Evaluation d'une méthode de flow optique sur des images de type PIV

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Fluid motion analysis: interests

Historically Optical-flow methods have been used in:

- Meteorology
  - ⋆ Tracking characteristics features (convective cells, vortices, ...)
  - ⋆ Weather forecasting
- Medical Imaging
  - ⋆ Blood flow monitoring
- Oceanography
  - ⋆ Tracking pollutant agents

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- Medical Imaging
  - ⋆ Blood flow monitoring
- Oceanography
  - ⋆ Tracking pollutant agents

$\Rightarrow$ Experimental fluid mechanics?

Outline

- Standard optical-flow
- Proposed estimator
- Results on a plane mixing layer
- Results on a circular cylinder wake
- Conclusion

Standard optical-flow methods

- Horn & Schunck (1981)
  - ⋆ Motion estimation: minimisation of a twofold energy function

$$\min_{\boldsymbol{v}} \mathcal{H}(E, \boldsymbol{v}) = \min_{\boldsymbol{v}} \mathcal{H}_{obs}(E, \boldsymbol{v}) + \min_{\boldsymbol{v}} \mathcal{H}_{reg}(\boldsymbol{v})$$

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⇒ OBSERVATION term:

Standard optical-flow methods

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⇒ OBSERVATION term: brightness constancy assumption

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⇒ OBSERVATION term: brightness constancy assumption

⇒ REGULARISATION term:

Standard optical-flow methods

- Horn & Schunck (1981)
  - ⋆ Motion estimation: minimisation of a twofold energy function

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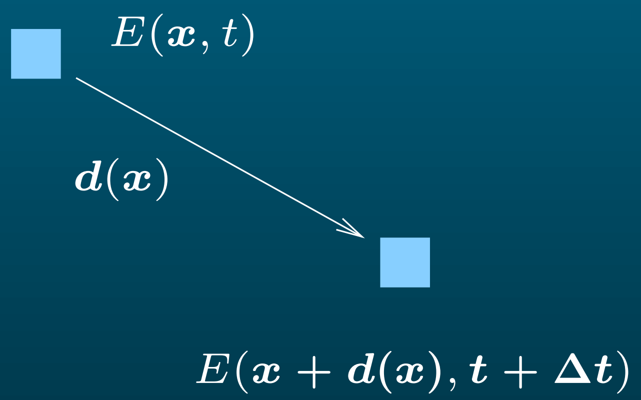
⇒ OBSERVATION term: brightness constancy assumption

⇒ REGULARISATION term: spatial motion field coherence

Standard motion estimation technique

- Observation

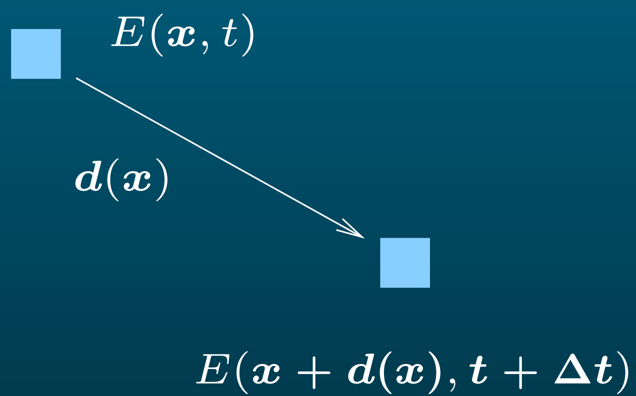


Optical-flow constraint equation

$$\underbrace{\iint_{\Omega} \left[\frac{\partial E(\boldsymbol{x}, t)}{\partial t} + \boldsymbol{\nabla} E(\boldsymbol{x}, t) . \boldsymbol{v}(\boldsymbol{x}, t) \right]^2 d\boldsymbol{x}}_{\mathcal{H}_{obs}(E, \boldsymbol{v})}$$

Standard motion estimation technique

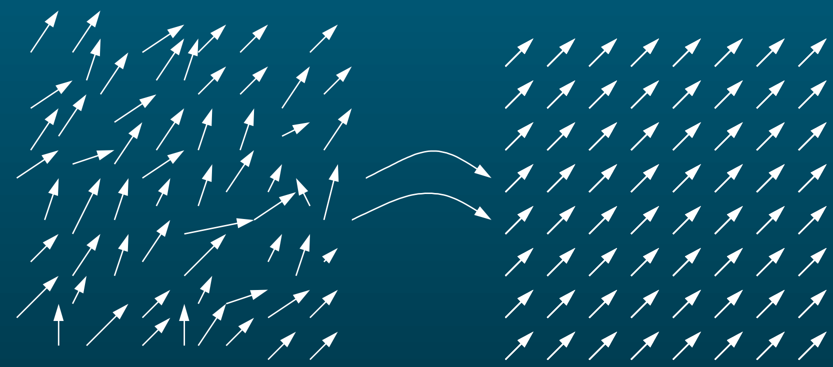
- Observation



Optical-flow constraint equation

$$\underbrace{\iint_\Omega \left[\frac{\partial E(\boldsymbol{x},t)}{\partial t} + \boldsymbol{\nabla} E(\boldsymbol{x},t).\boldsymbol{v}(\boldsymbol{x},t)\right]^2 d\boldsymbol{x}}_{\mathcal{H}_{obs}(E,\boldsymbol{v})}$$

- Regularization



First order smoothness term

$$\underbrace{\alpha \iint_\Omega [|\boldsymbol{\nabla} u(\boldsymbol{x},t)|^2 + |\boldsymbol{\nabla} v(\boldsymbol{x},t)|^2] d\boldsymbol{x}}_{\mathcal{H}_{reg}(\boldsymbol{v})}$$

α controls the balance between the observation term and the regularization term

Problem statement

- For fluid motion the brigthness is not always preserved

$\Rightarrow$ Brightness constancy assumption violated

- With classical regularization fluid is seen as a rigid body
  - $\star$ Motion field has the same smoothness as motion for rigid body
  - $\star$ Can we use a first order regularization for fluid motion?

$$\mathcal{H}_{reg}(\boldsymbol{v}) = \iint_{\Omega} [\text{curl}^2 u(\boldsymbol{x}, t) + \text{div}^2 v(\boldsymbol{x}, t)] d\boldsymbol{x}$$

$\Rightarrow$ Limits the amplitude of both divergence and vorticity

Proposed solution: integration of prior knowledge

Corpetti *et al.* (2002)

- Observation term
  - ⋆ Sticking on the continuity equation from fluid mechanics
  - ⇒ Expression of density conservation during the displacement of the fluid cell
- Regularization term
  - ⋆ Use of the Helmholtz decomposition
  - ⇒ Regularization based on the divergence and the vorticity

Continuity equation

- Mass conservation:

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \boldsymbol{v}) = 0$$

- Assumptions
  - ⋆ The luminance function is related to a passive quantity transported by the fluid
  - ⋆ Continuity equation holds for the bidimensional motion field captured by the image sequence

  $\Rightarrow$ The luminance needs to integrate the information of the 3D component
    - ∗ Medical tomography (Fitzpatrick (1988), Wildes *et al.* (1997))
    - ∗ Studied for infrared Meteosat images (Bereziat (2000), Zhou (2000))

Integration in a dense motion estimator

- Image intensity relative to fluid density

$$\frac{\partial E}{\partial t} + \text{div}(E\boldsymbol{v})$$

- New constraint

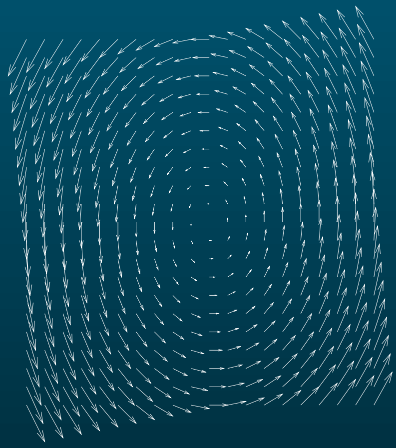
$$\begin{aligned}\mathcal{H}_{obs}(E, \boldsymbol{v}) &= \iint_{\Omega} \left(\frac{\partial E}{\partial t} + \text{div}(E\boldsymbol{v}) \right)^2 d\boldsymbol{x} \\ &= \iint_{\Omega} \left(\frac{dE}{dt} + E\,\text{div}(\boldsymbol{v}) \right)^2 d\boldsymbol{x}\end{aligned}$$

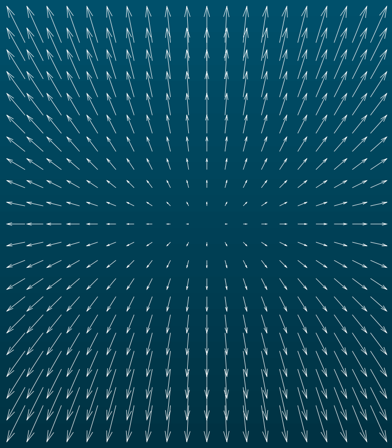
Div-Curl regularization

- Helmholtz decomposition

3 components: Irrotational, Solenoidal, Laminar

curl$(\boldsymbol{v}) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$



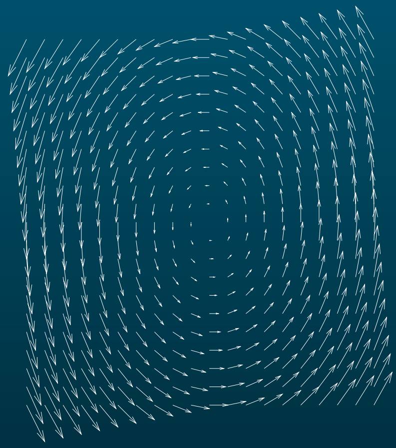


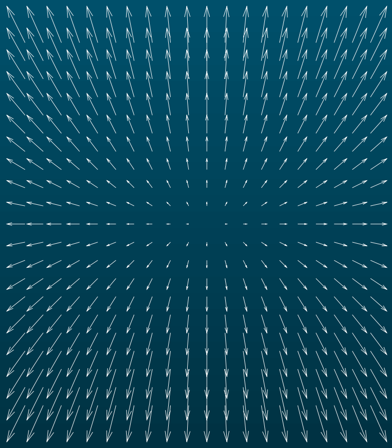
div$(\boldsymbol{v}) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$

