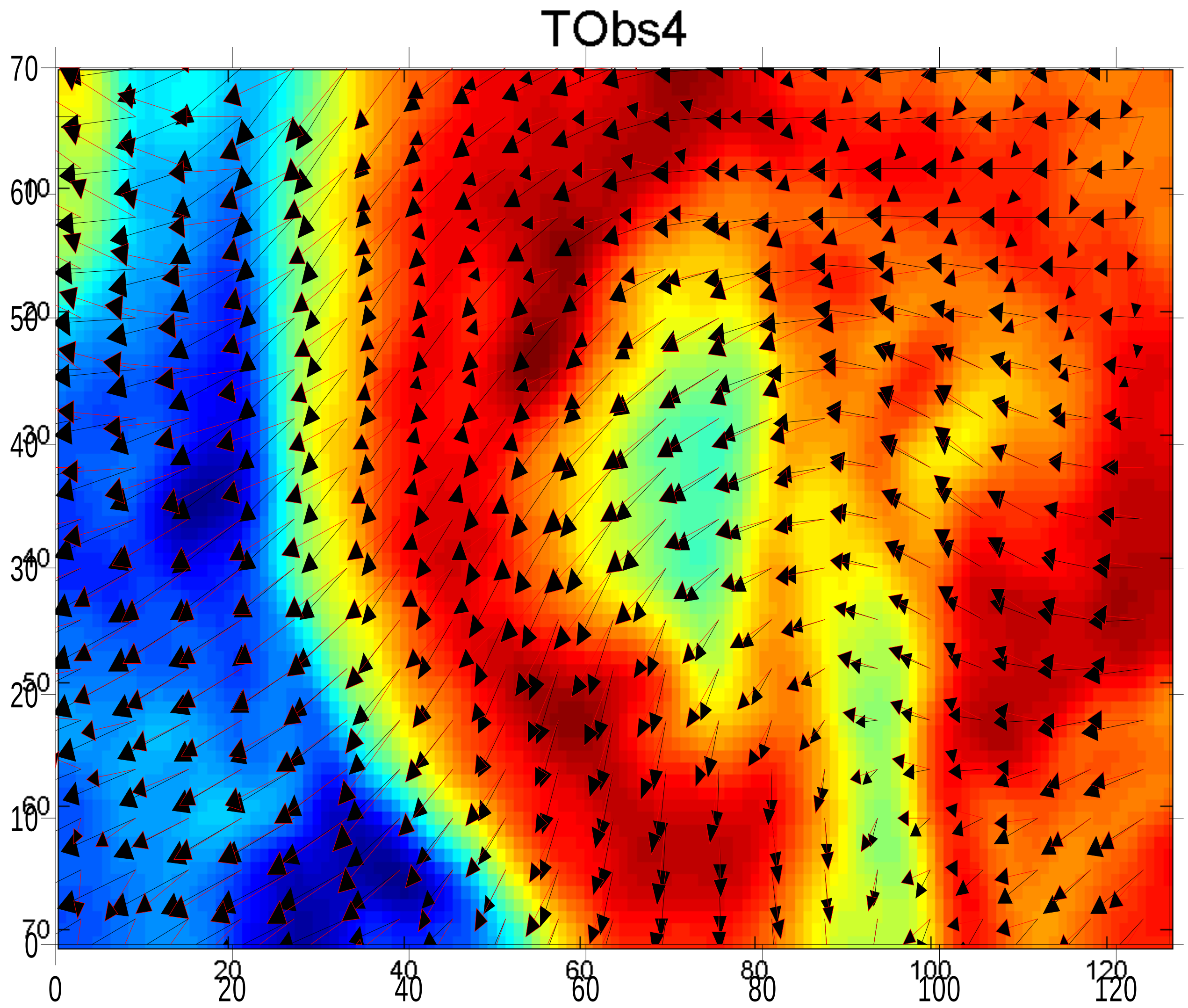
40
60
80
100
120



TObs2

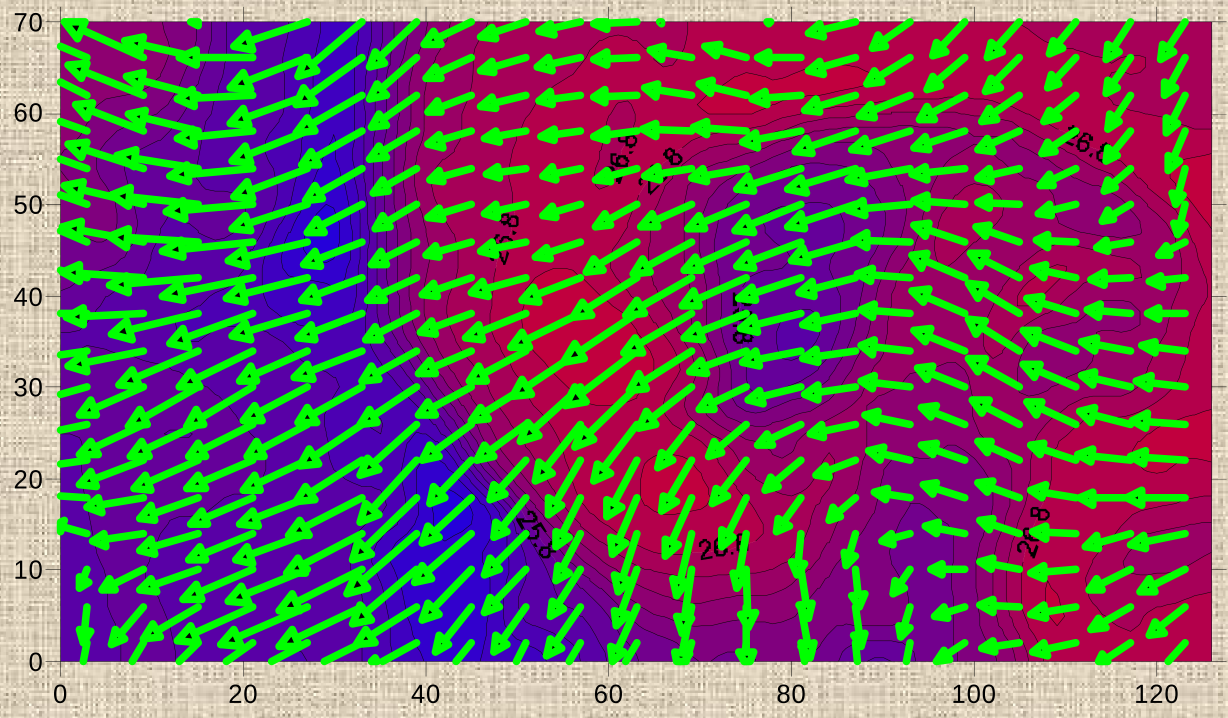
TObs3





TObs4
0
20
40
60
80
100
120
10
20
30
40
50
60
70

Processing of images of channel 4



70
60
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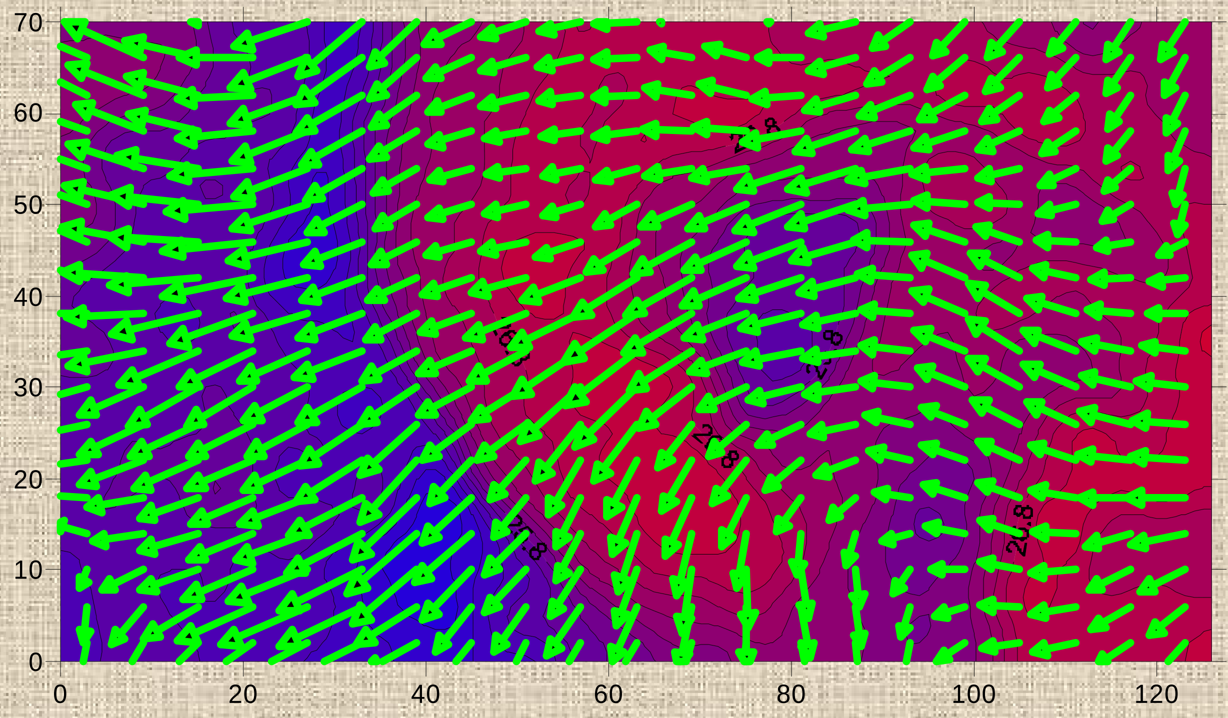
40
30
20
10
0
0
20
40
60
80
100
120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0
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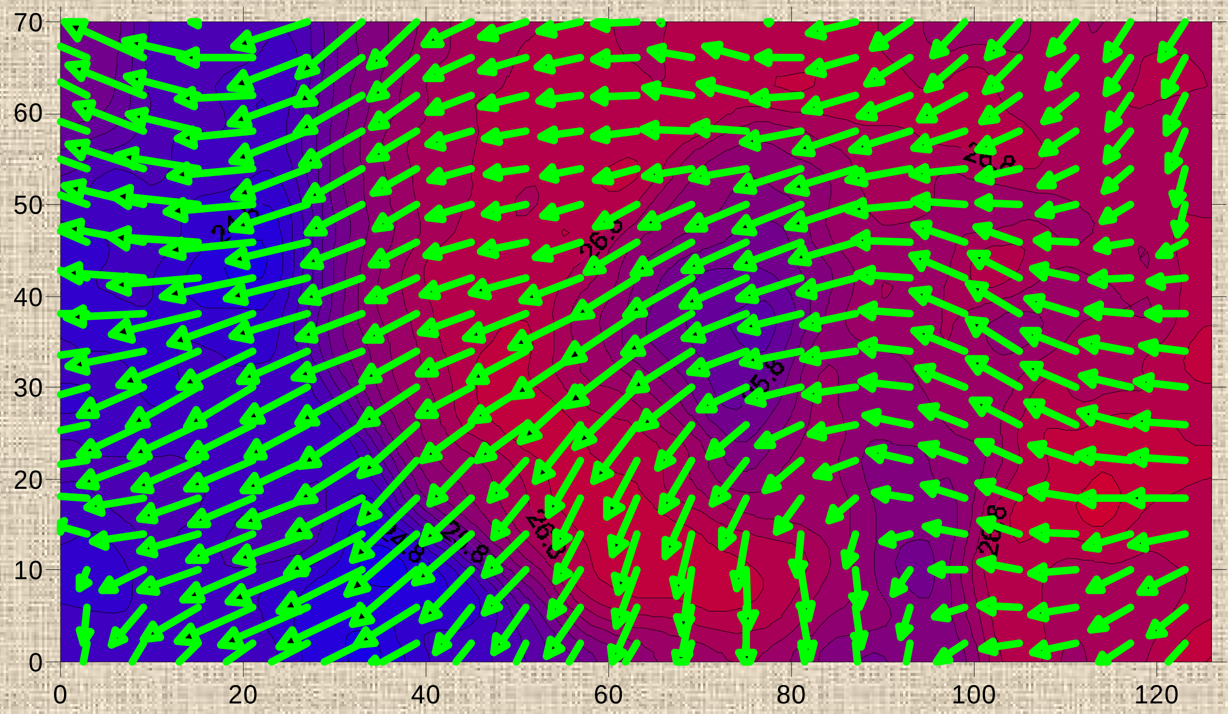
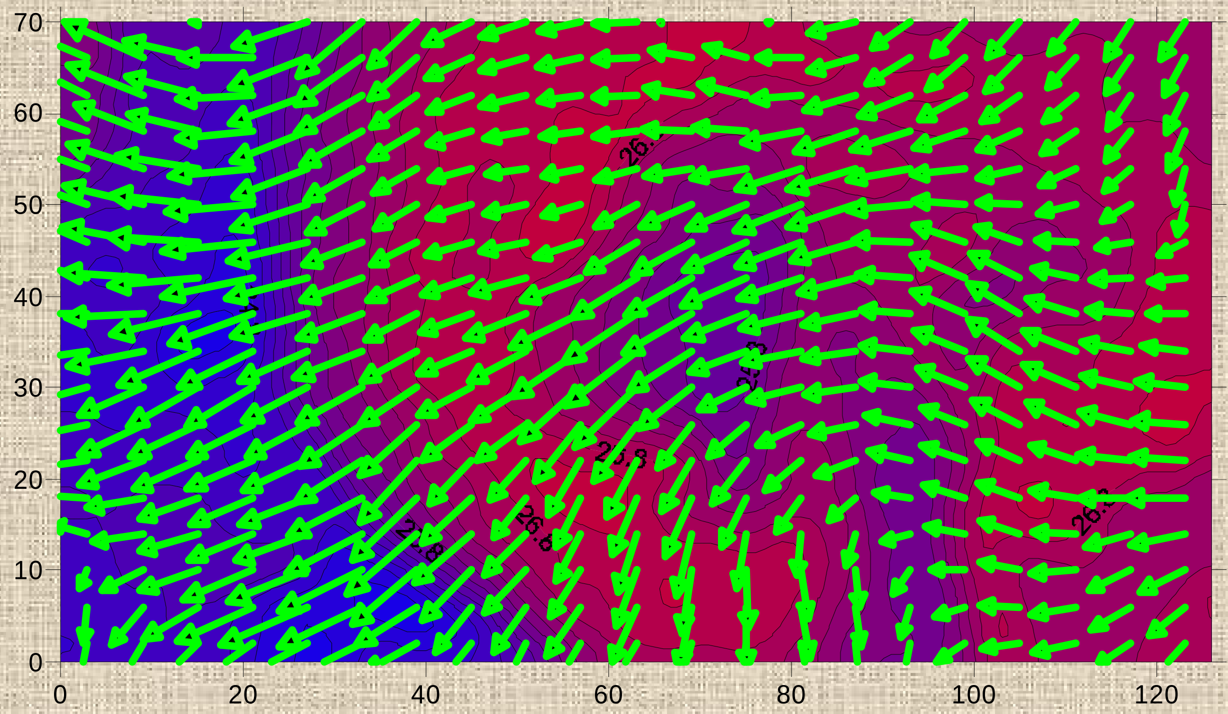
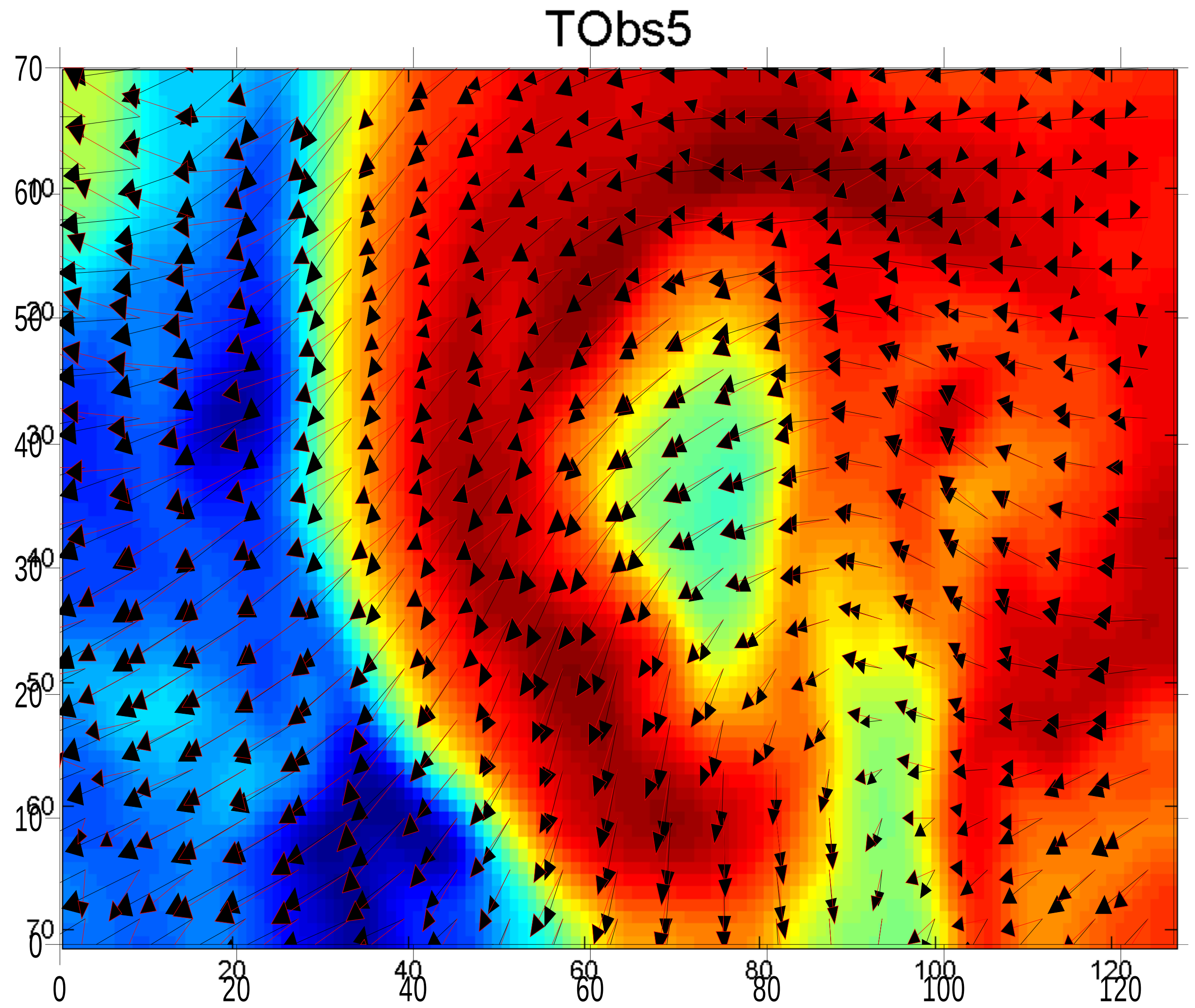
120



