Image-based modelling of ocean surface circulation from satellite acquisitions

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Objective

- Estimation of surface circulation (2D motion $\mathbf{w}(\mathbf{x}, t)$) from an image sequence $T(\mathbf{x}, t)$
- Data Assimilation

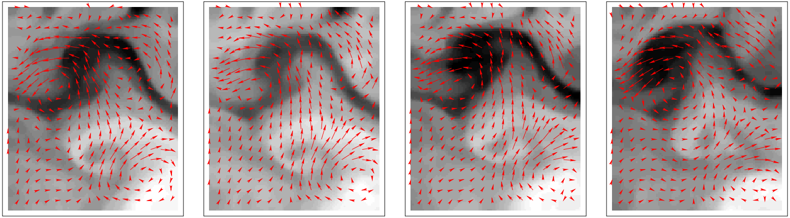


Figure : Satellite data acquired over the Black Sea. Circulation

- Important issue for pollutant transport and meteorology forecast

- Image structures as tracer of the sea surface circulation
- Optical flow methods:
  - They compute translational displacement between two observations
  - They are ill-posed (the Aperture Problem) and required spatial regularization
  - Validity of brightness constancy assumption?

$\Rightarrow$ not physically suited for ocean circulation

State-of-the-art

Ocean circulation models and images

- Circulation: advanced 3D oceanographic models are available (Navier-Stokes, see NEMO project (`http://www.nemo-ocean.net`) for instance)
- But: only a thin upper layer of ocean is observable (from satellite)
- From 3D Navier-stokes and various simplifications, a 2D ocean surface circulation model is derived → **shallow water** equations
- Compute an optimal solution w.r.t. the model and fitting observations: use of **Data Assimilation** techniques
- Need of an **observation model**: link between state vector and observations

- Shallow water model requires information on temperature and upper layer thickness
- Temperature: available from Sea Surface Temperature images (NOAA-AHVRR sensors)
- Layer thickness: not available from remote sensing

⇒ We propose a method to compute surface circulation from SST image and without information on the upper layer thickness

⇒ Use of a rough model, missing information will be represented in an additional model

⇒ Solution is computed using a weak 4D-Var formulation

- State vector is velocity, $\mathbf{w} = (u, v)^T$, surface temperature (T_s) and upper layer thickness (η):

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} + fv - g'\frac{\partial \eta}{\partial x} + K_{\mathbf{w}}\Delta u \quad (1)$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - fu - g'\frac{\partial \eta}{\partial y} + K_{\mathbf{w}}\Delta v \quad (2)$$

$$\frac{\partial \eta}{\partial t} = -\frac{\partial (u\eta)}{\partial x} - \frac{\partial (v\eta)}{\partial y} - \eta\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \quad (3)$$

$$\frac{\partial T_s}{\partial t} = -u\frac{\partial T_s}{\partial x} - v\frac{\partial T_s}{\partial y} + K_T\Delta T_s \quad (4)$$

- $K_{\mathbf{w}}$, K_T are diffusive constants, f the Coriolis parameter and g' the reduced gravity (see paper for details)
- Functions $\mathbf{w}$, T_s and η are defined on a space-time domain: $\Omega \times [0, \mathrm{T}], \Omega \subset \mathbb{R}^2$

- Geophysical forces (in red) are grouped

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} + fv - g'\frac{\partial \eta}{\partial x} + K_{\mathbf{w}}\Delta u \quad (5)$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - fu - g'\frac{\partial \eta}{\partial y} + K_{\mathbf{w}}\Delta v \quad (6)$$

$$\frac{\partial \eta}{\partial t} = -\frac{\partial (u\eta)}{\partial x} - \frac{\partial (v\eta)}{\partial y} - \eta\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \quad (7)$$

$$\frac{\partial T_s}{\partial t} = -u\frac{\partial T_s}{\partial x} - v\frac{\partial T_s}{\partial y} + K_T\Delta T_s \quad (8)$$

in a hidden part, $\mathbf{a} = (a_u, a_v)^T$, named "additional model"