Div-Curl regularization

- Helmholtz decomposition

3 components: Irrotational, Solenoidal, Laminar





curl$(\boldsymbol{v}) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ div$(\boldsymbol{v}) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$

- Euler Lagrange optimality condition yields:

$$\min_{\boldsymbol{v}} \iint_\Omega \left[|\nabla u|^2 + |\nabla v|^2\right] d\boldsymbol{x} = \min_{\boldsymbol{v}} \iint_\Omega \left[\text{div}^2 \boldsymbol{v} + \text{curl}^2 \boldsymbol{v}\right] d\boldsymbol{x}$$

- Idea: regularization based on the divergence and the vorticity

Div-Curl regularization

- Idea: Preserve the divergence and the vorticity
- Usual second order div-curl regularization

$$\iint_{\Omega} \left[|\nabla \mathrm{div} \boldsymbol{v}|^2 + |\nabla \mathrm{curl} \boldsymbol{v}|^2 \right] d\boldsymbol{x}$$

- Problems
  - ⋆ Numerical instabilities
  - ⋆ 4^{th} order PDE

Div-Curl regularization

- Proposed approach (Corpetti *et al.* 2002)
  - ⋆ Scalar intermediary fields ξ and ζ

$$\mathcal{H}_{reg}(\boldsymbol{v},\xi,\zeta) = \iint_{\Omega} |\text{div}(\boldsymbol{v}) - \xi|^2 + \lambda(|\nabla\xi|)^2 + \iint_{\Omega} |\text{curl}(\boldsymbol{v}) - \zeta|^2 + \lambda(|\nabla\zeta|)^2$$

  - ⋆ Advantages
    - ∗ Simpler resolution
    - ∗ Introduction of prior knowledge
  - ⋆ Solving alternatively w.r.t ξ, ζ and $\boldsymbol{v}$ until convergence

Robust estimators

- Problem: outlier data
  - ⋆ Occlusion, transparency, changes of illuminations ...
    $\longrightarrow$ Brigthness constancy assumption not valid
  - ⋆ Boundaries, motion discontinuities
    $\longrightarrow$ No motion field coherence

- Solution
  - ⋆ Not the same penalisation for such data (non quadratic)
    $\Rightarrow$ Use of robust estimators to preserve strong variations

$$\mathcal{H} = \underbrace{\iint_\Omega f_1\left(\frac{\partial E}{\partial t} + \mathrm{div}(E\boldsymbol{v})\right) d\boldsymbol{x}}_{Observation} + \underbrace{\alpha \iint_\Omega |\mathrm{div}\ \boldsymbol{v} - \xi|^2 + \lambda f_2(|\nabla \xi|) d\boldsymbol{x} + \alpha \iint_\Omega |\mathrm{curl}\ \boldsymbol{v} - \zeta|^2 + \lambda f_2(|\nabla \zeta|) d\boldsymbol{x}}_{Regularization}$$

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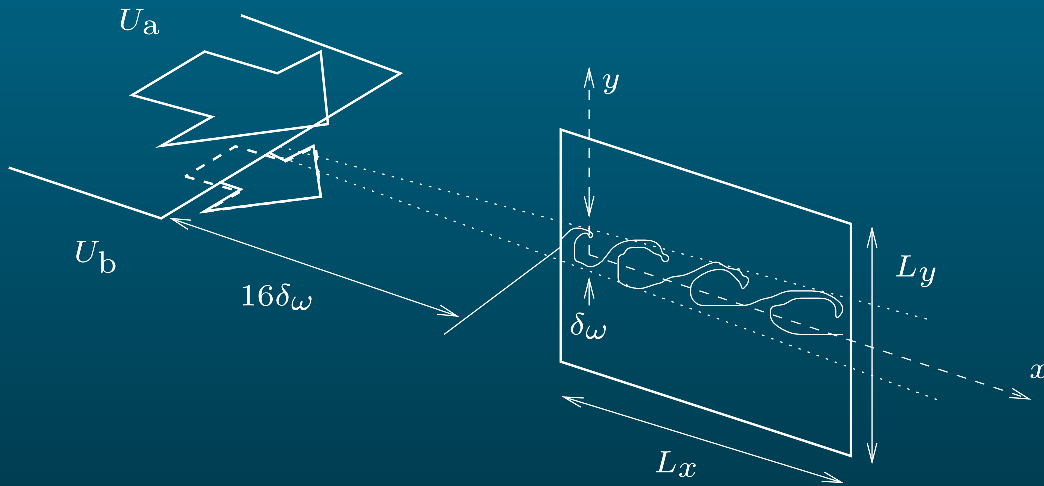
Plane mixing layer



$U_\mathrm{a} = 9$ m s^{-1}
$U_\mathrm{b} = 6$ m s^{-1}
$\delta_\omega = 15$ mm

$r = U_\mathrm{b}/U_\mathrm{a} = 0.67$
$\lambda = (1-r)/(1+r) = 0.2$
$Re_{\delta_\omega} = \Delta U \delta_\omega/\nu = 7000$

$L_x \times L_y = 84.5 \text{ mm} \times 82.5 \text{ mm} = 5.6\ \delta_\omega \times 5.5\ \delta_\omega$

Plane mixing layer - PIV settings

- Nd:YAG Laser system (30 mJ – $\lambda = 532\ \mathrm{nm}$ – Quantel)
- CCD Kodak camera of 1008×984 pixels – 8 bits dynamic range
- Synchronization and correlation: LaVision hardware and software (Davis)
- Two PIV treatments:

PIV I	PIV II
single-pass	multipass
32×32 pixels	32×32 to 16×16 pixels
50% overlap	50% overlap
grid spacing	grid spacing
16×16 pixels	8×8 pixels
$0.09\ \delta_\omega \times 0.09\ \delta_\omega$	$0.045\ \delta_\omega \times 0.045\ \delta_\omega$

Plane mixing layer - Optical-flow settings

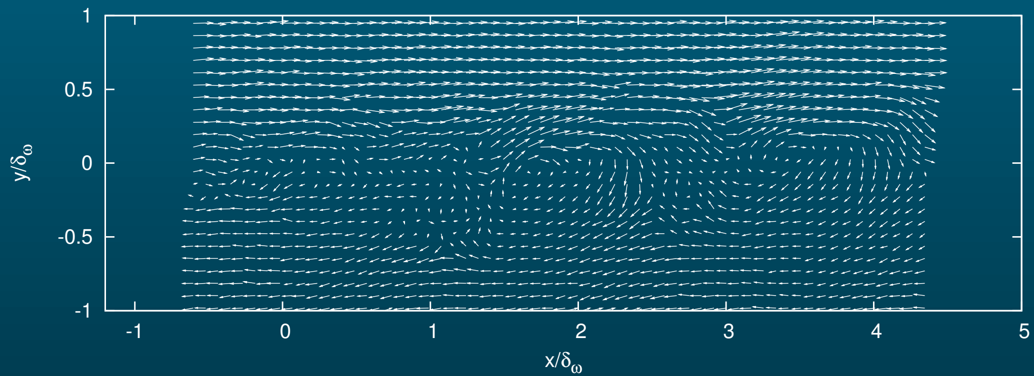
$$\mathcal{H} = \iint_\Omega f_1 \left(\frac{\partial E}{\partial t} + \text{div}(E\boldsymbol{v}) \right) d\boldsymbol{x} + \alpha \iint_\Omega |\text{div}\, \boldsymbol{v} - \xi|^2 + \lambda f_2(|\boldsymbol{\nabla} \xi|) d\boldsymbol{x}$$
$$+ \alpha \iint_\Omega |\text{curl}\, \boldsymbol{v} - \zeta|^2 + \lambda f_2(|\boldsymbol{\nabla} \zeta|) d\boldsymbol{x}$$

- Penalty functions:
  - ⋆ $f_1(x) = 1 - \exp(-\tau_1 x^2)$ with $\tau_1 = 1.6$ (Leclerc penalty function)
  - ⋆ f_2 quadratic penalty function
- Regularization parameters: $\alpha = 300$ and $\lambda = 300$
- Grid spacing of 1×1 pixel $= 0.08$ mm $\times\ 0.08$ mm $= 0.006\delta_\omega \times 0.006\delta_\omega$