0
10
20
30
40
50
60
70
0
20
40
60
80
100
120


70
60
50
40
30
20
10
0
0
20
40
60
80
100
120

Simulated velocities overlapped on the channel 2 images


TObs5
70
60
50
40
30
20
10
0
0
20
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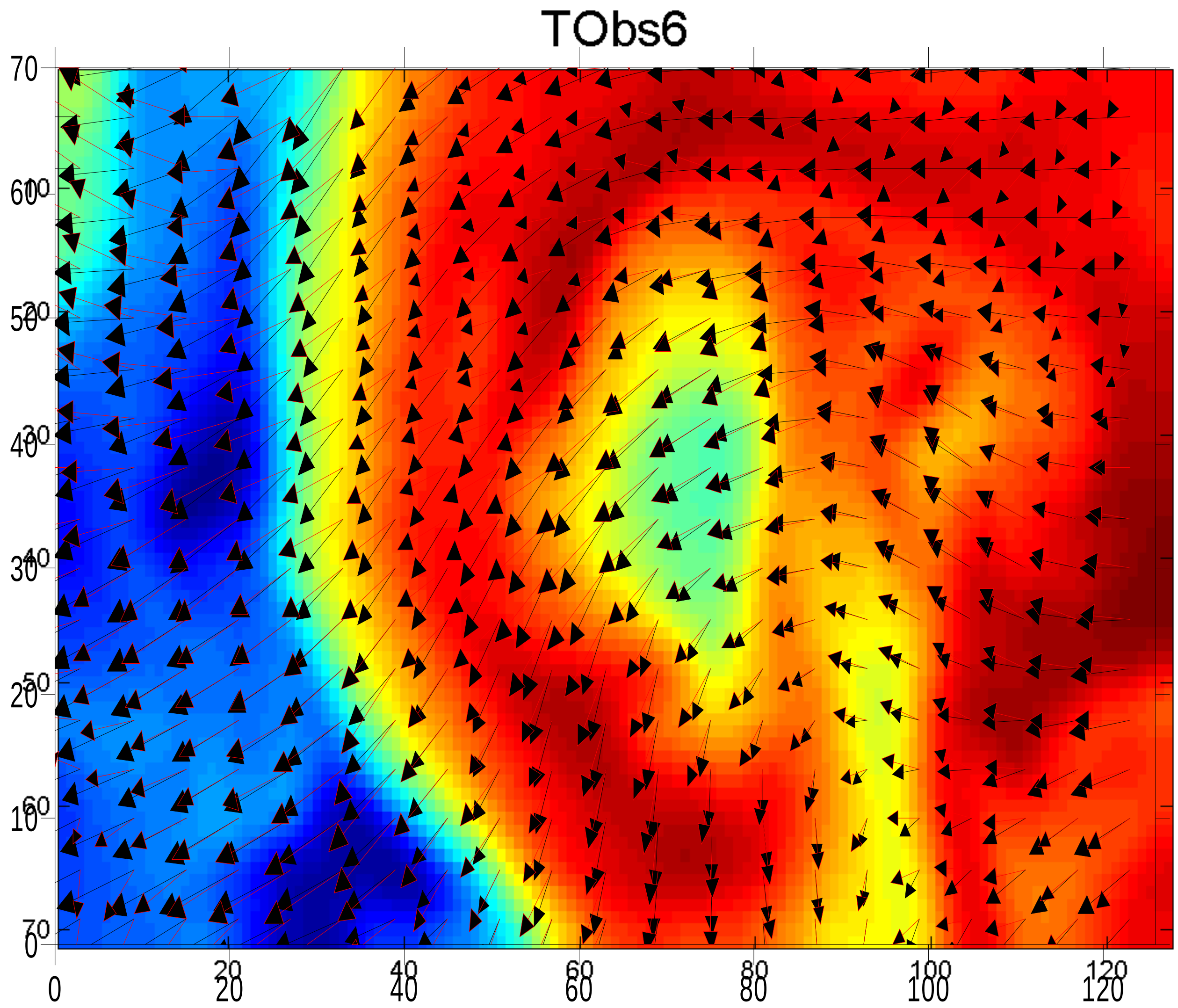
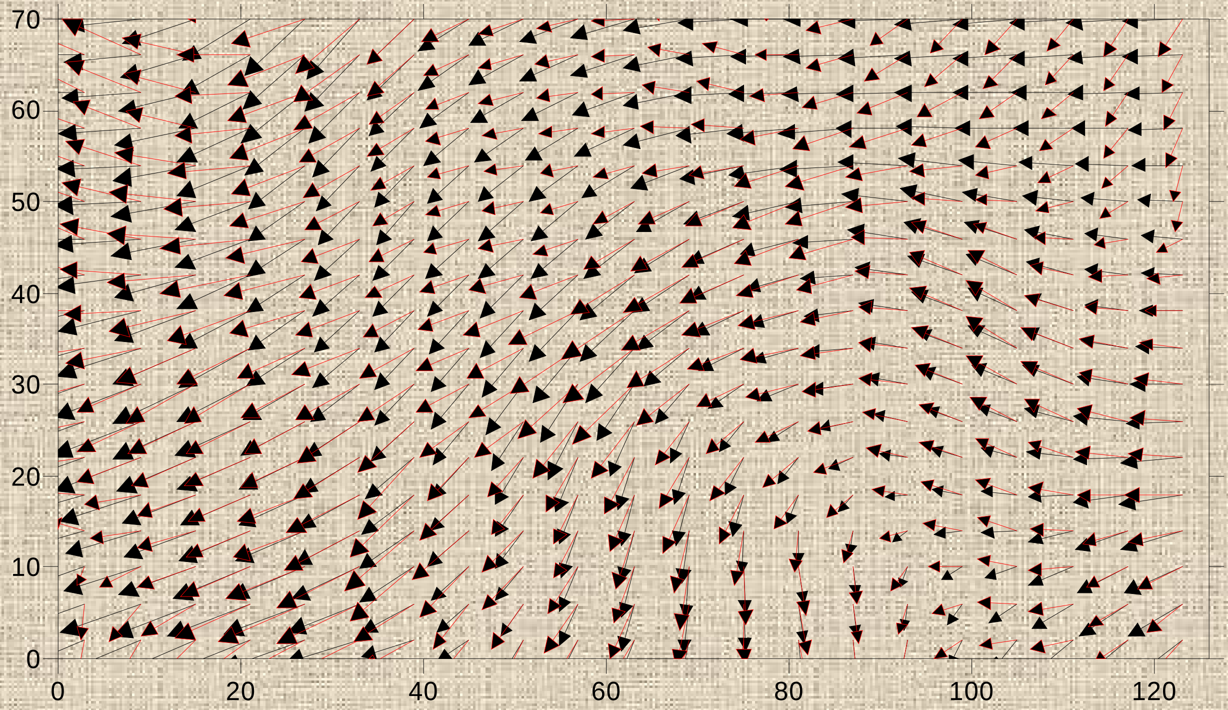
60
80
100
120

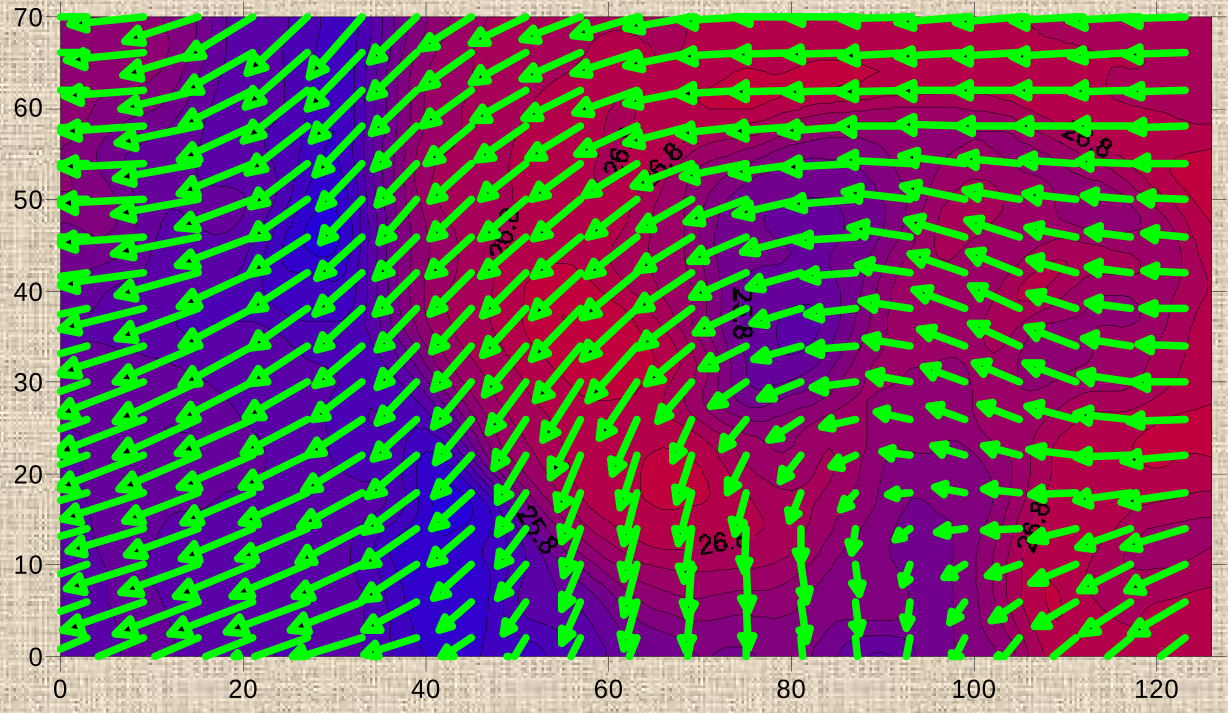


TObs6
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
0
20
40
60
80
100
120
20
40
60
80
100
120

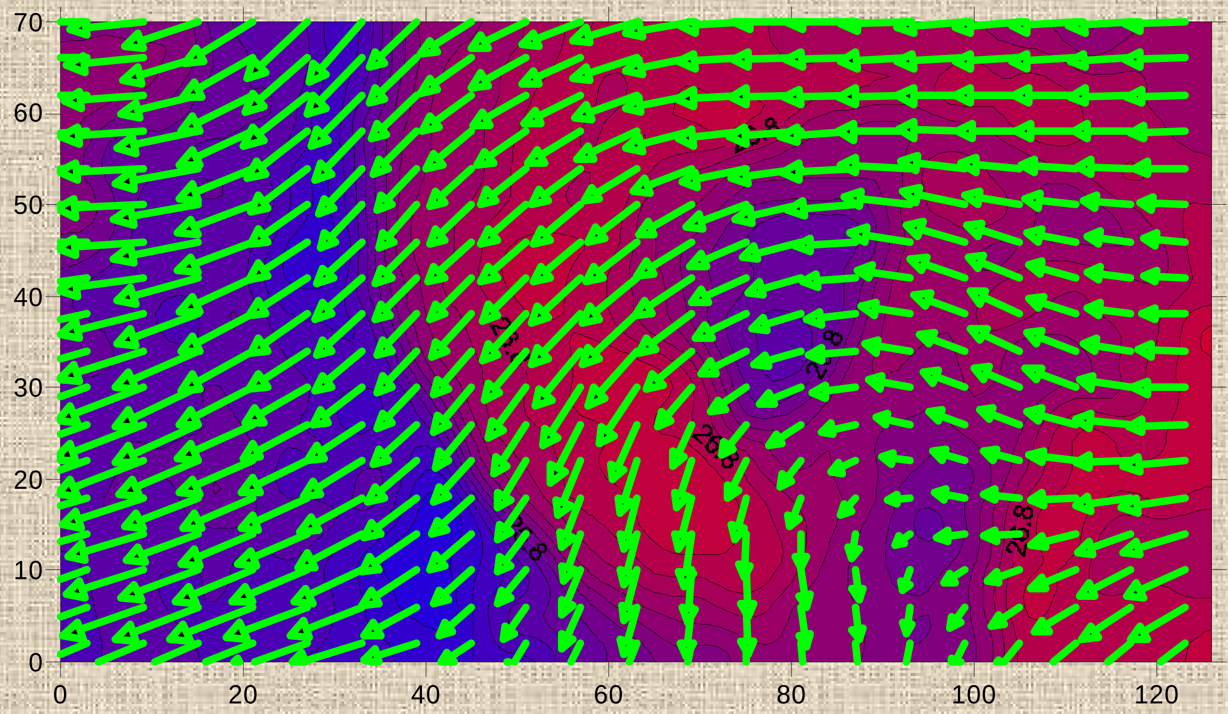
Velocity retrieved from the processing of one and two channel images



Processing of images of channels 2 and 4



70
60
50
40
30
20
10
0
0
20
40
60
80
100
120
25.8
26.1



70
60
50
40
30
20
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0
0
20
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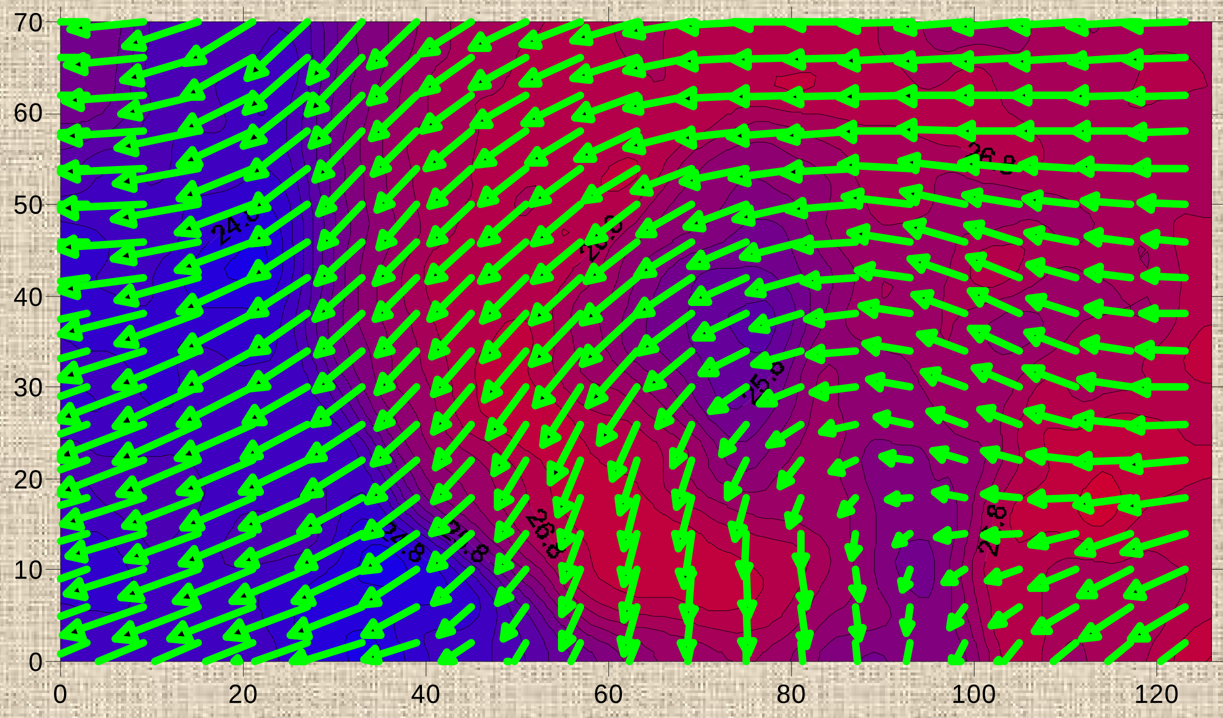
60
80
100
120



