Application of data assimilation to the reconstruction of the Black Sea climate and reanalysis of the Black Sea dynamics

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Optimal interpolation (OI) (since the work of Gandin on OI of meteorological fields)

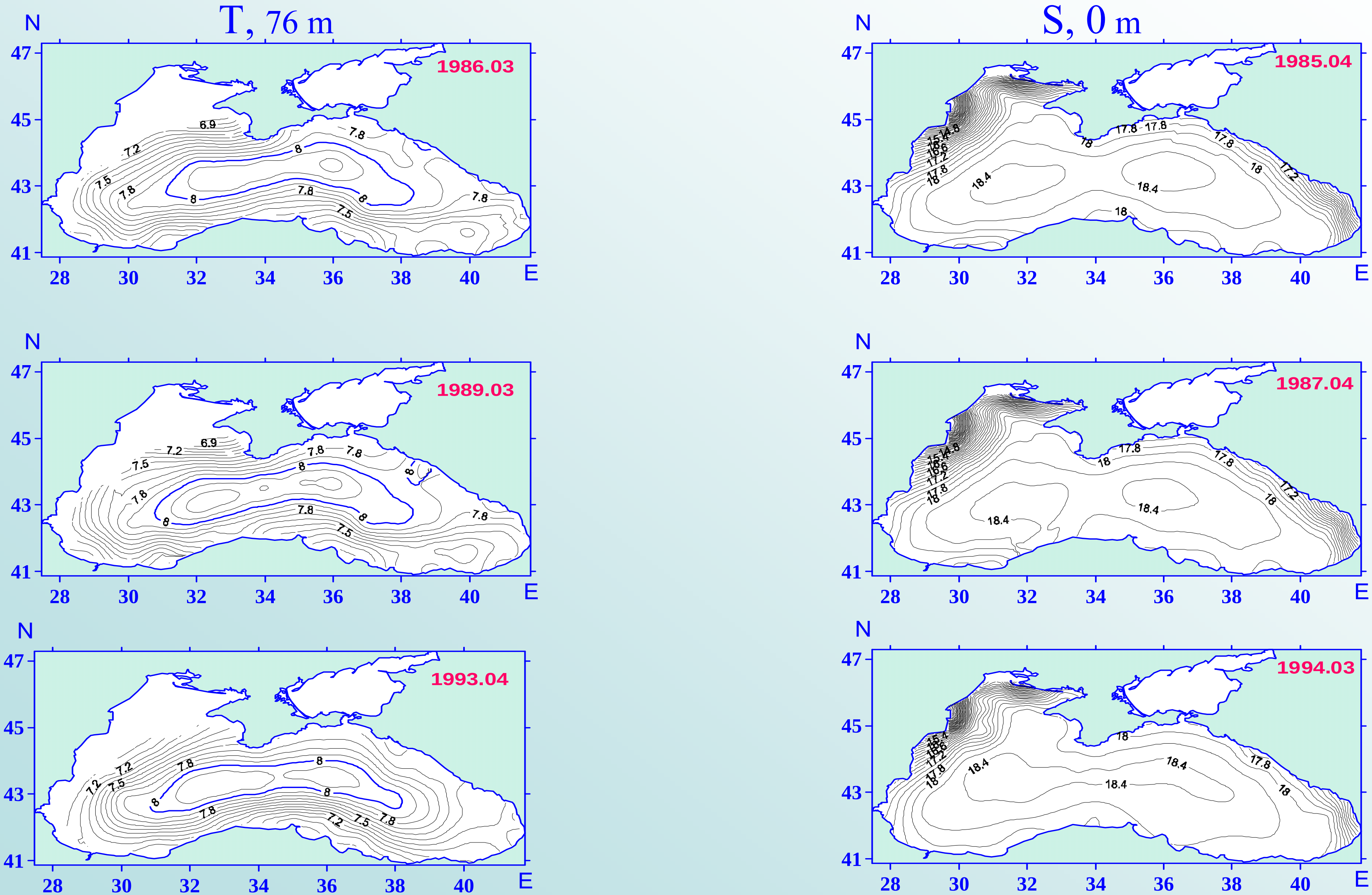
OI is the analysis technique in which optimal weights are calculating founding the minimum of r.m.s. in least square sense

For each gridpoint the value of the component of the state vector is

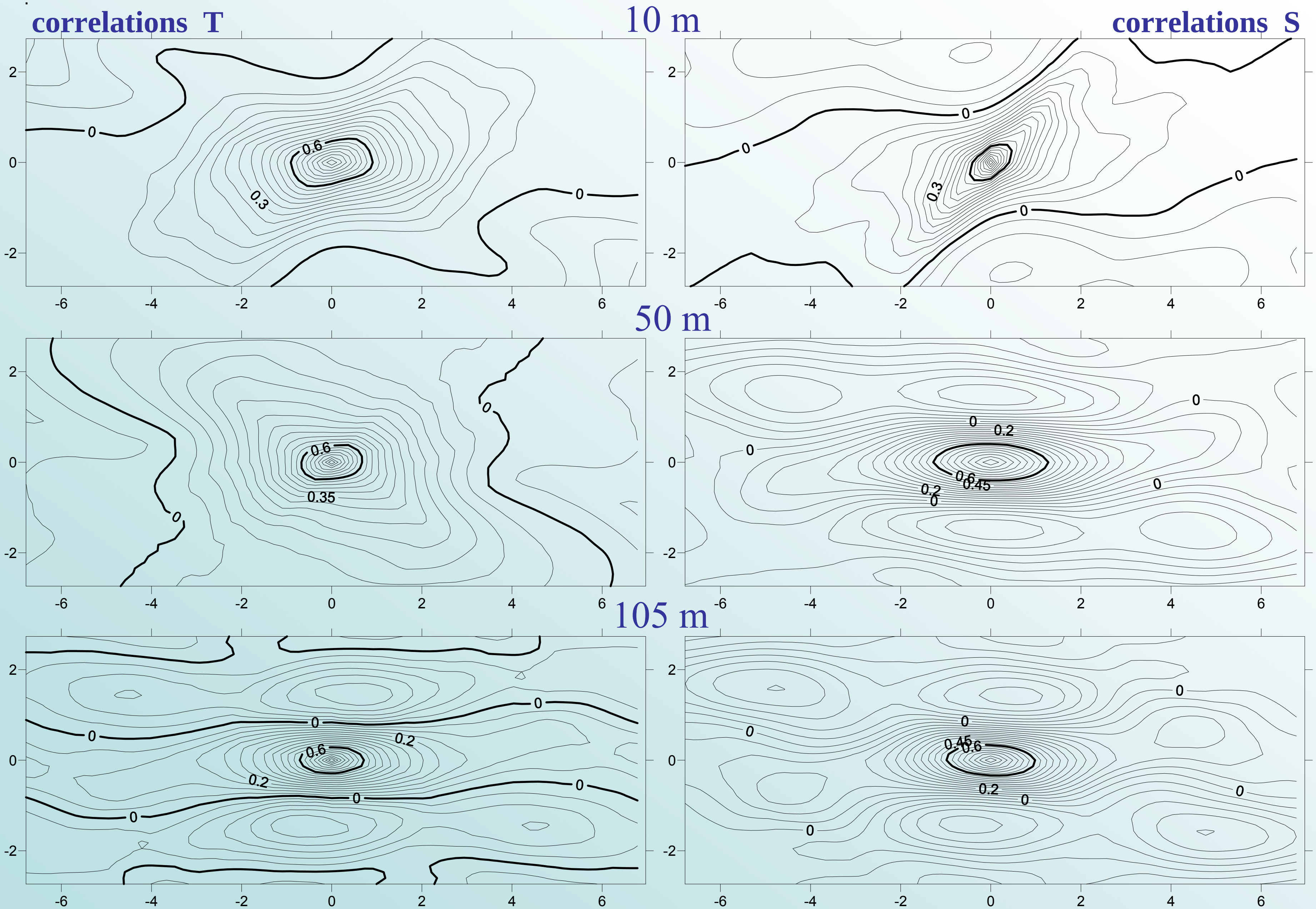
$$f'_0 = \sum_{\varsigma=1} p_\varsigma f'_\varsigma$$

f'_ς misfit background-observation

Monthly fields of temperature and salinity, retrieved by optimal interpolation (reanalysis)