Plane mixing layer - Instantaneous velocity field

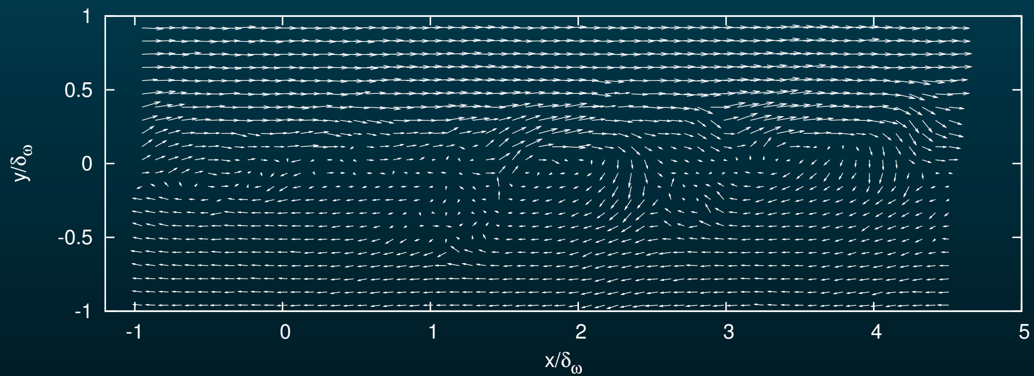
- Optical-flow

  1 vector out of 225



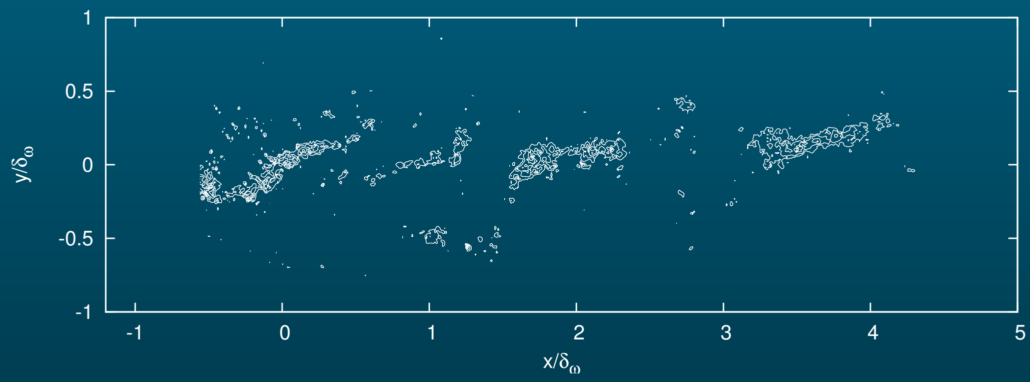
- PIV II

  1 vector out of 9

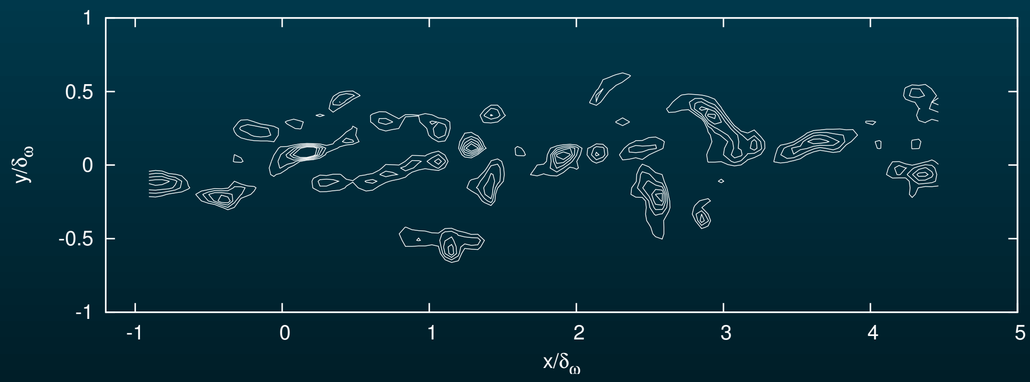


Plane mixing layer - Instantaneous iso-vorticity

- Optical-flow



- PIV II



isocontour of vorticity $\omega_z^\star = (\omega_z \Delta U)/\delta_\omega$ ($\omega^\star_{z_{\min}} = -9$, $\omega^\star_{z_{\max}} = -2$, $\Delta\omega_z^\star = 1$)

Plane mixing layer - Mean quantities

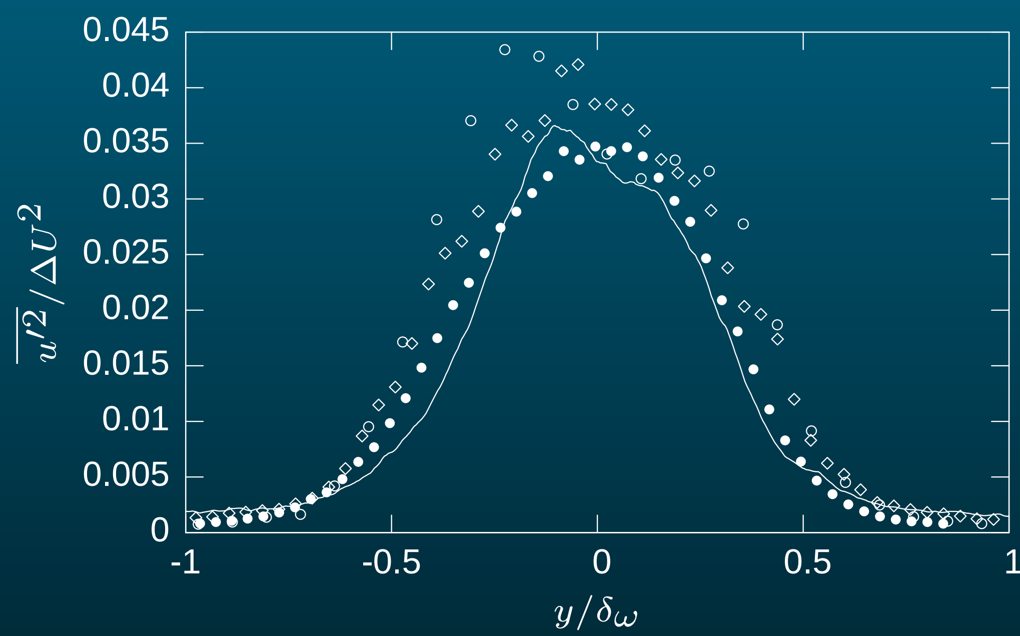
The mean streamwise velocity component can be expressed with the theoretical solution as:

$$\frac{\overline{U} - U_b}{U_a - U_b} = \frac{1}{2}(1 - \mathrm{erf}(\sigma\eta))$$

where $\eta = (y - y_\mathrm{o})/(x - x_\mathrm{o})$ with $(x_\mathrm{o}, y_\mathrm{o})$ the coordinate of virtual origin of the mixing layer and the spreading parameter σ is constant.

	Heitz 1999	PIV	optical-flow
σ	52.7	43.55	46.37
$d\delta_w/dx$	0.0336	0.0407	0.0382

Plane mixing layer - Reynolds stress



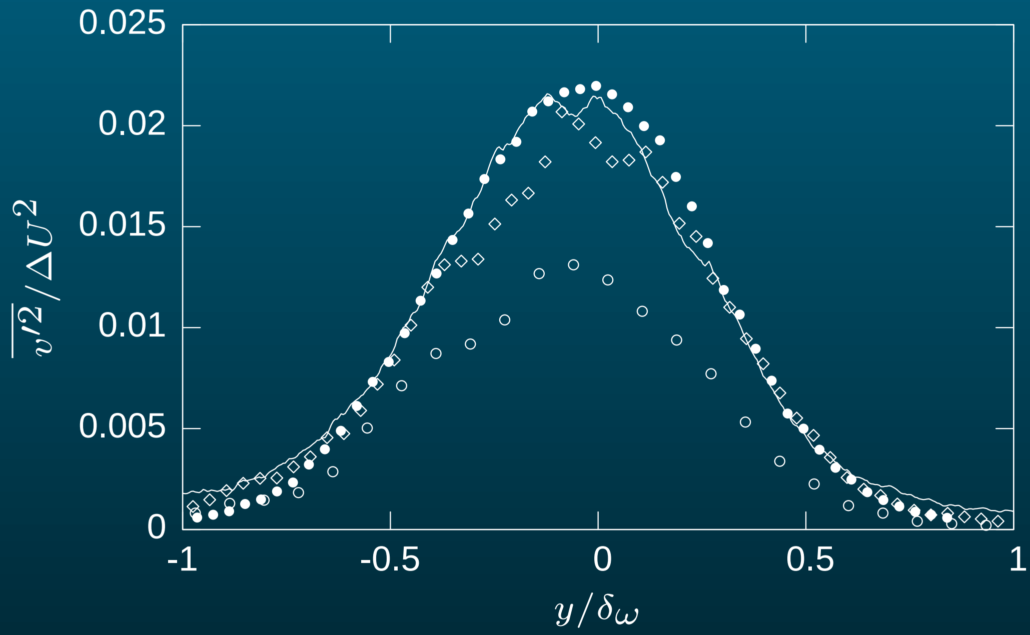
—, Optical-flow

◇, PIV II

○, PIVI

●, Hot-wire (Heitz (1999))

Plane mixing layer - Reynolds stress



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○, PIVI

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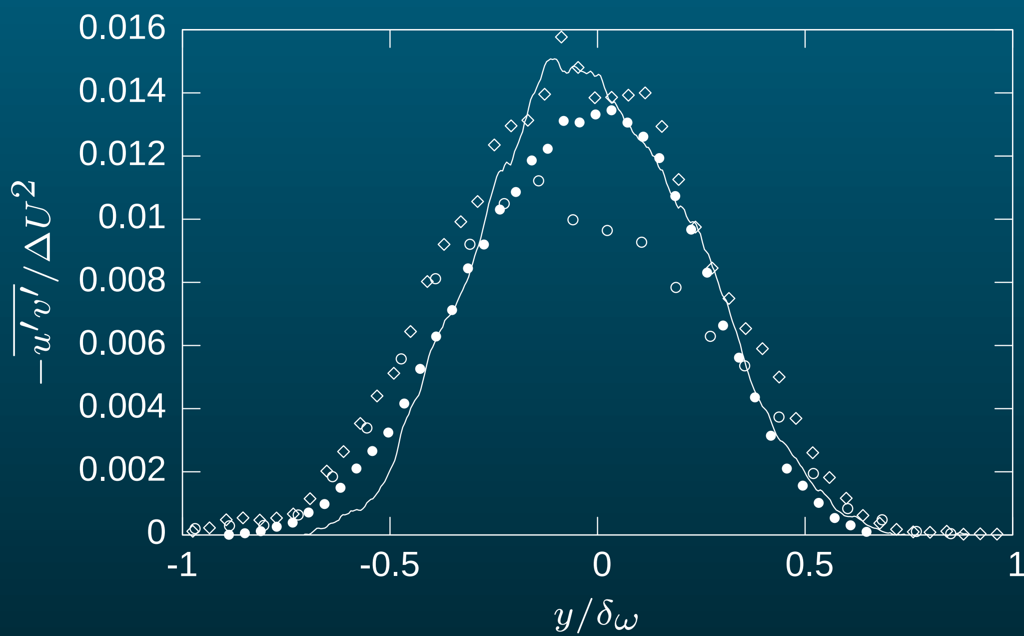
Plane mixing layer - Reynolds stress