70
60
50
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20
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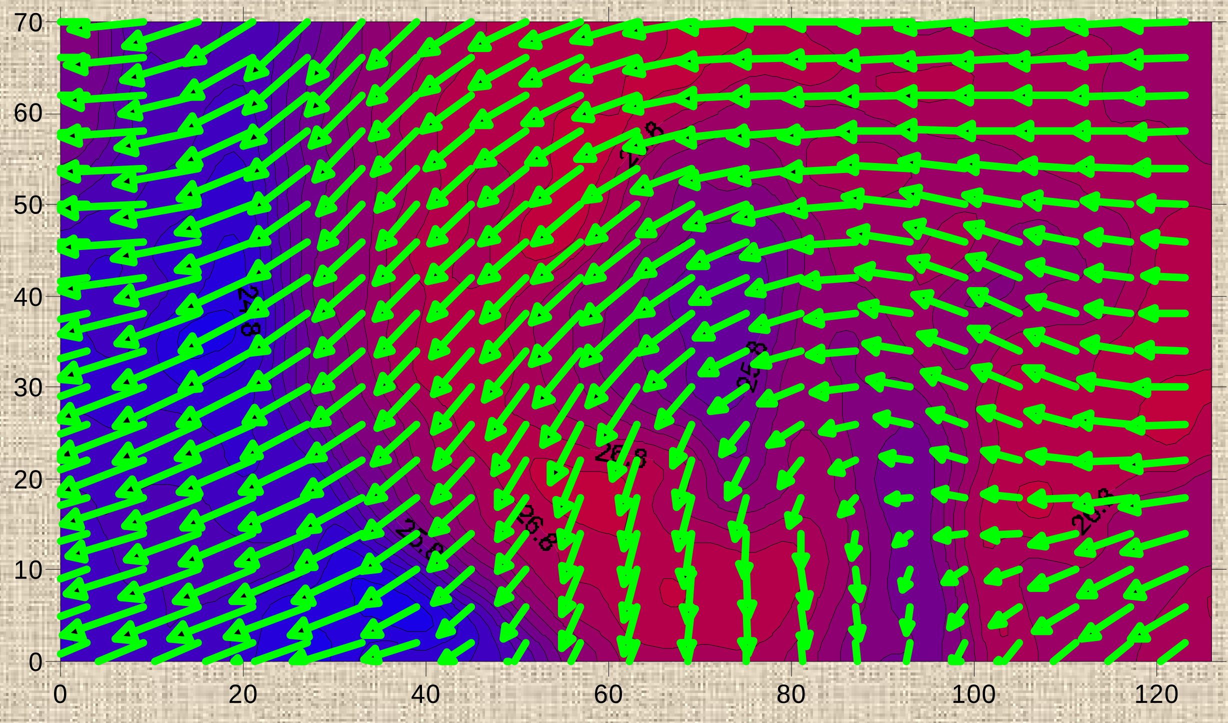
0
0
20
40
60
80
100
120



70
60
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40
30
20
10
0
0
20
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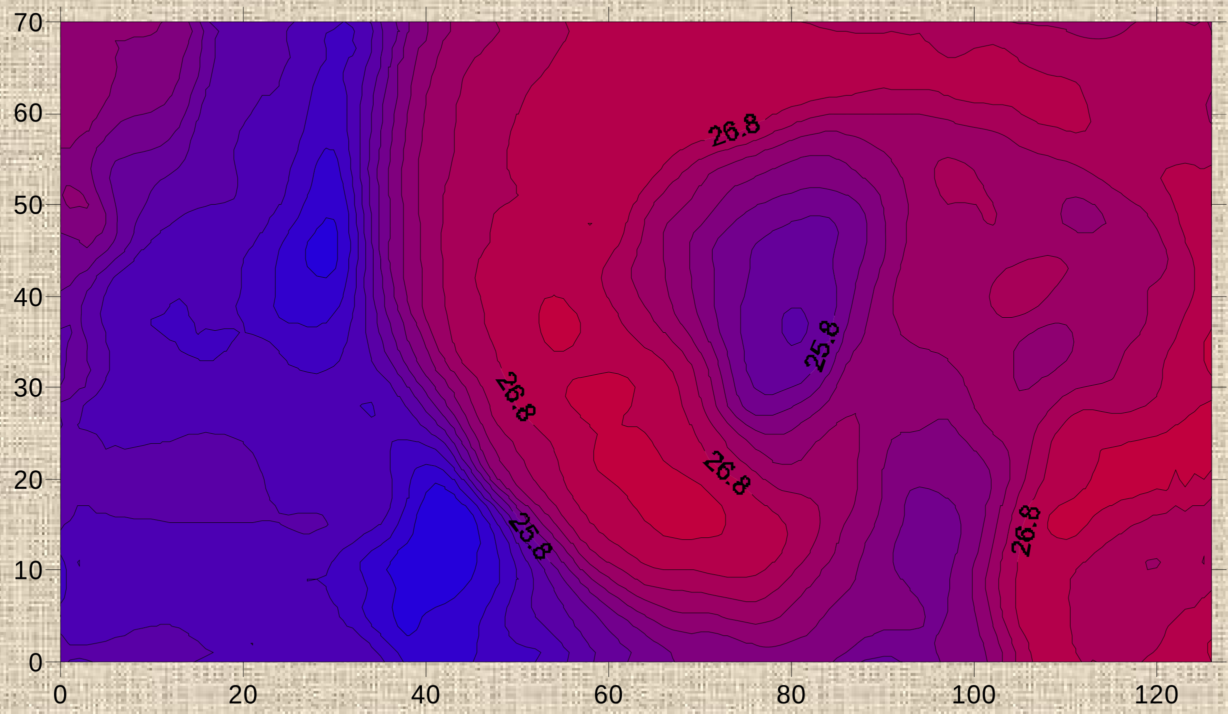
60
80
100
120

Simulated images: channel 4



70
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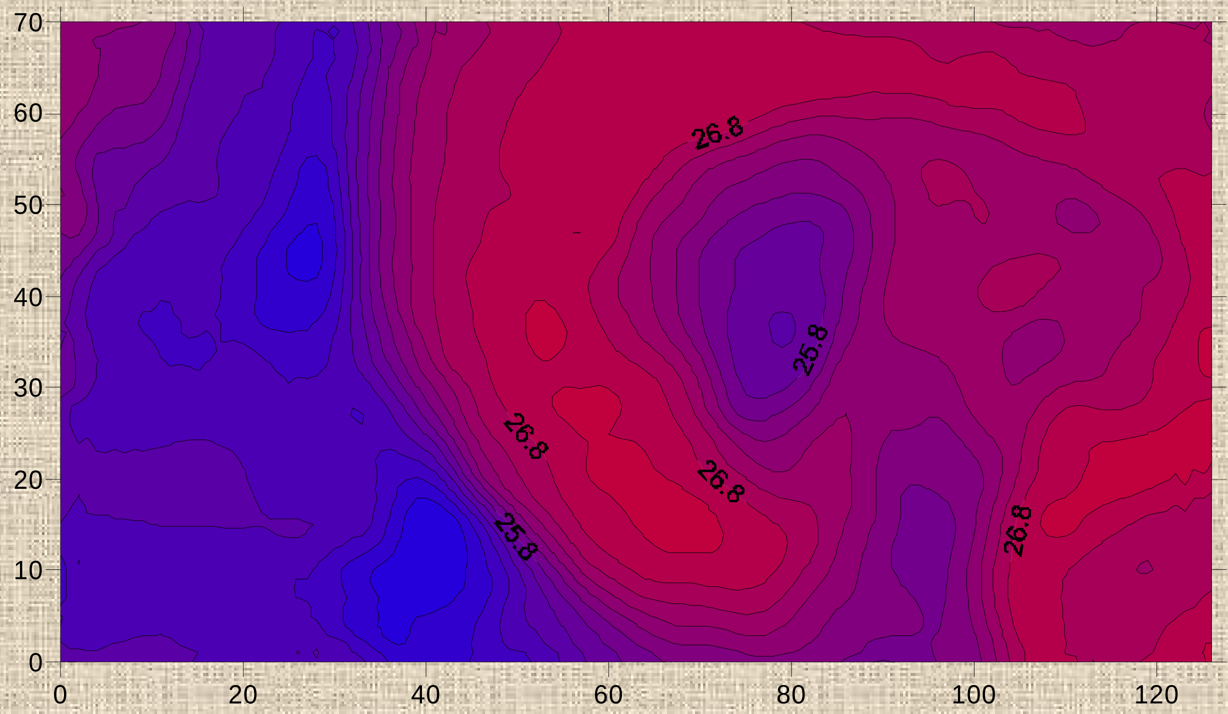
120
26.8
25.8
26.8
26.8
25.8
26.8



70
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40
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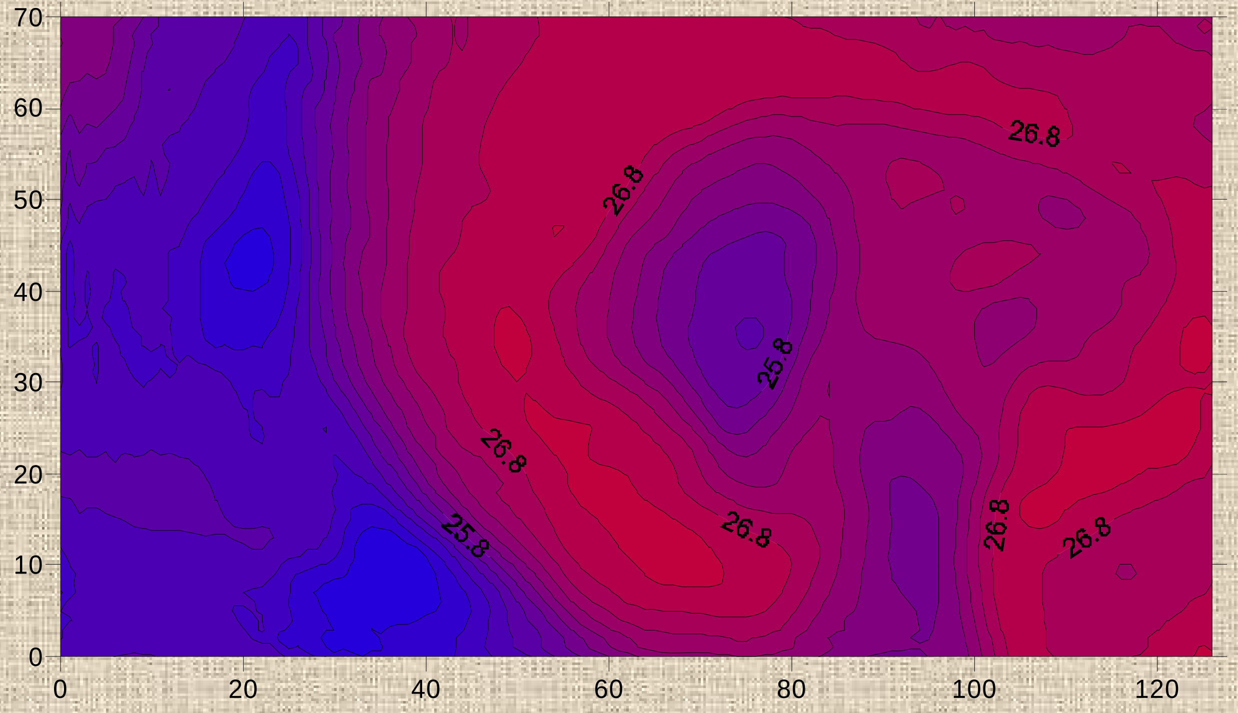
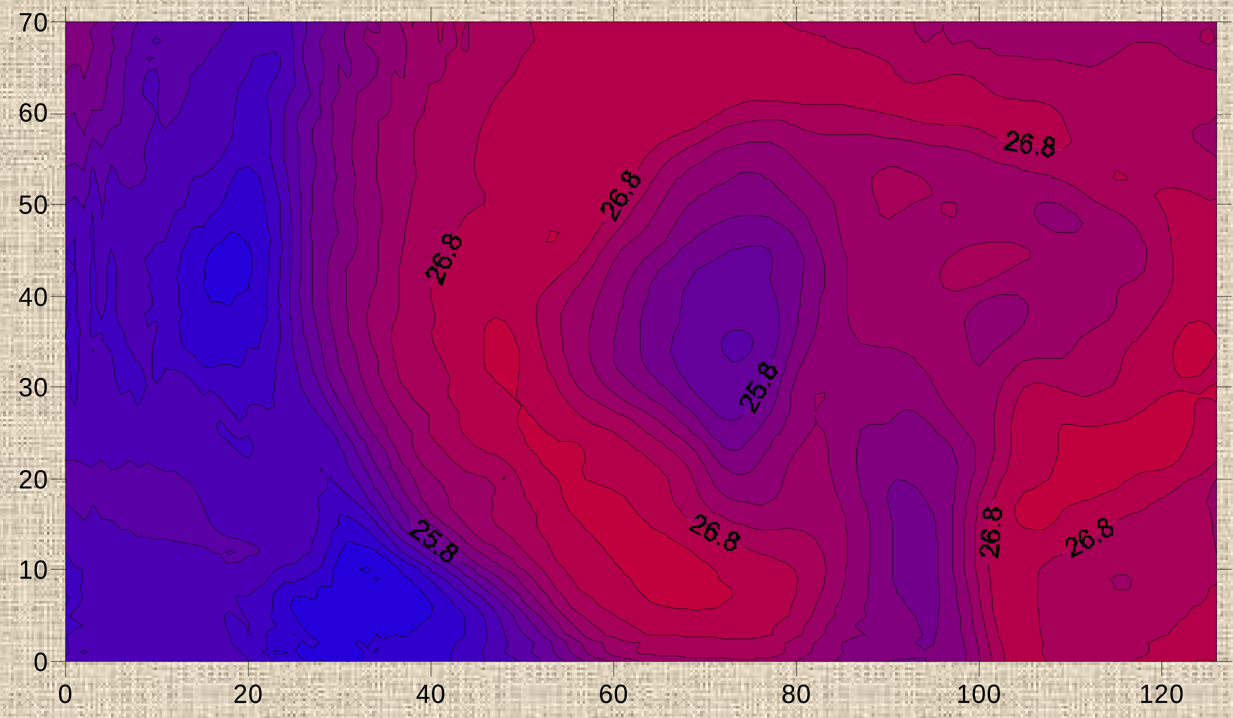
20
40
60
80
100
120
26.8
25.8
26.8
26.8
25.8
26.8