Proposed model

- The previous system is rewritten as:

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} + a_u \quad (9)$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} + a_v \quad (10)$$

$$\frac{\partial T_s}{\partial t} = -u\frac{\partial T_s}{\partial x} - v\frac{\partial T_s}{\partial y} + K_T \Delta T_s \quad (11)$$

- with

$$a_u = fv - g'\frac{\partial \eta}{\partial x} + K_{\mathbf{w}}\Delta u \quad (12)$$

$$a_v = -fu - g'\frac{\partial \eta}{\partial y} + K_{\mathbf{w}}\Delta v \quad (13)$$

where η verifies Eq.(3)

Proposed model

- The additional model $\mathbf{a}$ is now considered as an unknown that we want to retrieve
- The state vector is $\mathbf{X} = \begin{pmatrix} \mathbf{w} & T_s \end{pmatrix}^T$, and the model ruling the evolution in time of $\mathbf{X}$ is summarized as:

$$\frac{\partial \mathbf{X}}{\partial t}(\mathbf{x}, t) + \mathbb{M}(\mathbf{X})(\mathbf{x}, t) = \begin{pmatrix} \mathbf{a}(\mathbf{x}, t) \\ 0 \end{pmatrix} \quad \mathbf{x} \in \Omega, t \in (0, T] \tag{14}$$

with

$$\mathbb{M}(\mathbf{X}) = \begin{pmatrix} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \\ u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \\ u\frac{\partial T_s}{\partial x} + v\frac{\partial T_s}{\partial y} + K_T \Delta T_s \end{pmatrix}$$

Initialization and Observation model

- Need of an initial condition for $\mathbf{X}(0)$:

$$\mathbf{X}(\mathbf{x},0) = \mathbf{X}_b(\mathbf{x}) + \varepsilon_B(\mathbf{x}), \quad \mathbf{x} \in \Omega \tag{15}$$

  ε_B Gaussian with covariance matrix B

- Observations are SST images, $T(t_i)$, available at some given dates $t_1, \cdots, t_N$
- The observation operator $\mathbb{H}$ projects the state vector in the observation space. It is defined as:

$$\mathbb{H}(\mathbf{X}) = T_s \tag{16}$$

- Link between state vector and observation:

$$\mathbb{H}(\mathbf{X})(\mathbf{x},t_i) = T(\mathbf{x},t_i) + \varepsilon_R(t_i), \quad \mathbf{x} \in \Omega, i = 1, \cdots, N \tag{17}$$

  ε_R Gaussian with covariance matrix R

- To solve:

$$\begin{aligned}
\frac{\partial \mathbf{X}}{\partial t}(\mathbf{x},t) + \mathbb{M}(\mathbf{X})(\mathbf{x},t) &= \begin{pmatrix} \mathbf{a}(\mathbf{x},t) \\ 0 \end{pmatrix} \quad \mathbf{x} \in \Omega, t \in (0,\mathrm{T}] \\
\mathbb{H}(\mathbf{X})(\mathbf{x},t_i) &= T(\mathbf{x},t_i) + \varepsilon_R(t_i), \quad \mathbf{x} \in \Omega, i = 1,\cdots,N \\
\mathbf{X}(\mathbf{x},0) &= \mathbf{X}_b(\mathbf{x}) + \varepsilon_B(\mathbf{x}), \quad \mathbf{x} \in \Omega
\end{aligned} \tag{18}$$

  we minimize the cost function J:

$$\begin{aligned}
J(\varepsilon_B, \mathbf{a}(t_1),\cdots,\mathbf{a}(t_N)) = \left\langle \varepsilon_B, B^{-1}\varepsilon_B \right\rangle + \gamma \sum_{i=1}^{N} \|\nabla \mathbf{a}(t_i)\|^2 + \\
\sum_{i=1}^{N} \left\langle \mathbb{H}(\mathbf{X})(t_i) - T(t_i), R^{-1}(\mathbb{H}(\mathbf{X})(t_i) - T(t_i)\varepsilon_B \right\rangle
\end{aligned} \tag{19}$$

  under the constraint of Eq.(18)
- γ term is introduced to prevent numerical instabilities