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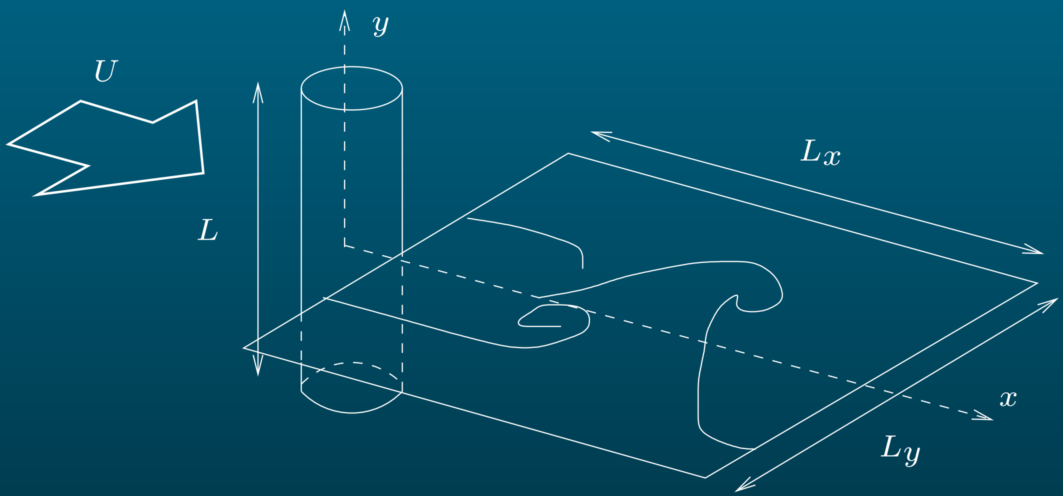
Circular cylinder near wake



$U = 4.5 \text{ m s}^{-1}$
$D = 10 \text{ mm}$
$L = 142 \text{ mm}$

$L/D = 14.2$
$Re = UD/\nu = 3000$

$L_x \times L_y = 98.7 \text{ mm} \times 83.4 \text{ mm} = 9.9\ D \times 8.3\ D$

Circular cylinder near wake - PIV settings

- Nd:YAG Laser system (30 mJ – $\lambda = 532\ \mathrm{nm}$ – New Wave)
- CCD PCO camera of 1280×1024 pixels – 12 bits dynamic range
- Synchronization and correlation: LaVision hardware and software (Davis)
- PIV treatments:

PIV
multipass
32×32 to 16×16 pixels
25% overlap
grid spacing
12×12 pixels
$0.008\ D \times 0.008\ D$

Circular cylinder near wake - Optical-flow settings

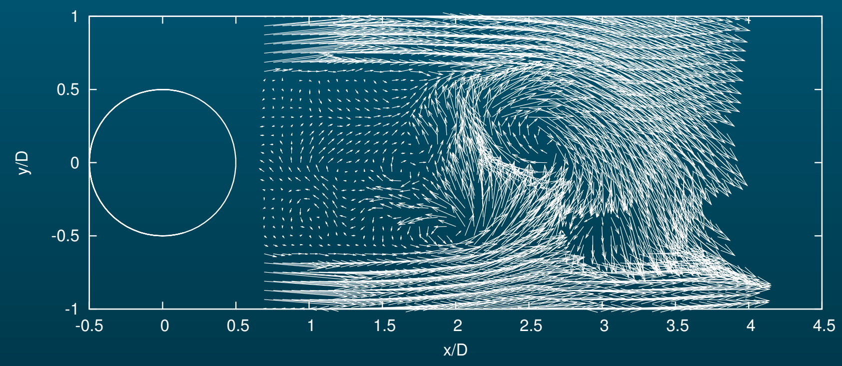
$$\mathcal{H} = \iint_\Omega f_1\left(\frac{\partial E}{\partial t} + \text{div}(E\boldsymbol{v})\right) d\boldsymbol{x} + \alpha \iint_\Omega |\text{div}\ \boldsymbol{v} - \xi|^2 + \lambda f_2(|\boldsymbol{\nabla}\xi|) d\boldsymbol{x}$$
$$+ \alpha \iint_\Omega |\text{curl}\ \boldsymbol{v} - \zeta|^2 + \lambda f_2(|\boldsymbol{\nabla}\zeta|) d\boldsymbol{x}$$

- Penalty functions:
  - ⋆ $f_1(x) = 1 - \exp(-\tau_1 x^2)$ with $\tau_1 = 1.6$ (Leclerc penalty function)
  - ⋆ f_2 quadratic penalty function
- Regularization parameters: $\alpha = 300$ and $\lambda = 300$
- Grid spacing of 1×1 pixel $= 0.08$ mm $\times$ 0.08 mm $= 0.008D \times 0.008D$

