ADAMS SECOND ANNUAL MEETING

Motion estimation using the variational data assimilation framework.
Comparison between Simple and Extended Image Model

October 29th, 2009

INSTITUT NATIONAL
DE RECHERCHE
EN INFORMATIQUE
ET EN AUTOMATIQUE

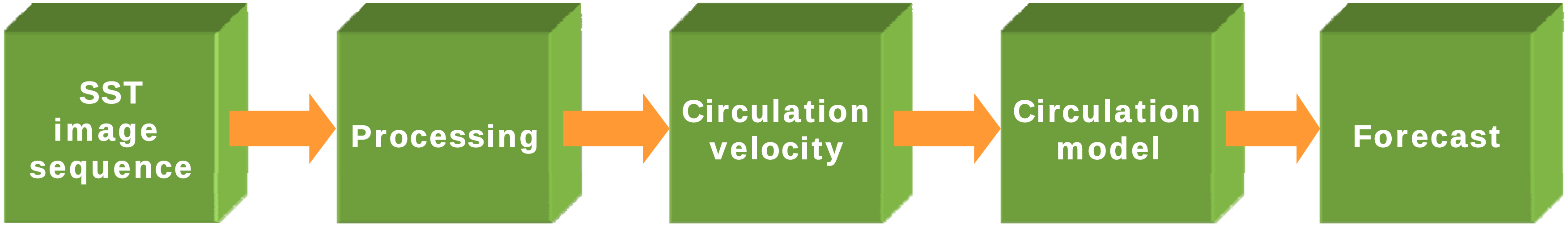
INRIA

centre de recherche
PARIS - ROCQUENCOURT

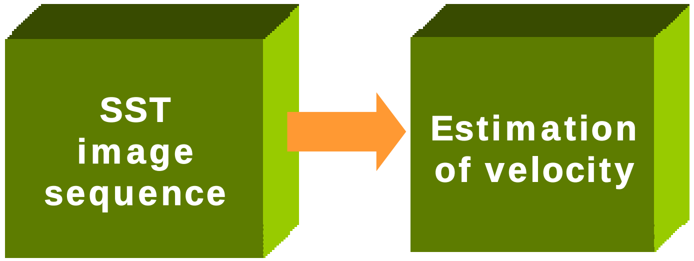
Context

1. Derive **pseudo-observations** of circulation velocity from sequence of Sea Surface Temperature (**SST**) image sequences.

2. These pseudo-observations are in turn **assimilated** in an ocean circulation model.



Estimation of the apparent velocity



- Common approach:
  - Image processing techniques (correlation, optical flow, …)
  - Problem: missing data, cloud coverage
    ➔ impossible to compute derivatives.

- Data assimilation approach:
  - Image Model: expression of the transport of temperature by surface velocity
  - Assimilation of SST within the Image Model
    ➔ estimation of initial velocity field even when data are missing.

Image Models

Simple Image Model

$$\begin{cases} \dfrac{\partial T}{\partial t} = -\mathbf{v} \cdot \nabla T + K_T \Delta T \\ \\ \dfrac{\partial u}{\partial t} = 0 \\ \\ \dfrac{\partial v}{\partial t} = 0 \end{cases}$$

Extended Image Model

$$\begin{cases} \dfrac{\partial T}{\partial t} = -\nabla T \cdot \mathbf{v} + K_T \Delta T \\ \\ \dfrac{\partial u}{\partial t} = -\nabla u \cdot \mathbf{v} + fv + g' \dfrac{\partial \eta}{\partial x} + K_{\mathbf{v}} \Delta u \\ \\ \dfrac{\partial v}{\partial t} = -\nabla v \cdot \mathbf{v} - fu + g' \dfrac{\partial \eta}{\partial y} + K_{\mathbf{v}} \Delta v \\ \\ \dfrac{\partial \eta}{\partial t} = -\dfrac{\partial (u\eta)}{\partial x} + \dfrac{\partial (v\eta)}{\partial y} - h_m \left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} \right) \end{cases}$$

Configuration of the assimilation process #1

- Perfect model
- No assumption on the *background* term
- Uncorrelated observations.
- Control variable = initial conditions

$$J_0(X_0) \;=\; \frac{1}{2}\int_{\mathcal{D}} (T(X_0) - T_{obs})^2 d\mathcal{D}$$

Configuration of the assimilation process #2

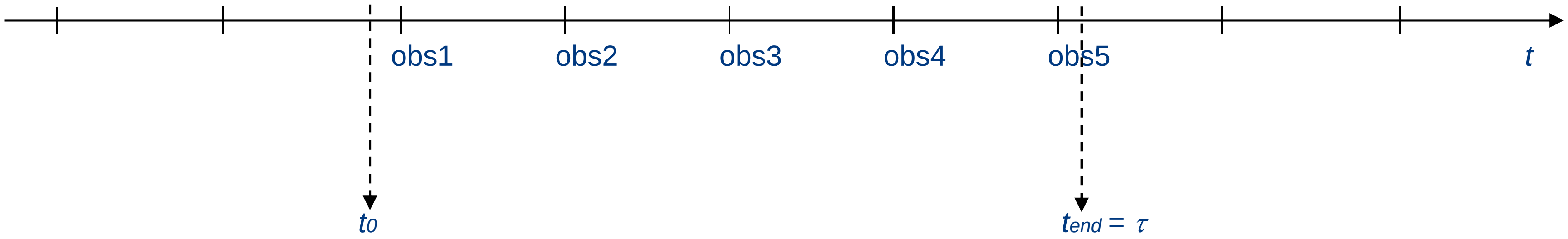
- Practical cost functions:

$$J_1 = \frac{1}{2}\int_{\mathcal{D}}[(T - T_{obs})^2 + \frac{\alpha}{2}(|\bigtriangledown u|^2 + |\bigtriangledown v|^2)]d\mathcal{D}$$

$$J_2 = \frac{1}{2}\int_{\mathcal{D}}[(T - T_{obs})^2 + \frac{\lambda}{2}(\alpha|\bigtriangledown \mathrm{div}\,\mathbf{v}|^2 + \beta|\bigtriangledown \mathrm{curl}\mathbf{v}|^2)]d\mathcal{D}$$

Configuration of the assimilation process #3

- Estimation of initial conditions



- Background terms
  - The background X_b is not used as a regularization term but it doesn't mean it isn't important.
  - $T_b = T_{obs1}$, (u_b,v_b) can be taken equal to zero (no assumptions) or computed by image processing softwares: such as Horn & Shunk method or GFME (Geofluid Motion Estimation).

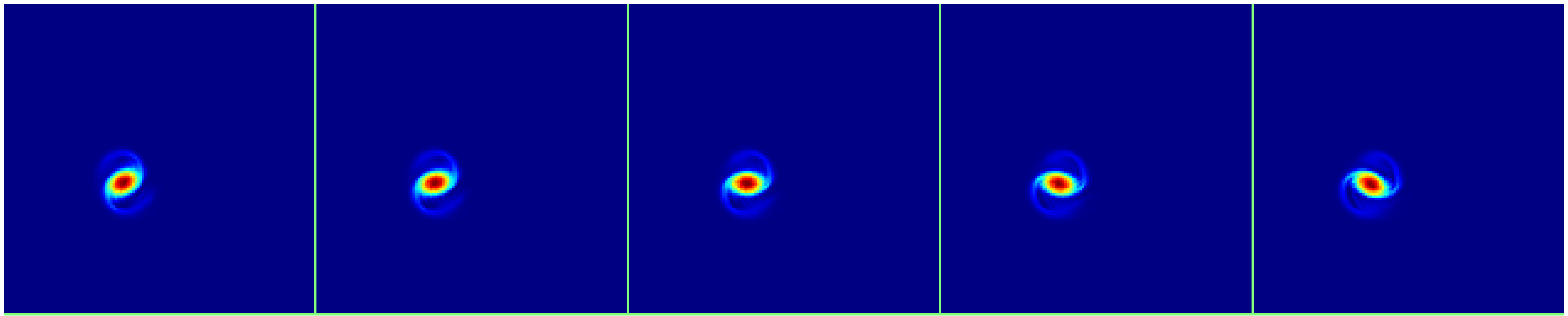
Notations

SIM We call here ***Simple Image Model*** (***SIM***) the assimilation software using the image model based on the *advection-diffusion of temperature* and *frozen velocity hypothesis.*

EIM We call here ***Extended Image Model*** (***EIM***) the assimilation software using the image model based on the *advection-diffusion of temperature* and the *shallow-water equations.*

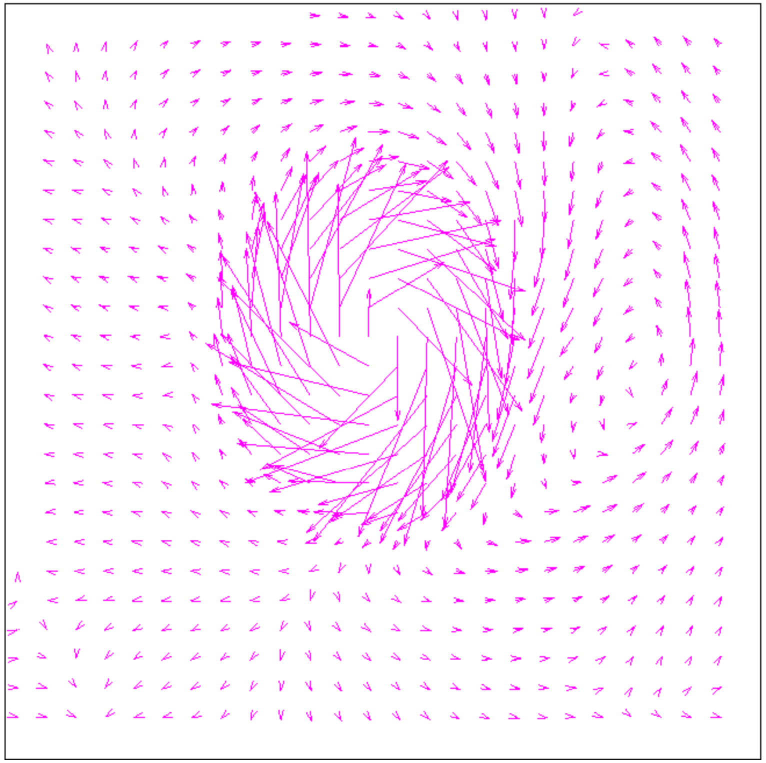
Parameters for the *SIM*

- The *SIM* depends on several parameters:
  - The choice of the cost function *(J1, J2, …)* and the value of their parameters,
  - The number of observation used,
  - The *background* values for T_b, u_b and,v_b.

