Minimax observability and optimality conditions for incorrect linear equations in Hilbert space

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CLIME Research project

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Outline

1. Problem formulation
   - Program or a priori estimations
   - Adaptive or a posteriori estimations
   - Minimax observability
2. Results for the linear-quadratic case
   - Minimax LQ a priori estimates
   - Minimax LQ a posteriori estimates
3. State estimation for linear differential-algebraic equations (DAEs)
   - Discrete time
   - Continuous time
4. Minimax observability
5. Example
6. Current work at CLIME

Ill-posed inverse problems for linear abstract equation

Let H_i be an abstract Hilbert space, $i = 1, 2, 3$ and take any linear closed operator L mapping $\mathscr{D}(L) \subset \mathrm{H}_1$ into H_2 so that the closure $\operatorname{cl} \mathscr{D}(L)$ coincides with H_1.

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Let $\varphi \in \mathscr{D}(L)$ be one of the solutions of the uncertain equation

$$L\varphi = f \qquad (1)$$

where $f \in \mathrm{H}_2$ **is not known exactly**.

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given information y in the form

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about some solution φ of (1) one needs to reconstruct the observed solution φ provided that

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A priori linear minimax estimates

Suppose that $f \in \mathscr{G}$ where $\mathscr{G}$ denotes given convex closed bounded subset of H_2 and

$$\mathrm{E}\eta = 0, \mathrm{E}\langle R\eta, \eta\rangle \leq 1$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in H_3, E denotes expected value and R is some positive self-adjoint linear bounded mapping with bounded inverse.

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We want to find an estimation of the projection of φ onto some direction $\ell \in \mathrm{H}_1 : \langle \ell, \varphi \rangle$.
We will look for the estimation in the class of all continuous linear functions $\langle u, \cdot \rangle$, $u \in \mathrm{H}_3$ defined on the measurements y: $\langle u, y \rangle$.

Definition

We define an minimax a priori estimation error as

$$\sigma(u,\ell) = \sup_{L\varphi\in\mathscr{G},\mathrm{E}\langle R\eta,\eta\rangle\leq 1} \mathrm{E}(\langle \ell,\varphi\rangle - \langle u,y\rangle)^2$$

Any solution $\hat{u}$ of the equation

$$\sigma(\hat{u},\ell) = \hat{\sigma}(\ell) := \inf_{u\in\mathrm{H}_3} \sigma(u,\ell)$$

is called minimax a priori estimation.

A posteriori minimax estimates

Suppose that the deterministic noise η and input f obey

$$(f, \eta) \in \mathscr{G}$$

Definition

A set

$$\mathscr{G}_y = \{\varphi : (L\varphi, y - H\varphi) \in \mathscr{G}\} \tag{3}$$

is called a posteriori set, (a, b) denotes a vector from $H_1 \times H_2$. We define an minimax a posteriori estimation error as

$$\rho(\ell, x) := \sup_{\psi \in \mathscr{G}_y} |\langle \ell, x \rangle - \langle \ell, \psi \rangle|$$

and minimax a posteriori estimation $\hat{\varphi}$ is set to be one of the solutions of the equation

$$\rho(\ell, \hat{\varphi}) = \hat{\rho}(\ell) := \inf_{\varphi \in \mathscr{G}_y} \rho(\ell, \varphi)$$

Minimax observable subspace

Linear quadratic case

Definition

The set

$$\mathcal{L} = \{\ell \in \mathrm{H}_1 : \inf \sigma(\ell, \varphi) < +\infty\}$$

is called a minimax observable subspace.

Informally the minimax observable subspace describes the observable part of the state φ with a respect to the signal y.

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Assumption 1

The linear sets $R(L)$ and $H(N(L))$ are closed.

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Assumption 2

The linear set $R(T) = \{(Lx, Hx), x \in \mathscr{D}(L)\}$ is closed.