Circular cylinder near wake - Instantaneous vector field

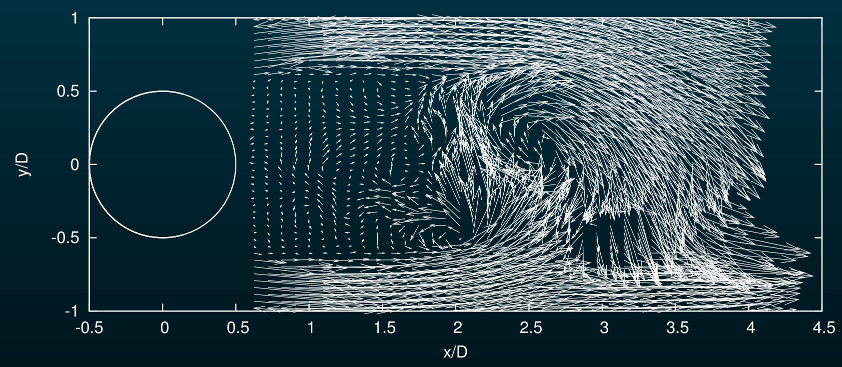
- Optical-flow

  1 vector out of 256



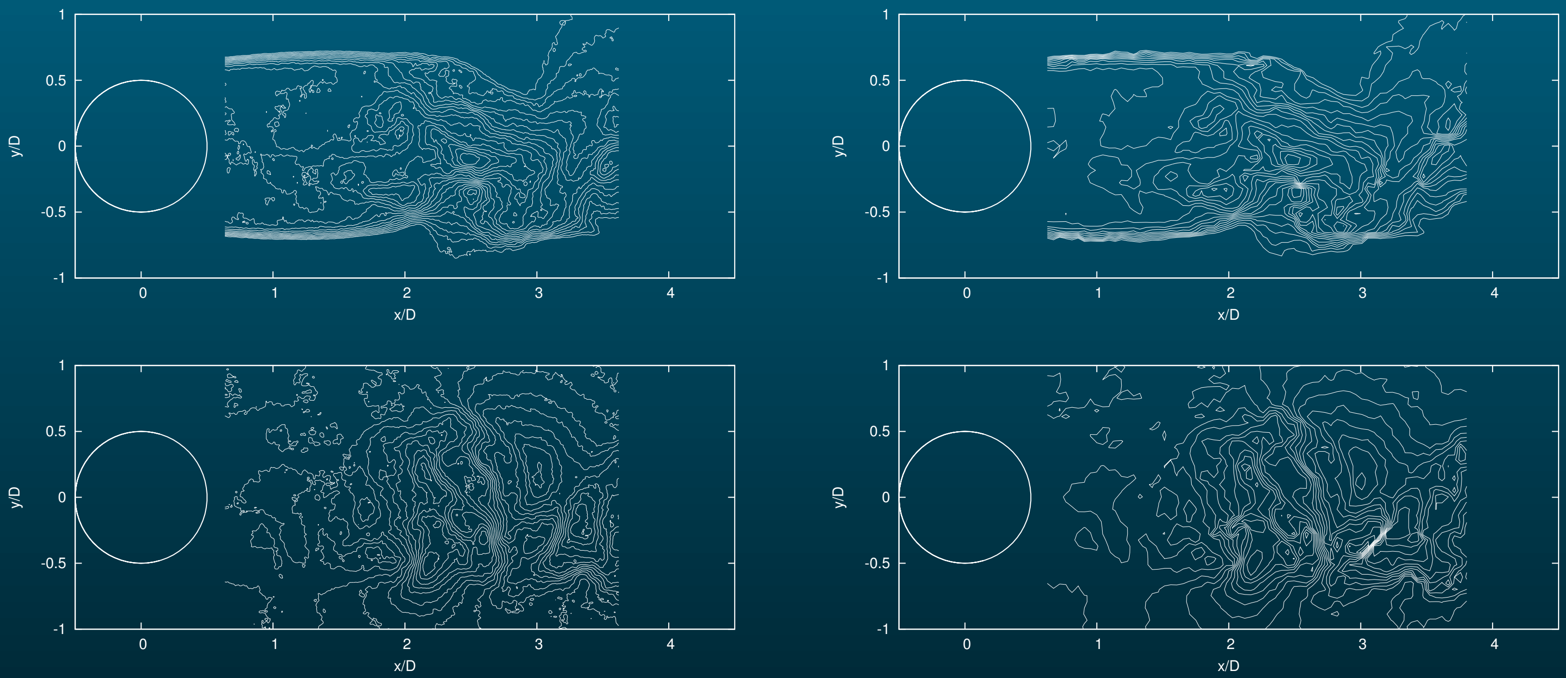
- PIV

  1 vector out of 4



Circular cylinder near wake - Instantaneous velocity

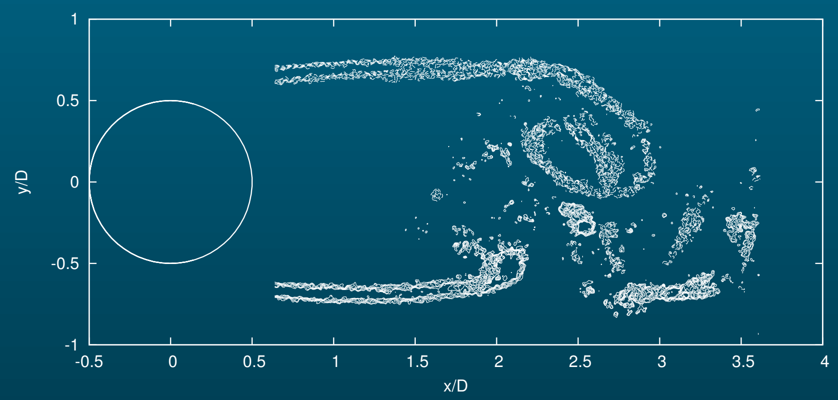
Optical-flow PIV



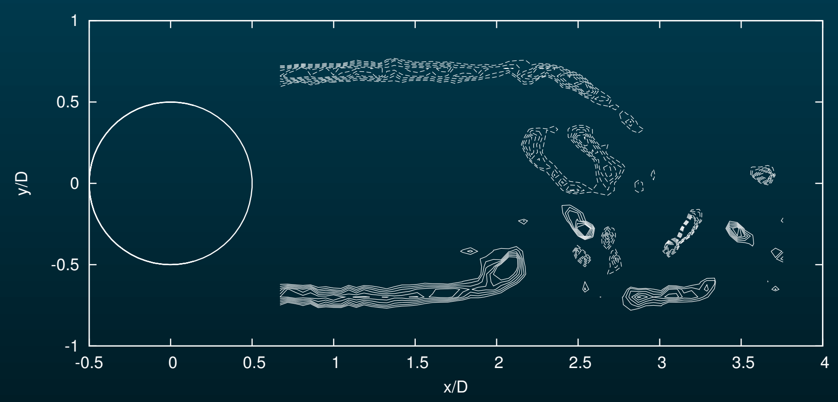
Top: 21 iso-contours of instantaneous streamwise velocity $u/U_\infty = -1, \ldots, 1$
Bottom: 21 iso-contours of instantaneous transverse velocity $v/U_\infty = -1, \ldots, 1$

Circular cylinder near wake - Instantaneous vorticity

- Optical-flow



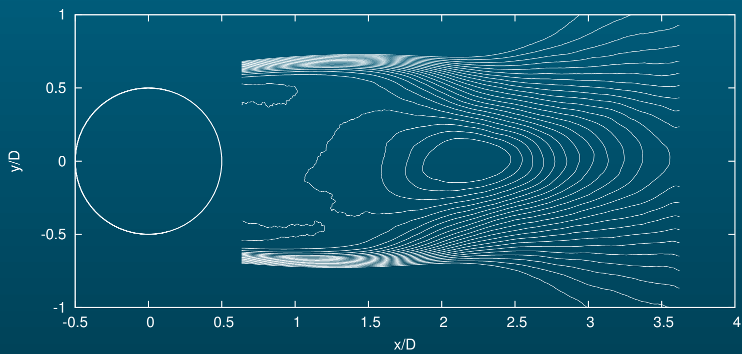
- PIV



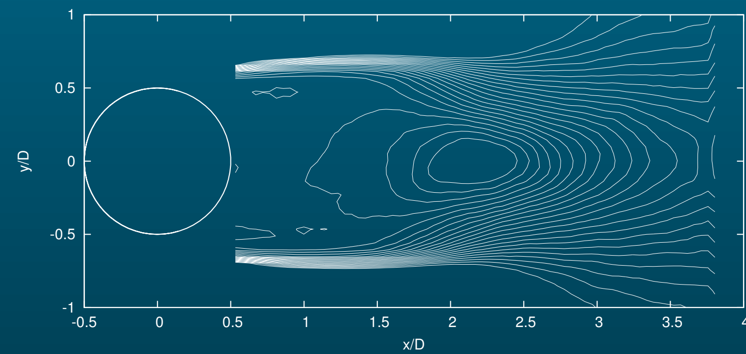
iso-contour of vorticity $\omega_z^\star = (\omega_z U)/D$ ($|\omega_{z_{\min}}^\star| = 0.5$, $|\omega_{z_{\max}}^\star| = 10$, $\Delta\omega_z^\star = 1$)

Circular cylinder near wake - Mean velocity

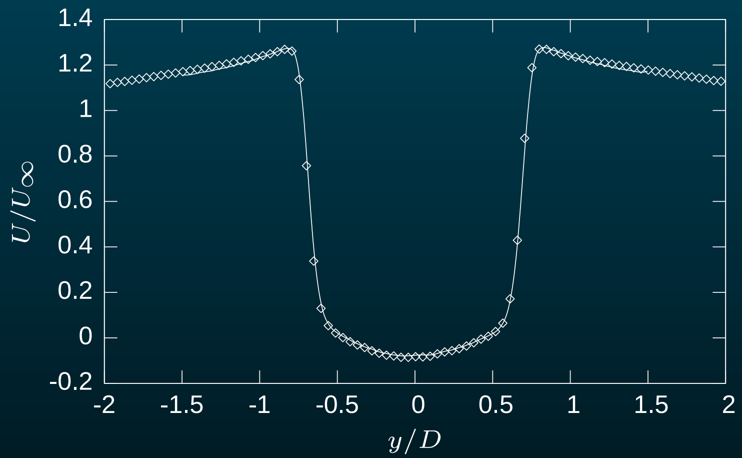
Optical-flow



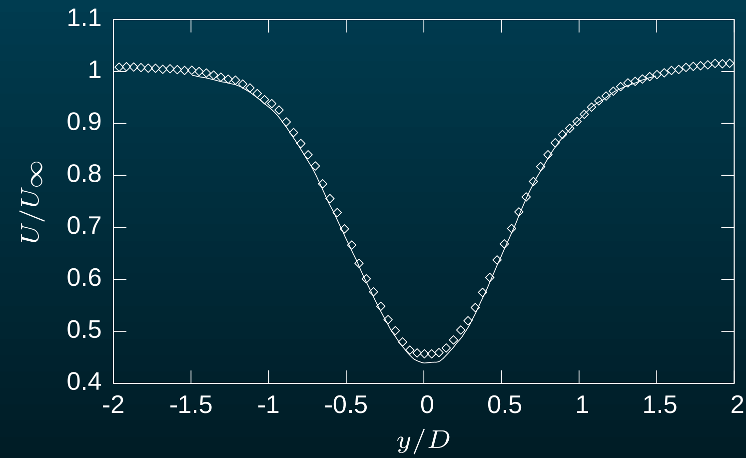
PIV



$x/D = 1.56$



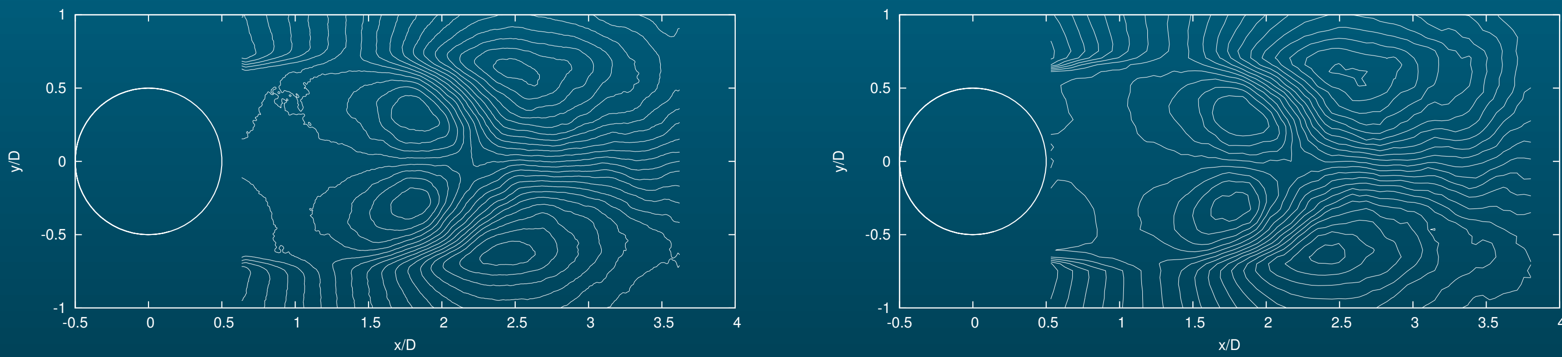
$x/D = 3.6$



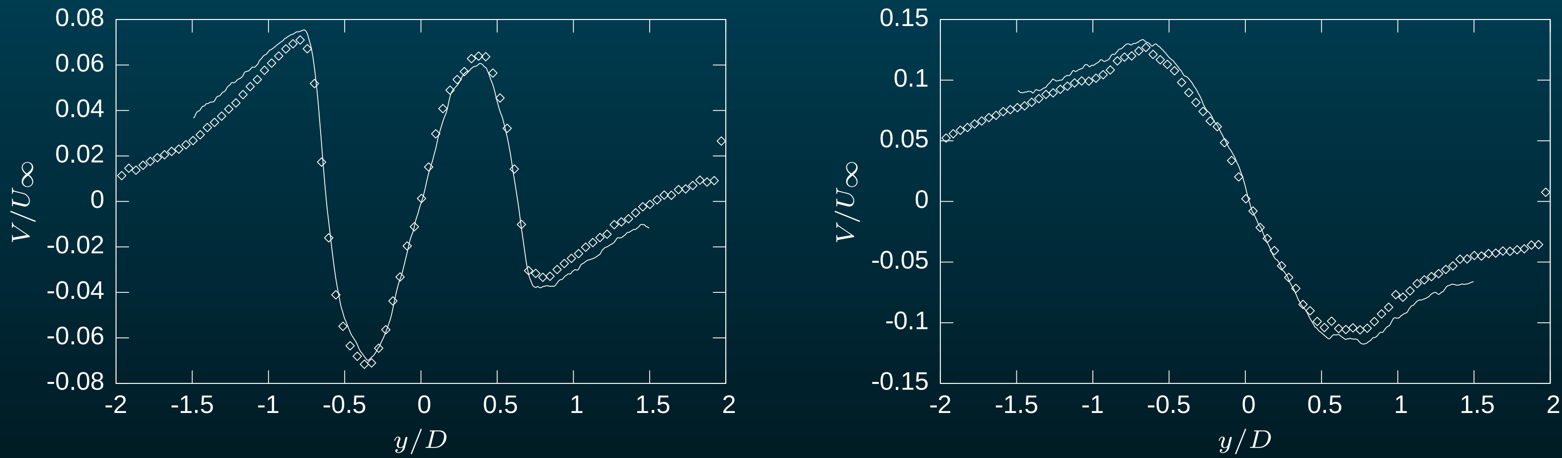
Top: 21 iso-contours of mean streamwise velocity $U/U_\infty = -0.2, \ldots, 1$.