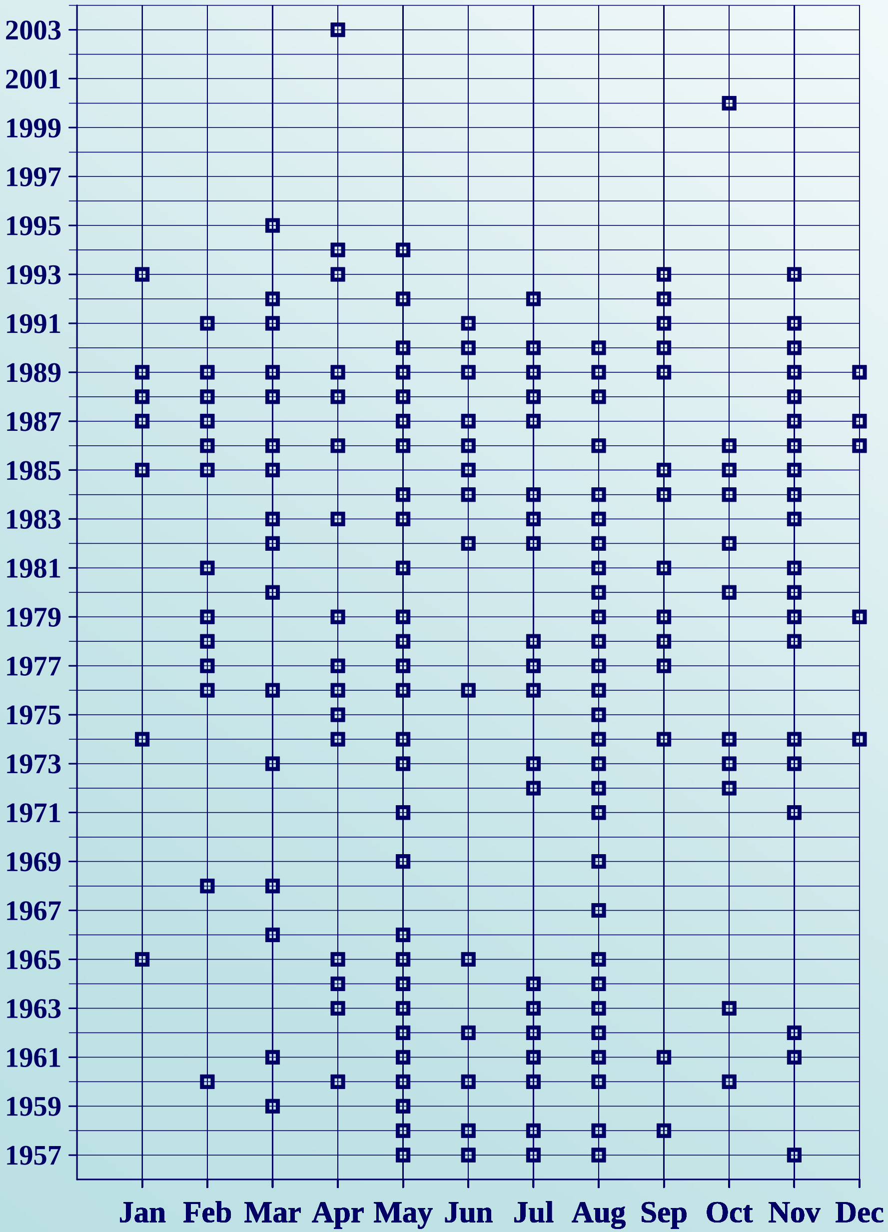


Distribution of spatial correlations of climatic fields of temperature and salinity in winter

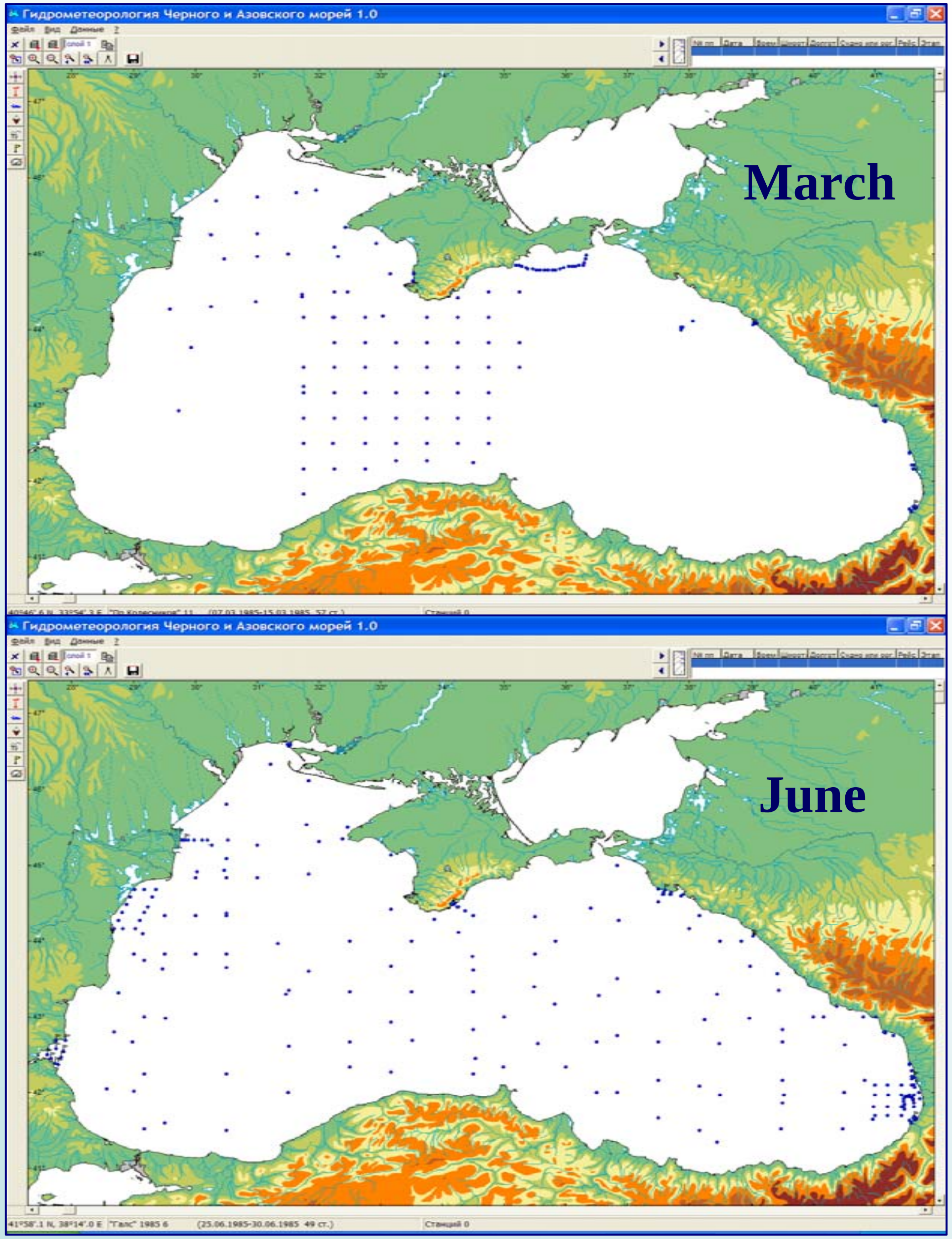


The interval between isolines is equal 0.05

Monthly distribution of hydrological data of the Black Sea over the period of 1957-2003 year