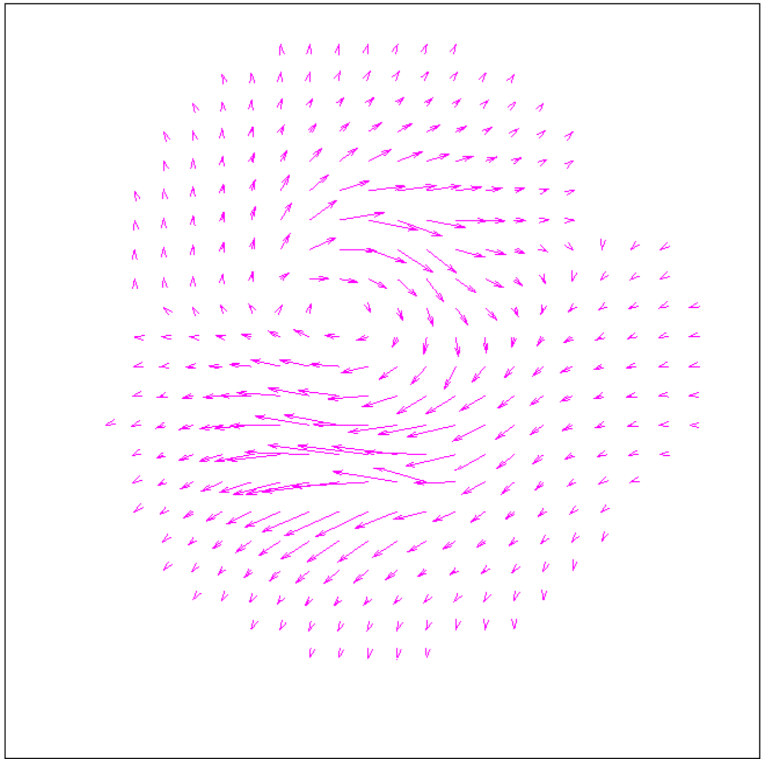


Simulation of the experiemental Coriolis platform.

