Surface circulation from satellite images: reduced model of the Black Sea

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Oceanography: Black Sea circulation

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Objective

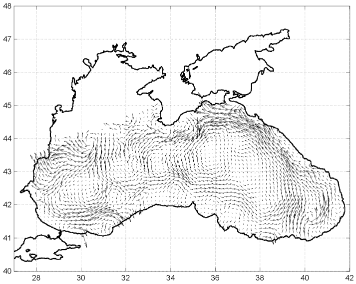
Estimation of motion $\mathbf{W}(\mathbf{x}, t)$ on an image sequence $I(\mathbf{x}, t)$ by Data Assimilation.

Input Data

Sequence of satellite images I:

Result of estimation

Estimation of apparent velocity $\mathbf{W}$:



Data assimilation (4D-Var)

$\mathbf{X}(t)$ state vector, $\mathbf{Y}(t)$ observation vector.

Find $\mathbf{X}$ such as :

$$
\begin{aligned}
\frac{\partial \mathbf{X}}{\partial t}(t) + \mathbb{M}(\mathbf{X})(t) &= 0 & (1)\\
\mathbf{X}(0) &= \mathbf{X}_b + \mathcal{E}_b & (2)\\
\mathbb{H}(\mathbf{X}, \mathbf{Y})(t) &= \mathcal{E}_O(t) & (3)
\end{aligned}
$$

- $\mathbf{X}(t)$ depends only from $\mathbf{X}(0)$ and the integration from (1)
- The problem is then to minimize the two error terms $\mathcal{E}_b$ and $\mathcal{E}_O$. This is solved using a steepest descent method (L-BFGS).

Model

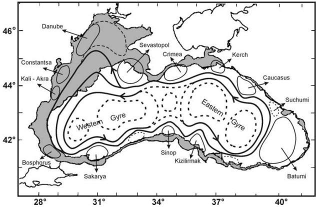
- Lagrangian constancy of the velocity $\mathbf{w}(t)$

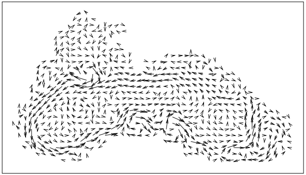
$$\frac{d\mathbf{w}}{dt} = \frac{\partial \mathbf{w}}{\partial t} + (\mathbf{w}.\nabla)\mathbf{w} = 0$$

- Optical flow on the pseudo image $I_s(t)$

$$\frac{\partial I_s}{\partial t} + \mathbf{w}.\nabla I_s = 0$$

Reduction





- Reduced state: less memory
- Regularity: applied on basis elements
- Boundary conditions: imposed to the basis elements
- Numerical schemes: ODE vs PDE

Full and reduced models

Full model

$$\begin{cases} \frac{\partial \mathbf{w}}{\partial t}(\mathbf{x},t) + (\mathbf{w}\cdot\nabla)\,\mathbf{w}(\mathbf{x},t) = 0 \\ \frac{\partial I_s}{\partial t}(\mathbf{x},t) + \mathbf{w}\cdot\nabla I_s(\mathbf{x},t) = 0 \end{cases}$$

Reduced model

$$\begin{cases} \frac{da_k}{dt}(t) + a^T B(k) a = 0,\ k = [\![1,K]\!] \\ \frac{db_l}{dt}(t) + a^T G(l) b = 0,\ l = [\![1,L]\!] \end{cases}$$

$$\begin{cases} \mathbf{w}(\mathbf{x},t) \approx \sum_{k=1}^{K} a_k(t)\phi_k(\mathbf{x}) \\ I_s(\mathbf{x},t) \approx \sum_{l=1}^{L} b_l(t)\psi_l(\mathbf{x}) \end{cases}$$

$$B(k)_{i,j} = \frac{\langle(\phi_i\nabla)\phi_j,\phi_k\rangle}{\langle\phi_k,\phi_k\rangle}$$