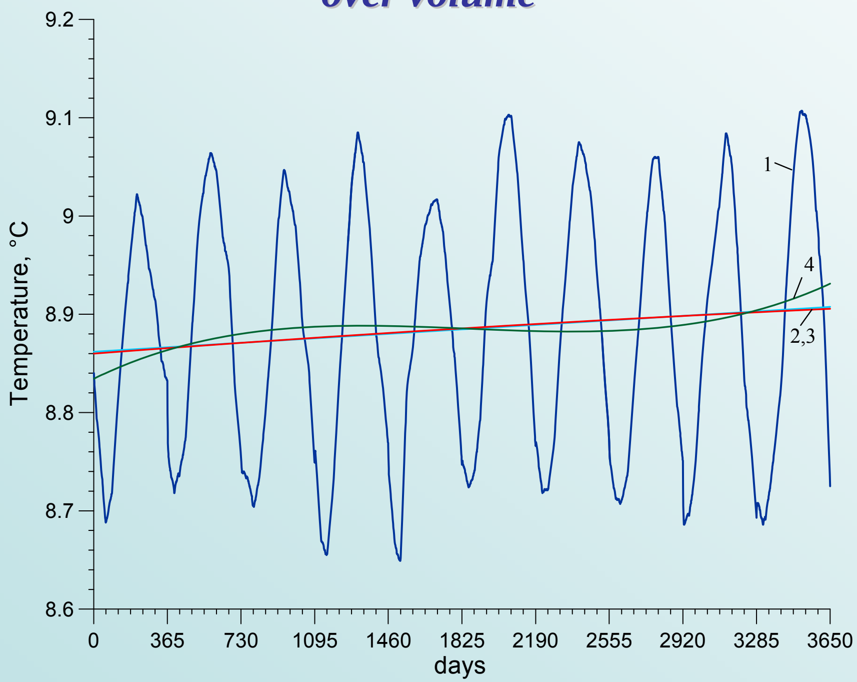


Example of spatial schemes of hydrological shoots in March and June in 1985.

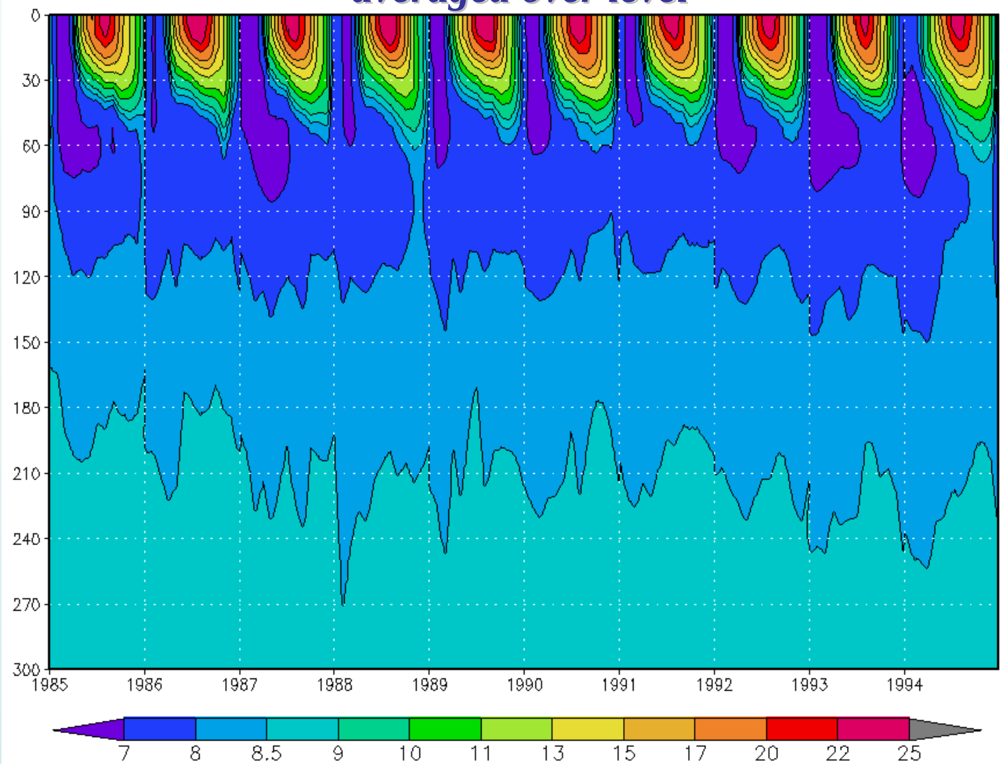


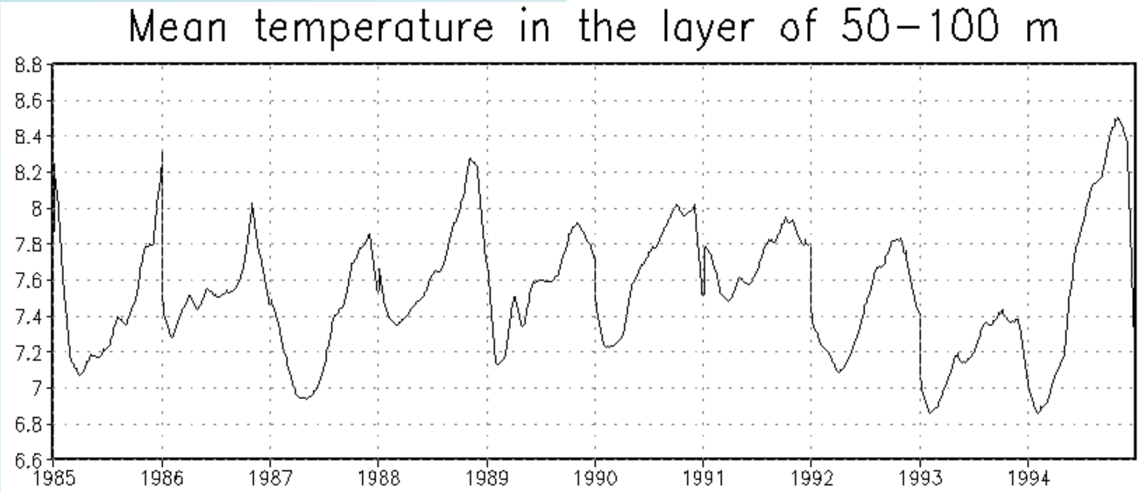
Interanual variability of temperature over a period of 1985-1994 yy.