Theorem 1

Let $\lambda(\mathbf{x}, t)$ be an auxiliary variable (named adjoint variable) as solution of:

$$\lambda(T) = 0 \quad (20)$$

$$-\frac{\partial \lambda(t)}{\partial t} + \left(\frac{\partial \mathbb{M}}{\partial \mathbf{X}}\right)^* \lambda = \mathbb{H}^T R^{-1}[\mathbb{H}\mathbf{X}(t) - T(t)] \quad t = t_i \quad (21)$$

$$= 0 \quad t \neq t_i \quad (22)$$

Then, gradient of J is:

$$\frac{\partial J}{\partial \varepsilon_B} = 2\left(B_I^{-1}\varepsilon_B + \lambda(0)\right) \quad (23)$$

$$\frac{\partial J}{\partial \mathbf{a}(t_i)} = 2\left(-\gamma \Delta \mathbf{a}(t_i) + \lambda(t_i)\right) \quad (24)$$

Weak 4D-Var formulation

Algorithm

1. *Forward pass:* integrate forward in time $\mathbf{X}(t)$, compute J
2. *Backward pass:* integrate backward in time $\lambda(t)$, compute ∇J
3. Perform a steepest descent using numerical solver and get new values for $\mathbf{X}(0)$ and $\mathbf{a}(t_i), i = 1 \cdots N$
4. Repeat steps 1, 2, 3 up to convergence

Implementation

- Eq. (18) must be discretized:
  - In time: Euler scheme
  - In space:
    - components u and v are transported by a non linear advection (Burger equations): Godunov scheme
    - component I_s is transport by a linear advection: a first order up-wind
- Adjoint model: operator $\left(\frac{\partial \mathbb{M}}{\partial \mathbf{X}}\right)^*$ in Eq. (21) is formally defined as a dual operator:

$$\left\langle \phi, \left(\frac{\partial \mathbb{M}}{\partial \mathbf{X}}\right)^* \psi \right\rangle = \left\langle \left(\frac{\partial \mathbb{M}}{\partial \mathbf{X}}\right) \phi, \psi \right\rangle$$

  It must be determined from the discrete model $\mathbb{M}_{\text{dis}}$: we use an automatic differentiation software,
  Tapenade [Hascoët and Pascual, 2013]
- Steepest descent is performed by BFGS solver [Byrd et al., 1995]

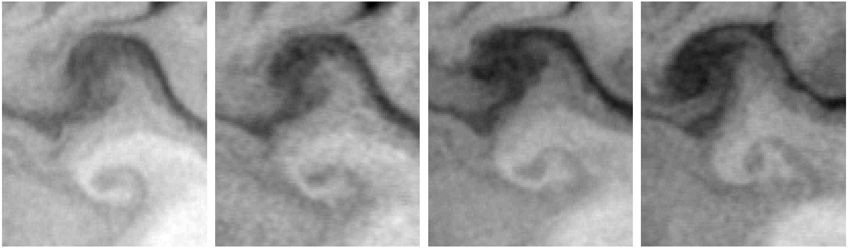
Data assimilation setup

- Model: temperature diffusion is neglected $(K_T = 0)$
- Initial condition $\mathbf{X}_b = \begin{pmatrix}\mathbf{w}_b & I_b\end{pmatrix}^T$:
  - no information available on initial velocity, we set $\mathbf{w}_b = \vec{0}$
  - I_b is initialized to the first available observation $I(t_1)$
- Covariance matrix B: without information on initial velocity we choose $\varepsilon_B = \begin{pmatrix}0 & 0 & \varepsilon_{B_{I_s}}\end{pmatrix}$ and we have:

$$\left\langle \varepsilon_B, B^{-1}\varepsilon_B \right\rangle = \left\langle \varepsilon_{B_{I_s}}, B_{I_s}^{-1}\varepsilon_{B_{I_s}} \right\rangle$$