Minimax LQ-estimates

Theorem (a priori estimates)

Let

$$\mathscr{G} = \{f : \langle Q_1 f, f\rangle \le 1\}, \eta \in \{\eta : \mathrm{E}\langle Q_2\eta, \eta\rangle \le 1\}, \tag{4}$$

and suppose that Assumption 1 or 2 holds. Then the minimax observable subspace is given by

$$\mathcal{L} = R(L^*) + R(H^*) \tag{5}$$

If $\ell \in \mathcal{L}$ *then* $\hat\sigma(\ell) = \langle \ell, \hat p\rangle^{\frac{1}{2}}$ *and the unique minimax a priori estimation is given by* $\hat u = Q_2 H\hat p$*,* $\hat p$ *is any solution of the equation*

$$\begin{aligned} L^*\hat z &= \ell - H^* Q_2 H\hat p, \\ L\hat p &= Q_1^{-1}\hat z \end{aligned} \tag{6}$$

Theorem (a posteriori estimates)

Let

$$\mathscr{G} = \{(f, v) : \langle Q_1 f, f\rangle + \langle Q_2\eta, \eta\rangle \leq 1\} \tag{7}$$

and suppose that Assumption 1 or 2 holds. Then the minimax observable subspace is given by

$$\mathcal{L} = R(L^*) + R(H^*) \tag{8}$$

If $\ell \in \mathcal{L}$ then the minimax a posteriori error is given by

$$\hat{\rho}(\ell) = (1 - \langle Q_2 y, y - H\hat{\varphi}\rangle)^{\frac{1}{2}}\hat{\sigma}(\ell) \tag{9}$$

and minimax a posteriori estimation is given by any solution $\hat{\varphi}$ of the equation

$$\begin{aligned} L^*\hat{q} &= H^* Q_2(y - H\hat{\varphi}), \\ L\hat{\varphi} &= Q_1^{-1}\hat{q} \end{aligned} \tag{10}$$

Corollary

Minimax a priori and a posteriori estimates coincide

$$\langle \ell, \hat{\varphi} \rangle = \langle \hat{u}, y \rangle \tag{11}$$

If $\hat{\rho}(\ell) < +\infty$ *for any* ℓ *then*

$$\inf_{\varphi \in G_y} \sup_{x \in G_y} \|\varphi - x\| = \sup_{x \in G_y} \|\hat{\varphi} - x\| = (1 - (Q_2 y, y - H\hat{\varphi})^{\frac{1}{2}} \max_{\|\ell\|=1} \hat{\sigma}(\ell) \tag{12}$$

so that $\hat{\varphi}$ *is the center of the a posteriori set* $\mathscr{G}_y$.

Minimax estimation for non-causal DAEs

Suppose that the signal y^*_k is being observed in the form

$$y_k = H_k x_k + g_k, k = 0, 1, \dots \tag{13}$$

with $x_k = x^*_k$ and $g_k = g^*_k$ provided that the state x^*_k is derived from

$$F_{k+1} x_{k+1} - C_k x_k = f_k, F_0 x_0 = q \tag{14}$$

with input $f_k = f^*_k$, initial condition $q = q^*$ and

$$\langle Sq, q \rangle + \sum_0^\infty \langle S_i f_i, f_i \rangle + \langle R_i g_i, g_i \rangle \leq 1 \tag{15}$$

where $F_k, C_k \in \mathbb{R}^{m \times n}$, $H_k \in \mathbb{R}^{p \times n}$, S, $S_k \in \mathbb{R}^{m \times m}$, $R_k \in \mathbb{R}^{p \times p}$ are symmetric and positive-definite.

The a posteriori set $\mathscr{G}(\tilde{y})$

The a posteriori set $\mathscr{G}(\tilde{y})$ generated by $\tilde{y} = \{y_k^*\}$ consists of all $x = \{x_k\}$ derived from (14) with some $[q, \{f_k\}]$ provided that the output $y^1 = \{y_k\}$ derived from (13) with some $\{g_k\}$ coincides with $\tilde{y}$ and $(q, \{f_k\}, \{g_k\})$ obey (15). Therefore the state x_τ^* of (14) at instant $k = \tau$ belongs to the cross section $X(\tau)$ of a posteriori set $\mathscr{G}(\tilde{y})$ at instant $k = \tau$.

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Our aim here is

to describe the evolution in τ of the set $X(\tau)$ containing all possible states x_τ of the state model consistent with online measurements y_k. In terms of the optimal control theory this is the reachability set of the state model consistent with measurements.