Temporal changes of temperature averaged over volume



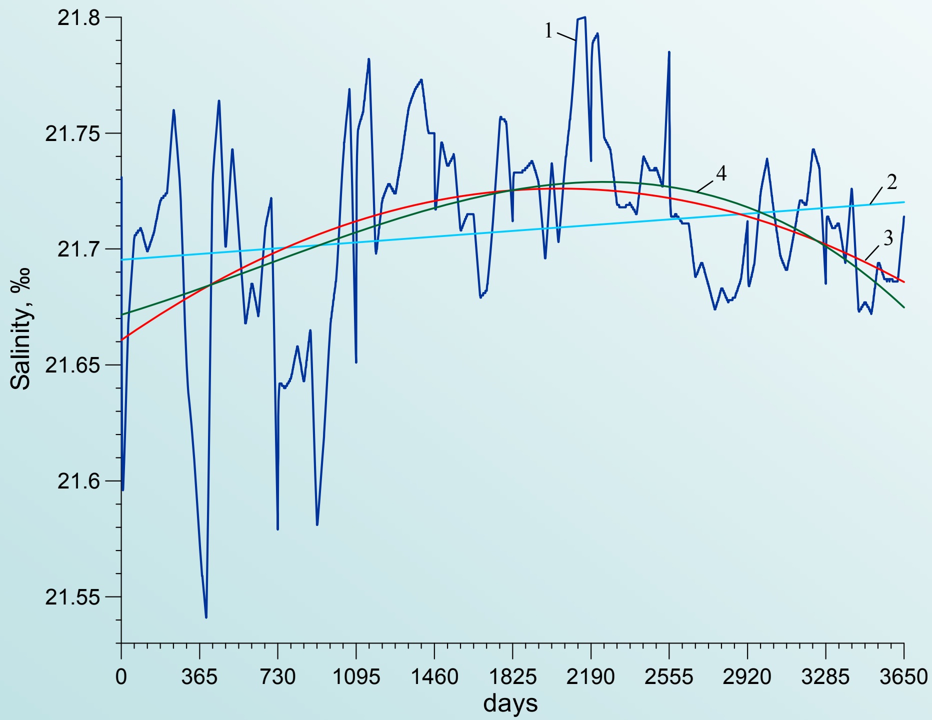
Time-depth diagram of variability of temperature averaged over level



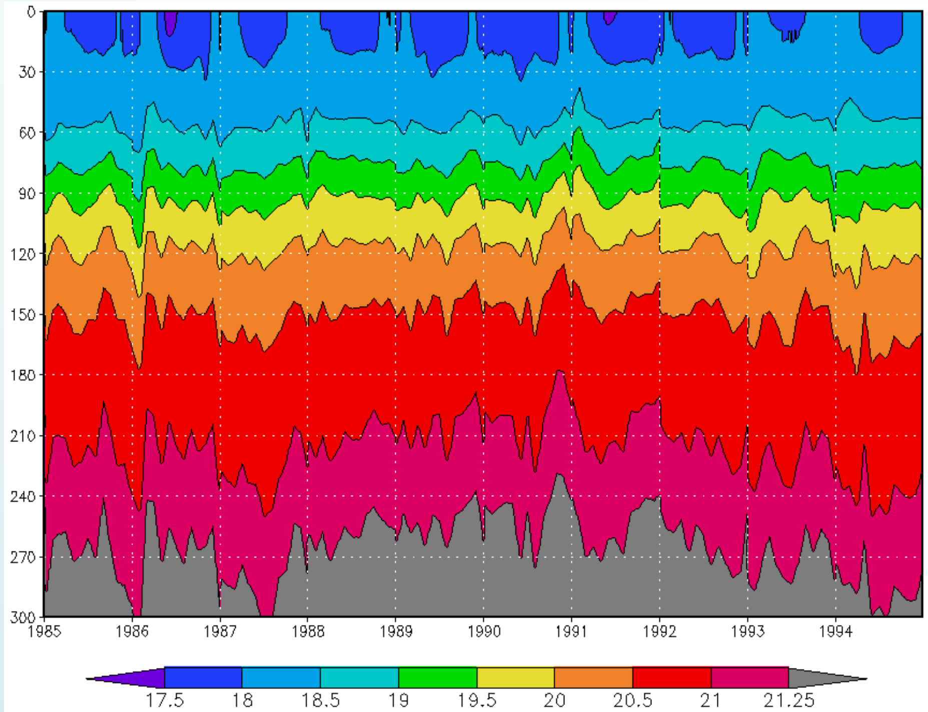


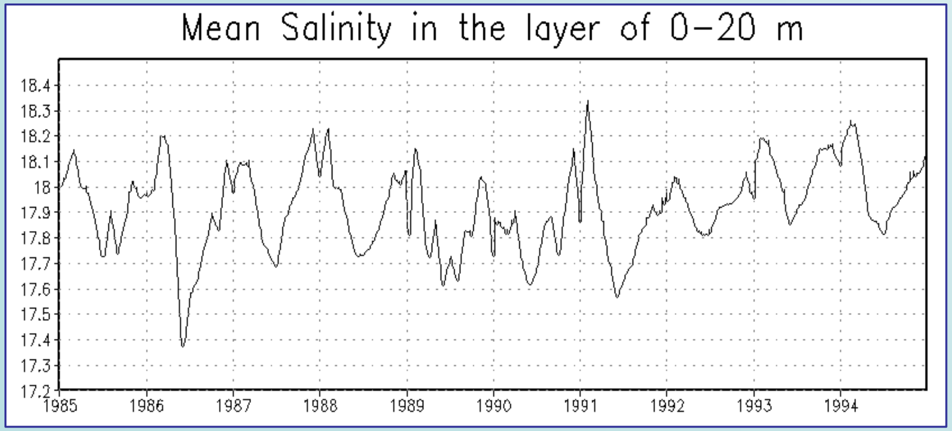
Interanual variability of salinity over a period of 1985-1994 yy.

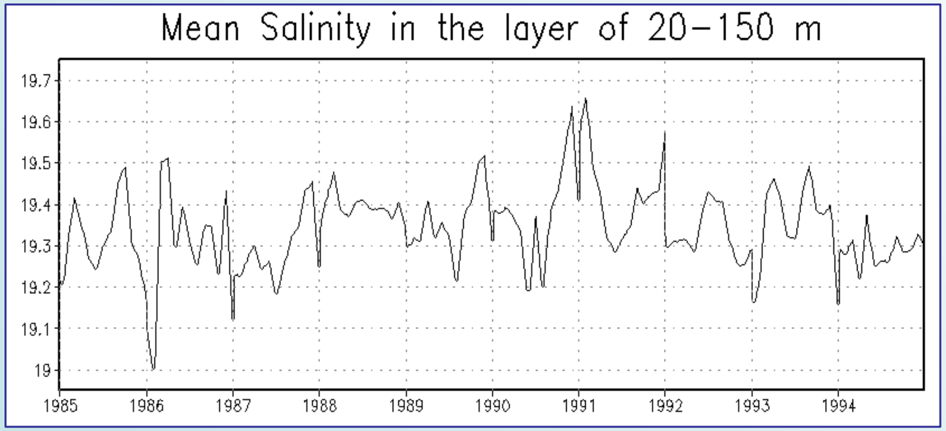
Temporal changes of salinity averaged over volume