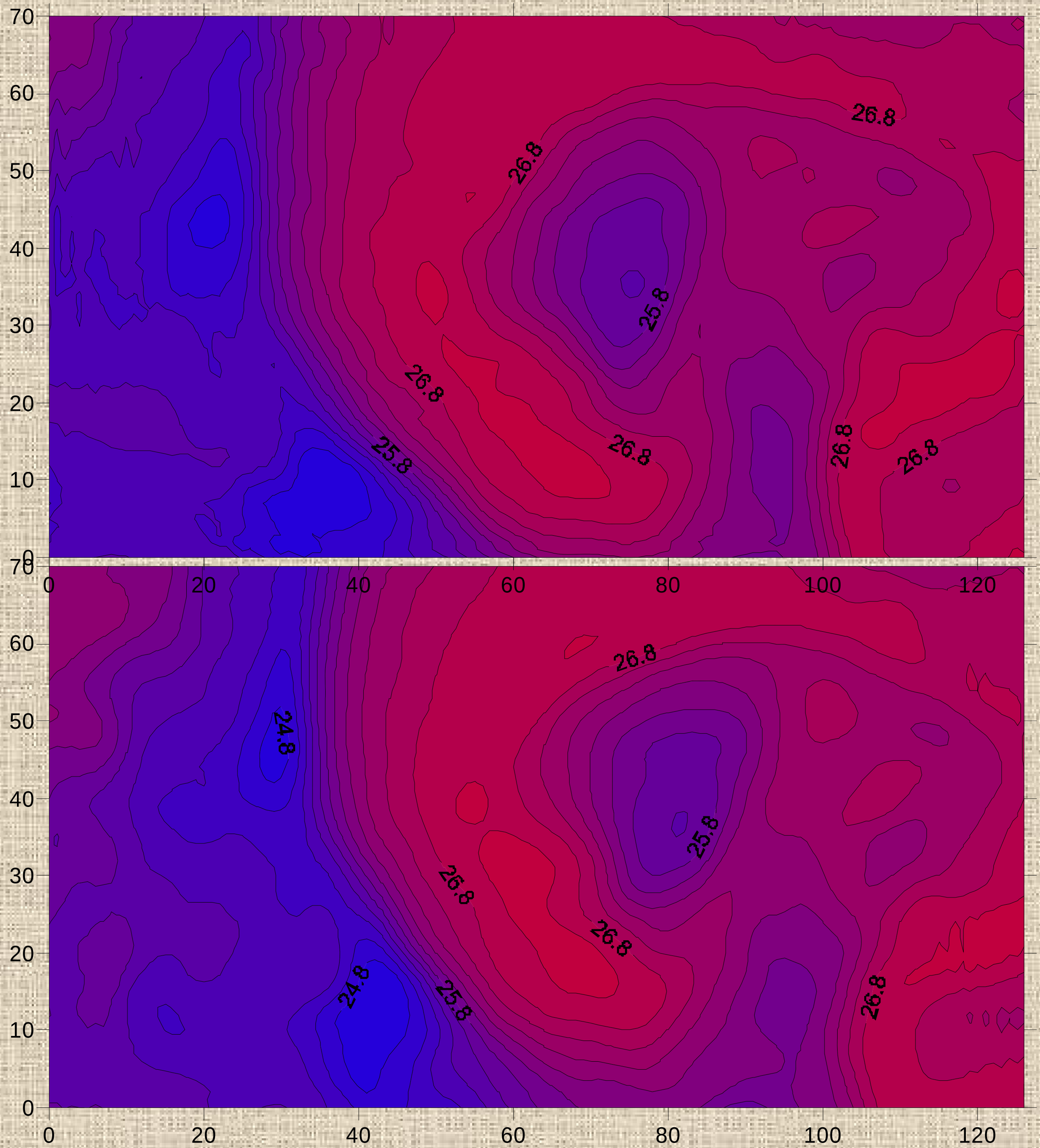


70
60
50
40
30
20
10
0
0
20
40
60
80
100
120
26.8
26.8
25.8
26.8
25.8
26.8
26.8
26.8

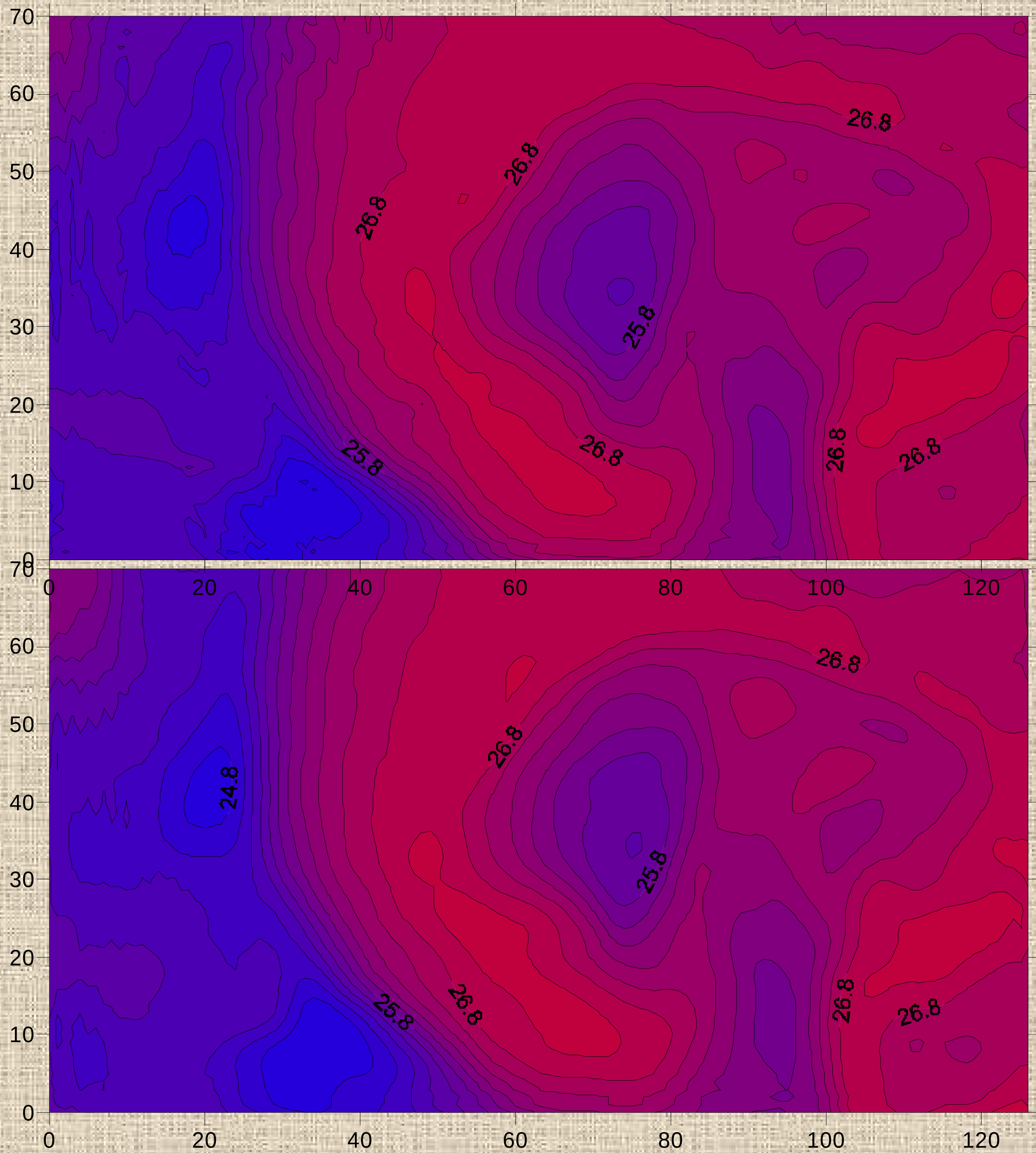


70
60
50
40
30
20
10
0
0
20
40
60
80
100
120
26.8
26.8
26.8
25.8
25.8
26.8
26.8
26.8

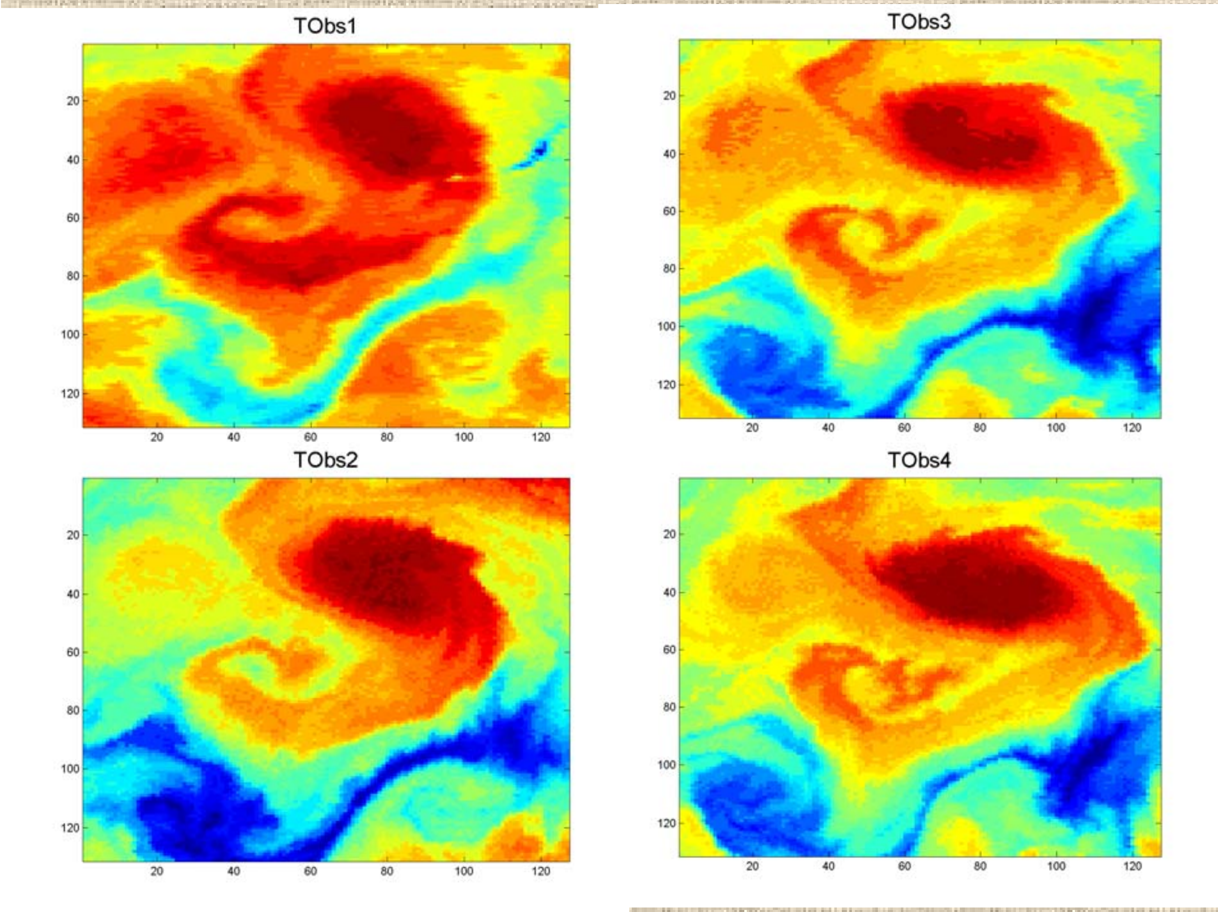
Simulated images: channel 2 and 4

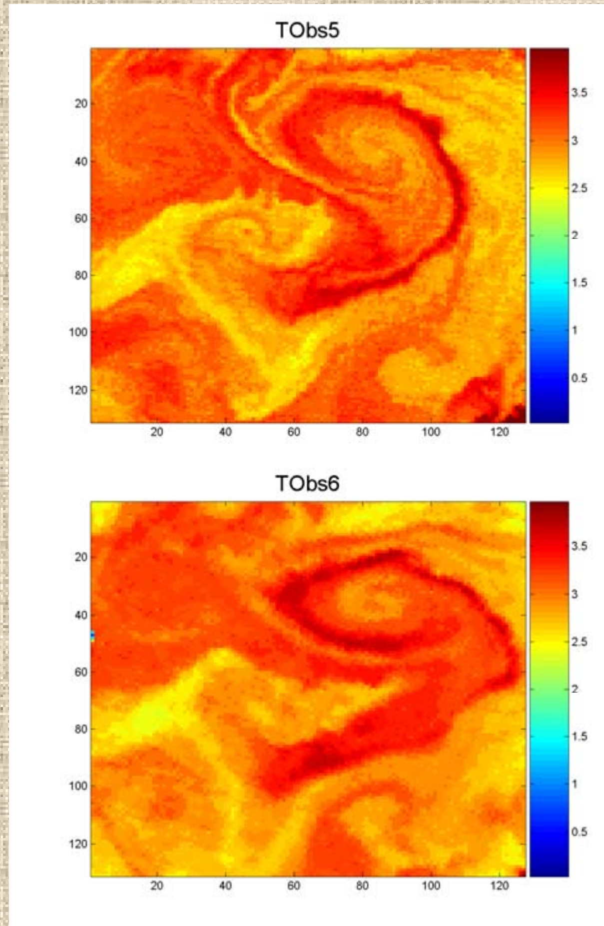


70
60
50
40
30
20
10
0
0
20
40
60
80
100
120
26.8
25.8
24.8


70
60
50
40
30
20
10
0
26.8
26.8
26.8
25.8
25.8
26.8
26.8
26.8
70
60
50
40
30
20
10
0
20
40
60
80
100
120
24.8
26.8
25.8
26.8
25.8
26.8
26.8
26.8
0
20
40
60
80
100
120

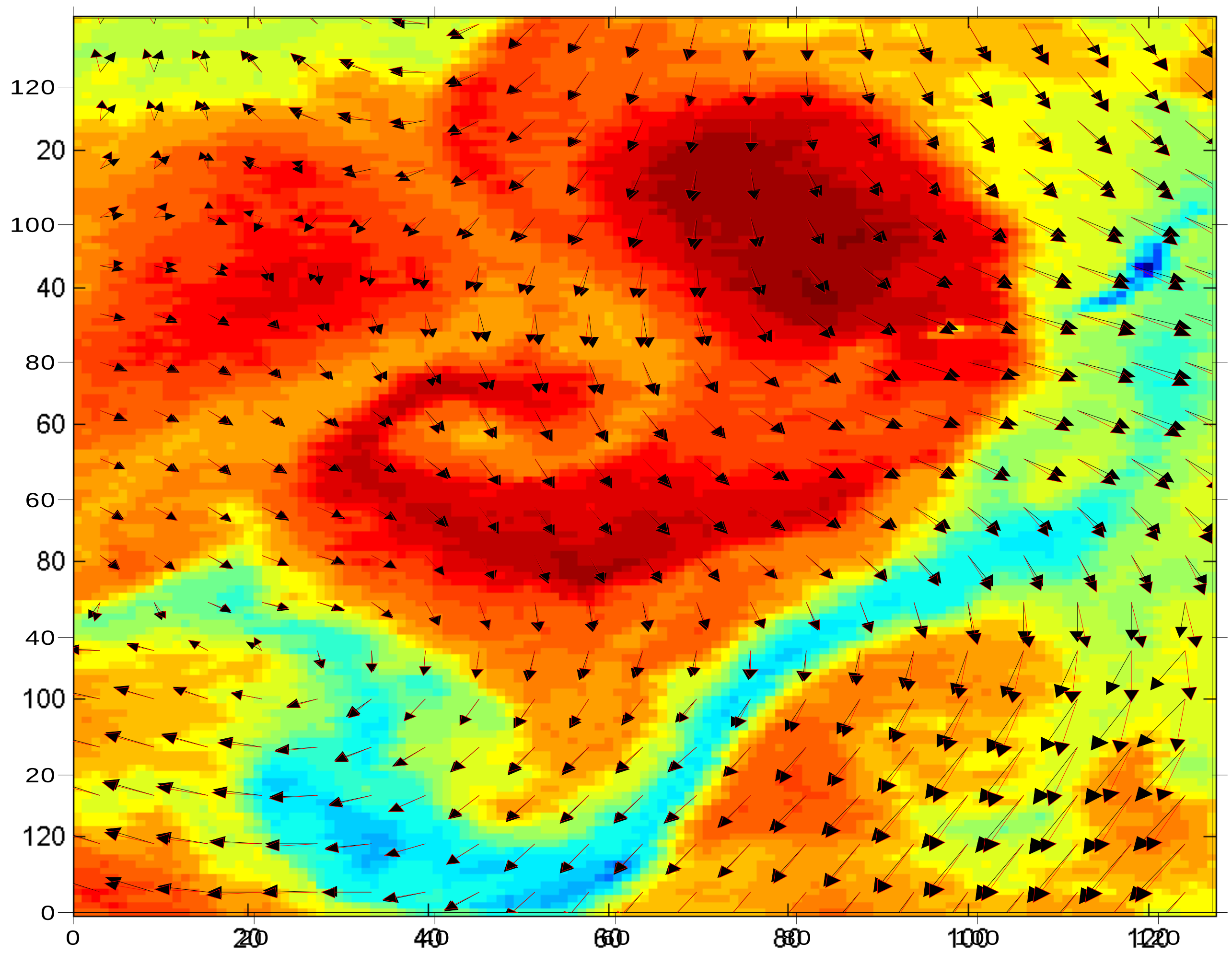
Example 2. Processing of four IR images and two visible band images