$X(\tau)$ is a set describing all possible states

Theorem

The minimax observable subspace for the model (14) *is given by* $\mathcal{L}(\tau) = \{\ell : P_\tau^+ P_\tau \ell = \ell\}$ *and*

$$X(\tau) = \{x \in \mathbb{R}^n : (P_\tau(x - \hat{x}_\tau), x - \hat{x}_\tau) \leq \hat{\beta}_\tau\} \quad (16)$$

where $\hat{x}_\tau = P_\tau^+ r_\tau$, $\hat{\beta}_\tau := 1 - \alpha_\tau + (P_\tau \hat{x}_\tau, \hat{x}_\tau)$

$$P_k = H_k' R_k H_k + F_k'[S_{k-1} - S_{k-1} C_{k-1} B_{k-1}^+ C_{k-1}' S_{k-1}] F_k,$$
$$P_0 = F_0' S F_0 + H_0' R_0 H_0, B_k = P_k + C_k' S_k C_k,$$

$\alpha_i = \alpha_{i-1} + (R_i y_i, y_i) - (B_{i-1}^+ r_{i-1}, r_{i-1})$, $\alpha_0 = (R_0 y_0, y_0)$ *and*

$$r_k = F_k' S_{k-1} C_{k-1} B_{k-1}^+ r_{k-1} + H_k' R_k y_k, r_0 = H_0' R_0 y_0$$

If $\ell \in \mathcal{L}(\tau)$ *then the worst-case estimation error in the direction* ℓ *at instant* $k = \tau$ *is given by* $\hat{\rho}(\ell, \tau) = \hat{\beta}_\tau^{\frac{1}{2}} (P_\tau^+ \ell, \ell)^{\frac{1}{2}}$

A priori minimax estimates for noncausal DAEs

In this talk we focus on the following case: x denotes some solution of the linear differential-algebraic equation (DAE)

$$\frac{d}{dt}Fx(t) = C(t)x(t) + f(t), Fx(t_0) = x_0, \tag{17}$$

where $x(t) \in \mathbb{R}^n$, $f(t) \in \mathbb{R}^m$, $F \in \mathbb{R}^{m\times n}$, $t \mapsto C(t) \in \mathrm{C}^{m\times n}(t_0, T)$, $f(\cdot) \in \mathbb{L}_2(t_0, T)$.

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$$(x_0, f(\cdot)) \in G = \{(x_0, f(\cdot)) : (Q_0 x_0, x_0) + \int_{t_0}^{T} (Q(t)f(t), f(t)) \leq 1\},$$

where $Q_0 \in \mathbb{R}^{n\times n}$, $Q_0 = Q_0' > 0$, $Q(t) \in \mathbb{R}^{m\times m}$, $Q = Q' > 0$, $Q^{-1}(t)$ is continuous function of t on $[t_0, T]$.

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A priori minimax estimates for noncausal DAEs

Suppose that the information y about some DAE solution $x(\cdot)$ is given by

$$y(t) = H(t)x(t) + \eta(t), t \in [t_0, T], \tag{18}$$

where $y(t) \in \mathbb{R}^p$, $t \mapsto H(t) \in \mathrm{C}^{p\times n}(t_0, T)$, $t_0, T \in \mathbb{R}$, $\eta(\cdot)$ is a realization of the random process η with zero mean satisfying

$$\eta \in W = \{\eta : \mathrm{E} \int_{t_0}^{T} (R(t)\eta(t), \eta(t)) \leq 1\} \tag{19}$$

$R(t) \in \mathbb{R}^{p\times p}$, $R' = R > 0$.

Definitions

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in the class of all bounded linear functionals

$$u(y) = \int_{t_0}^{T} (u(t), y(t))dt, u(\cdot) \in \mathbb{L}_2(t_0, T)$$

defined on the realizations of the random process y representing an available information about DAE solution $x(\cdot)$ in the form of

$$y(t) = H(t)x(t) + \eta(t), t \in [t_0, T],$$

Minimax observability

Definition

A function $\hat{u}$ is called minimax a-priori estimation iff

$$\hat{u} \in \mathrm{Arginf} \sup_{(x_0, f(\cdot)) \in G, \eta \in W} E[\ell(x) - u(y)]^2$$

Minimax estimation error is given by

$$\sigma(\hat{u}, \ell) := \sup_{(x_0, f(\cdot)) \in G, \eta \in W} E[\ell(x) - u(y)]^2$$

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Definition

Linear subspace

$$\mathcal{L}(\tau) := \{\ell \in \mathbb{R}^n : \inf_{u(\cdot) \in \mathbb{L}_2(t_0, T)} \sigma(u, \ell) < +\infty\}$$

is called minimax observability subspace for the pair of models (17),(18).

Weak solutions for DAE

Definition

We say that $x(\cdot)$ is a solution of

$$\frac{d}{dt}Fx(t) = C(t)x(t) + f(t), Fx(t_0) = x_0,$$

if $Fx(\cdot) \in \mathbb{W}_2^m(t_0, T)$, the derivative of $Fx(\cdot)$ coincides with the right side of (17) almost everywhere and $Fx(t_0) = x_0$ holds.