Estimation with *J*1

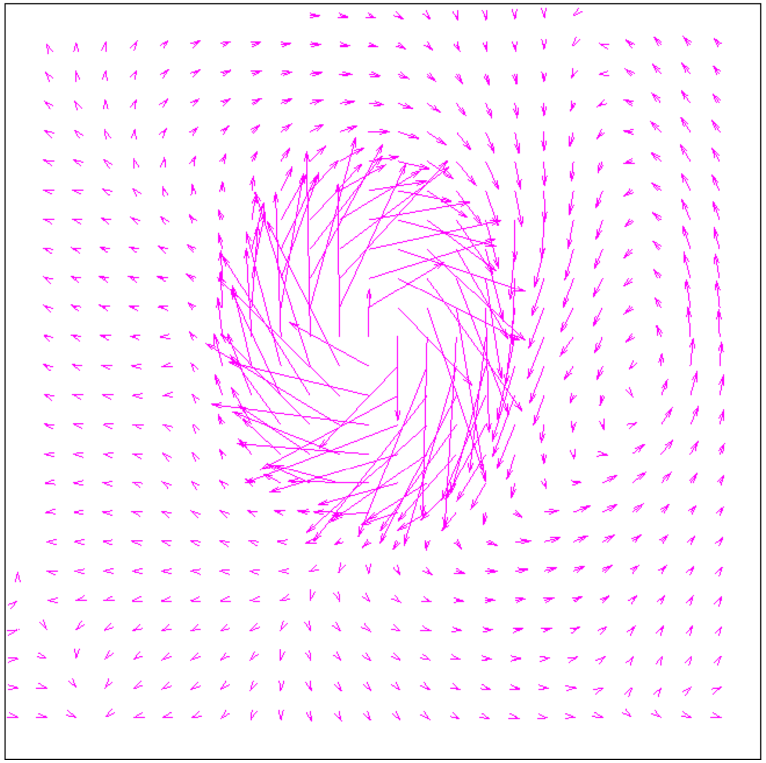


Real velocity field

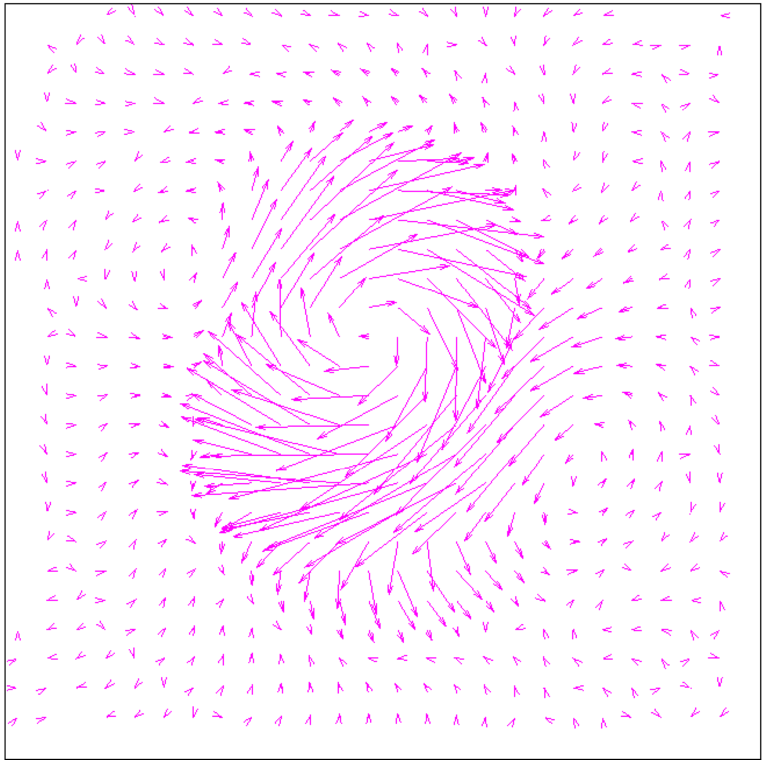


Velocity field estimated with *J*1

Estimation with *J*2

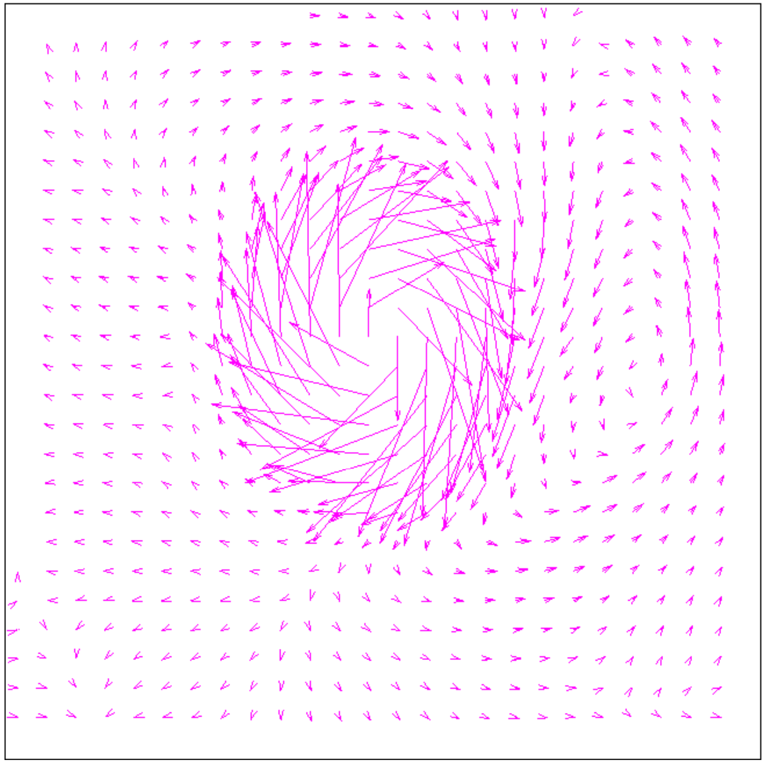


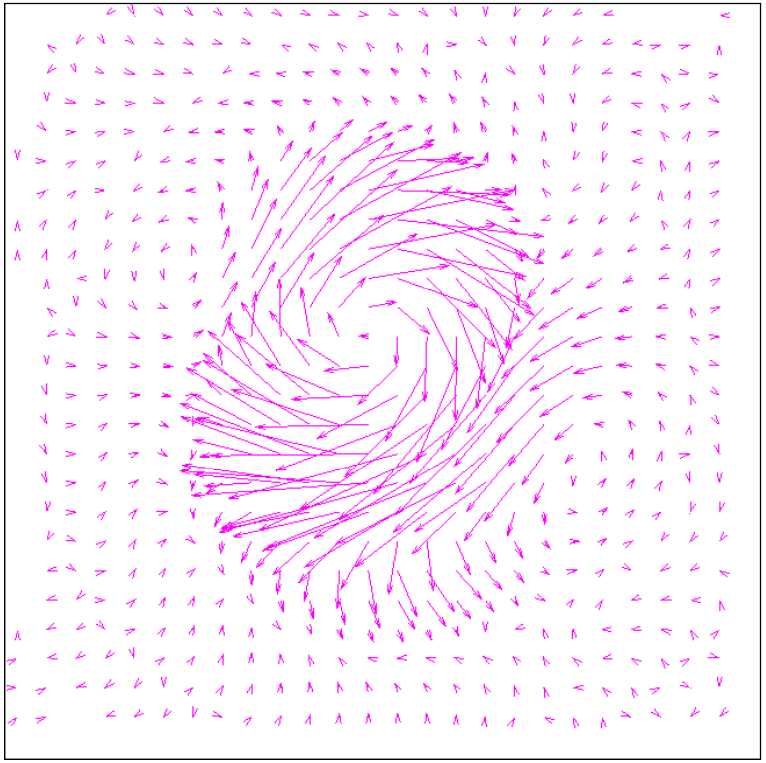
Real velocity field



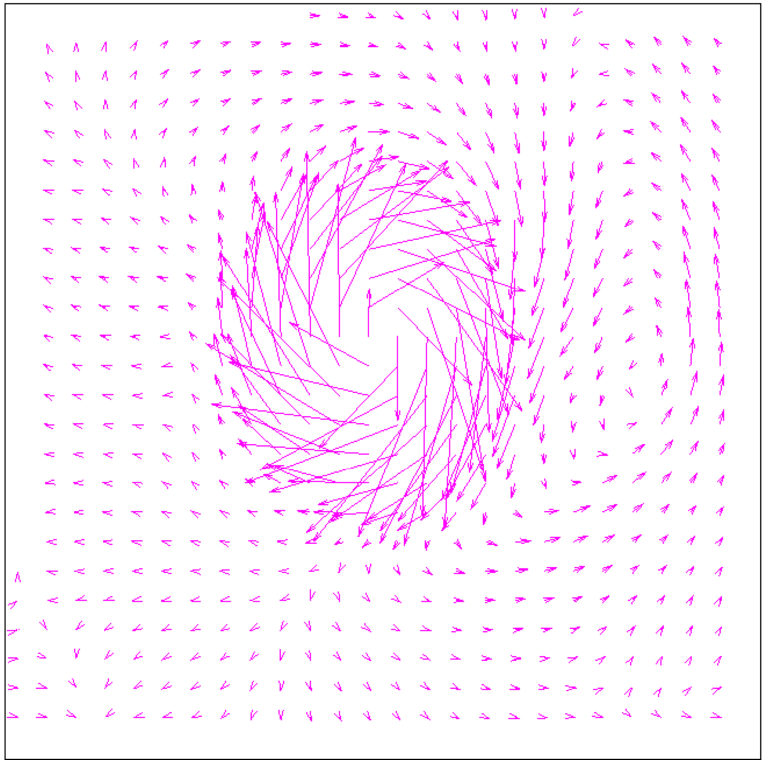
Velocity field estimated with *J*2

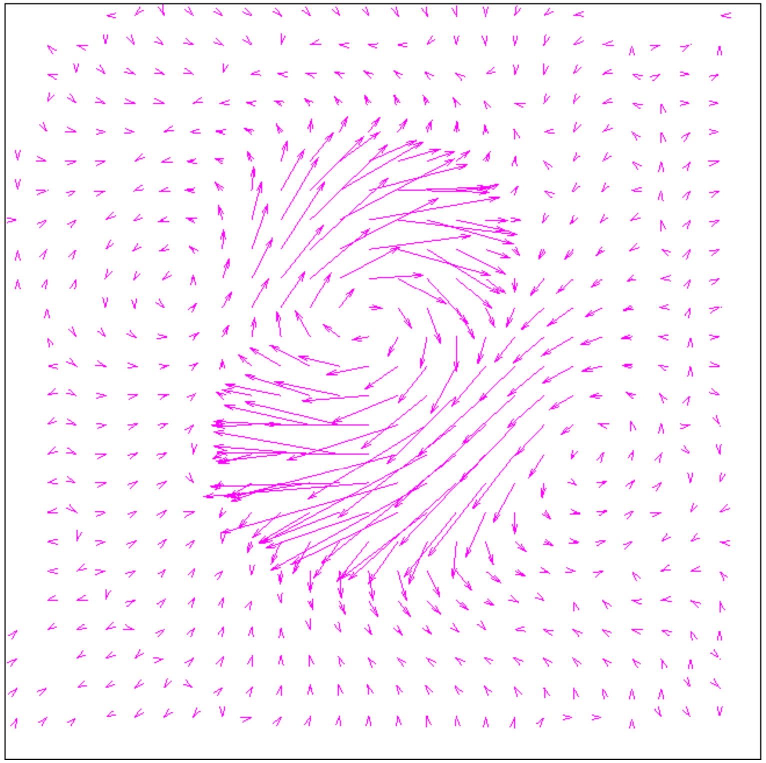
Estimation with 5 images



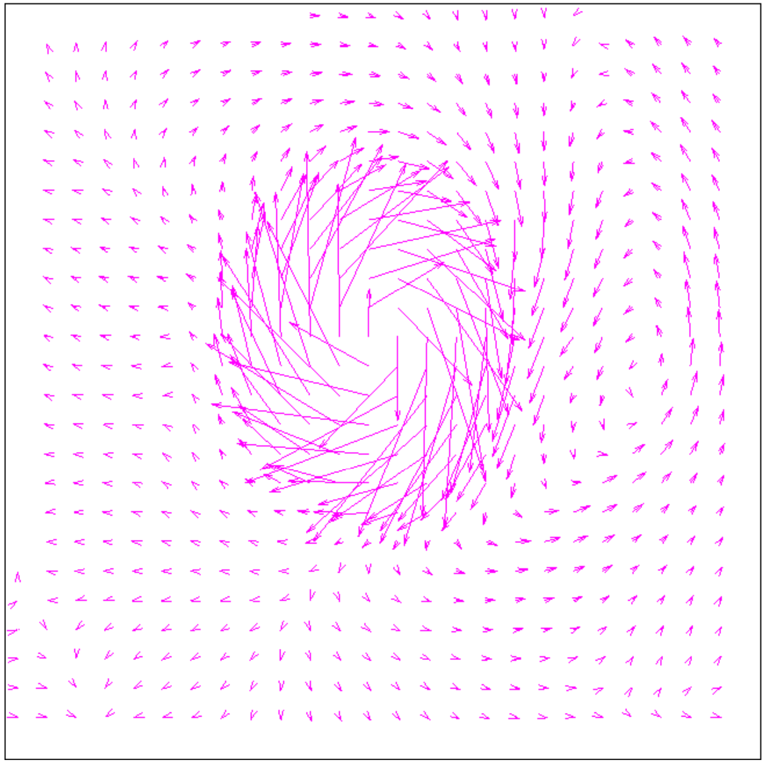


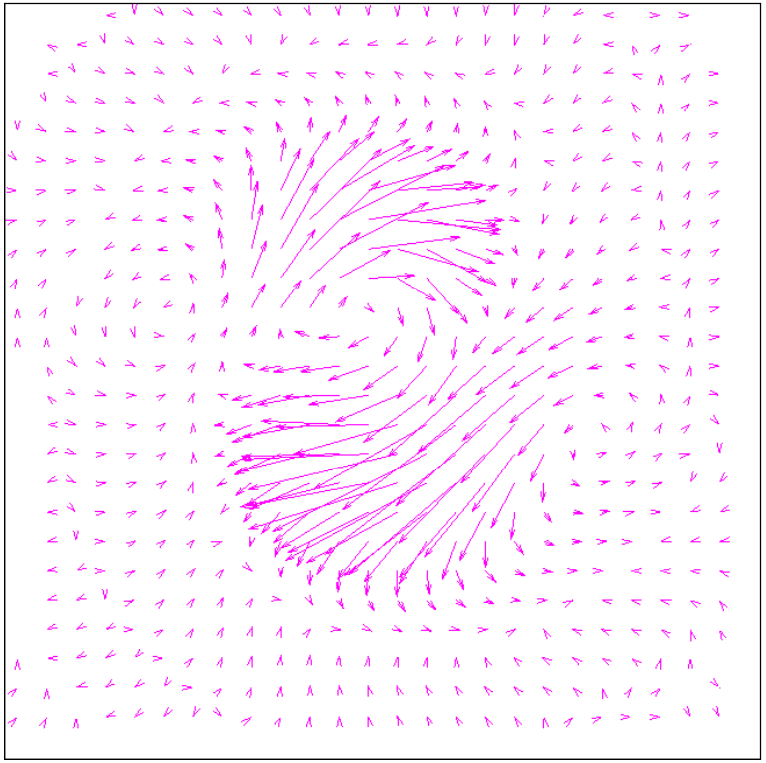
Estimation with 4 images



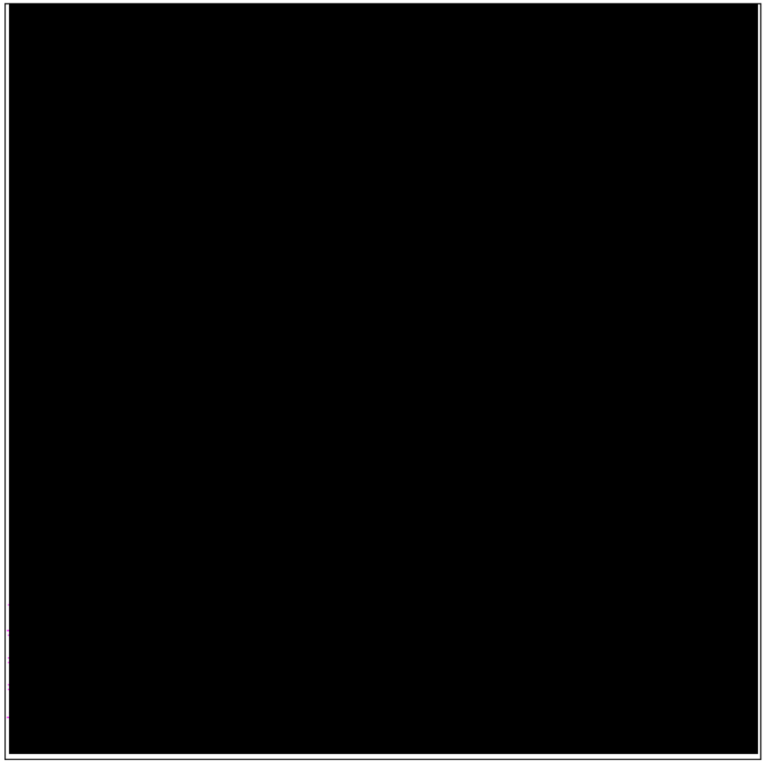


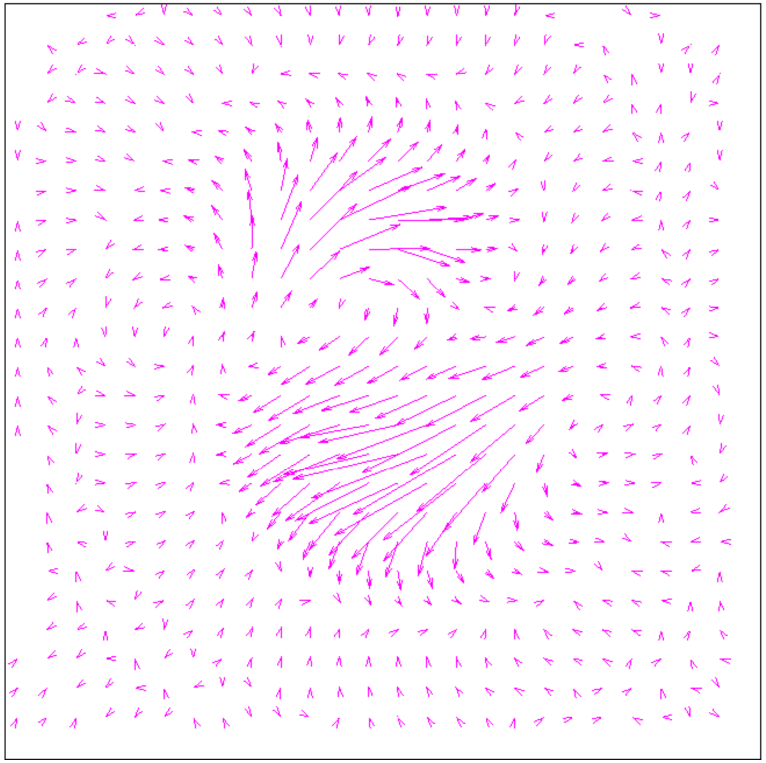
Estimation with 3 images



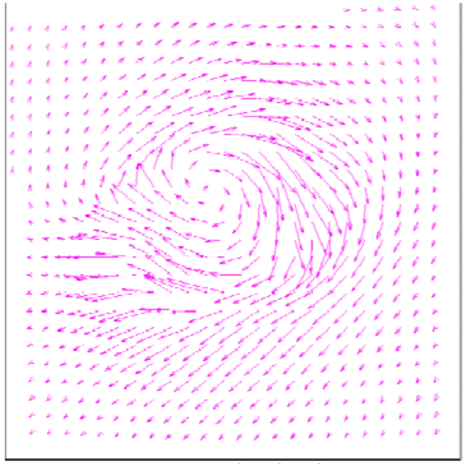


Estimation with 2 images