Time-depth diagram of variability of temperature averaged over level

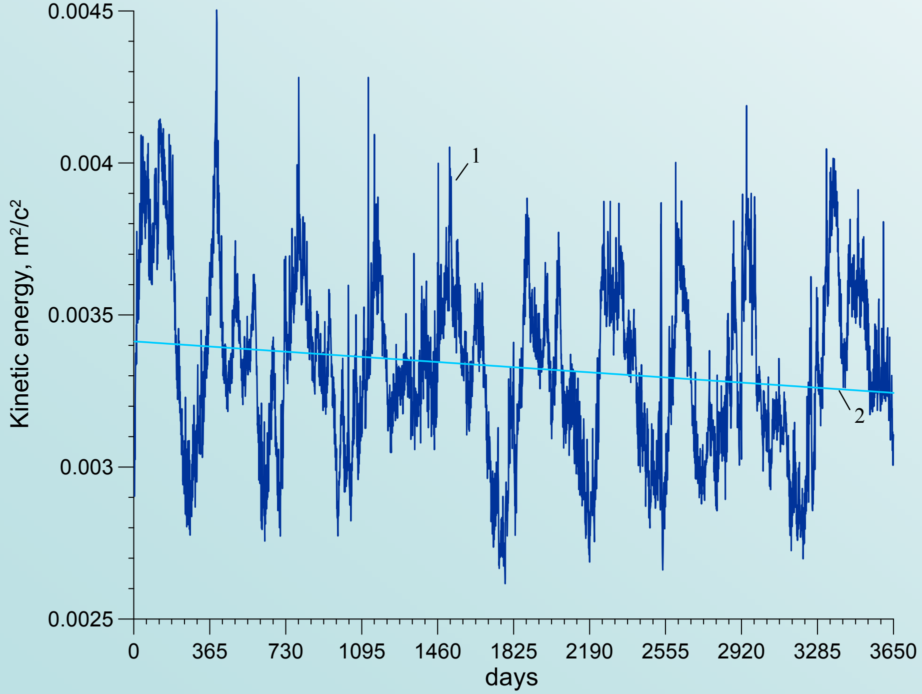


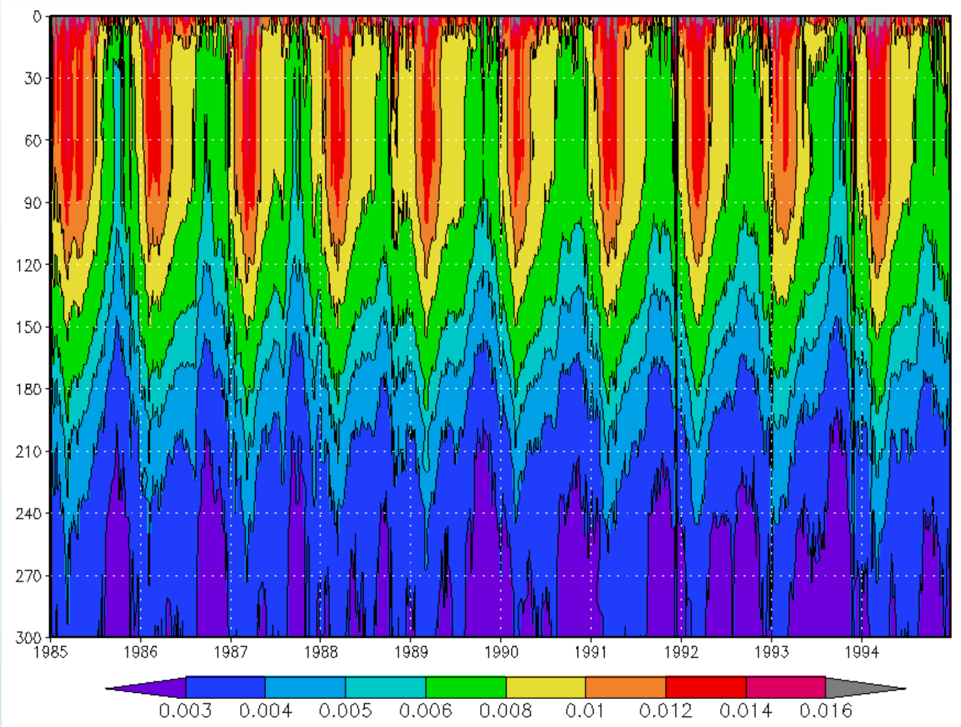




Interanual variability of kinetic energy over a period of 1985-1994 yy.

Temporal changes of kinetic energy averaged over volume





Time-depth diagram of variability of kinetic energy averaged over level

Basic concepts for designing algorithms of climatic temperature and salinity assimilation

Let denote $\vec{U}(\vec{x},t) = \left\| u(\vec{x},t), v(\vec{x},t), T(\vec{x},t), S(\vec{x},t), \zeta(x,y,t) \right\|$ – state vector, where

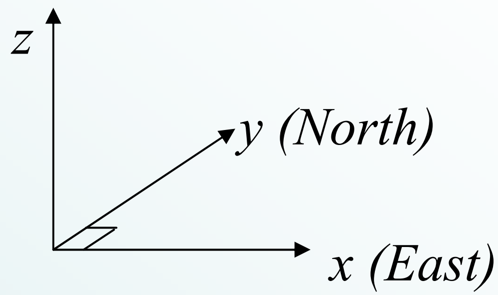
u, v – horizontal components of velocity;

T, S – temperature and salinity;

ζ – sea level;

t – time

$\vec{x} = (x, y, z),$



Neglecting the model errors for linearized dynamics model

$$\frac{\partial \overset{\frown}{\vec{U}}(\vec{x},t)}{\partial t} = L_{\vec{x}} \overset{\frown}{\vec{U}}(\vec{x},t),$$

$$\frac{\partial P(\vec{x},\vec{x}',t)}{\partial t} = L_{\vec{x}} P(\vec{x},\vec{x}',t) + \left[L_{\vec{x}'} P(\vec{x}',\vec{x},t) \right]^T$$

where

$P(\vec{x},\vec{x}',t) = E\left\{ \delta\vec{U}(\vec{x},t) \cdot \delta\vec{U}^T(\vec{x},t) \right\}$ background error covariation function

$\delta\vec{U}(\vec{x},t) = \vec{U}(\vec{x},t) - \overset{\frown}{\vec{U}}(\vec{x},t)$ background error

$L_{\vec{x}}, L_{\vec{x}'}$ linearized dynamics operators

In the case of getting “measurements” of climatic temperature and salinity on a discrete time step t_l in N points estimation of temperature and salinity are corrected according to formulas

$$\widehat{T}(\vec{x},t_l^+) = \widehat{T}(\vec{x},t_l^-) + \sum_{r=1}^{N} \left[\Delta_r^{3T}(\vec{x},t_l^-)\delta T(\vec{x}_{r,}t_l^+)\ \right]$$

$$\widehat{S}(\vec{x},t_l^+) = \widehat{S}(\vec{x},t_l^-) + \sum_{r=1}^{N} \left[\Delta_r^{4S}(\vec{x},t_l^-)\delta S(\vec{x}_{r,}t_l^+)\ \right]$$

where

$$\Delta^{3T}(\vec{x},t_l^-) = \left[\ P_T(\vec{x}_r,\vec{x}_p,t_l^-) + R_T(\vec{x}_r,\vec{x}_p,t_l)\ \right]^{-1} \times P_T(\vec{x},\vec{x}_r,t_l^-),$$

$$\Delta^{4S}(\vec{x},t_l^-) = \left[\ P_S(\vec{x}_r,\vec{x}_p,t_l^-) + R_S(\vec{x}_r,\vec{x}_p,t_l)\ \right]^{-1} \times P_S(\vec{x},\vec{x}_r,t_l^-),$$

$$\delta T(\vec{x}_r,t_l^+) = T(\vec{x}_r,t_l) - \widehat{T}(\vec{x}_r,t_l^-), \qquad \delta S(\vec{x}_r,t_l^-) = S(\vec{x}_r,t_l) - \widehat{S}(\vec{x}_r,t_l^-),$$

«–» , «+» represents function values before and after correction;

$P_T(\cdot), P_S(\cdot)$ background error covariation functions for temperature and salinity

$R_T(\cdot), R_S(\cdot)$ observation error covariation functions for temperature and salinity

In the moment of data assimilation corresponding forecast error covariation functions for temperature and salinity must be corrected

$$P_T(\vec{x},\vec{x}',t_l^+) = P_T(\vec{x},\vec{x}',t_l^-) - \sum_{r=1}^{N} \Delta_r^{3T}(\vec{x},t_l^-)P_T(\vec{x}_r,\vec{x}',t_l^-).$$