Circular cylinder near wake - Mean velocity

Optical-flow PIV



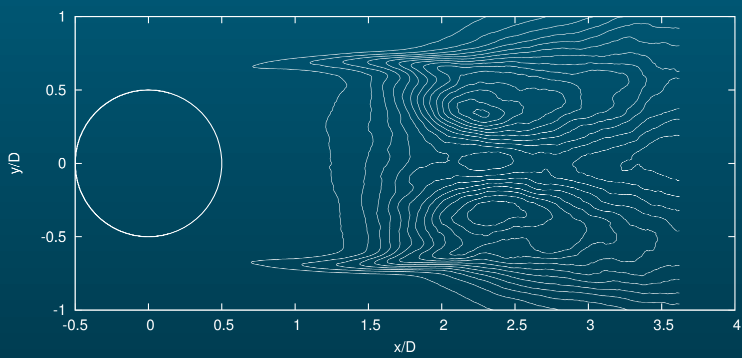
$x/D = 1.56$ $x/D = 3.6$



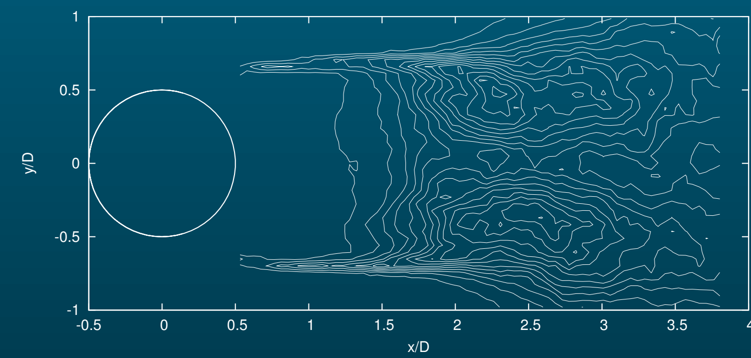
Top: 21 iso-contours of mean transverse velocity $V/U_\infty = -0.25, \ldots, 0.25$

Circular cylinder near wake - $\overline{u'^2}$

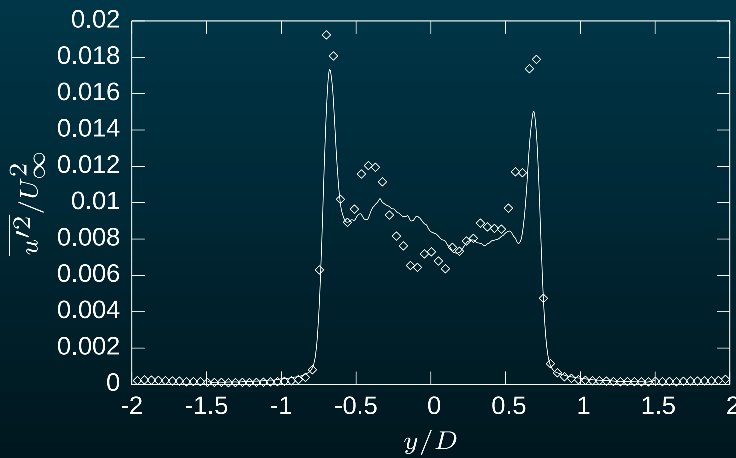
Optical-flow



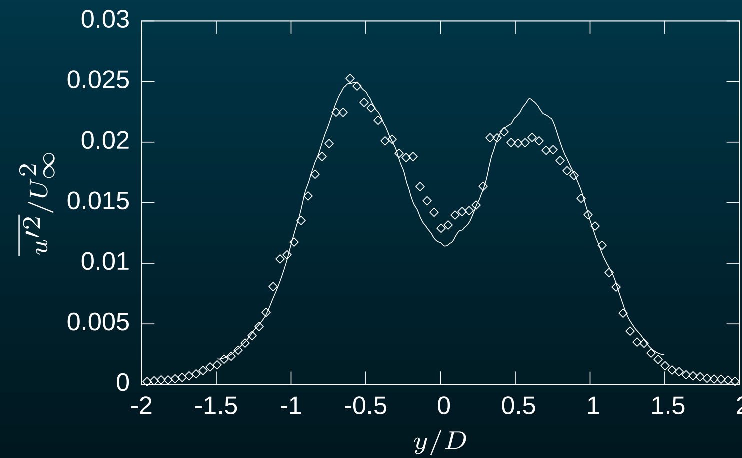
PIV



$x/D = 1.56$



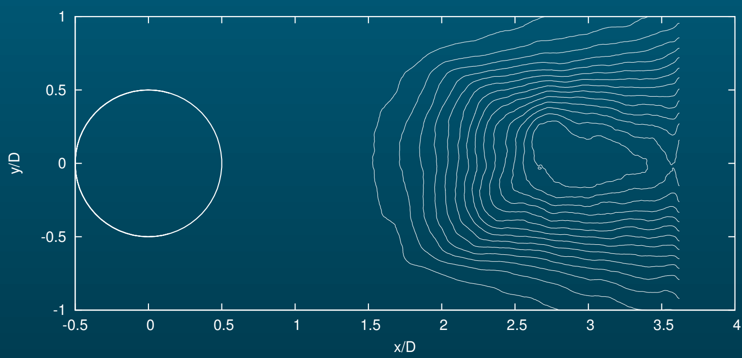
$x/D = 3.6$



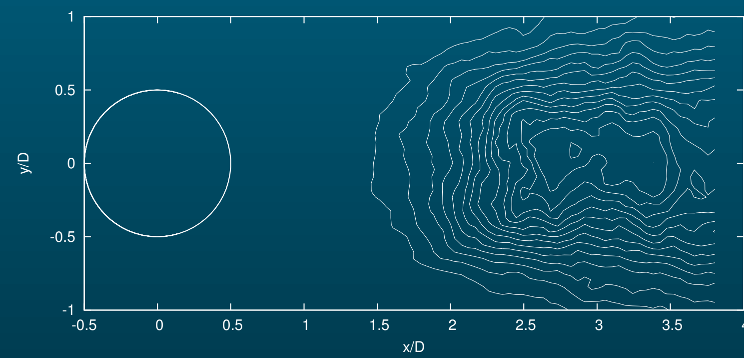
Top: 15 iso-contours of $\overline{u'^2}/U_\infty^2 = 0, ..., 0.05$.

