On the Assimilation of High Resolution tracer images in mesoscale ocean models using Lagrangian Coherent Structures maps

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ANR ADDISA Project

Project-Team INRIA MOISE

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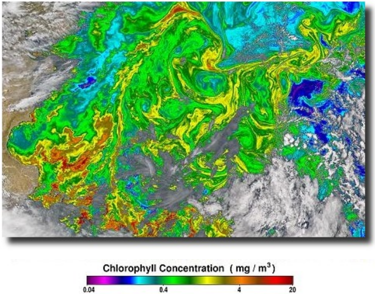
Phytoplankton bloom (Malvinas currents)

December 6, 2006
MODIS Aqua

True Colors



- Ocean Color images show **patterns** that are not only due to biochemical processes. They are also **linked with the system dynamic** through shear and mixing processes.

Direct Image Sequences Assimilation

Lagrangian Coherent Structures

The use of FSLE / FTLE maps in Image Structures Assimilation

Direct Image Sequences Assimilation

A new methodology

$$J(V) = \underbrace{\frac{1}{2}\int_0^T \|\mathcal{H}[\mathbf{X}] - \mathbf{y}_{obs}\|_{\mathcal{O}}^2 dt}_{\text{Classical term}} + \frac{1}{2}\int_0^T \| \underbrace{\mathcal{H}_{\mathcal{X}\to\mathcal{S}}[\mathbf{X}]}_{\text{Structures Observation Operator}} - \underbrace{\mathbf{y}_s}_{\text{Image Structures}} \|_{\mathcal{S}}^2 dt + \text{reg.}$$

- $\mathbf{y}_s$ is the **observed structures** in the image
- $\mathbf{y}_s$ is **modelled** in a **normed** structure space $\mathcal{S}$
  - Frequency characteristics (e.g. through mutli-scale image transform)
  - Geometrical patterns (edges, regions of interest ...)
- The pixel value (linked with some physical properties via radiances) is not so important here
- $\mathcal{H}_{\mathcal{X}\to\mathcal{S}}$: maps the state variable space onto the space of structured images $\mathcal{S}$

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Example 1: Computation of a **synthetic image sequences** of the observed process: Innocent's talk

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Example 1: Computation of a **synthetic image sequences** of the observed process: Innocent's talk
Example 2: Computation of **F[TS]LE maps** from velocity field

Finite Lyapunov Exponents (definition)

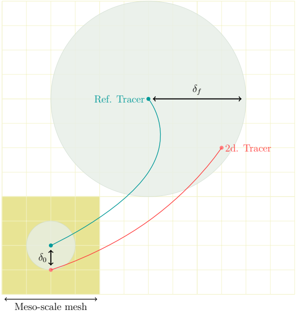
Finite Lyapunov exponents caracterize the evolution of the relative separation between two particles advected by the flow

They give a kind of measure of the lagrangian dynamic of the stirring rate of a tracer

Finite **Size** Lyapunov Exponents:

$$\lambda(\mathbf{x}, t, \delta_0, \delta_f) = \frac{1}{\tau} \log \frac{\delta_f}{\delta_0}$$

$$\delta_f = \delta_0 \exp(\tau\lambda)$$

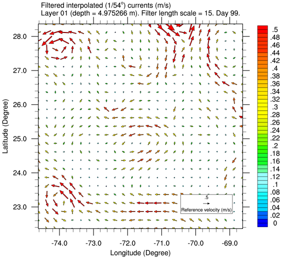


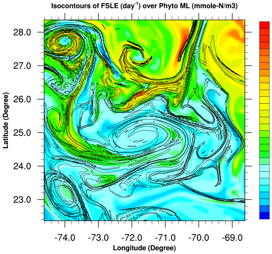
- $(\mathbf{x}, t)$: trajectory starting point of the reference particle
- δ_0 : initial distance between second particle and reference particle
- δ_f : final separation distance (a priori fixed for Finite **Size** LE)
- τ : advection time (a priori fixed for Finite **Time** LE)

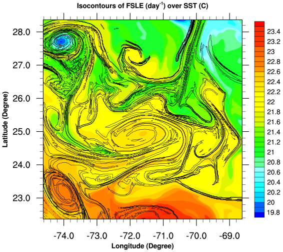
(Aurell *et. al*, 1997), (Artale *et. al.*, 1997), (Mancho *et. al*, 2004), (D'Ovidio *et. al*, 2004), (Lehahn *et. al.*, 2007)