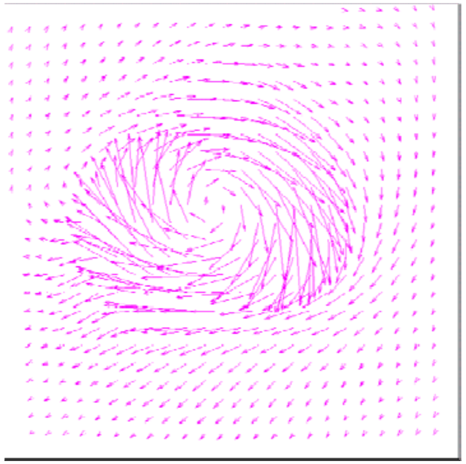




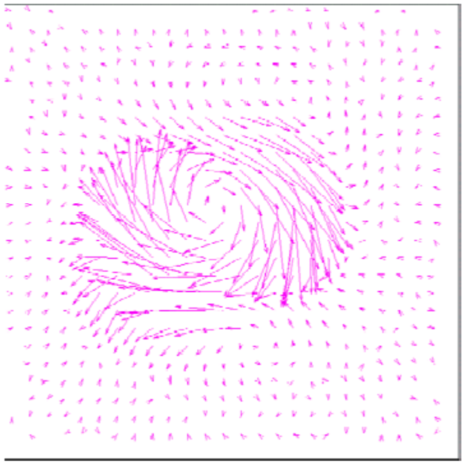
Background velocity



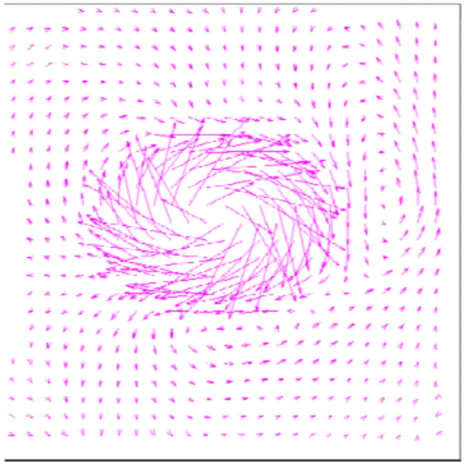
Horn and Schunk



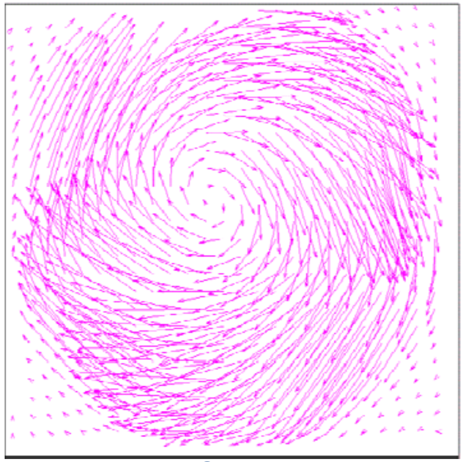
SIM and background velocity = Horn & Schunk



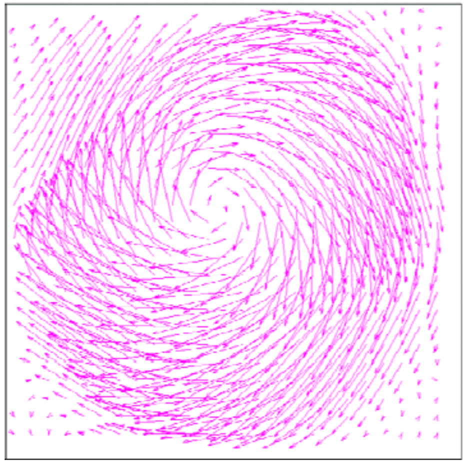
SIM and background velocity = 0



ground truth

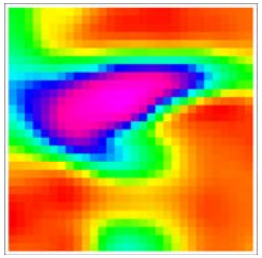


GFME

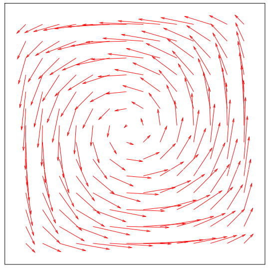


SIM and background velocity = GFME

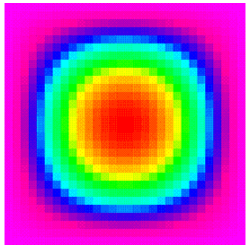
Comparison between the two models



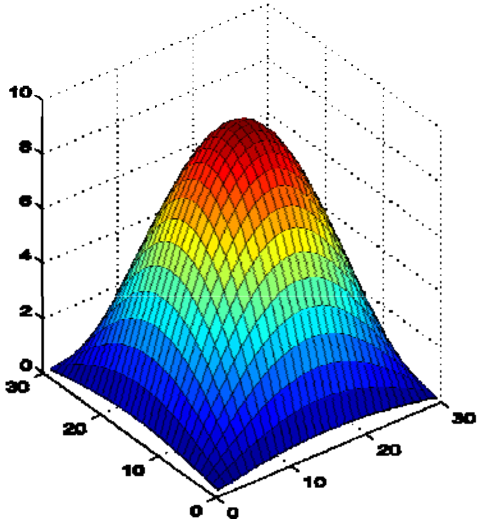
Initial Temperature

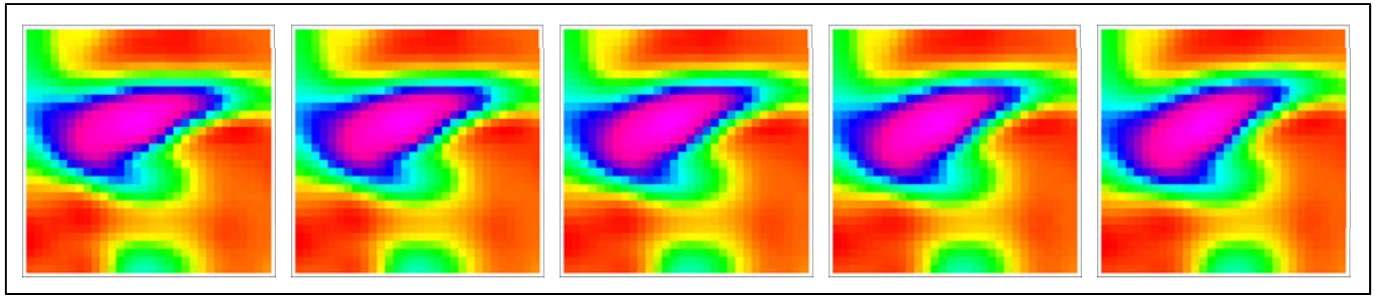


Initial velocity



Initial η





Five observation computed by the Extended Image Model (*shallow water*)