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The set of solutions of

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$$\frac{d}{dt}Fx(t) = C(t)x(t), Fx(t_0) = 0,$$

may contain subspace of infinite dimension. If $F = I$ then the minimax observable subspace is trivial $\mathcal{L}(\tau) = \mathbb{R}^n$.

Theorem

If

$$\sup_{\varepsilon\in[-1,1],t\in[t_0,T]} \|(\varepsilon^2+C_4'(t)C_4'(t)+H_2(t)'H_2(t)+H_4'(t)H_4(t)^{-1}C_2'(t)\| < +\infty,$$

where C_2, C_4, H_2, H_4 *are blocks of* C *and* H *respectively induced by the SVD-decomposition of* F *then* $\mathcal{L}(T)$ *is non-empty and BVP*

$$\frac{d}{dt}F'q(t) = -C'(t)q(t) + H'(t)QHp(t), F'q(T) = \ell,$$
$$\frac{d}{dt}Fp(t) = C(t)p(t) + Q^{-1}(t)q(t), Fp(t_0) = Q_0^{-1}F'z(t_0),$$

is solvable for $\ell \in \mathcal{L}(T)$. *In this case minimax estimation is given by* $\hat{u} = QHp(t)$.

Example

Consider the DAE

$$\frac{d}{dt}Fx(t) = C(t)x(t) + f(t), Fx(t_0) = x_0, \tag{20}$$

with matrices

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{21}$$

and

$$C = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} \tag{22}$$

Also let

$$y(t) = Hx(t) + \eta(t)$$

with

$$H = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{23}$$

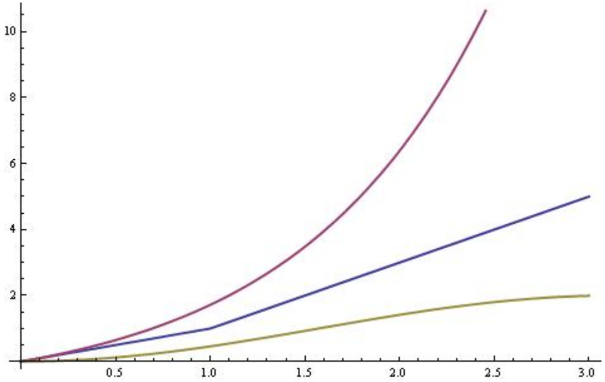
and set

$$\eta = (\eta_1, \eta_2), \eta_1(t) = \eta_2(t) = \sin 50t \cos 20t \tag{24}$$

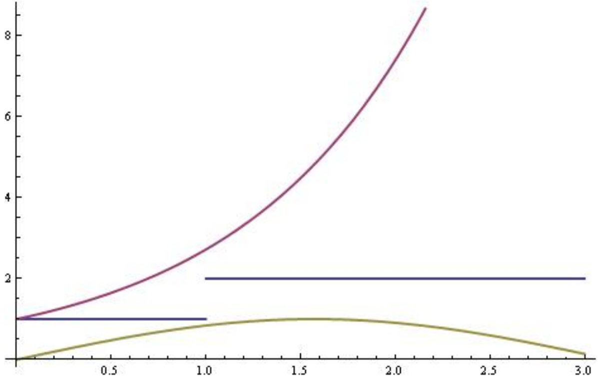
After simple algebra one obtains that (20) is equal to

$$\begin{aligned} x_2 &= -x_3 \\ x_2' &= x_5 \\ x_1 &= 0 \\ x_3 &= x_4 \\ x_1(t_0) &= 0 \\ x_2(t_0) &= 0 \end{aligned} \tag{25}$$