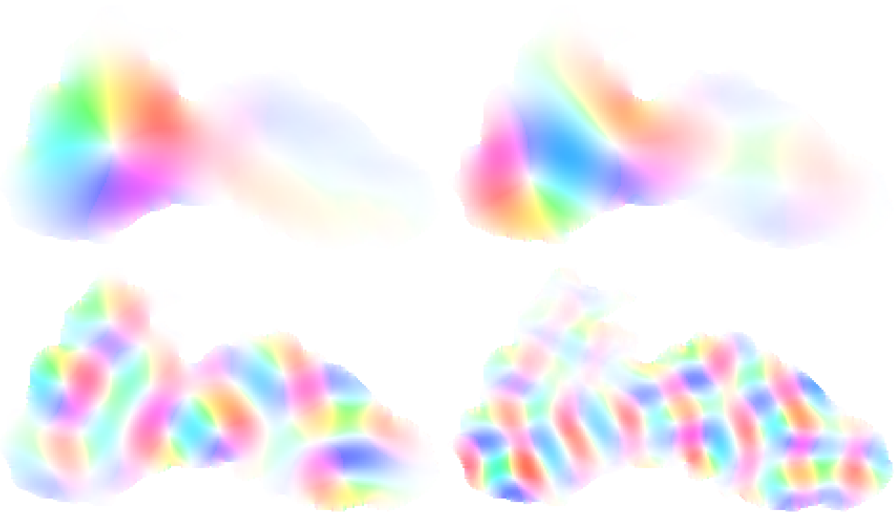
$$G(l)_{i,j} = \frac{\langle\phi_i\cdot\nabla\psi_j,\psi_l\rangle}{\langle\psi_l,\psi_l\rangle}$$

Motion basis

ϕ_i are obtained by sequentially solving systems S_i:

$$S_i = \begin{cases} \phi_i = \min_{\mathbf{f} \in L_2(\Omega)^2} \langle \nabla \mathbf{f}, \nabla \mathbf{f} \rangle \\ \text{div}\ (\phi_i(\mathbf{x})) = 0 \quad \forall \mathbf{x} \in \Omega \\ \phi_i(\mathbf{x}) \cdot \mathbf{n}(\mathbf{x}) = 0 \quad \forall \mathbf{x} \in \partial\Omega \\ \langle \phi_i, \phi_k \rangle = \delta_{i,k}, \quad k \in [\![1, i]\!] \end{cases} \tag{4}$$

Motion basis



Motion basis

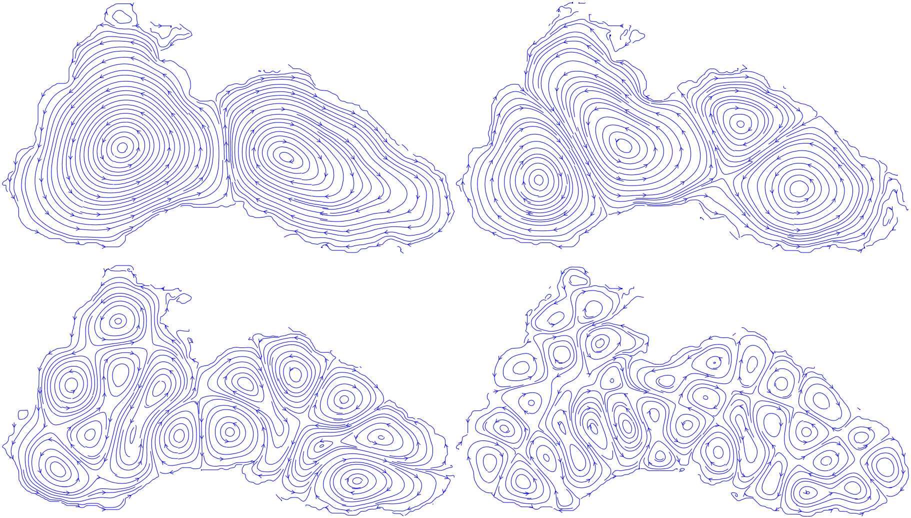
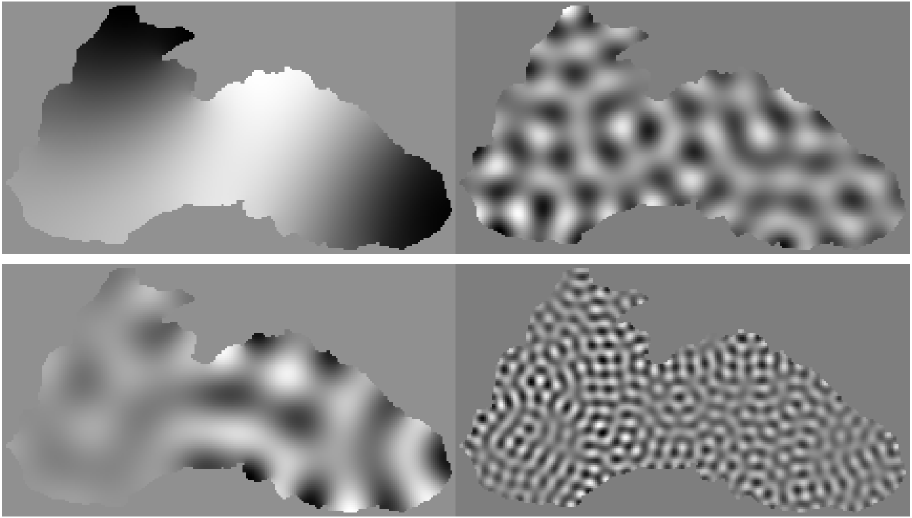


