Image assimilation for the analysis of geophysical flows

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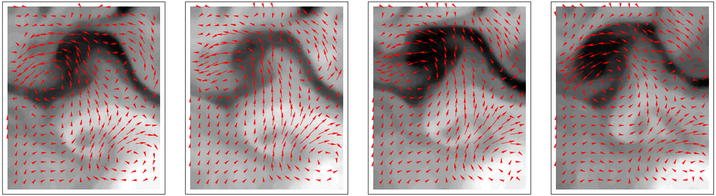
Why coupling models and images?

- Whatever model's resolution, images of higher resolution.
- Deriving characteristics from acquisitions, further assimilated as pseudo-observations. Atmospheric Motion Vectors. Ocean surface motion.
- Direct assimilation of new high-level data. Gradient maps. Wavelets or curvlets coefficients.
- Control of structures positions.



Satellite acquisitions of Black Sea and estimated motion

Which research themes?

- Empirical models from image data. Describing objects evolution: pollutant spills, ocean or meteorological structures. Major interest for nowcasting.
- Coupling models and images of different resolutions. Subgrid parameterization. High resolution coastal currents.
- Optimal bases for image and model reduction. Crisis management.

Identification of operational needs

- Short-term photovoltaic production forecast. EDF R&D in the test side of Reunion Island.
- Pollutant transport and littoral monitoring.
- Monitoring of offshore equipments.
- To be discussed in SAMA.

Actions in Clime in the last 4 years

- State estimation with 4D-Var data assimilation. Observation equations for image data, observation error covariance matrix. Motion estimation, inpainting, structures tracking.
- Model error. Image models being obtained from heuristics, estimation of their error allows assessing the dynamics.
- Model reduction. Sliding windows method for long sequences and POD reduction. Div-free motion from vorticity on sine basis. Computation of basis from motion properties (domain shape, boundary conditions).
- Ensemble methods. Definition of an ensemble from optical flow methods.

Highlight1

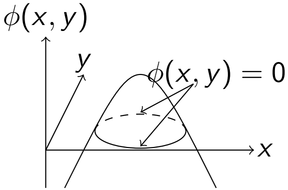
Image Model for Motion Estimation and Structure Tracking

Highlight1

Image Model for Motion Estimation and Structure Tracking

State vector $\mathbf{X}(x,y,t) = \left(\mathbf{w}(x,y,t)^T \quad I_s(x,y,t) \quad \Phi(x,y,t)\right)^T$

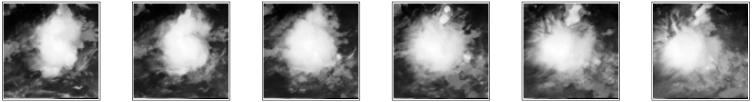
- Lagrangian constancy of velocity
  $$\frac{\partial \mathbf{w}}{\partial t} + (\mathbf{w} \cdot \nabla)\,\mathbf{w} = 0$$
- Transport of image function
  $$\frac{\partial I_s}{\partial t} + \mathbf{w} \cdot \nabla I_s = 0$$
- Advection of Φ
  $$\frac{\partial \Phi}{\partial t} + \mathbf{w} \cdot \nabla \Phi = 0$$



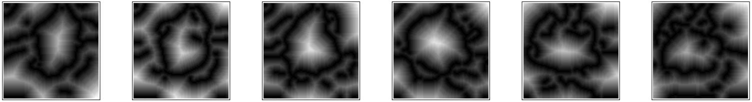
Motion Estimation and Structure Tracking

Observations

Satellite images $I(t_i)$ acquired by satellite at dates t_i



Distance to contours points $D_C(t_i)$ computed on the images

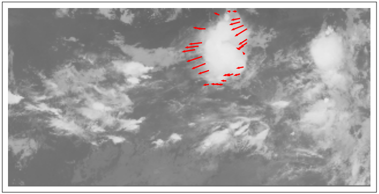


Definition of H :