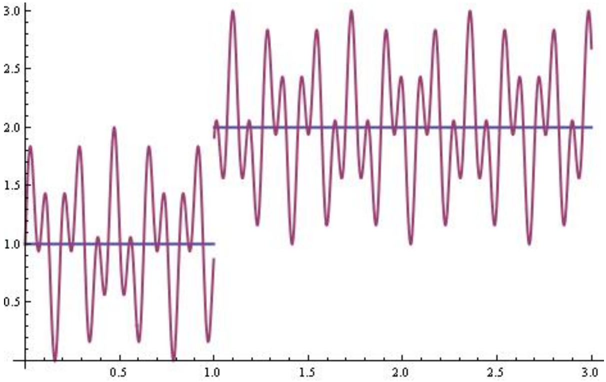
$x_2(t)$



$x_5(t)$



$x_5(t), y_1(t)$



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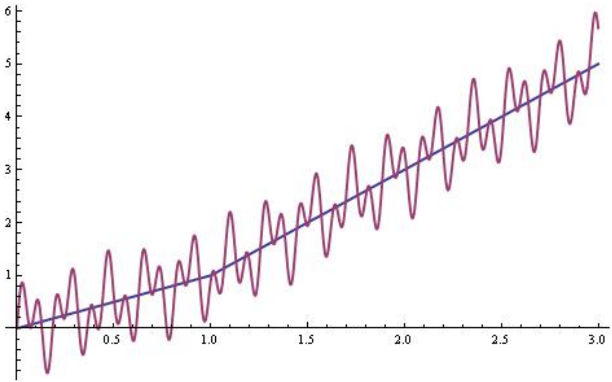
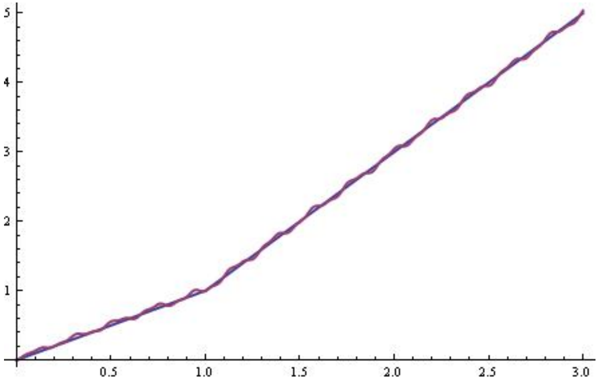
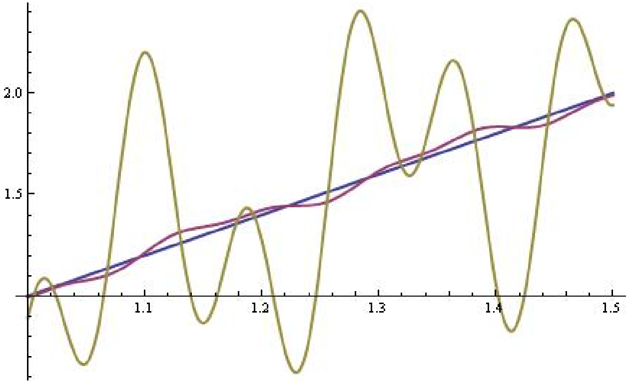


$x_2, \hat{x}_2(t)$



$x_2, \hat{x}_2(t), y_1(t)$



Current work at CLIME

Minimax estimation algorithm for state estimation of the nonlinear evolution equation

$$\frac{d}{dt}v(t) + M(v) = f(t)$$

with implicit observation model

$$L(I, v) = g(t)$$

where $t \mapsto v(t) \in \mathrm{H}_1$ is strongly absolutely continuous and strongly differentiable, M is Lipshits-continuous operator in H_1, $f(t) \in \mathbb{L}_2((t_0, T), \mathrm{H}_1$.

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$$\mathrm{H}_1 = L_2(t_0, T), M(v) = H(Dv(t, x)) - \nabla^2 v(t, x),$$

$$L(I, v) = \frac{\partial}{\partial t}I(t, x) + (v(t, x), \frac{\partial}{\partial x}I(t, x))$$

Selected scientific publications

S. Zhuk, *Recursive state estimation for noncausal discrete-time linear descriptor systems*	preprint, 2009, submitted to Automatica
S. Zhuk, *State estimation for dynamic system described by linear equation with unknown parameters*	Ukrainian mathematical journal, 61(2):178-194, 2009
S. Zhuk, *Recursive set-membership state estimation for linear noncausal time-variant differential-algebraic equation with continuous time*	preprint, 2009, submitted to SICON
S. Zhuk, *Closedness and normal solvability of an operator generated by a degenerate linear differential equation with variable coefficients*	Nonlinear Oscillations, 10(4):118, 2007

Conference presentations

IFAC Workshop on Control Applications of Optimisation	Jyvaskyla, Finland, 2009
International Conference "Differential Equations and Topology" dedicated to the centenary of L.Pontryagin	Moscow, 2008
International Conference "Differential and Difference Equations and Applications", (CDDEA 2008)	Strechno, Slovakia, 2008
International Conference "Modern analysis and applications" dedicated to the centenary of M.Krein	Odessa, Ukraine, 2007
Annual International Conference "Problems of Decision Making under Uncertainty"	Ukraine, 2003-2008