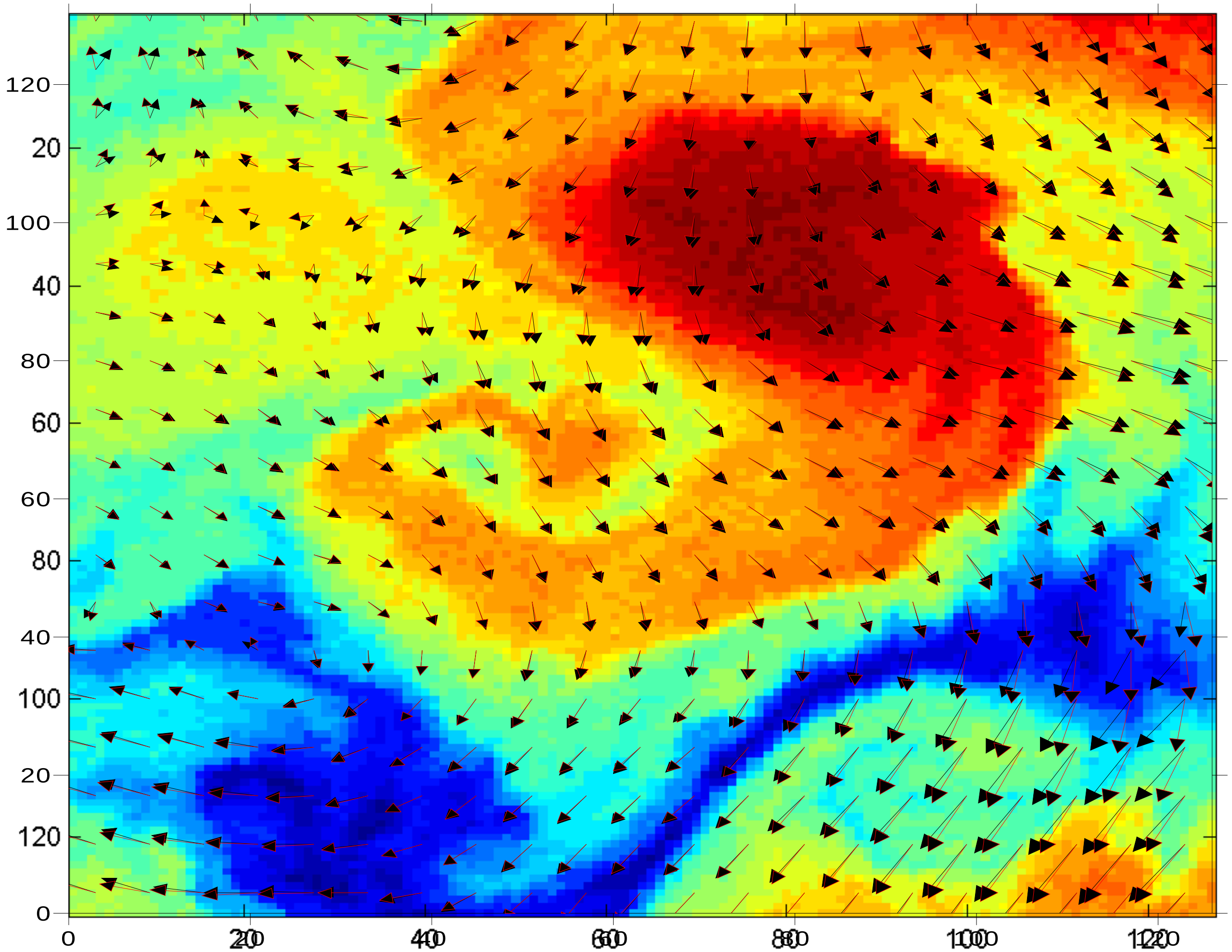


Simulated velocities overlapped on the IR images

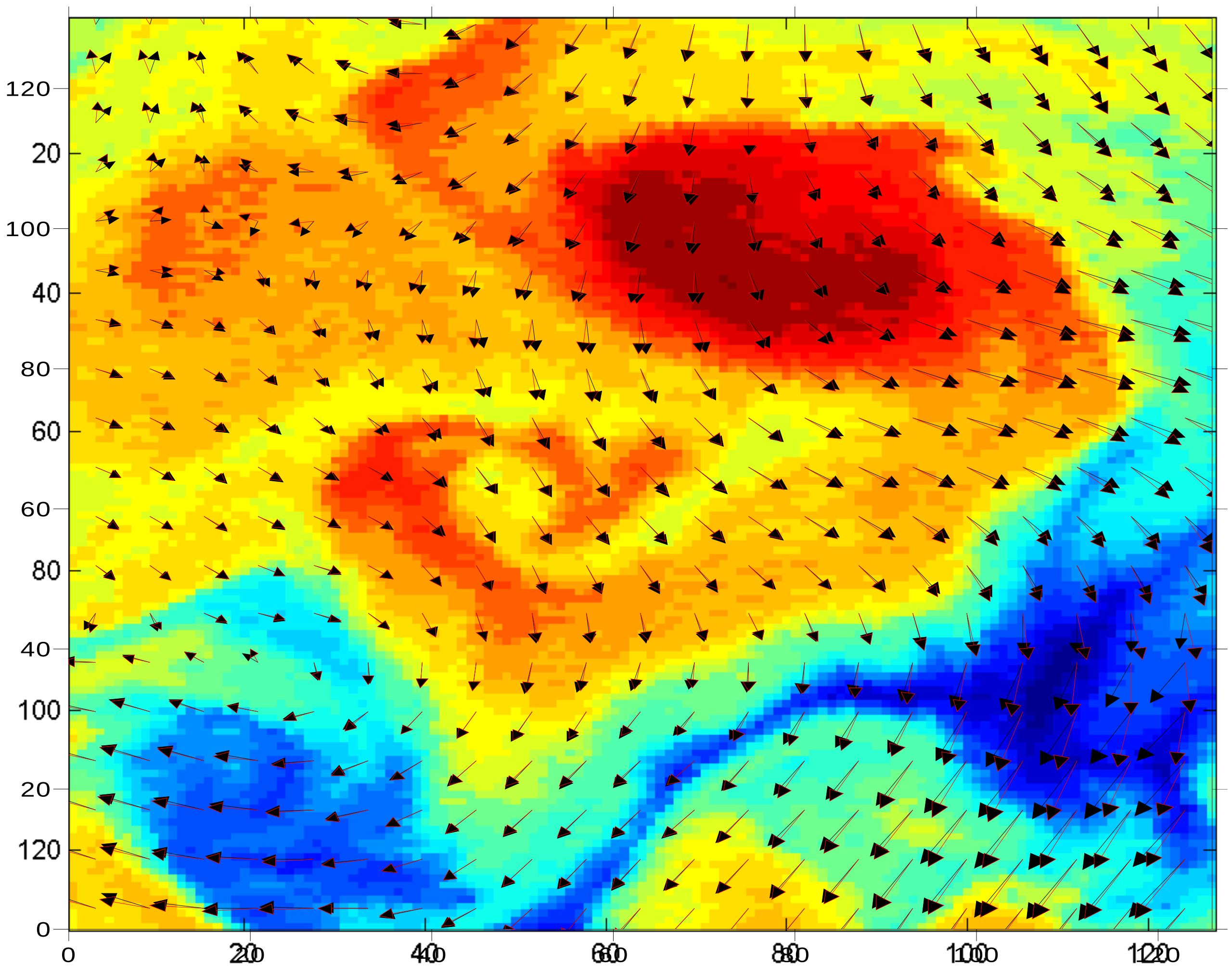
TObs1



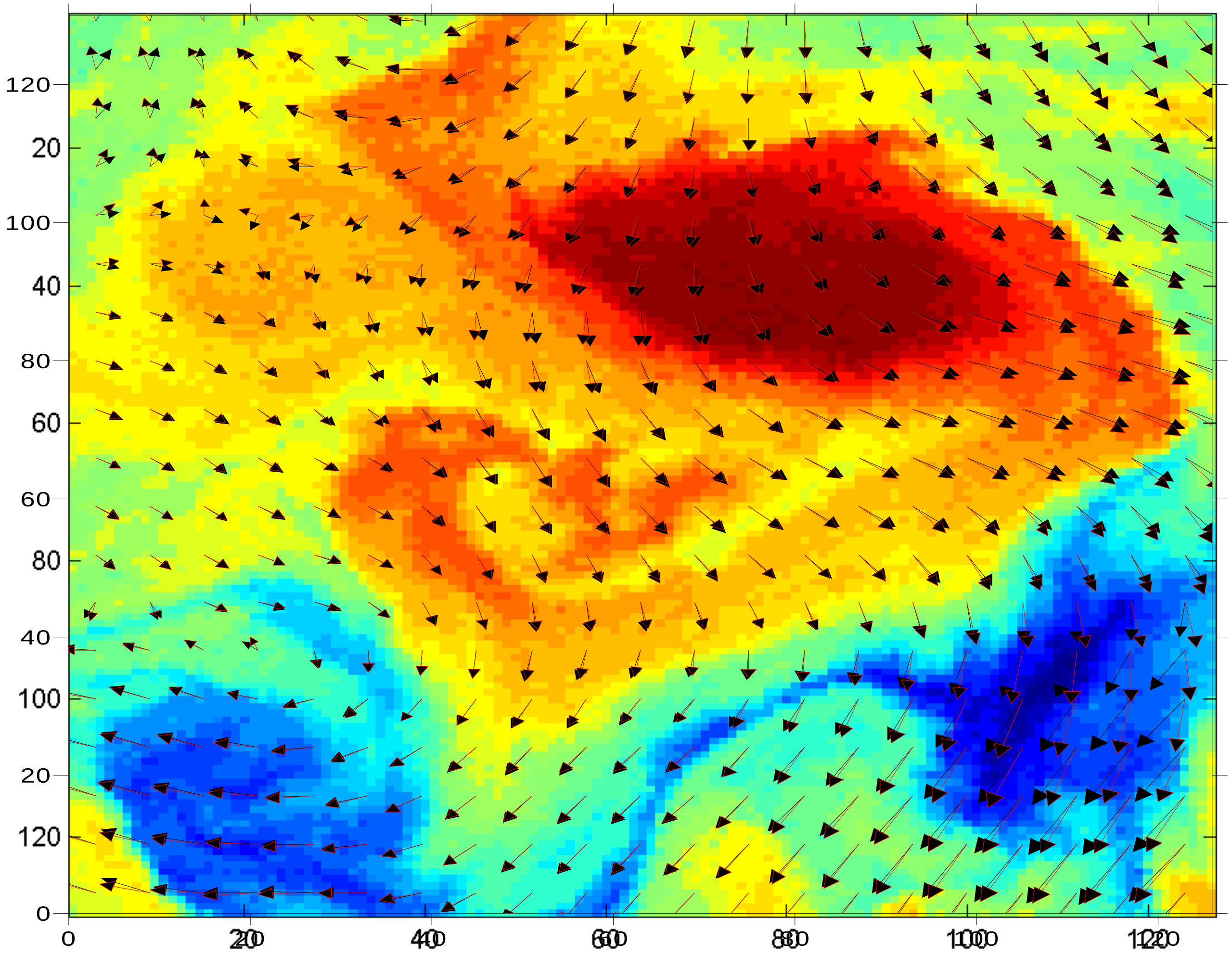
TObs2



TObs3

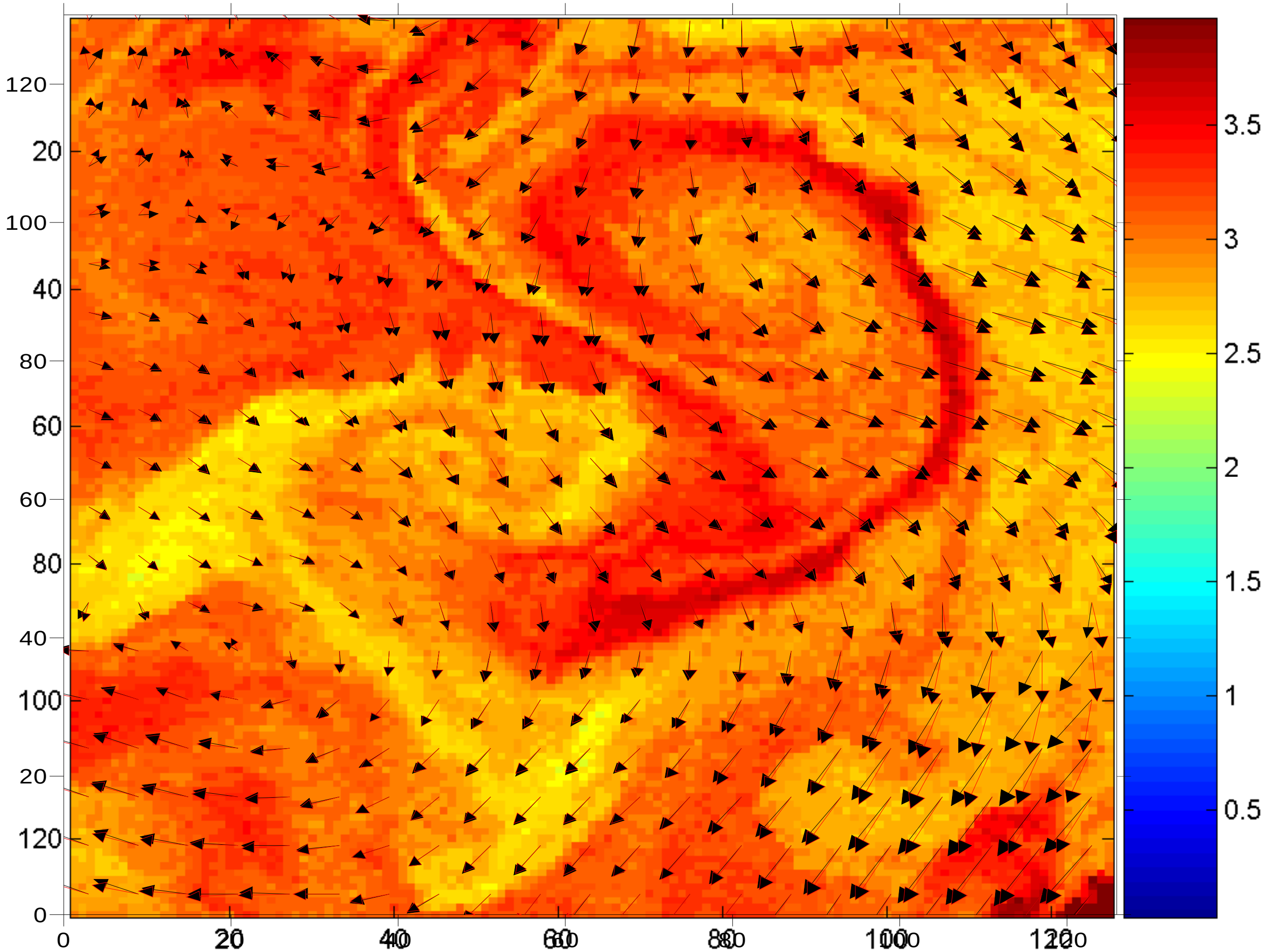


TObs4



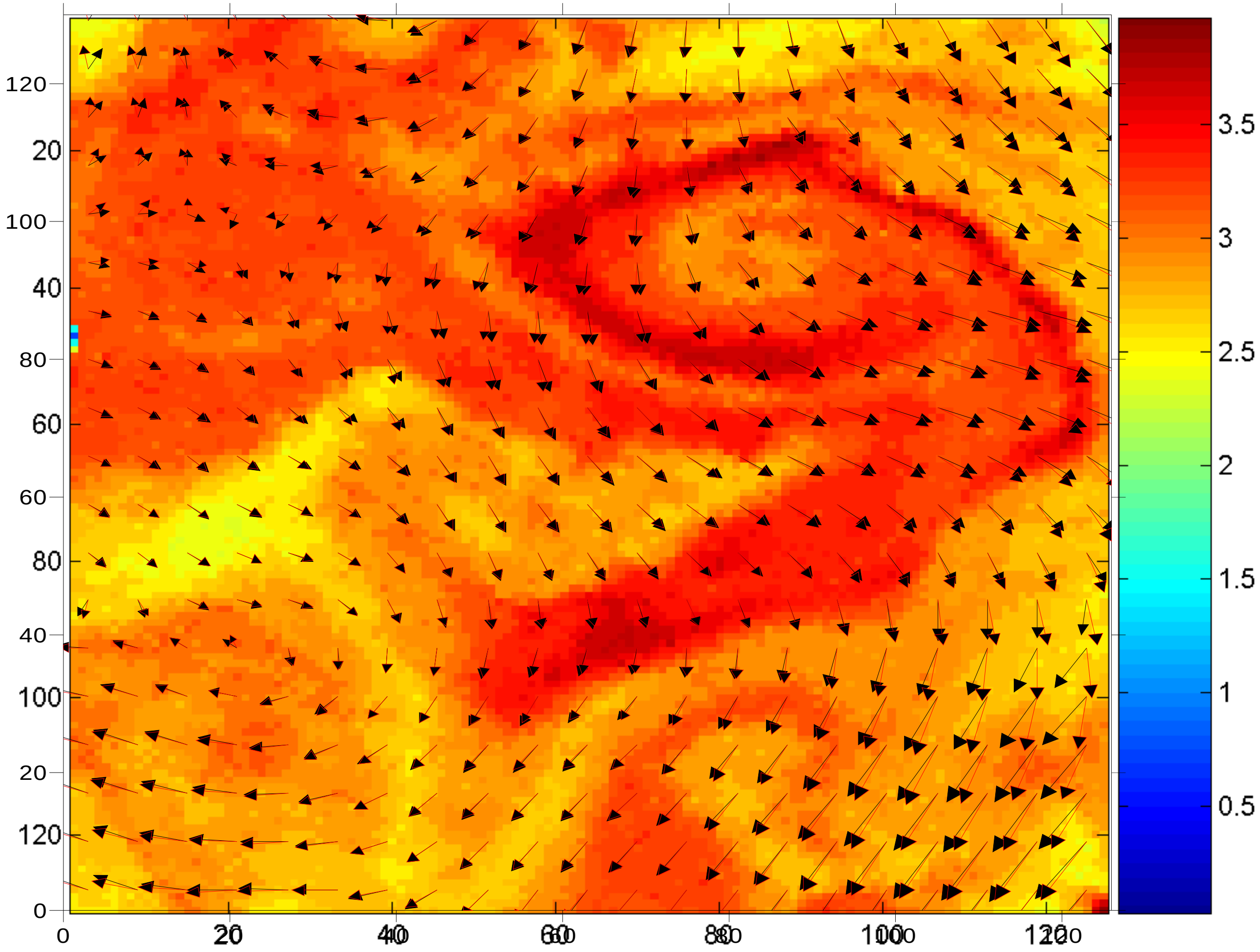
Simulated velocities overlapped on the visible band images

TObs5