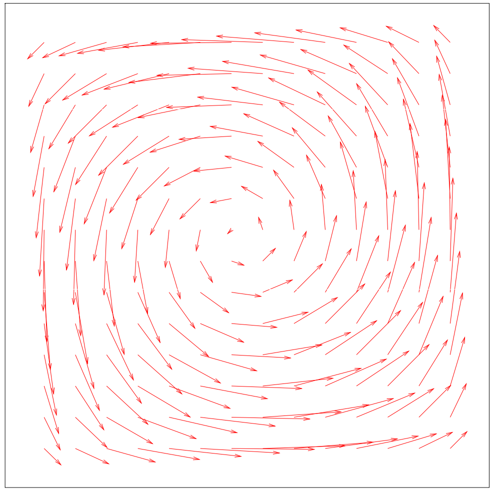
SIM

J2 + 5 images

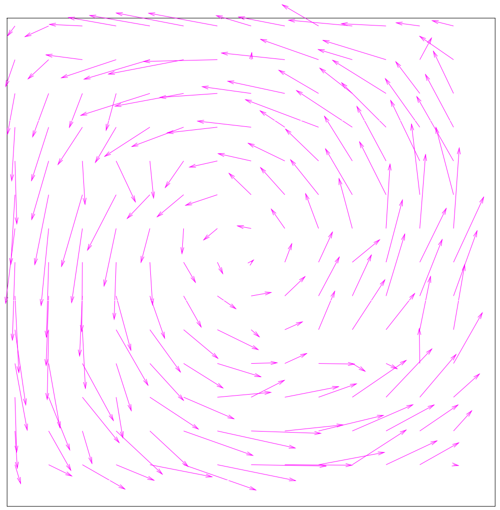
Background conditions:

$T_b = T_{obs1}$

$u_b = v_b = 0$



Ground truth



Estimation

EIM (twin experiment 1)

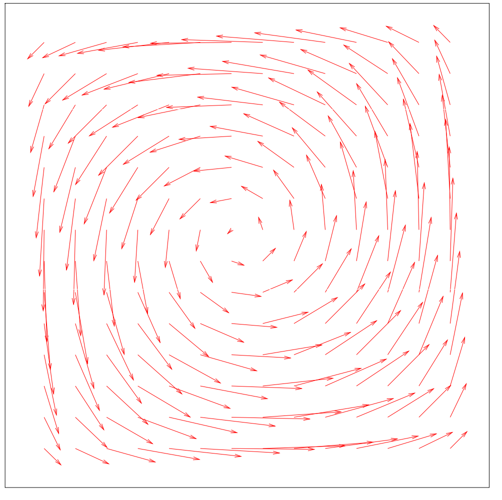
J2 + 5 images

Background conditions:

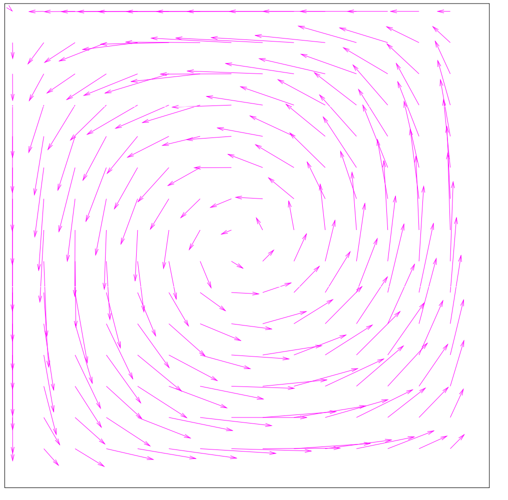
$T_b = T_{obs1}$

$u_b = v_b = 0$

$\eta_b = \eta_0$ exact.



Ground truth



Estimation

EIM (twin experiment 2)

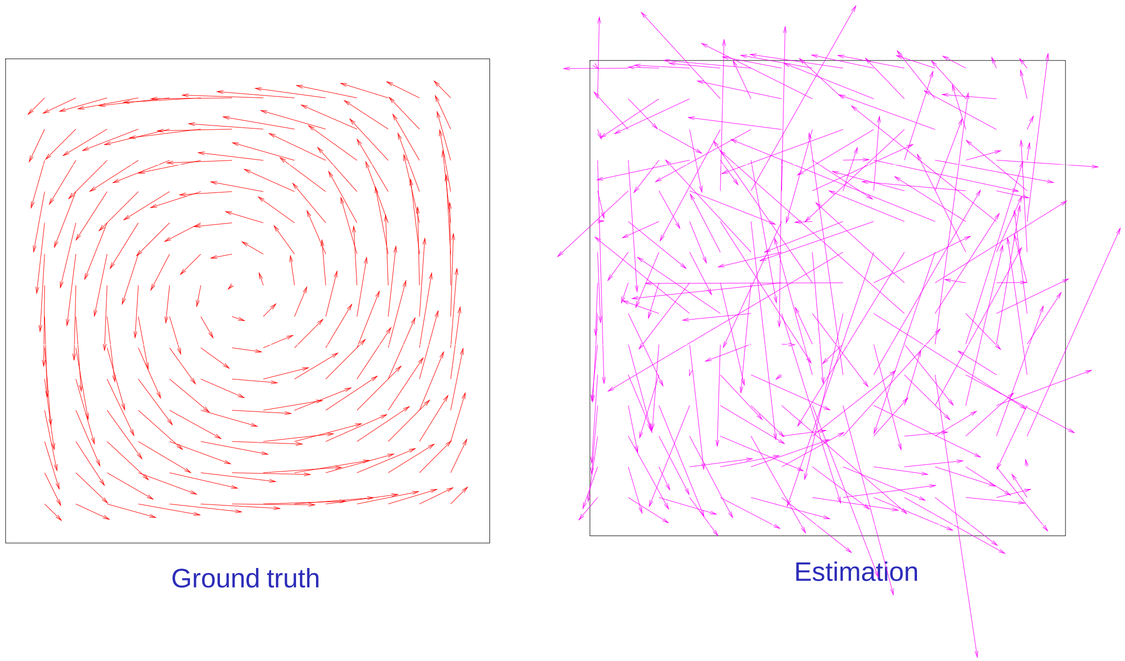
J2 + 5 images

Background conditions:

$T_b = T_{obs1}$

$u_b = v_b = 0$

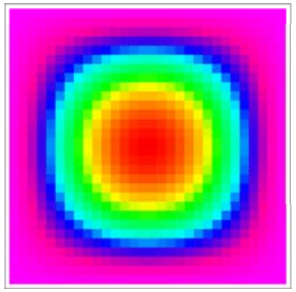
$\eta_b = 0.$

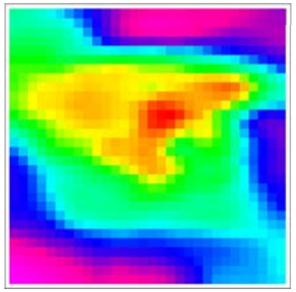


EIM (twin experiment 2)

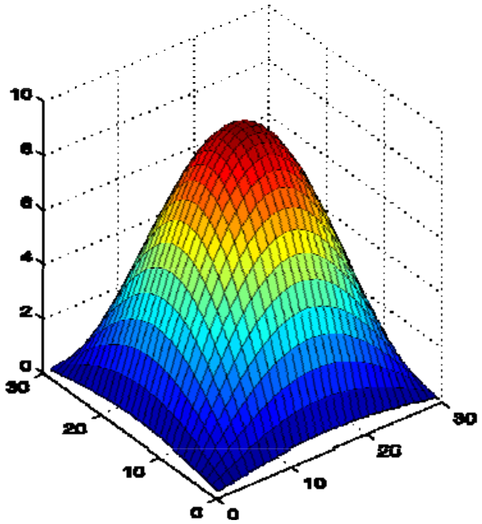
Estimation of η_0:

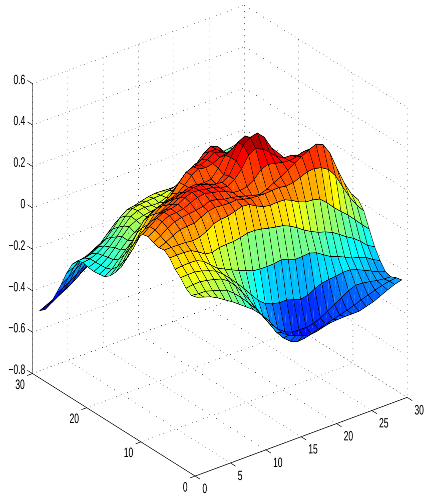
Ground truth for η





Estimated η





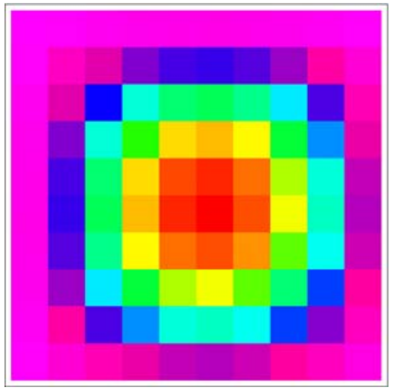
EIM (twin experiment 3)