Circular cylinder near wake - $\overline{v'^2}$

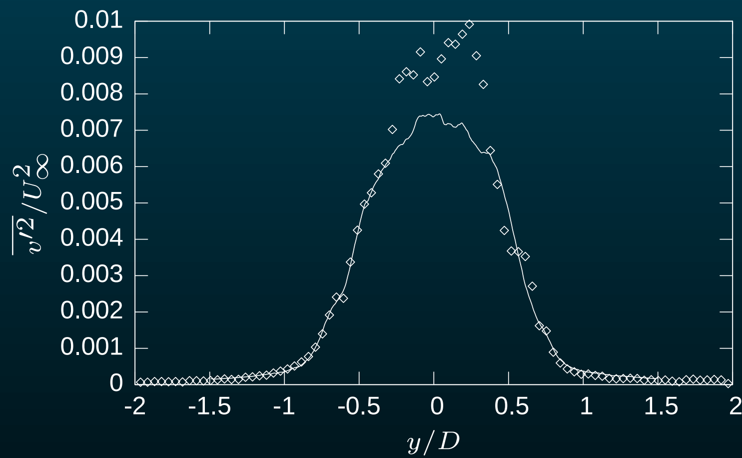
Optical-flow



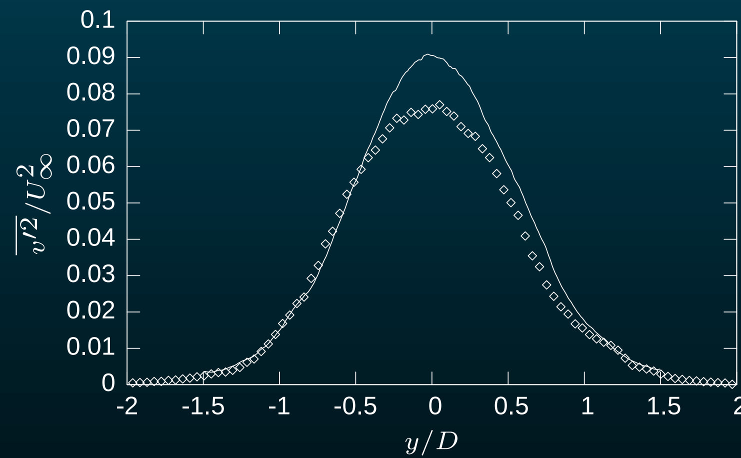
PIV



$x/D = 1.56$



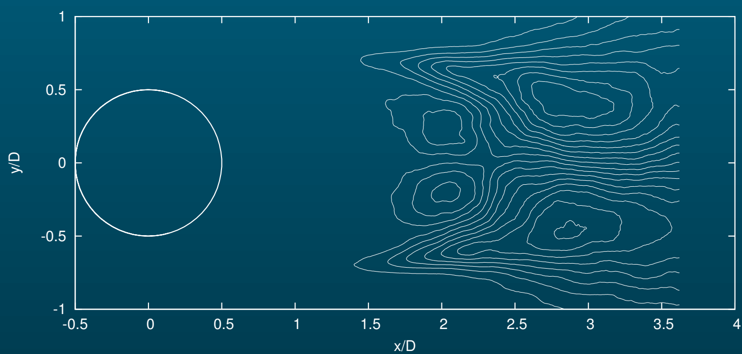
$x/D = 3.6$



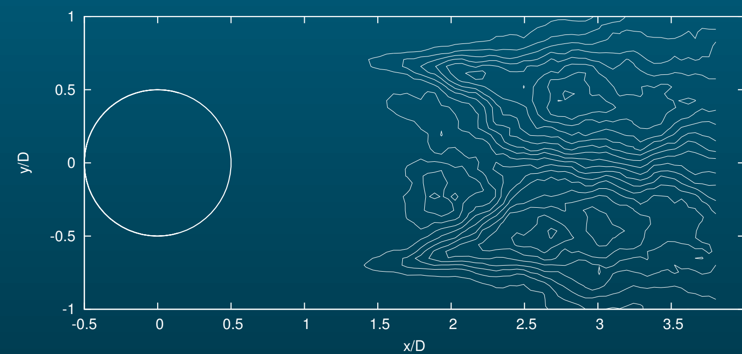
Top: 15 iso-contours of $\overline{v'^2}/U_\infty^2 = 0, ..., 0.1$.

Circular cylinder near wake - $\overline{u'v'}$

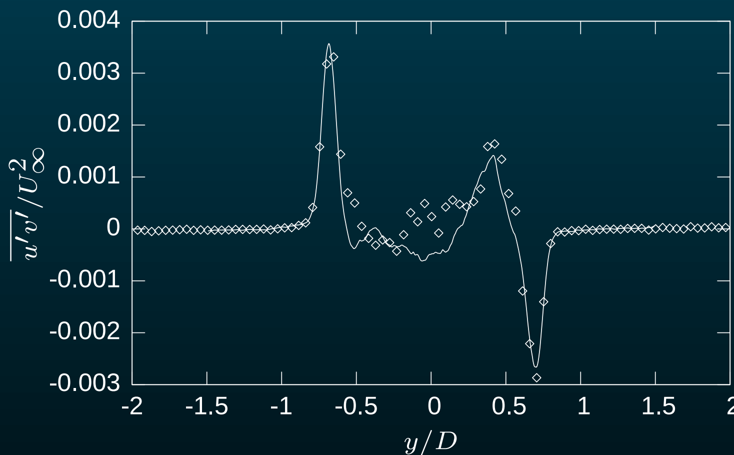
Optical-flow



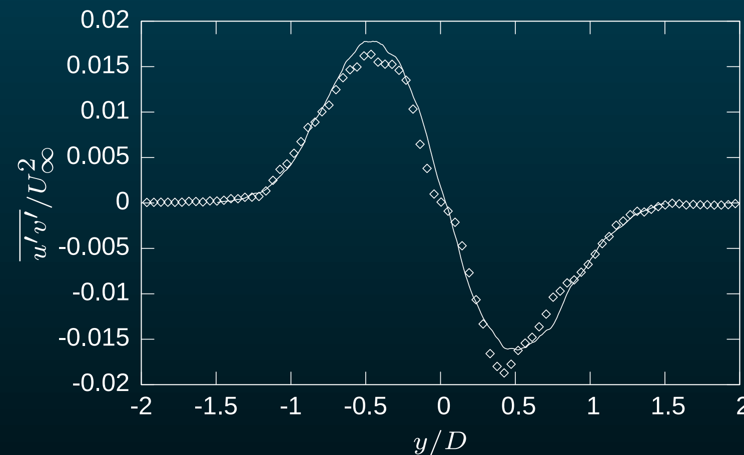
PIV



$x/D = 1.56$



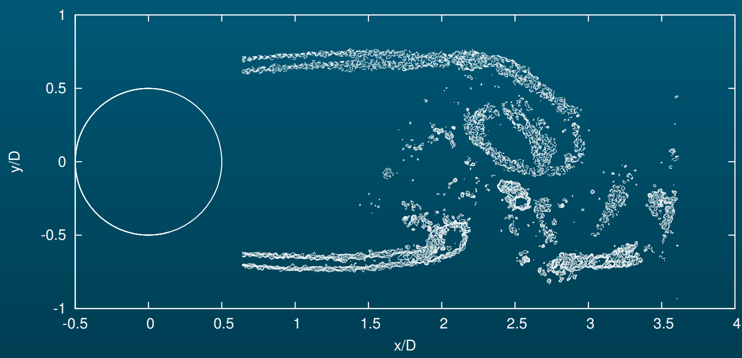
$x/D = 3.6$



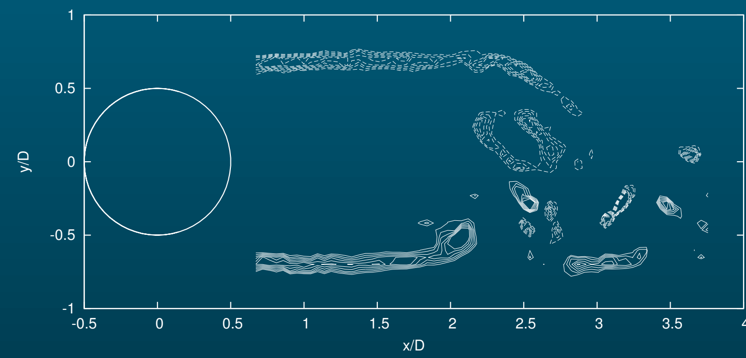
Top: 15 iso-contours of $\overline{u'v'}/U_\infty^2 = -0.025, ..., 0.025$.

Circular cylinder near wake - Formation length

Optical-flow



PIV



	Hot-wire	PIV	Optical-flow
$\overline{u'^2} + \overline{v'^2}$	2.8	2.70	2.80

Conclusion

A new optical-flow estimator based on continuity equation and div $-$ curl regularization has been proposed:

- This optical-flow approach can be applied to PIV images
- Gives similar results compared to PIV with different flow typology (mixing layer - circular cylinder near wake)
- Provides dense information (1 vector per pixel)
- Dense information needs to be validated with dedicated experiments (Characterize accuracy and dynamic range)
- Calibration of the parameters in fluid mechanics framework

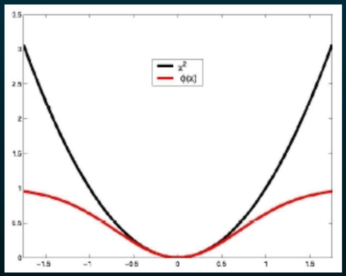
Robuts estimator

- Non quadratic penalisation

$$\mathcal{H}(E,\boldsymbol{v},\xi,\zeta) = \iint_\Omega \Psi\left(\frac{\partial E(\boldsymbol{x},t)}{\partial t} + \text{div}(E(\boldsymbol{x},t)\boldsymbol{v}(\boldsymbol{x},t))\right) d\boldsymbol{x} +$$

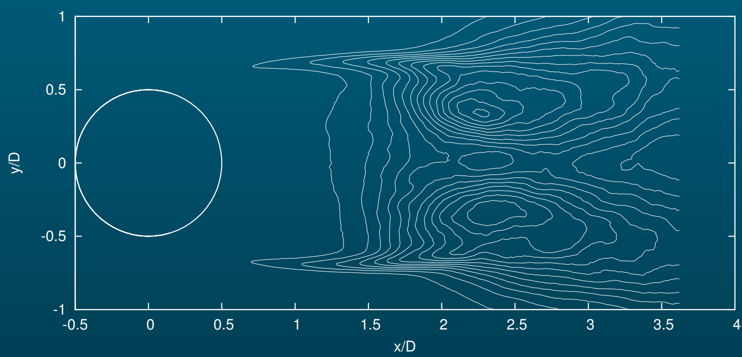
$$\iint_\Omega |\text{div}\ \boldsymbol{v}(\boldsymbol{x},t) - \xi(\boldsymbol{x},t)|^2 + \lambda\Psi(|\boldsymbol{\nabla}\xi(\boldsymbol{x},t)|)d\boldsymbol{x} +$$

$$\iint_\Omega |\text{curl}\ \boldsymbol{v}(\boldsymbol{x},t) - \zeta(\boldsymbol{x},t)|^2 + \lambda\Psi(|\boldsymbol{\nabla}\zeta(\boldsymbol{x},t)|)d\boldsymbol{x}$$