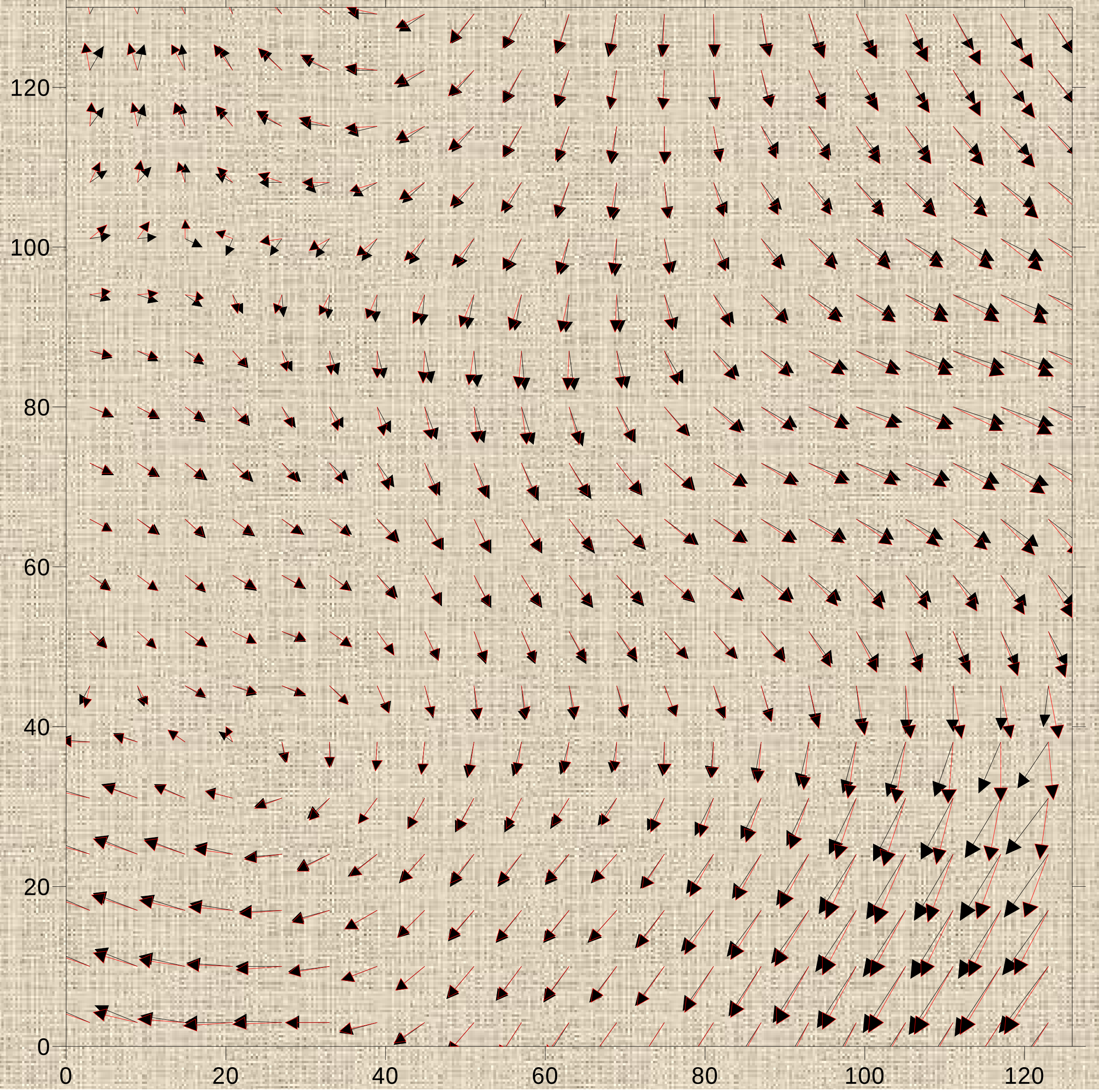


120
20
100
40
80
60
60
80
40
100
20
120
0
0
20
40
60
80
100
120
3.5
3
2.5
2
1.5
1
0.5

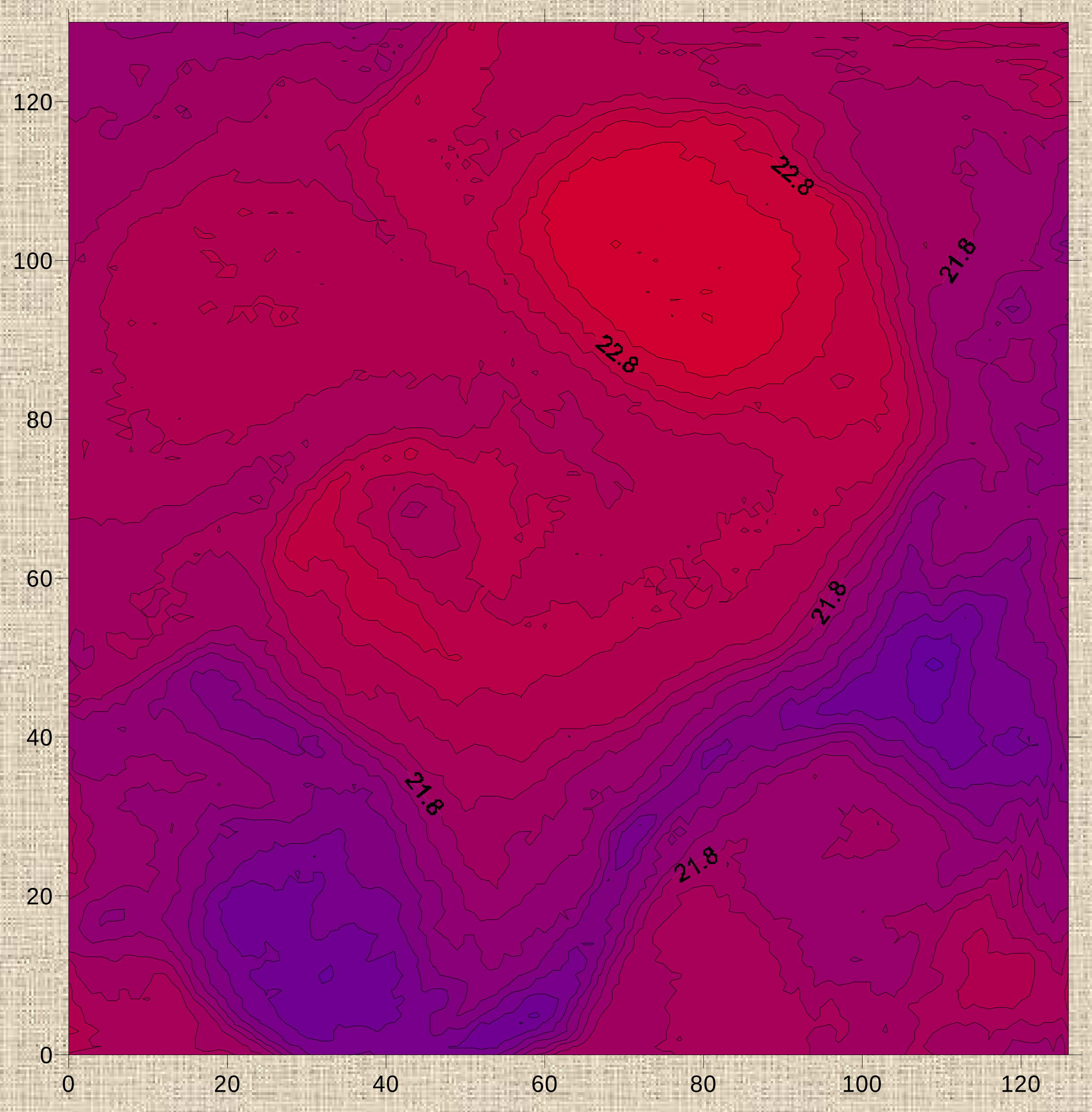
TObs6



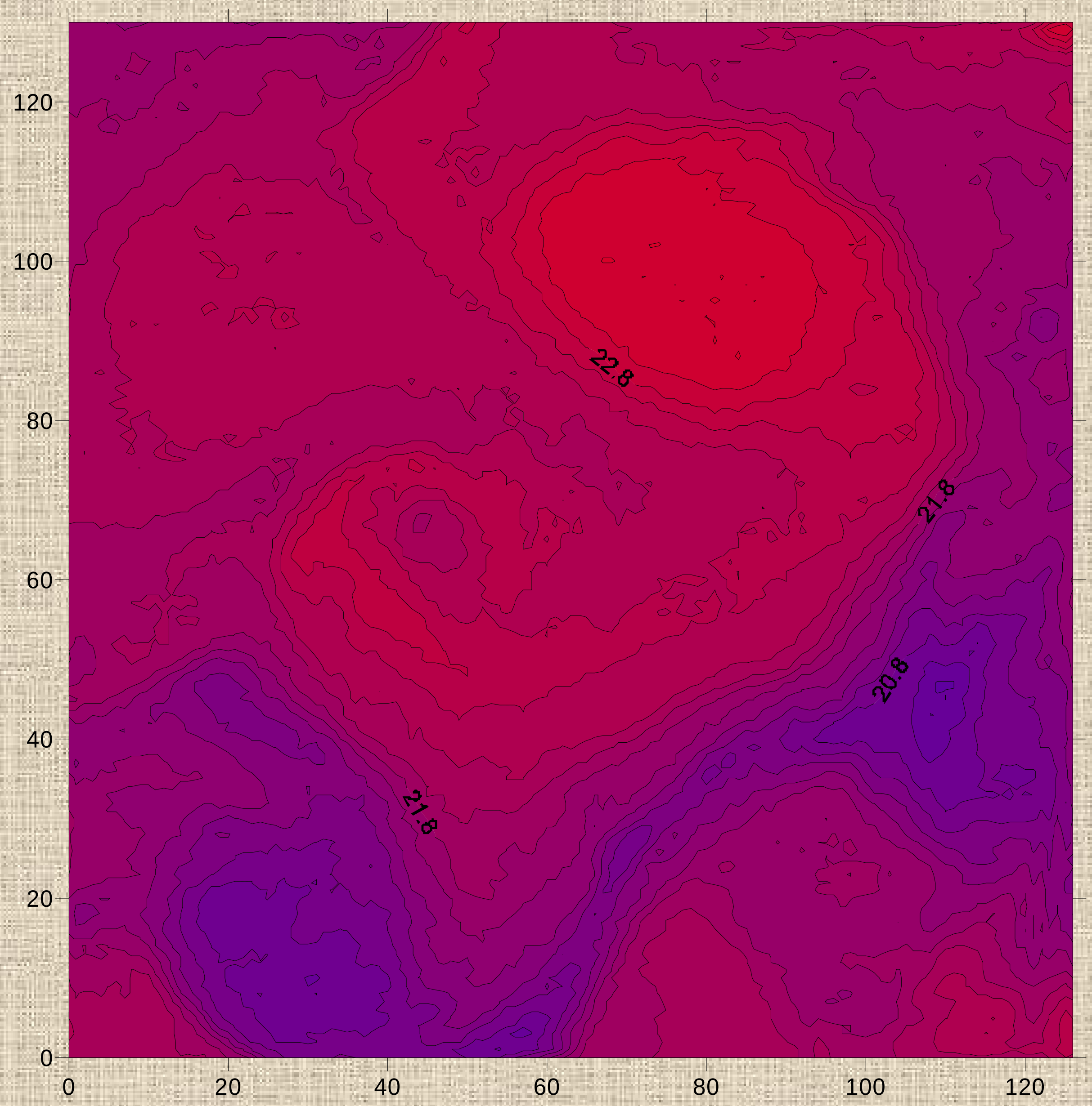
Simulated velocity based on the processing of IR or IR and visible band images



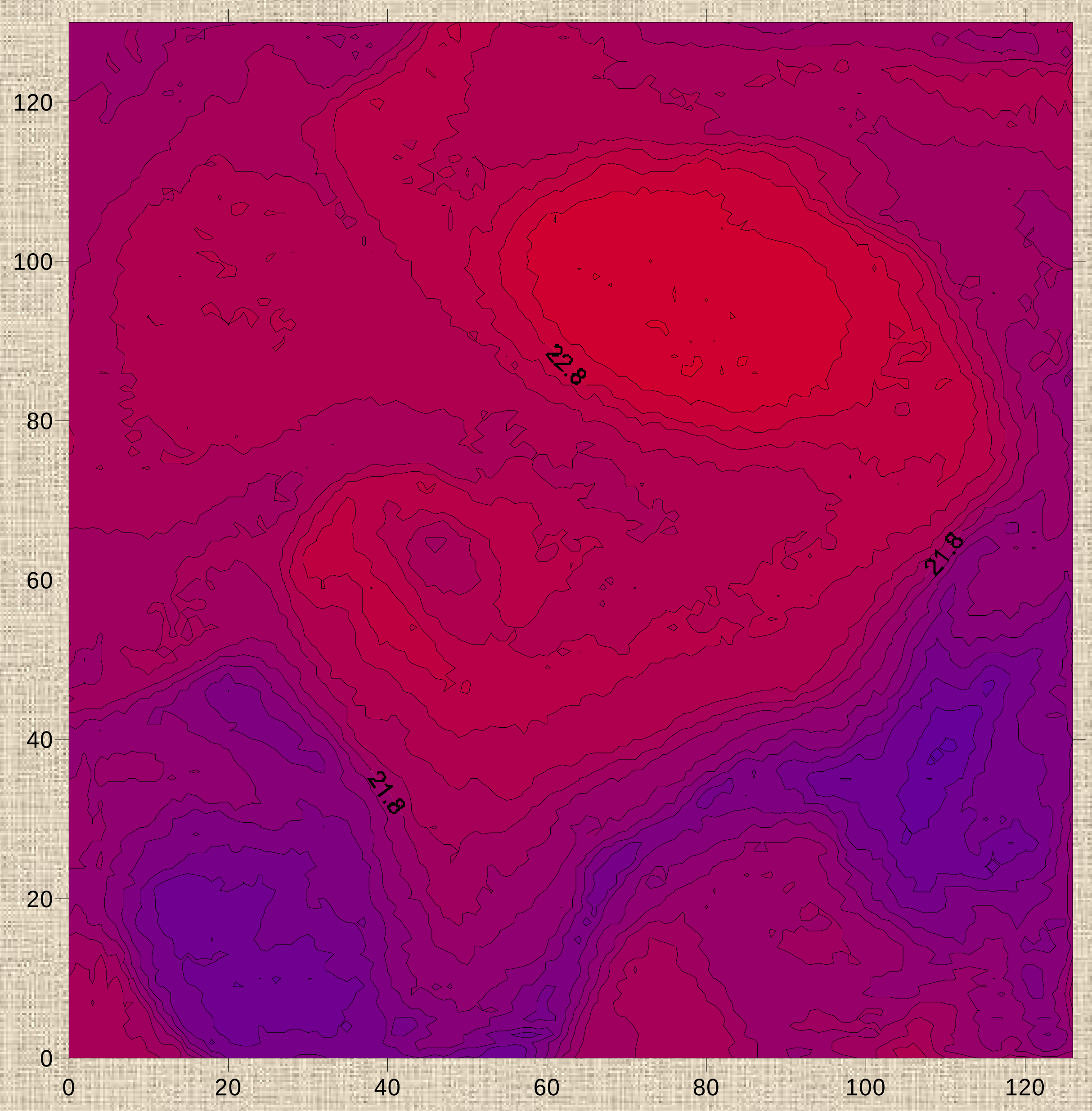
Simulated images: assimilated only IR data