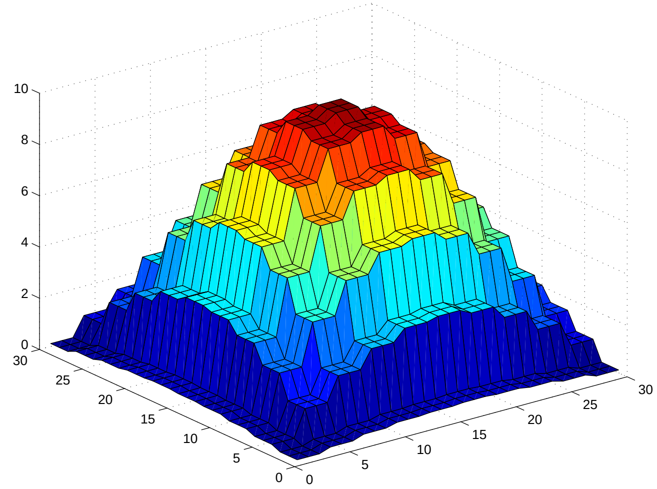
Background conditions:

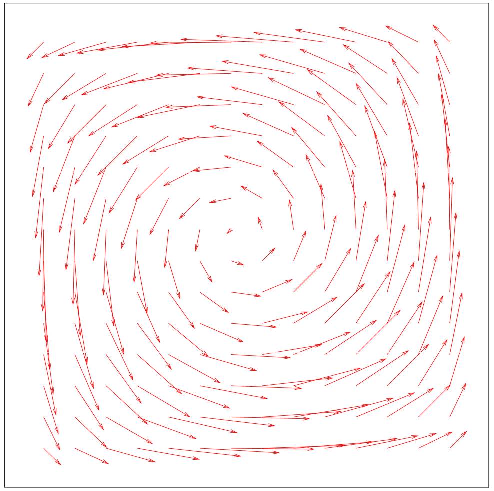
$T_{\mathrm{b}} = T_{\mathrm{obs1}}$

$u_{\mathrm{b}} = v_{\mathrm{b}} = 0$

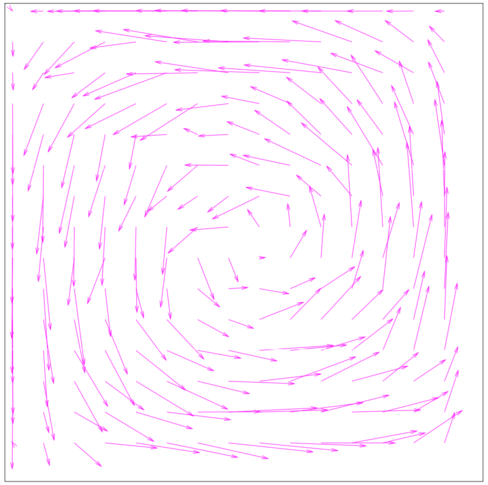
$\eta_{\mathrm{b}} = \eta_0$ degraded $\rightarrow$





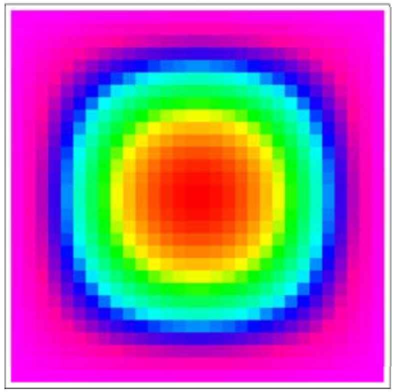


Ground truth

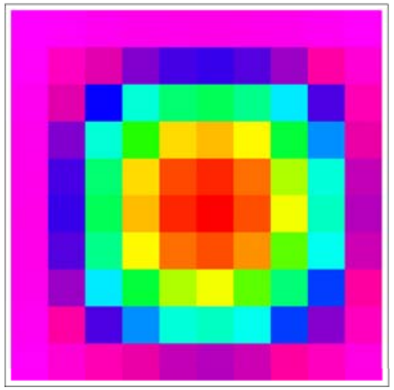


Estimation

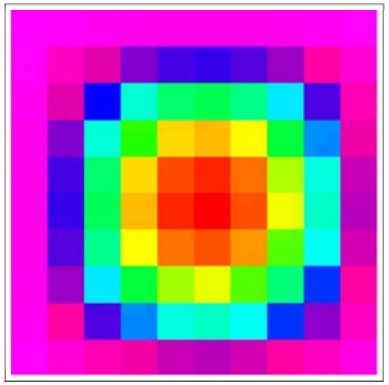
EIM (twin experiment 3)



η_0 ground truth



η_b



η_0 estimated

Introduction of a *weighting* coefficient

To reduce the influence on the η component against other components of the state variable, we introduced a coefficient w into the image model:

$$\begin{cases} \dfrac{\partial T}{\partial t} = -\nabla T \cdot \mathbf{v} + K_T \Delta T \\[2ex] \dfrac{\partial u}{\partial t} = -\nabla u \cdot \mathbf{v} + f v + \dfrac{g'}{w}\dfrac{\partial \eta}{\partial x} + K_{\mathbf{v}} \Delta u \\[2ex] \dfrac{\partial v}{\partial t} = -\nabla v \cdot \mathbf{v} - f u + \dfrac{g'}{w}\dfrac{\partial \eta}{\partial y} + K_{\mathbf{v}} \Delta v \\[2ex] \dfrac{\partial \eta}{\partial t} = -\dfrac{\partial (u\eta)}{\partial x} + \dfrac{\partial (v\eta)}{\partial y} - h_m \times w \left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} \right) \end{cases}$$

EIM (twin experiment 2bis)

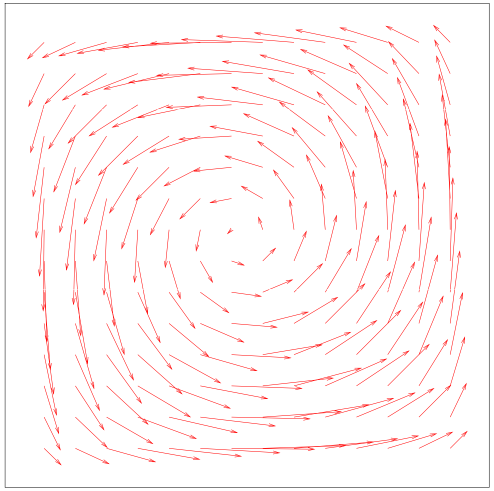
Using the weghting coefficient:

Background conditions:

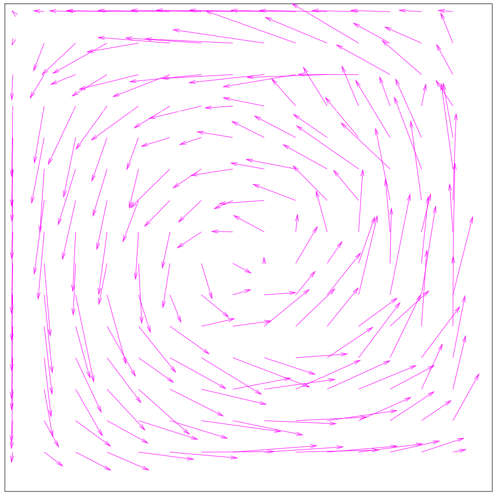
$T_{\mathrm{b}} = T_{\mathrm{obs1}}$

$u_{\mathrm{b}} = v_{\mathrm{b}} = 0$

$\eta_{\mathrm{b}} = 0.$



Ground truth

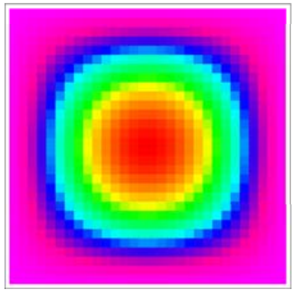


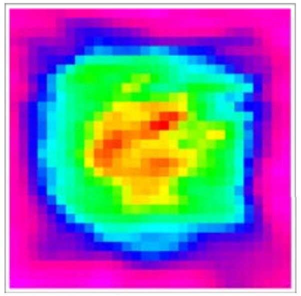
Estimation

EIM (twin experiment 2bis)

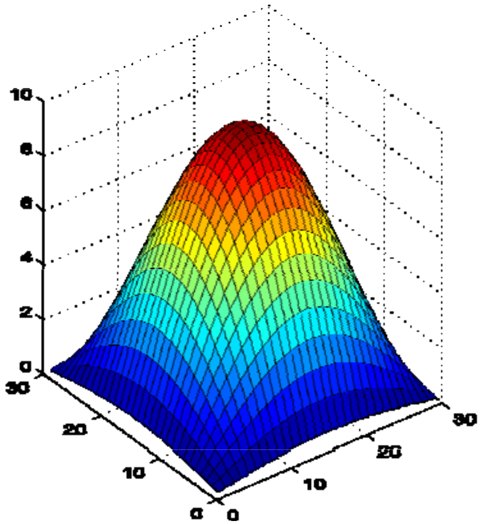
Estimation of η_0:

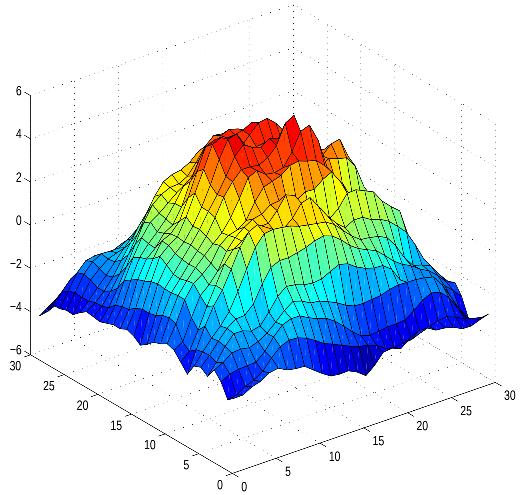
Ground truth for η





Estimated η





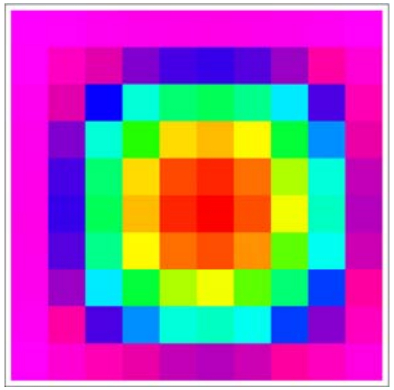
EIM (twin experiment 3bis)