The main difficulty in the practical implementation of KF is calculating covariation functions of the Kalman gain (P-matrix). Thus, the approximation is used

$$P_T(\vec{x},\vec{x}',t) \approx \sigma_T(\vec{x},t)\sigma_T(\vec{x}',t)P_T^{н}(|x-x'|,|y-y'|,z)$$

$$P_S(\vec{x},\vec{x}',t) \approx \sigma_S(\vec{x},t)\sigma_S(\vec{x}',t)P_S^{н}(|x-x'|,|y-y'|,z)$$

$\sigma_T(\cdot),\sigma_S(\cdot)$ r.m.s of forecast errors of temperature and salinity respectively

$P^{н}(\cdot)$ normilized autocorrelation function of corresponding fields of temperature and salinity

Assume, that climatic data on temperature and salinity was interpolated to points of the model grid, then covariation functions are reduced to the corresponding variances and correction is made according to

$$T(\vec{x},t_l^+) = T(\vec{x},t_l^-) + \frac{\sigma_T^2(\vec{x},t_l^-)}{\sigma_T^2(\vec{x},t_l^-) + \sigma_{T_m}^2(\vec{x})}\left[T^{cl}(\vec{x},t_l^-) - T(\vec{x},t_l^-)\right]$$

$$S(\vec{x},t_l^+) = S(\vec{x},t_l^-) + \frac{\sigma_S^2(\vec{x},t_l^-)}{\sigma_S^2(\vec{x},t_l^-) + \sigma_{S_m}^2(\vec{x})}\left[S^{cl}(\vec{x},t_l^-) - S(\vec{x},t_l^-)\right]$$

$T(\cdot),\ S(\cdot)$ model temperature and salinity

$\sigma_{T_m}^2(\vec{x}),\ \sigma_{S_m}^2(\vec{x})$ measurement error variances

To prevent the discontinuity in numerical solution, an assimilation terms are added to the right-hand side of transport-diffusion equations for heat and salinity:

$$Q_T(\vec{x},t_l) = \sigma_T^2(\vec{x},t_l) / REL\left[\sigma_T^2(\vec{x},t_l) + \sigma_{T_m}^2(\vec{x})\right]\left[T^{cl}(\vec{x},t_l) - T(\vec{x},t_l)\right]$$

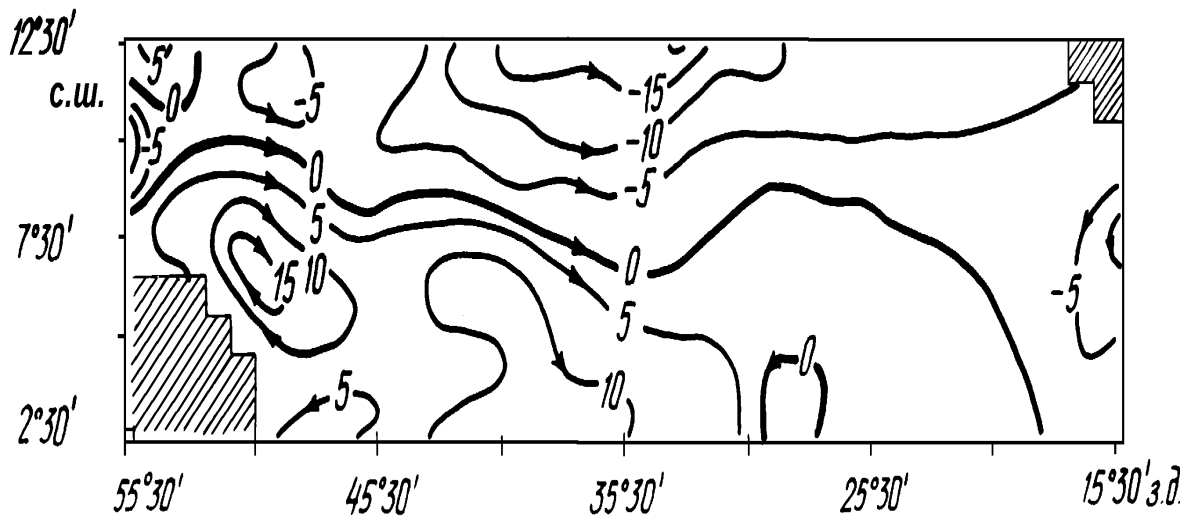
$$Q_S(\vec{x},t_l) = \sigma_S^2(\vec{x},t_l) / REL\left[\sigma_S^2(\vec{x},t_l) + \sigma_{S_m}^2(\vec{x})\right]\left[S^{cl}(\vec{x},t_l) - S(\vec{x},t_l)\right]$$

REL – relaxation parameter, t_l – moment of addition of terms

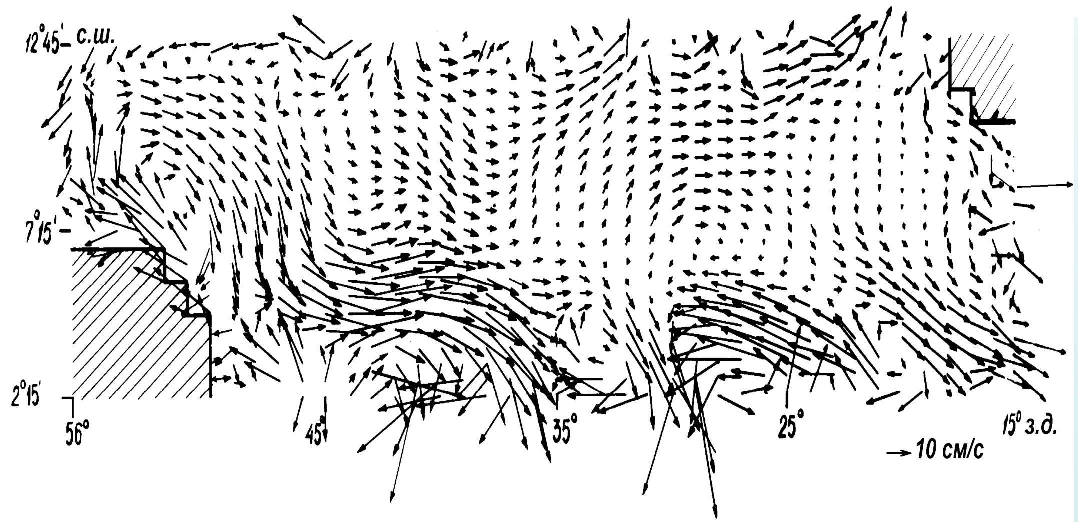
Differential equation for background error variance evolution is

$$\frac{\partial \sigma_S^2}{\partial t} + u\frac{\partial \sigma_S^2}{\partial x} + v\frac{\partial \sigma_S^2}{\partial y} + w\frac{\partial \sigma_S^2}{\partial z} = \frac{\partial}{\partial z} K_V \frac{\partial\ \sigma_S^2}{\partial z} + K_H \Delta \sigma_S^2$$ for salinity background error variance

Sea level topography, calculated in model using data assimilation



Current field on 100 m depth, calculated in model using data assimilation



The expressions for assimilation terms in equations of transport-diffusion of heat and salinity

$$Q_T(\vec{x},t_l) = \sigma_T^2(\vec{x},t_l) / REL\left[\sigma_T^2(\vec{x},t_l) + \sigma_{T_m}^2(\vec{x})\ \right]\left[\ T^{cl}(\vec{x},t_l) - T(\vec{x},t_l)\ \right]$$

$$Q_S(\vec{x},t_l) = \sigma_S^2(\vec{x},t_l) / REL\left[\sigma_S^2(\vec{x},t_l) + \sigma_{S_m}^2(\vec{x})\ \right]\left[\ S^{cl}(\vec{x},t_l) - S(\vec{x},t_l)\ \right]$$