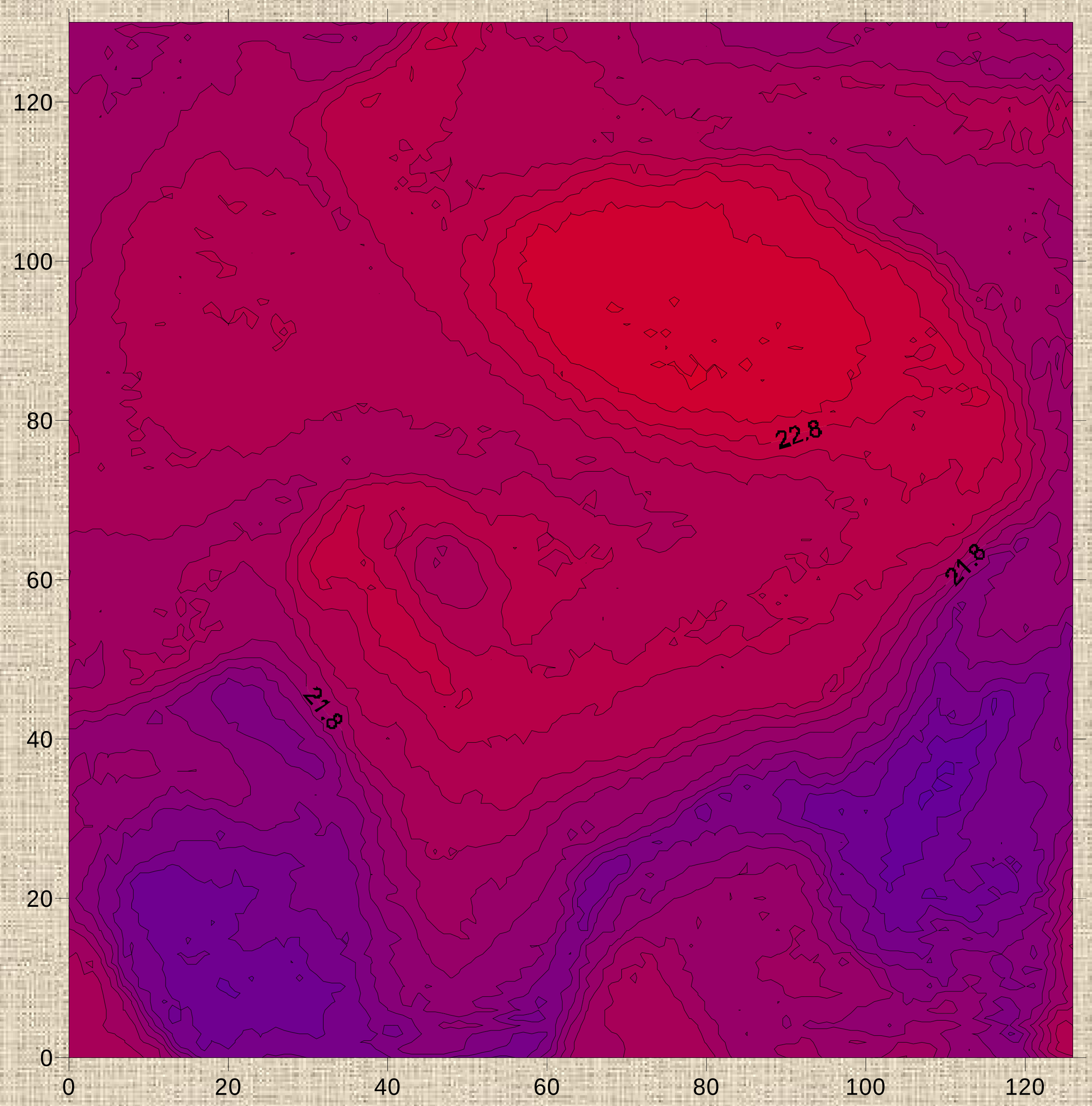
120
100
80
60
40
20
0
0
20
40
60
80
100
120
22.8
22.8
21.8
21.8
21.8
21.8



120
100
80
60
40
20
0
0
20
40
60
80
100
120
22.8
21.8
20.8
21.8

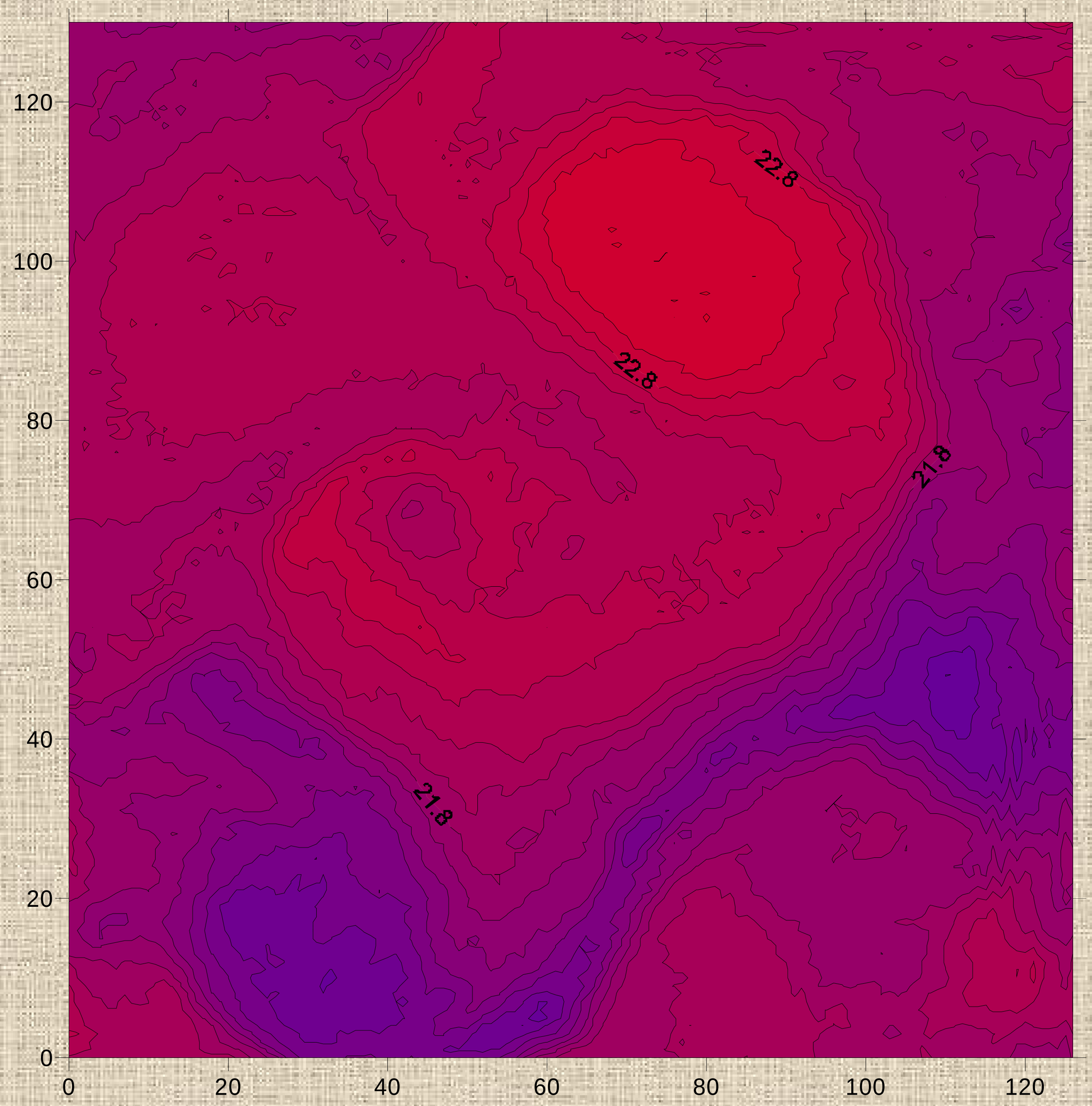


120
100
80
60
40
20
0
0
20
40
60
80
100
120
22.8
21.8
21.8

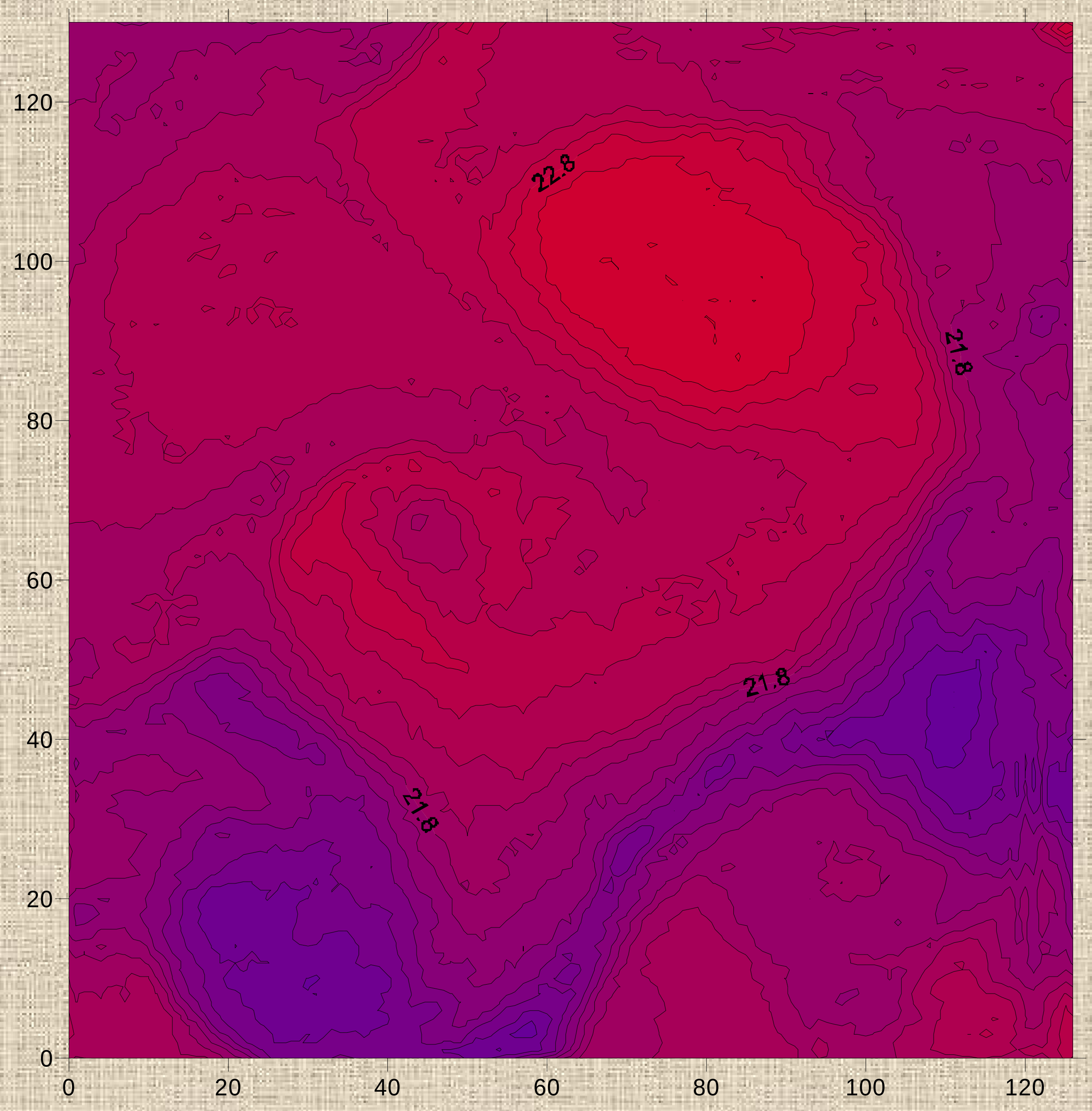


120
100
80
60
40
20
0
0
20
40
60
80
100
120
22.8
21.8
21.8

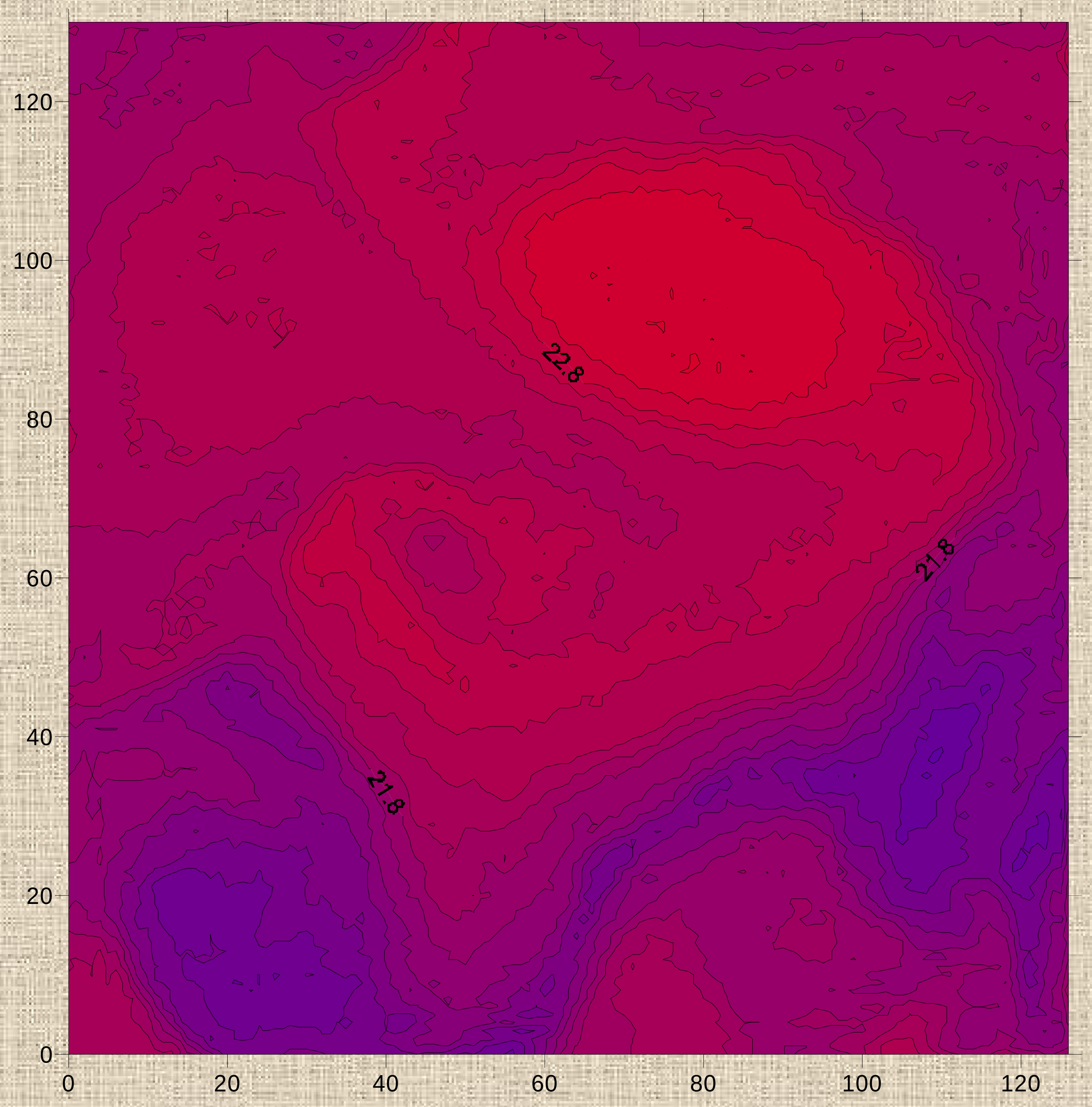
Simulated images: assimilated IR and visible band data