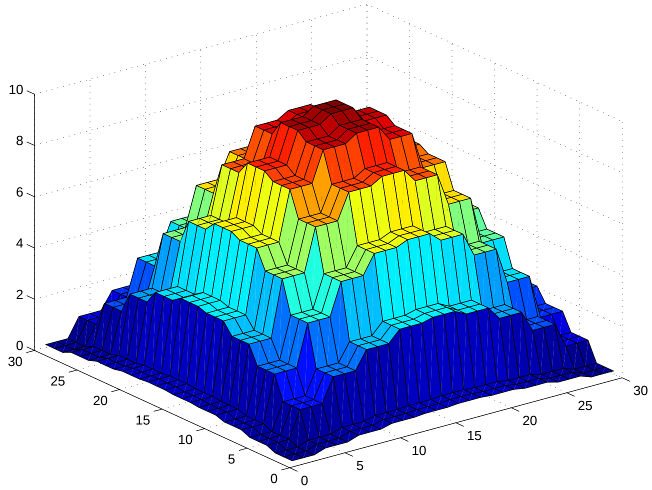
Background conditions:

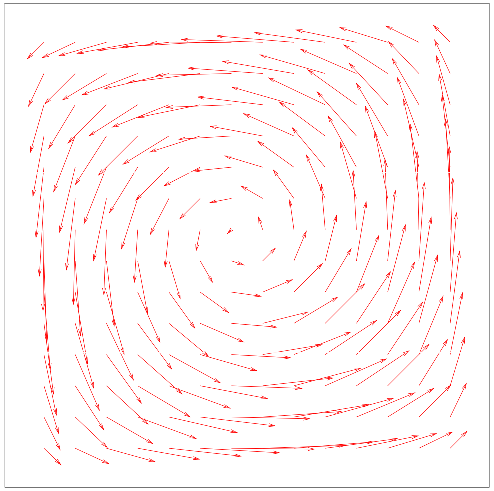
$T_{\mathrm{b}} = T_{\mathrm{obs1}}$

$u_{\mathrm{b}} = v_{\mathrm{b}} = 0$

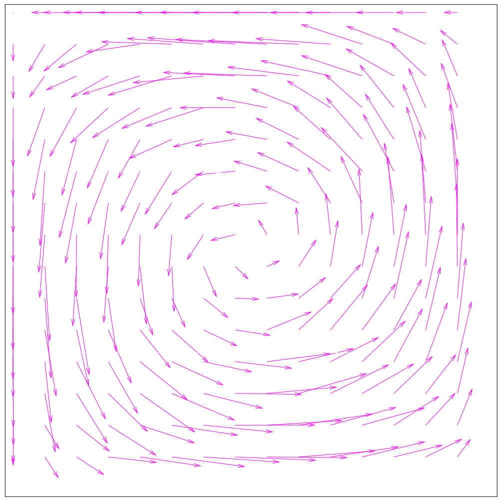
$\eta_{\mathrm{b}} = \eta_0$ degraded $\rightarrow$







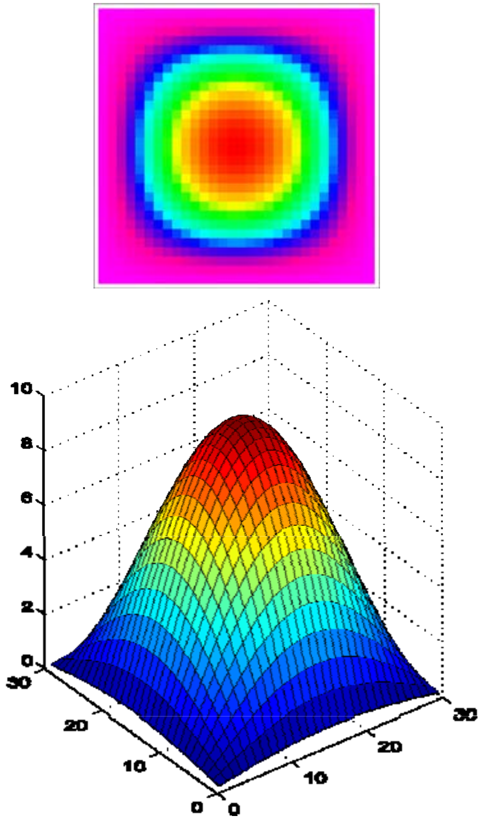
Ground truth



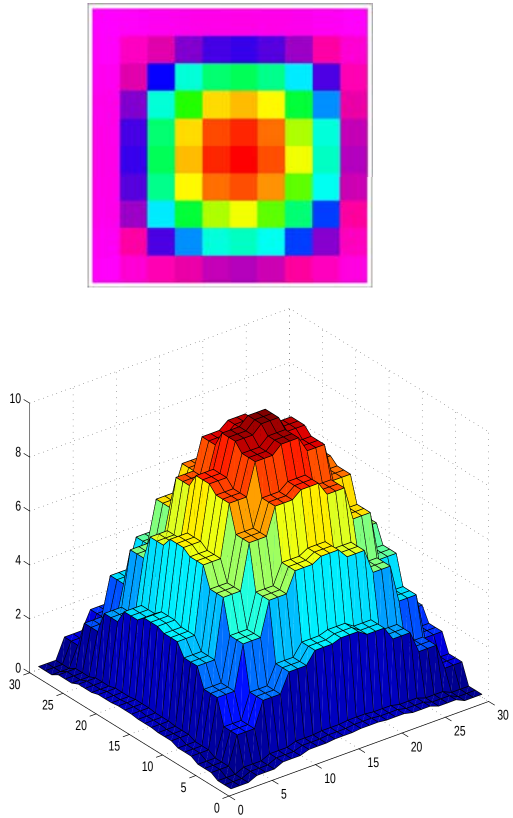
Estimation

EIM (twin experiment 3bis)