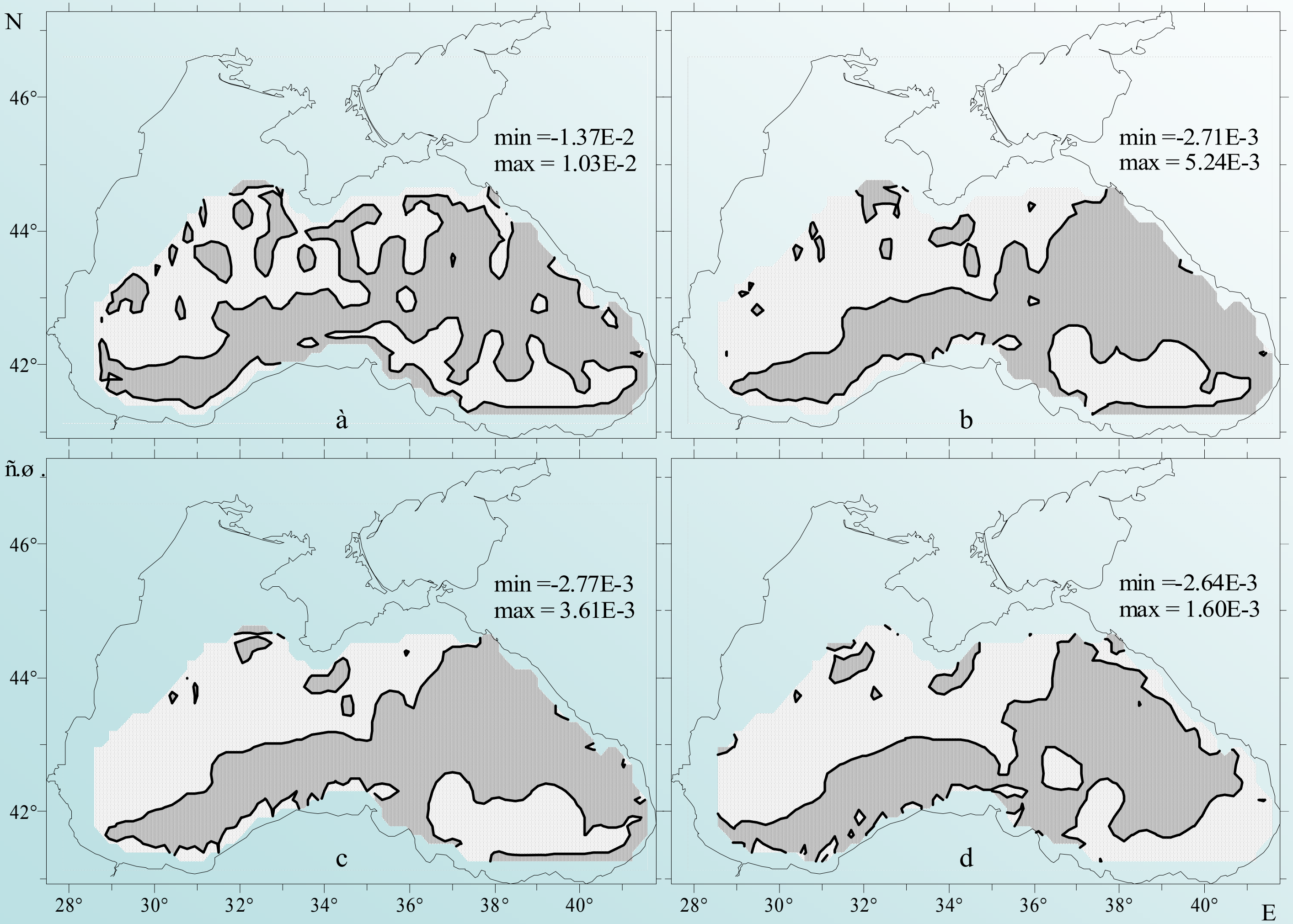
can be simplified by dividing numerator and denominator by $\sigma_T^2(\vec{x},t_l)$ $(\sigma_S^2(\vec{x},t_l))$ respectively. Denote new dimensionless quantity of measurement error, which depends only on z $\eta^2(z) = \sigma_{T_m}^2 / \sigma_T^2 = \sigma_{S_m}^2 / \sigma_S^2$, and equal both for temperature and salinity. Hence

$$Q_T(\vec{x},t_l) = 1/REL\left[\ 1+\eta^2(z)\ \right]\left[\ T^{cl}(\vec{x},t_l) - T(\vec{x},t_l)\ \right]$$

$$Q_S(\vec{x},t_l) = 1/REL\left[\ 1+\eta^2(z)\ \right]\left[\ S^{cl}(\vec{x},t_l) - S(\vec{x},t_l)\ \right]$$

This scheme was called “simplified data assimilation scheme“.

The algorithm of periodic data assimilation was designed on basis of simplified scheme of data assimilation