Example 2: Observation operator based on F[ST]LE maps

FLSE isocontours computed from **mesoscale** ($1/4^\circ$) velocity fields more or less match **submesocale** ($1/54^\circ$) **Phytoplankton and SST patterns**







F. D'Ovidio code Lamta 0.2 and Gyre / Lobster data of M. Lévy

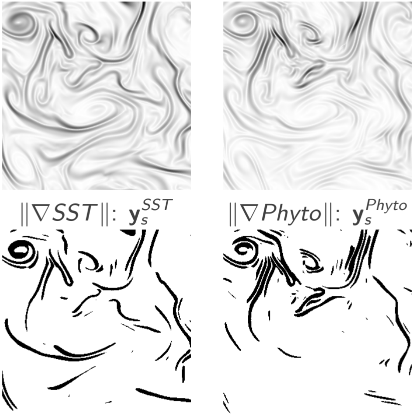
(F. d'Ovidio *et. al.*, 2004), (Y. Lehahn *et. al.*, 2007), (F. d'Ovidio *et. al.*, 2009)

Basic structure extraction from images

Observed structures are extracted using a **binarization of the gradient norm** of the image:

$$\mathbf{y_s} = \begin{cases} 1 & \text{if} \quad \|\nabla \mathbf{y_s}\| > \sigma, \\ 0 & \text{else.} \end{cases}$$

where the threshold σ is chosen such as a given percent (says 80%) of pixels of $\|\nabla \mathbf{y_s}\|$ are kept.

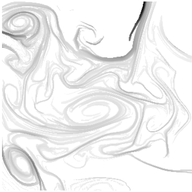


Binarized images

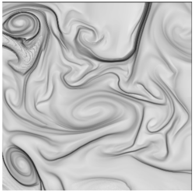
Structure Observation Operator

$\mathcal{H}_{\mathcal{X}\to\mathcal{S}}[\mathbf{X}]$ may be defined using FSLE or FTLE maps:

- Binarisation of the FSLE map
- Binarisation of the gradient norm of the FTLE map



FSLE map



FTLE map

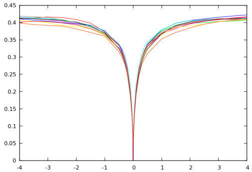
Bin. FSLE

Bin. $\|\nabla \mathrm{FTLE}\|$

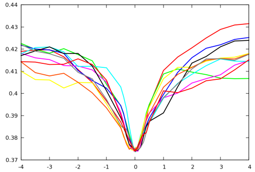
Sensitivity of the cost function with respect to velocity perturbations

Let $\mathbf{S} = (\mathbf{u}^{(1)}|\mathbf{u}^{(2)}|\cdots|\mathbf{u}^{(r)})$ where $(\mathbf{u}^{(l)})_{l=1}^{r}$ are the first r EOFs of an ensemble of velocity field. We consider velocity errors with zero mean and covariance of the sequence:

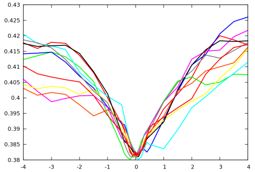
$$\delta_{\mathbf{u}} \sim \mathcal{N}(0, \mathbf{P} = \mathbf{S}\mathbf{S}^{\mathbf{T}}). \quad \delta_{\mathbf{u}} = \sum_{l=1}^{r} \mathbf{u}^{(l)} \delta x_l \quad \text{with} \quad \delta x_l \sim \mathcal{N}(0,1)$$



$\mathbf{y}_s = \text{FSLE}$



$\mathbf{y}_s = \|\nabla SST\|$



$\mathbf{y}_s = \|\nabla Phytoplankton\|$

Sensitivity of the cost function $J(\lambda) = \|\mathcal{H}_{\mathcal{X}\to\mathcal{S}}[\mathbf{u} + \frac{\lambda}{100}\delta\mathbf{u}] - \mathbf{y}_s\|_{\mathcal{S}}^2$ for 10 random perturbations $\delta\mathbf{u}$ in the reduced rank EOF space

- As expected J saturates with SST and Phyto. images: a part of the minimum error represents the quantity of information the image contains that is not due to pure meso-scale dynamics

Conclusions

- F[ST]LE maps can be used to defined an image observation operator for direct assimilation of images
- SST and Phytoplankton images contain information about the dynamic of the underlying system that can be exploited
- F[ST]LE maps link model mesoscale information with **observation submesoscale information** contained in images

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