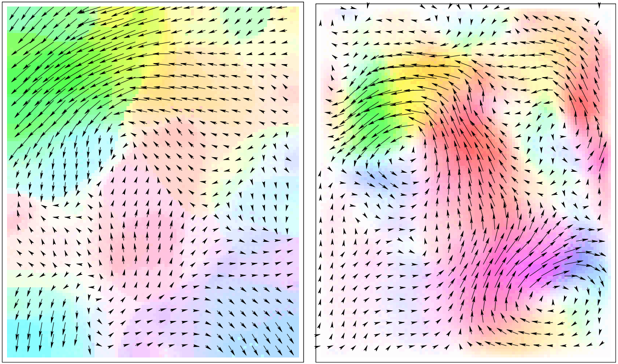
  - Matrix B_{I_s}: chosen diagonal, each element is set to 1 (1 Celsius degree means 25 % of image dynamics)
- Covariance matrix R: chosen diagonal, each element is set to 1
- γ : empirically fixed

- A sequence of four SST images was acquired over Black Sea on October 10th, 2007



(a) 30*min* (b) 6*h* (c) 15*h* (d) 30*h*

Figure : October 10th 2007, over Black Sea

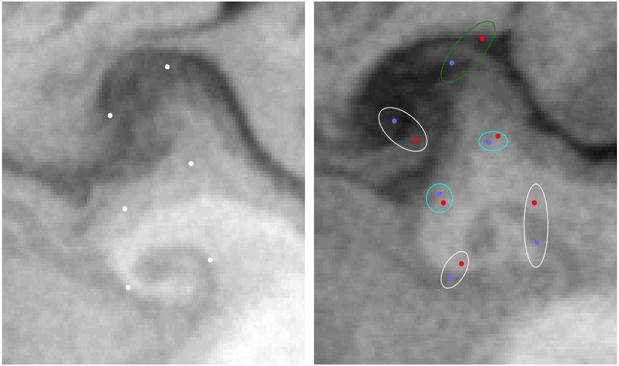


(a) [Sun et al., 2010] (b) Proposed method

Figure : Motion computed between the first and second observation

Quantitative evaluation on satellite images

- No ground-truth on satellite data, how to evaluate?
- We propose to compute the trajectory of some characteristic points in order to evaluate algorithms in term of transport
- A comparison with state-of-the-art is performed
- We proceed as follow:
  - Manual selection of a characteristic point in the first observation
  - A map of signed distance is computed
  - Distance map is transported by the velocity field that we want to evaluate [Lepoittevin et al., 2013]
  - Computation of local maximum in transported map gives the characteristic point position along the sequence



(a) First observation

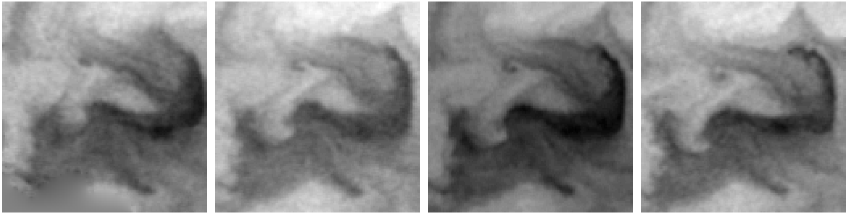
(b) Last observation. Blue = our method, red = Sun et al.