0
20
40
60
80
100
120
22.8
22.8
21.8
21.8



0
20
40
60
80
100
120
22.8
21.8
21.8
21.8



120
100
80
60
40
20
0
0
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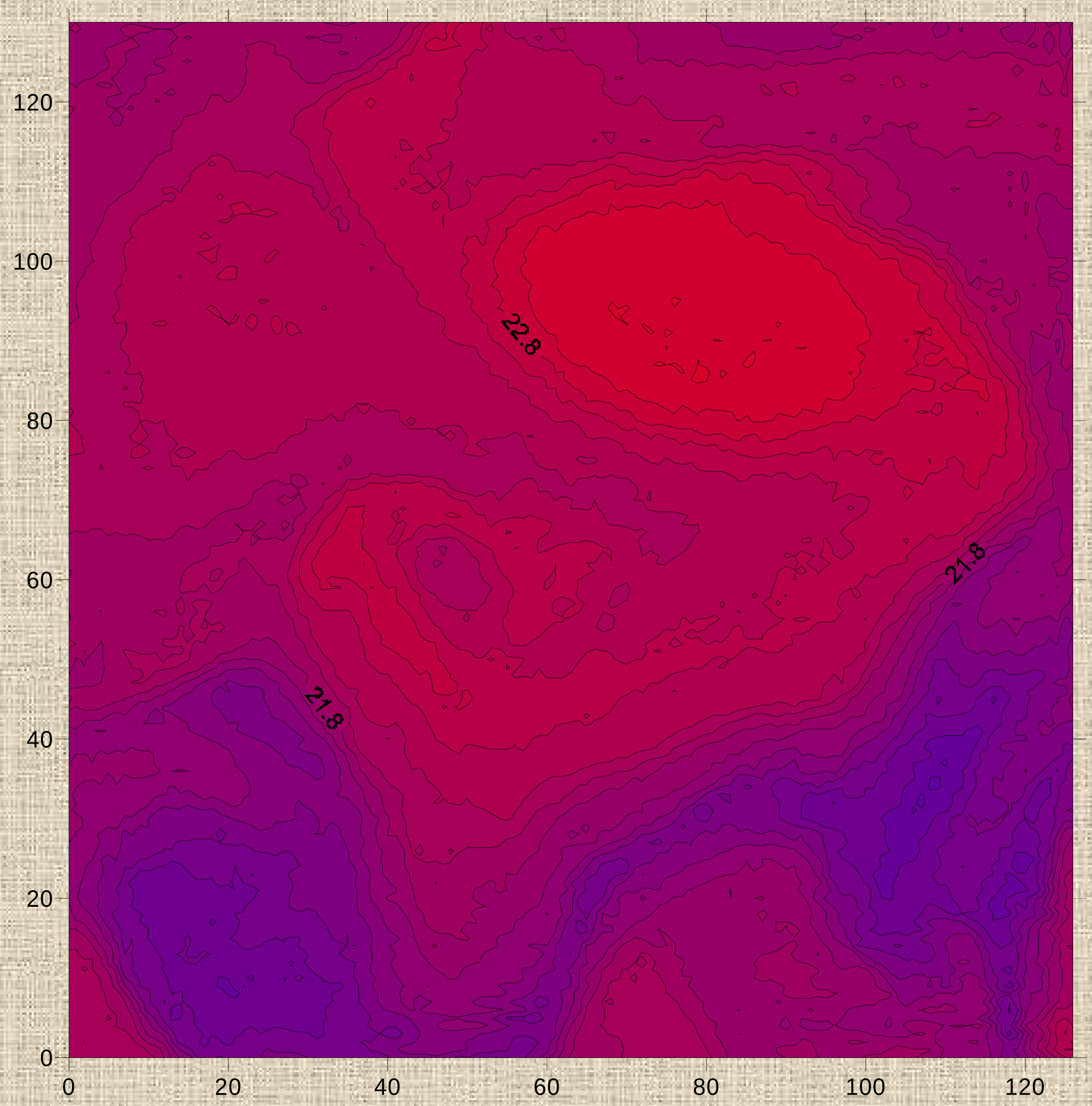
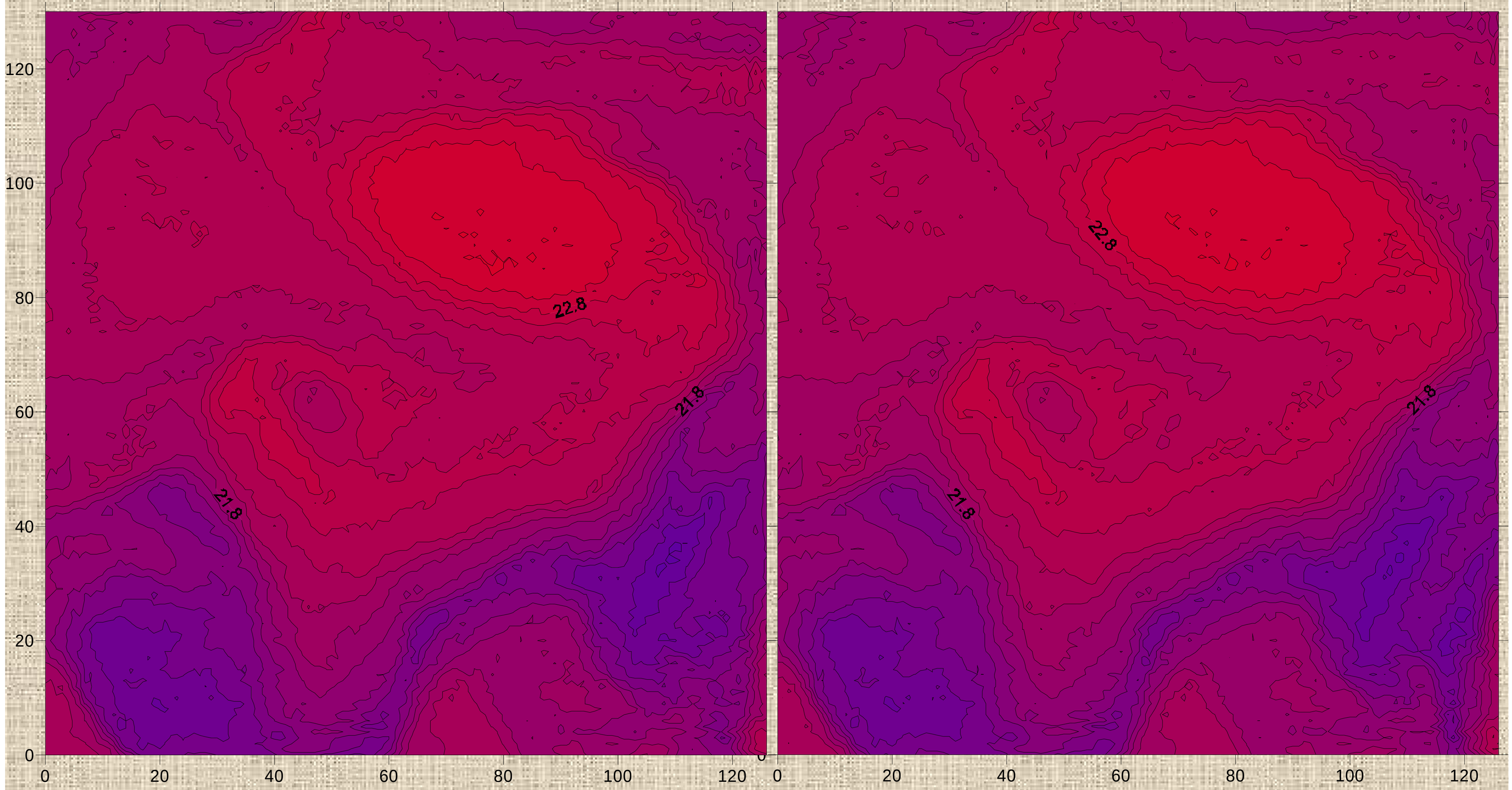
40
60
80
100
120
22.8
21.8
21.8



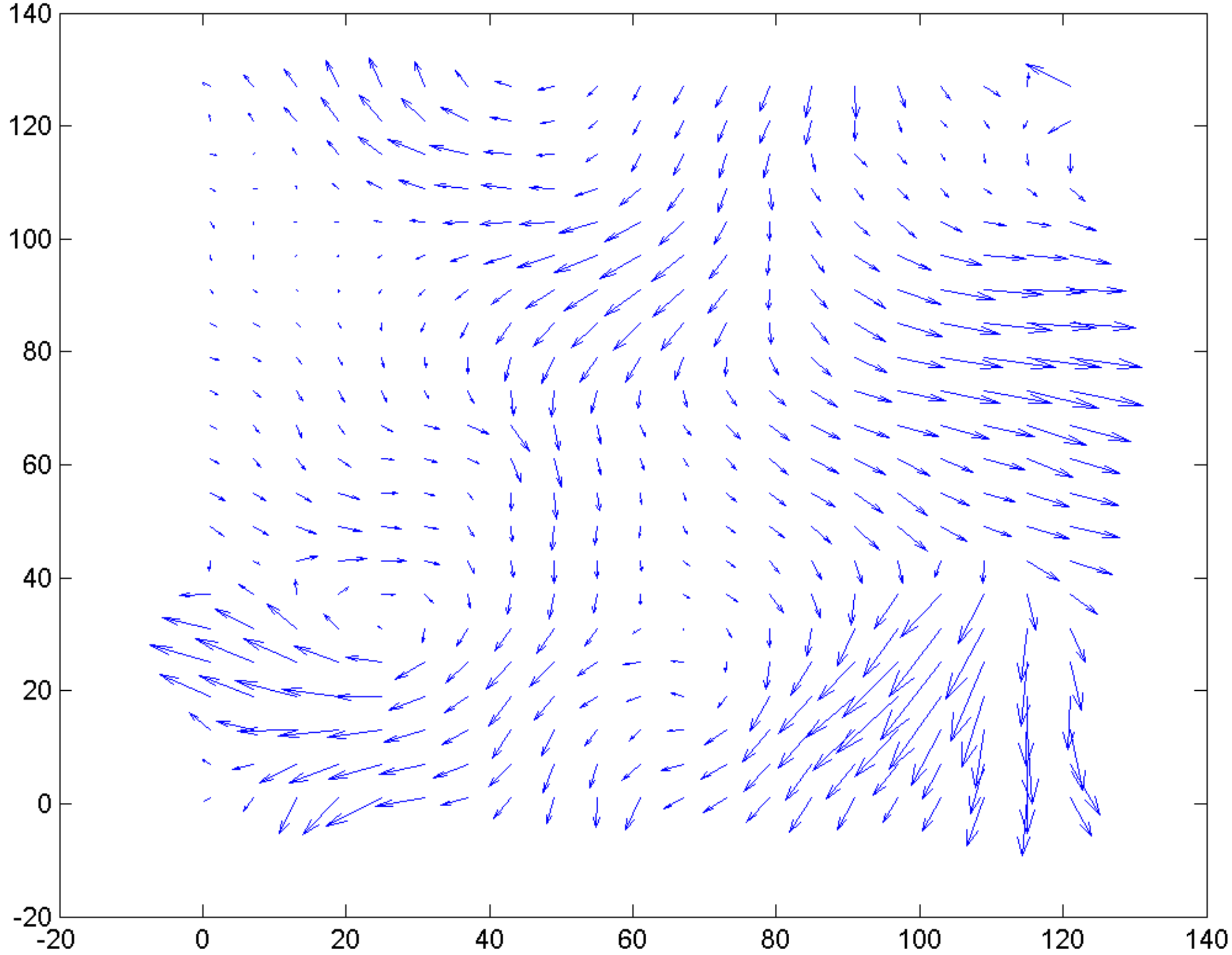
120
100
80
60
40
20
0
0
20
40
60
80
100
120
22.8
21.8
21.8

Simulated last pictures after assimilation of IR or IR and visible band images

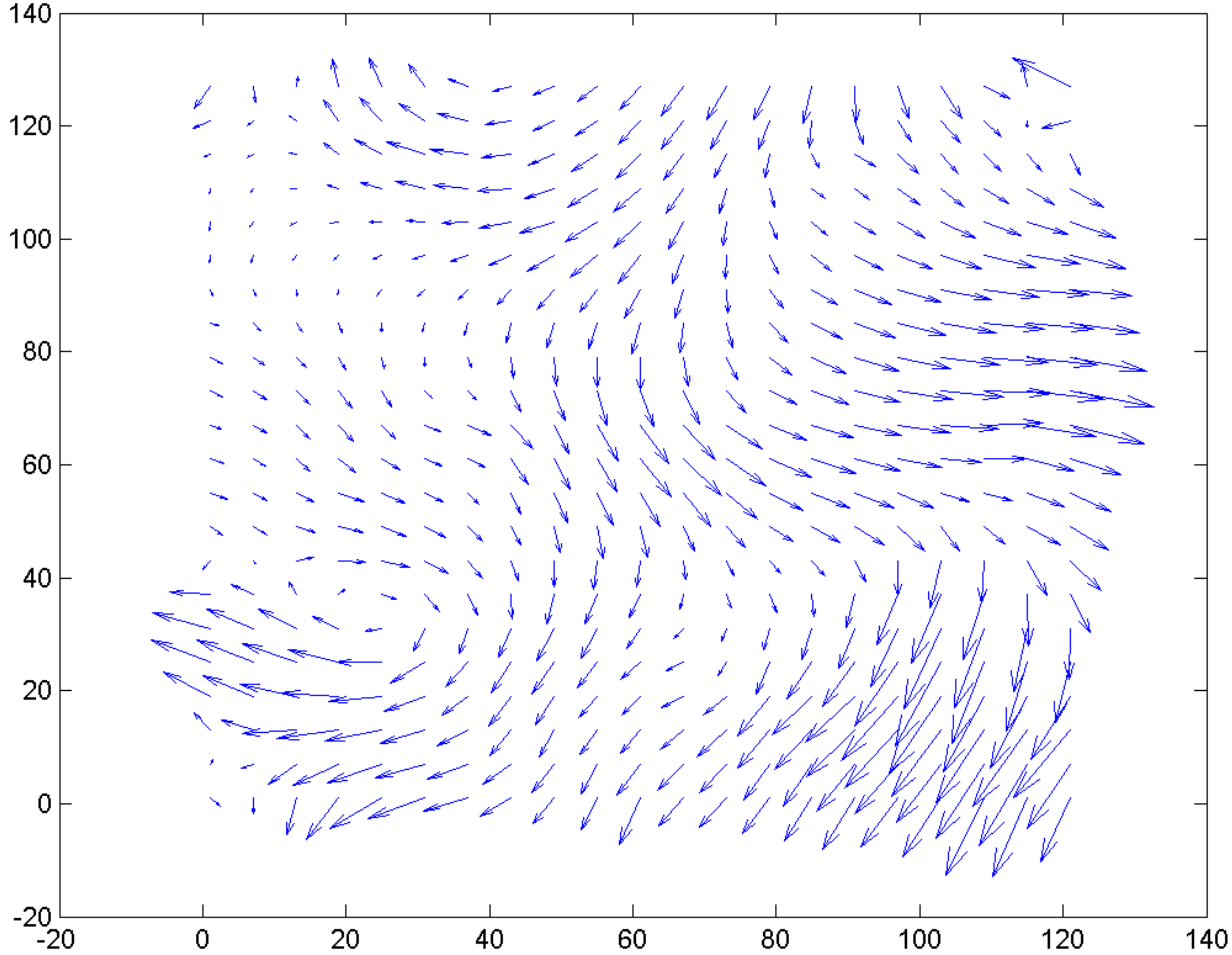


Another weighting

Calculated start velocity



Calculated start velocity



CONCLUSIONS

- Multichannel images can be efficiently processed
- More images can be collected during one-two days
- Algorithm looks more stable when more images is processed
- Prospect of the atmospheric correction