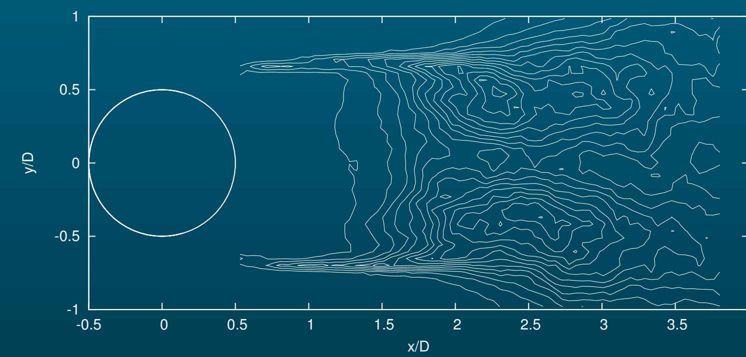


Circular cylinder near wake - $\overline{u'^2}$

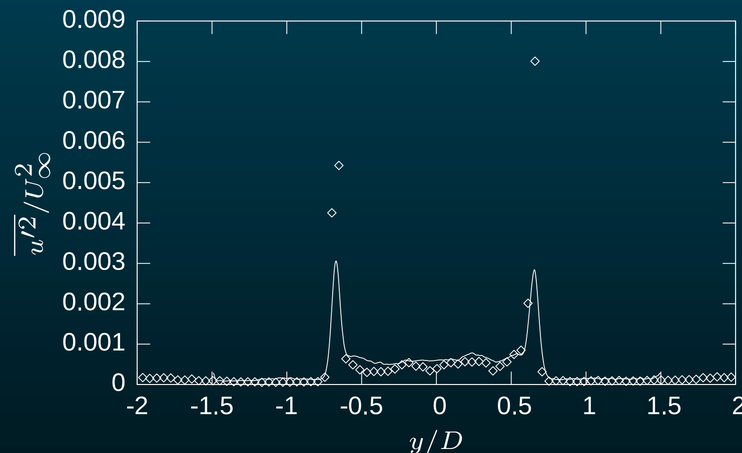
Optical-flow



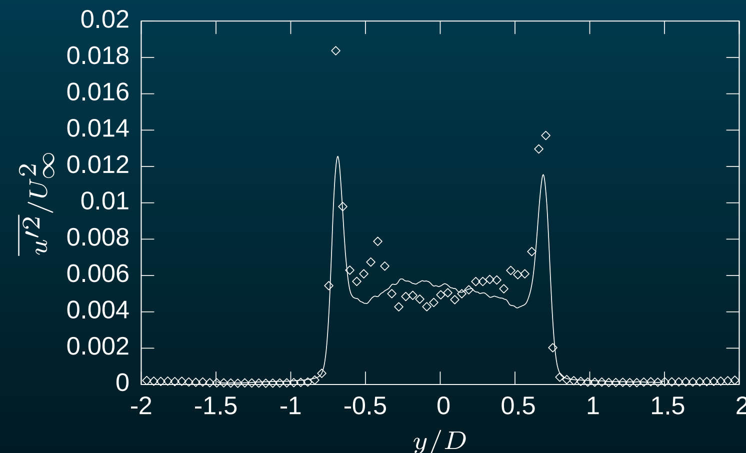
PIV



$x/D = 0.64$

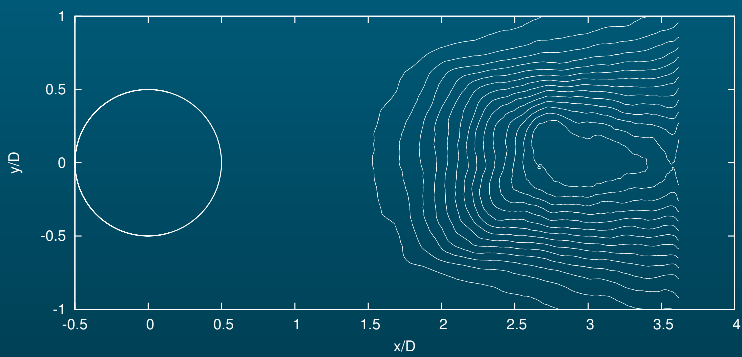


$x/D = 1.41$

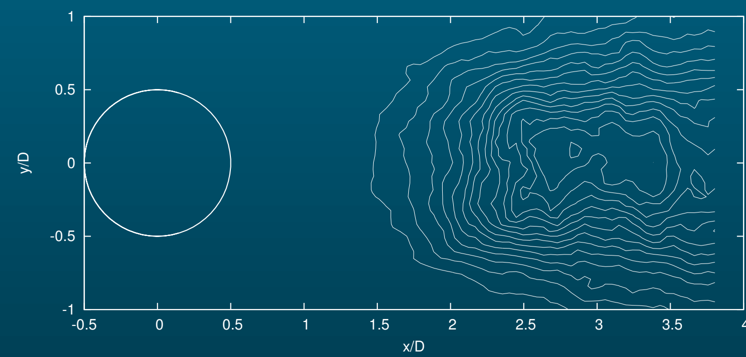


Circular cylinder near wake - $\overline{v'^2}$

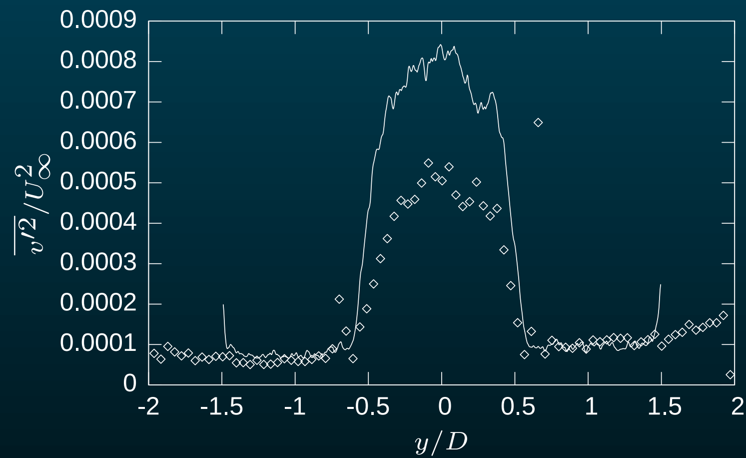
Optical-flow



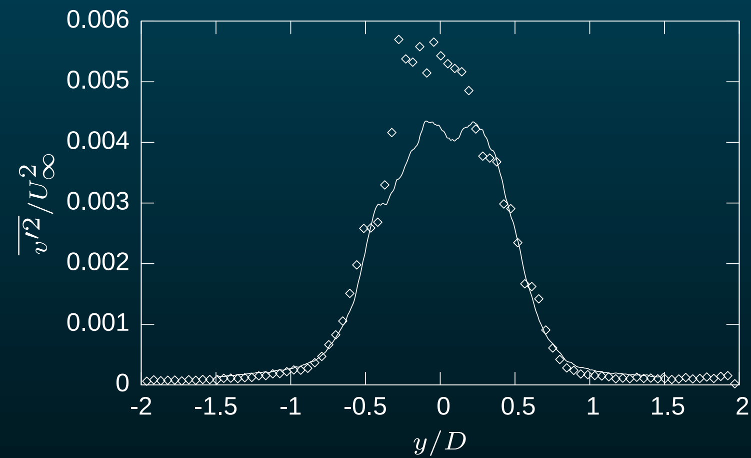
PIV



$x/D = 0.64$

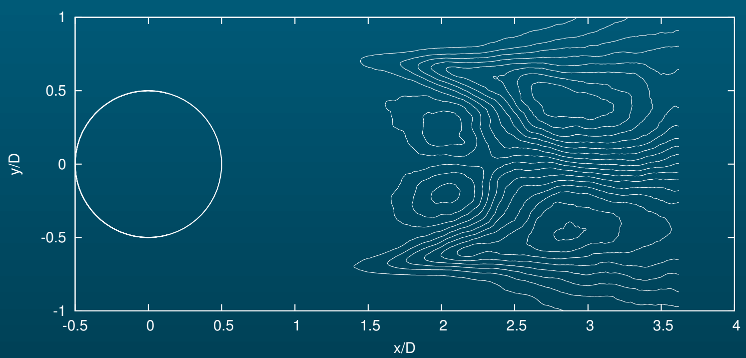


$x/D = 1.41$

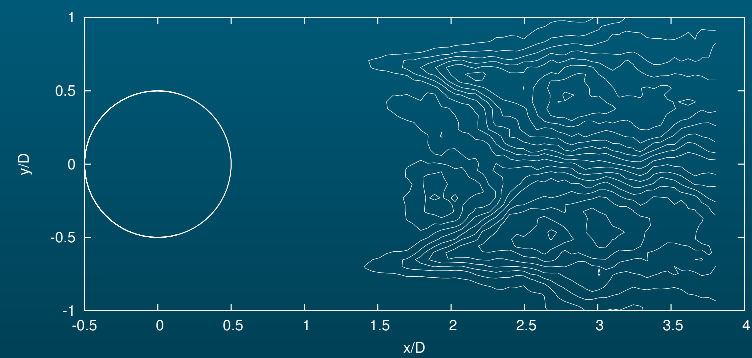


Circular cylinder near wake - $\overline{u'v'}$

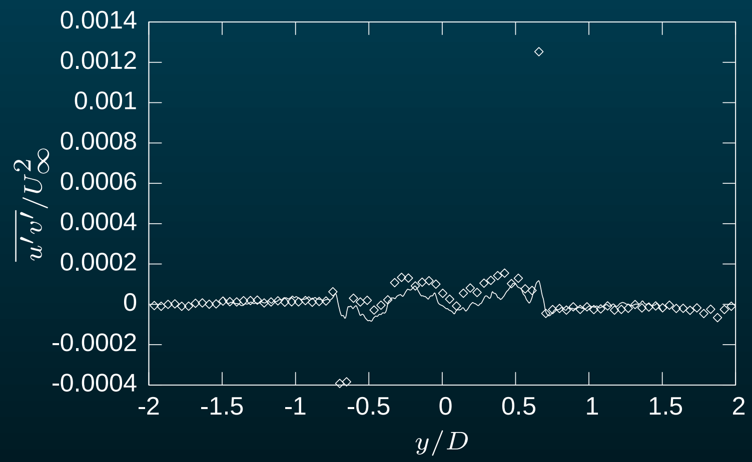
Optical-flow



PIV



$x/D = 0.64$



$x/D = 1.41$