Figure : Evolution of some characteristic points

Satellite Experiment #2

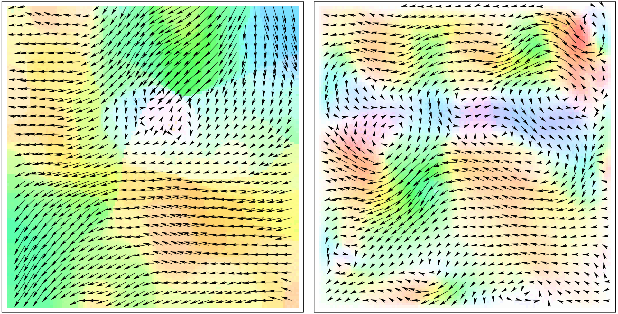
Observations

- Sequence acquired on October 8th, 2005



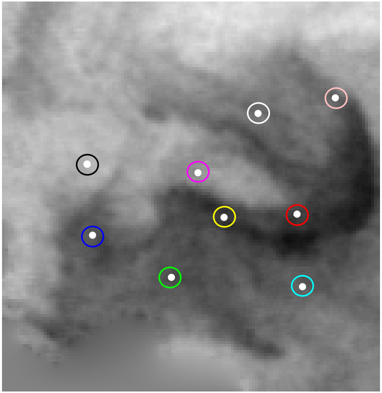
(a) 30min (b) 10h30min (c) 12h (d) 15h30min