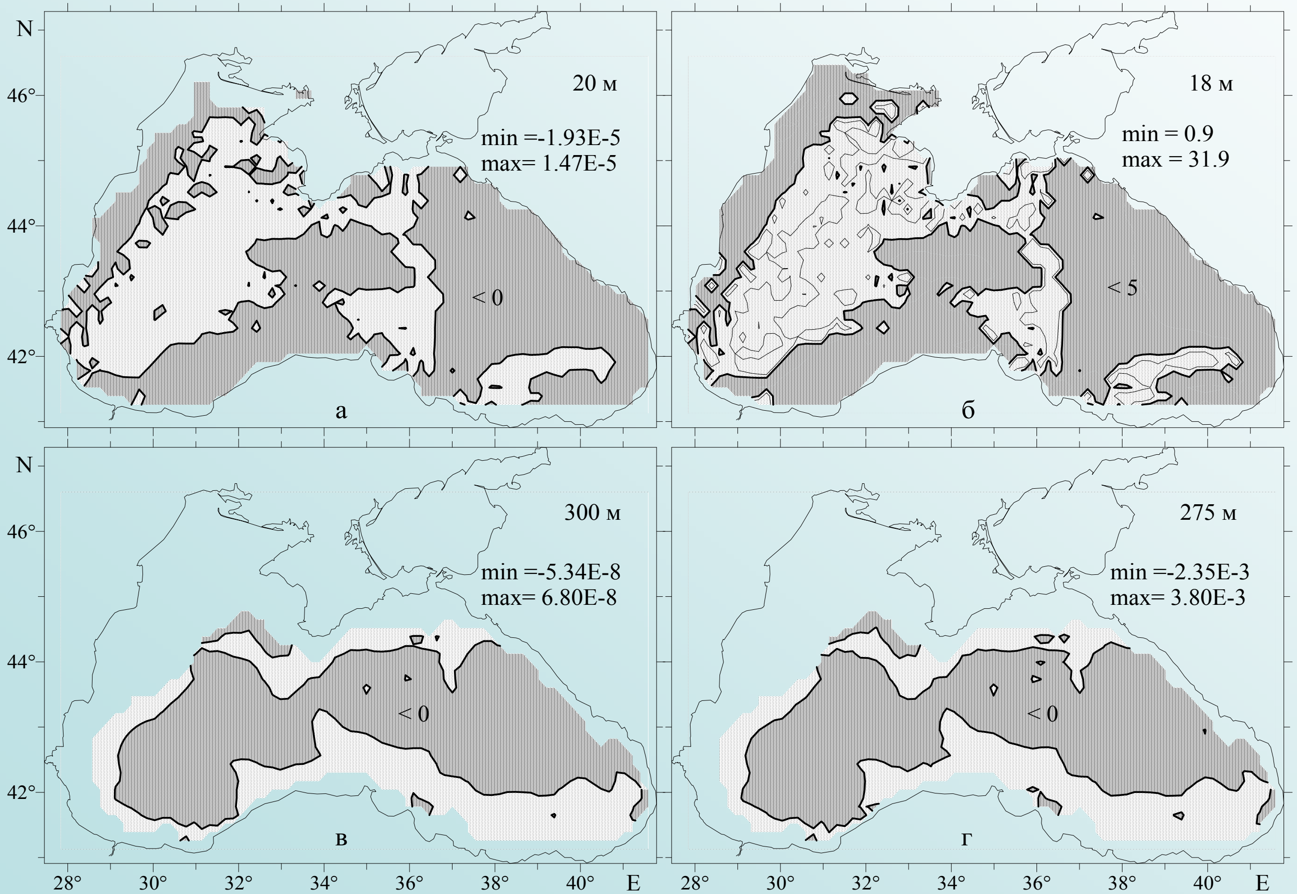


Vertical velocity fields reconstructed with simplified DA scheme and periodic DA

a – simplified DA scheme
b – 1,5 hour period
c – 3 hour period
d – 1 day period

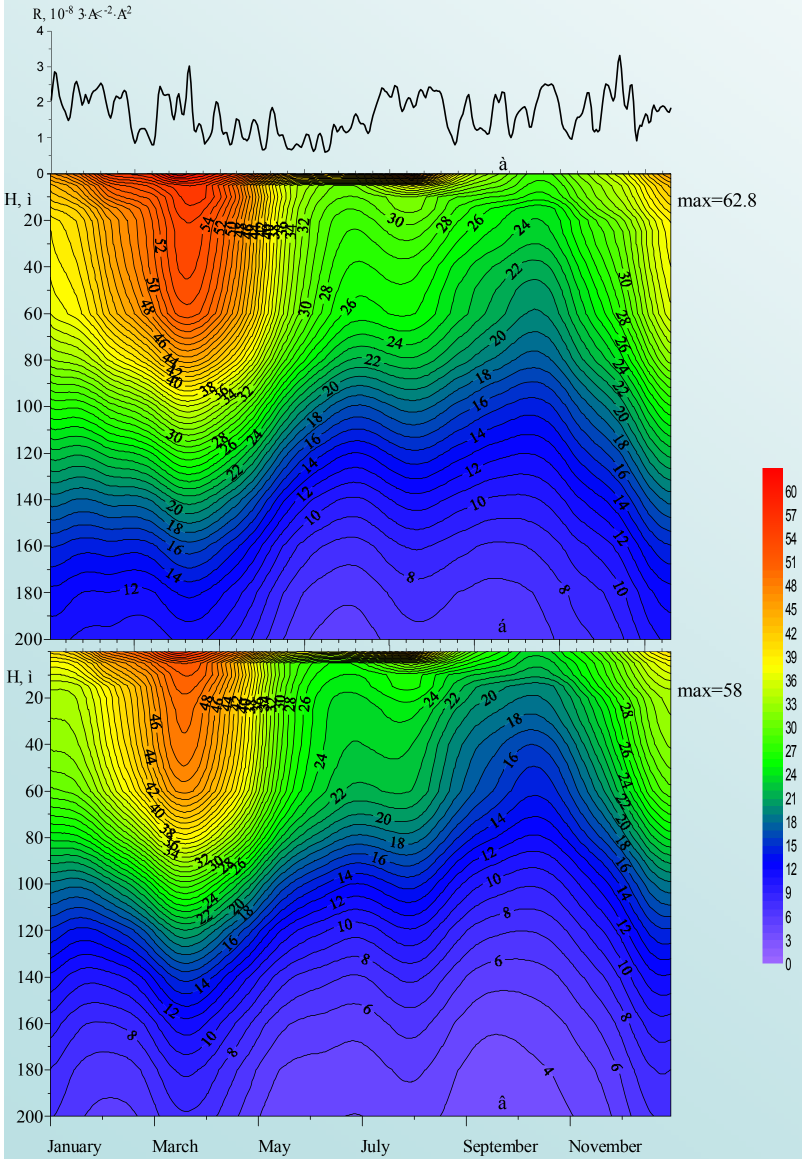
The adaptive statistics method. Differential equation for background error variance evolution with a nudging-term

$$\frac{\partial \sigma_S^2}{\partial t} + u\frac{\partial \sigma_S^2}{\partial x} + v\frac{\partial \sigma_S^2}{\partial y} + w\frac{\partial \sigma_S^2}{\partial z} = \frac{\partial}{\partial z} K_V \frac{\partial\ \sigma_S^2}{\partial z} + K_H \Delta \sigma_S^2 + \frac{1}{REL}(\sigma_{*S}^2 - \sigma_S^2).$$

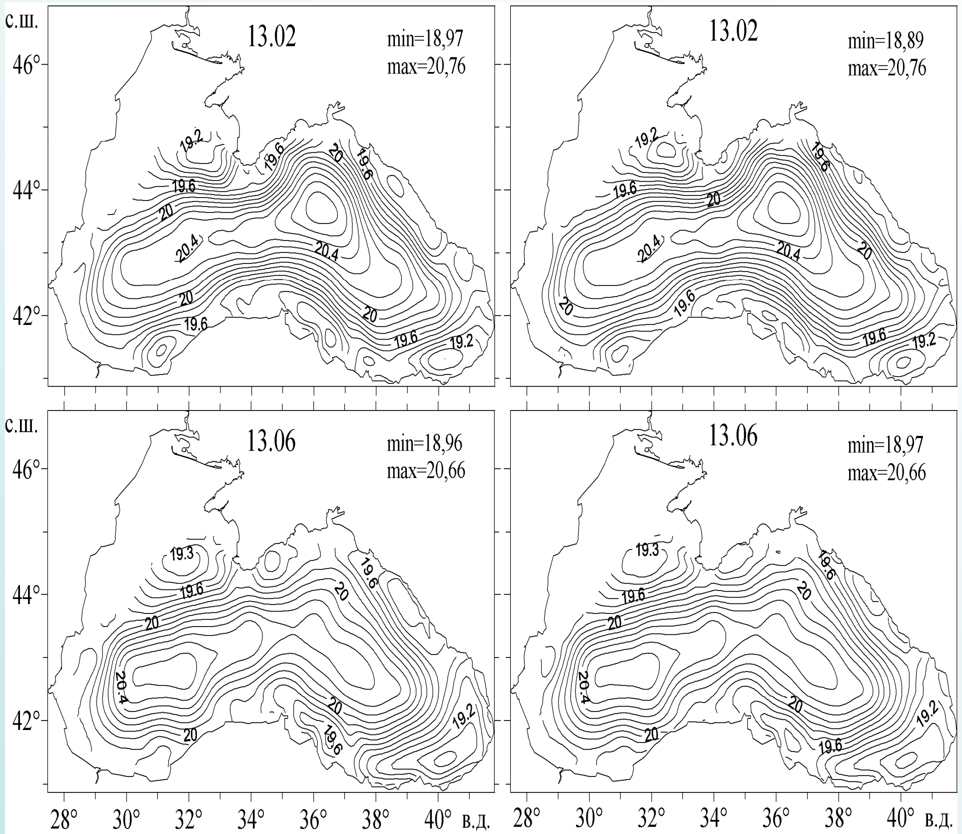


The simplified and adaptive statistics algorithms was implemented in a numerical ocean circulation model, developed in MHI (Demyshev, Knysh, Korotayev)

Time-depth diagram of kinetic energy averaged over a volume



Temperature fields, reconstructed using different DA schemes



Thanks for attention