Image Basis

ψ_i are obtained by sequentially solving systems S_i :

$$S_i = \begin{cases} \psi_i = \min\limits_{\mathbf{f} \in L_2(\Omega)} \langle \nabla \mathbf{f}, \nabla \mathbf{f} \rangle \, d\mathbf{x} \\ \nabla \psi_i(\mathbf{x}) \cdot \mathbf{n}(\mathbf{x}) = 0 \quad \forall \mathbf{x} \in \partial\Omega \\ \langle \psi_i, \psi_k \rangle = \delta_{i,k}, \quad k \in [\![1, i]\!] \end{cases} \tag{5}$$

Image basis



Data assimilation system

State vector: $\mathbf{X}(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$

with:

$$a(t) = (a_1(t), \ldots, a_K(t))^T$$
$$b(t) = (b_1(t), \ldots, b_L(t))^T$$

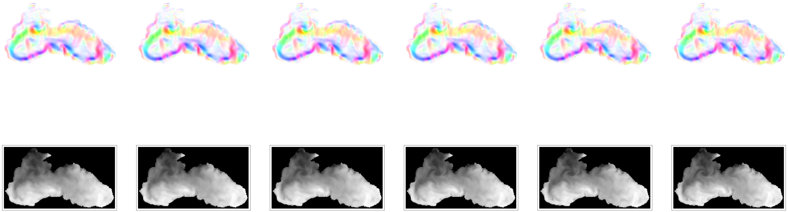
- Background equation: $b(0) - b^{obs}(t_1) = \mathcal{E}_B$
- Observation equation: $b(t_k) - b^{obs}(t_k) = \mathcal{E}_R(t_k)$.

Energy function:

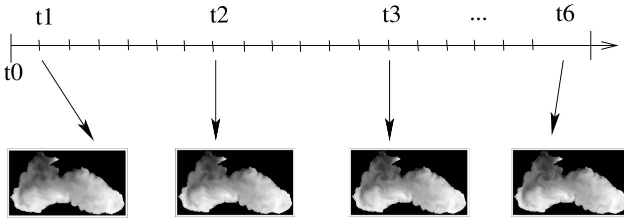
$$J(\mathbf{X}(0)) = \int \mathcal{E}_R(t)^T R^{-1}(t) \mathcal{E}_R(t) dt + \mathcal{E}_B^T B^{-1} \mathcal{E}_B$$

Whole Black Sea bassin

Simulation of full model from chosen initial conditions:



Observation dates chosen:



Assimilation of Black Sea simulations

Ground-truth:

Results of Assimilation in the reduced model:

Satellite images

SST images of the Black Sea acquired with NOAA/AVHRR between May 14th, 2005 and May 15th, 2005

Satellite images

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Characteristic points

Ground-truth

Reduced model
Full model
Sun

Assimilation of satellite images on the whole basin

Assimilation of satellite images on the whole basin

Conclusions and perspective

Advantages and drawbacks

- Reduction
- Boundary conditions applied to the basis elements
- Choice of coefficients number linked to studied spatial scale

Perspectives

- Adaptative waveforms
- POD or seasonal POD
- Krylov approximation