Figure : October 8th 2005, over Black Sea



(a) [Sun et al., 2010] (b) Proposed method

Figure : Motion computed between the first and second observation

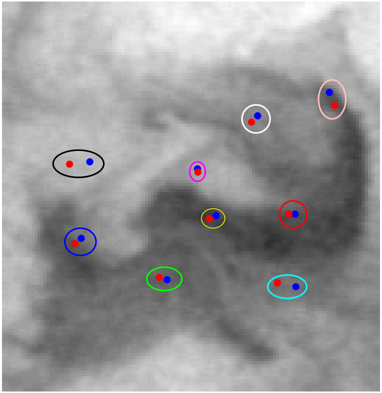


(a) First observation

Figure : Evolution of some characteristic points

Satellite Experiment #2

Characteristic points

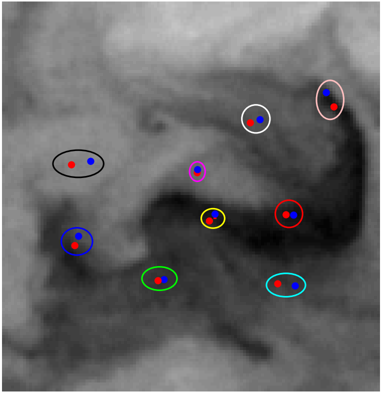


(a) Second observation

Figure : Evolution of some characteristic points

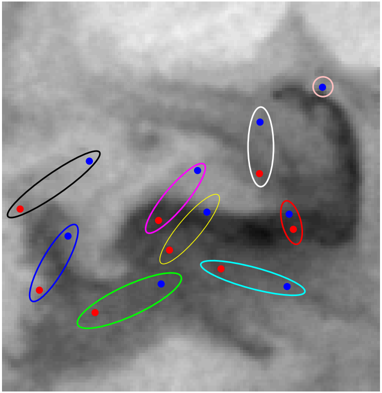
Satellite Experiment #2

Characteristic points



(a) Third observation

Figure : Evolution of some characteristic points

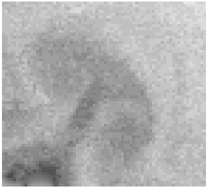


(a) Last observation

Figure : Evolution of some characteristic points

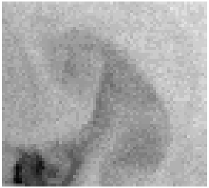
Satellite Experiment #3
Observations

- Sequence acquired on July 27^{th}, 2007

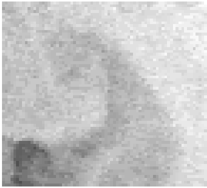


(a) $30min$

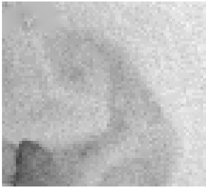
(b) $8h15min$



(c) $13h$

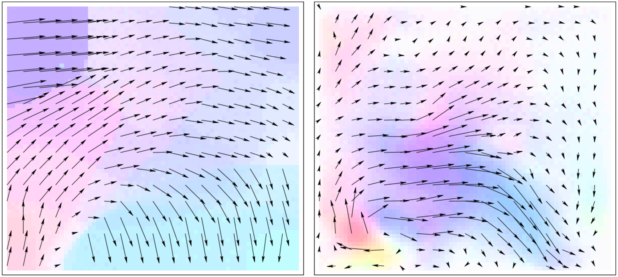


(d) $22h30min$



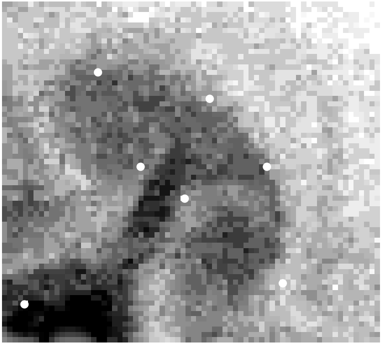
(e) $24h30min$

Satellite Experiment #3
Motion results



(a) [Sun et al., 2010] (b) Proposed method

Figure : Motion computed between the first and second observation

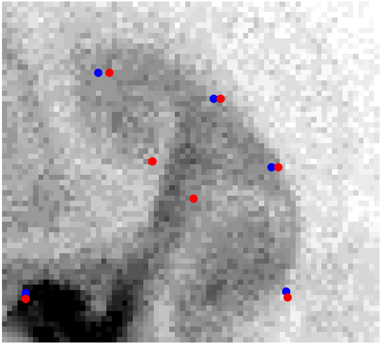


(a) First observation

Figure : Evolution of some characteristic points

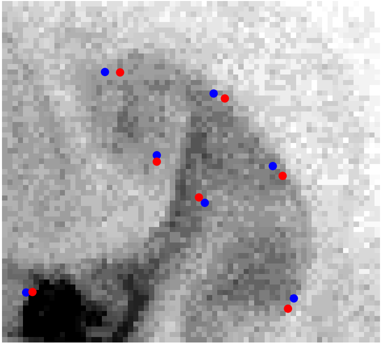
Satellite Experiment #3

Characteristic points



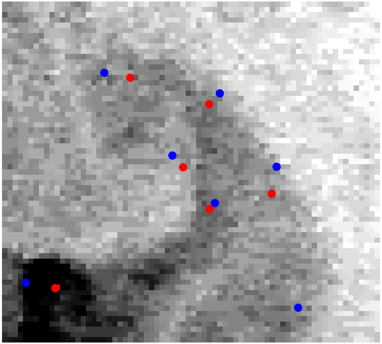
(a) Second observation

Figure : Evolution of some characteristic points



(a) Third observation

Figure : Evolution of some characteristic points

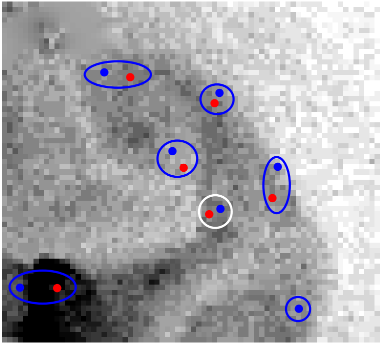


(a) Fourth observation

Figure : Evolution of some characteristic points

Satellite Experiment #3

Characteristic points

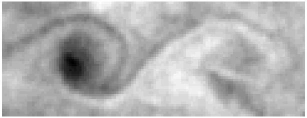


(a) Last observation

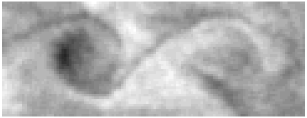
Figure : Evolution of some characteristic points