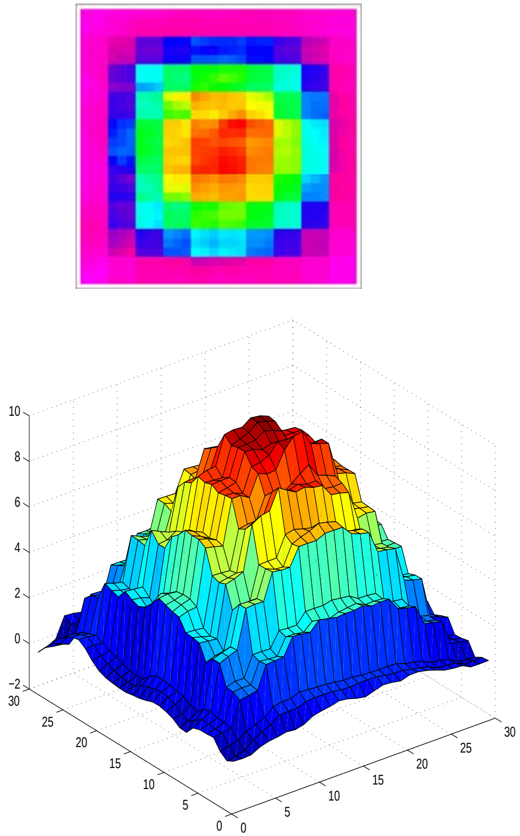
Ground truth for η



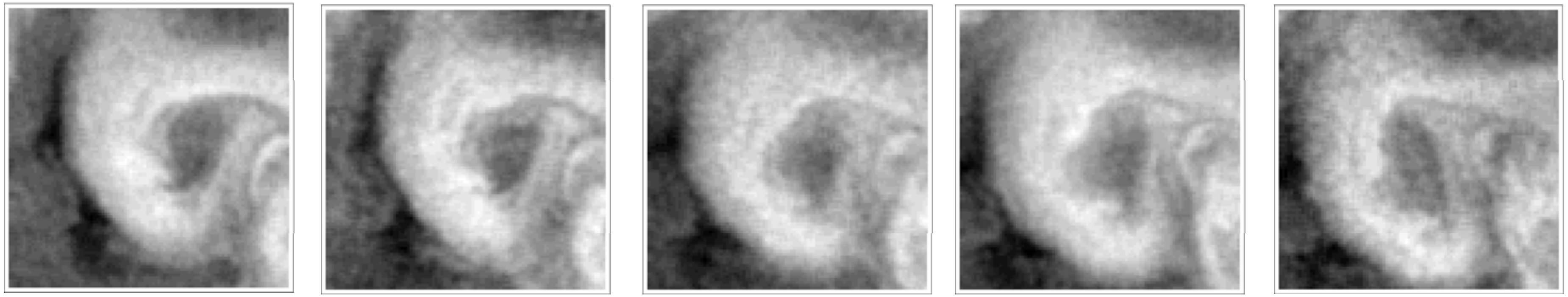
η_b



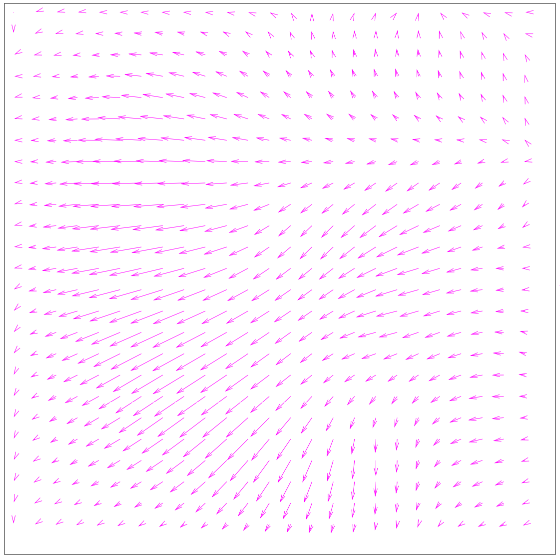
Estimated η



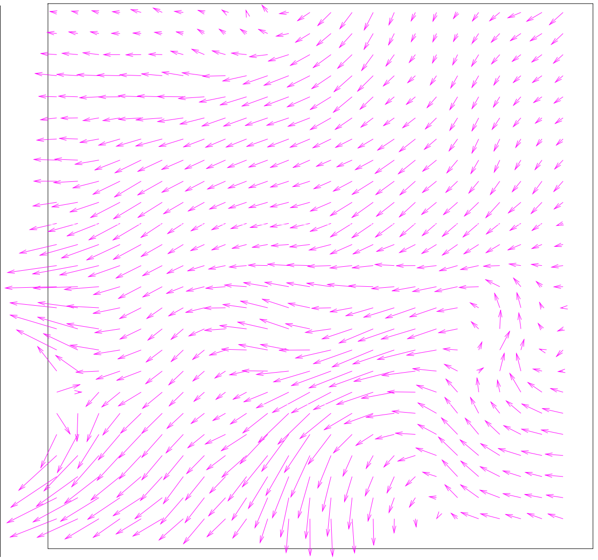
Satellite images



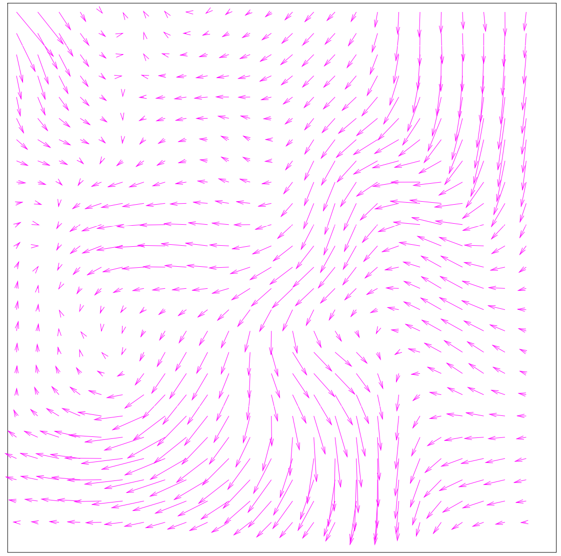
SIM results



Estimation using
J1



Estimation
computed at MHI



Estimation using
J1 + background
from GFME

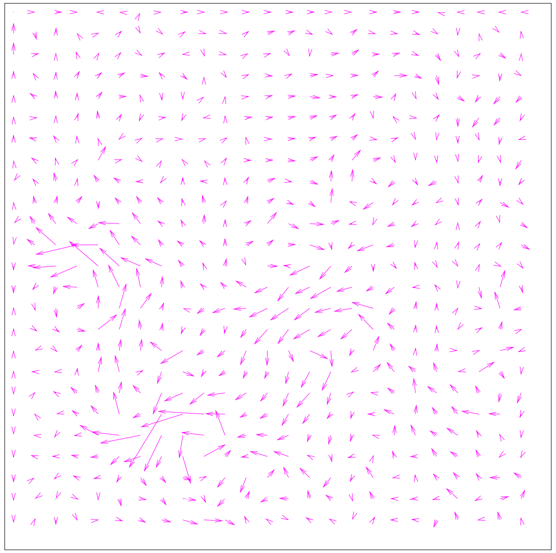
EIM results

background:

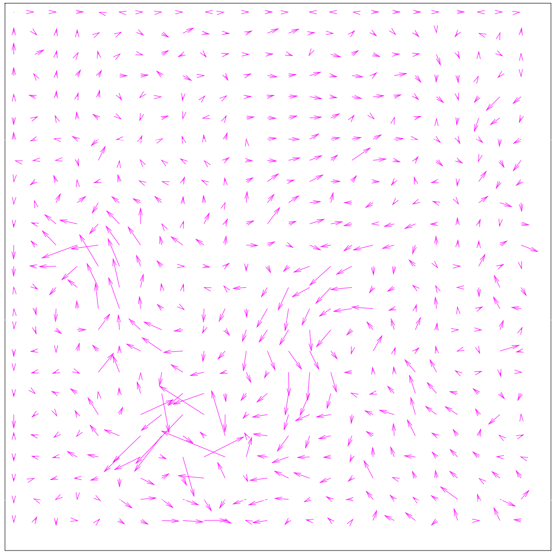
$T_\mathrm{b} = T_\mathrm{obs1}$

$u_\mathrm{b} = v_\mathrm{b} = 0$

$\eta_\mathrm{b} = 0.$



Estimation using
J1



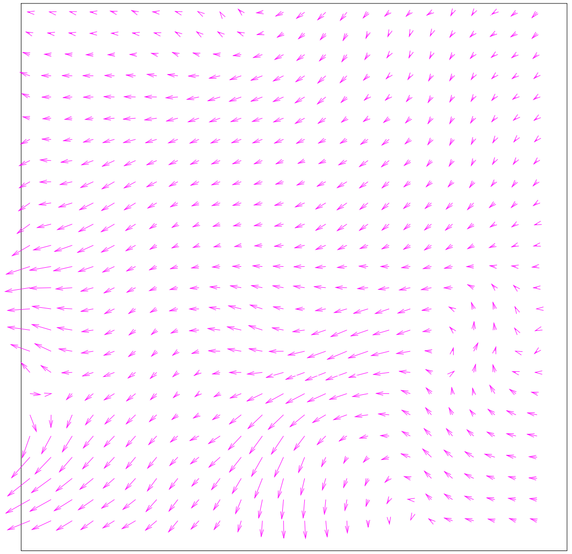
Estimation using
J2

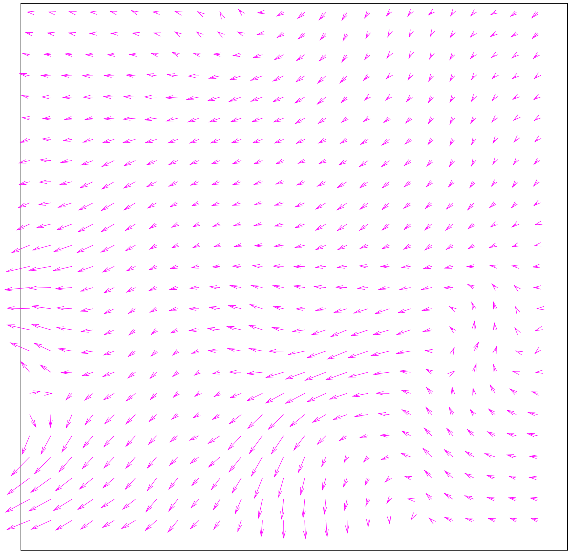
background:

$T_\mathrm{b} = T_\mathrm{obs1}$

$u_\mathrm{b} = v_\mathrm{b}$: computed with *SIM*

$\eta_\mathrm{b} = 0.$





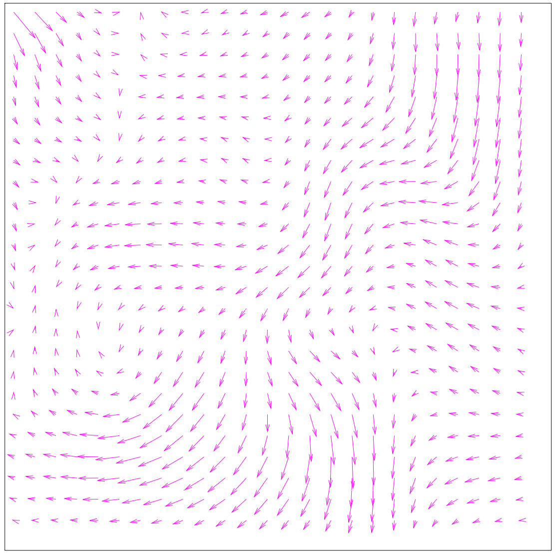
EIM results

background:

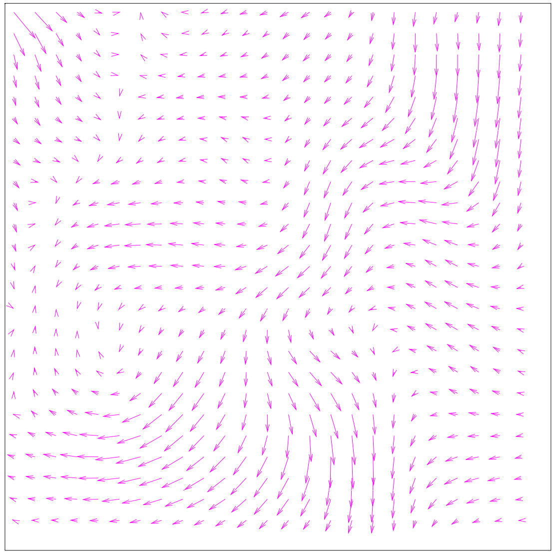
$T_{\rm b} = T_{\rm obs1}$

$u_{\rm b} = v_{\rm b}$ computed with GFME

$\eta_{\rm b} = 0$.



Estimation using *J1*



Estimation using *J2*

Conclusion and perspectives

- The quality of the estimation strongly depends on the *background* conditions – especially for the η component – even with the regularization term.

- How to improve the approach?
  - By taking into account the $\mathbf{R}$ and $\mathbf{B}$ matrices in the assimilation process.
  - By taking into account a non-perfect *Image Model*.