Satellite Experiment #4
Observations

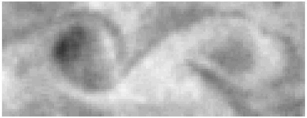
- Sequence acquired on May 14th, 2005



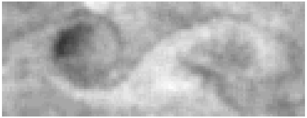
(a) 30min



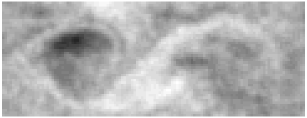
(b) 2h55min



(c) 5h15min



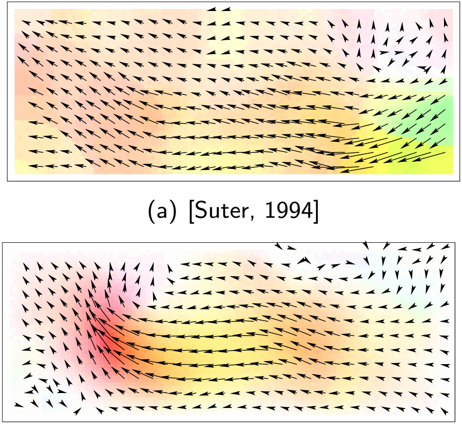
(d) 7h15min



(e) 16h15min

Satellite Experiment #4

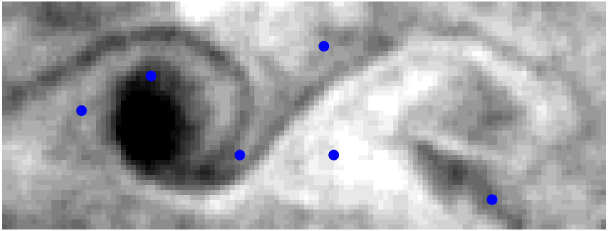
Motion results



(a) [Suter, 1994]

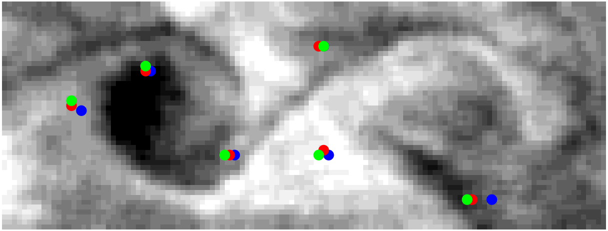
(b) Proposed method

Figure : Motion computed between the first and second observation



(a) First observation

Figure : Evolution of some characteristic points

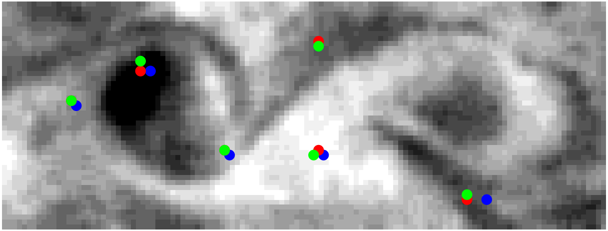


(a) Second observation

Figure : Evolution of some characteristic points

Satellite Experiment #4

Characteristic points

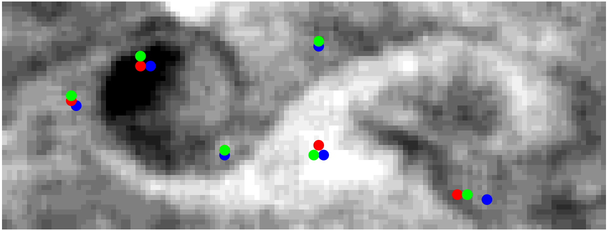


(a) Third observation

Figure : Evolution of some characteristic points

Satellite Experiment #4

Characteristic points

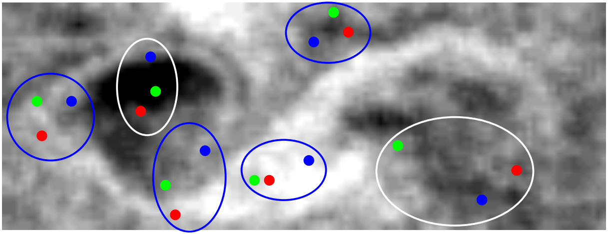


(a) Fourth observation

Figure : Evolution of some characteristic points

Satellite Experiment #4

Characteristic points



(a) Last observation

Figure : Evolution of some characteristic points

Concluding remarks

- Determination of surface circulation using a rough model $\mathbb{M}$
- Dynamics not modelled by $\mathbb{M}$ is captured in **a**
- Analysis of **a** retrieved and comparison with a shallow water model
- Experiments on synthetic models (Temperature and upper layer thickness) with ground truth

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