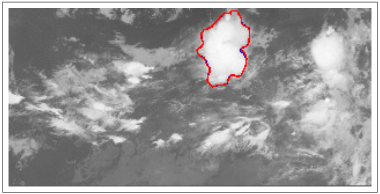
$$H(\mathbf{X},\mathbf{Y}) = I - I_s$$

$$H_\Phi(\mathbf{X},\mathbf{Y}) = (D_C - |\Phi|)\mathbb{1}_{|\Phi|<s}$$

Motion Estimation and Structure Tracking

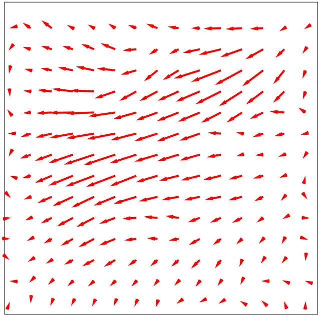


Motion Field

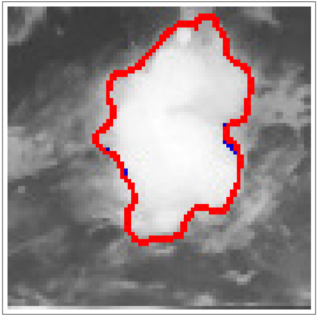


with contour points
without contour points

Motion Estimation and Structure Tracking



Motion Field

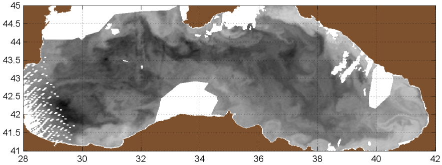


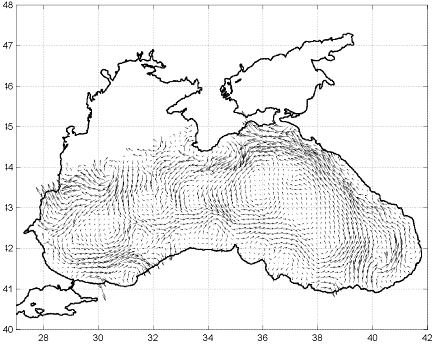
with contour points

without contour points

Highlight2

Spirit of model reduction

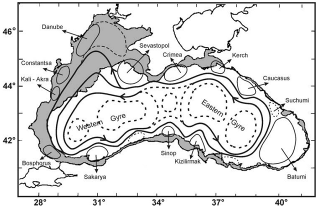


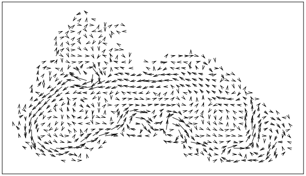


Courtesy: Marine Hydrophysical Institute, Ukrainian Academy of Sciences, Sevastopol

Highlight2

Spirit of model reduction





- Reduced state: less memory
- Regularity: applied on basis elements
- Boundary conditions: imposed to the basis elements
- Numerical schemes: ODE vs PDE

Full and reduced models

Full model

$$\begin{cases} \dfrac{\partial \mathbf{w}}{\partial t}(\mathbf{x},t) + (\mathbf{w}\cdot\nabla)\,\mathbf{w}(\mathbf{x},t) = 0 \\ \dfrac{\partial \mathbf{I}_s}{\partial t}(\mathbf{x},t) + \mathbf{w}\cdot\nabla\mathbf{I}_s(\mathbf{x},t) = 0 \end{cases}$$

$$\begin{cases} \mathbf{w}(\mathbf{x},t) \approx \displaystyle\sum_{k=1}^{K} a_k(t)\phi_k(\mathbf{x}) \\ I_s(\mathbf{x},t) \approx \displaystyle\sum_{l=1}^{L} b_l(t)\psi_l(\mathbf{x}) \end{cases}$$

Reduced model

$$\begin{cases} \dfrac{da_k}{dt}(t) + a^T B(k) a = 0,\ k = [\![1,K]\!] \\ \dfrac{db_l}{dt}(t) + a^T G(l) b = 0,\ l = [\![1,L]\!] \end{cases}$$

$$B(k)_{i,j} = \frac{\langle(\phi_i\nabla)\phi_j,\phi_k\rangle}{\langle\phi_k,\phi_k\rangle}$$

$$G(l)_{i,j} = \frac{\langle\phi_i\cdot\nabla\psi_j,\psi_l\rangle}{\langle\psi_l,\psi_l\rangle}$$

Motion basis

ϕ_i are obtained by sequentially solving systems S_i:

$$S_i = \begin{cases} \phi_i = \min_{\mathbf{f} \in L_2(\Omega)^2} \langle \nabla \mathbf{f}, \nabla \mathbf{f} \rangle \\ \text{div}\ (\phi_i(\mathbf{x})) = 0 \quad \forall \mathbf{x} \in \Omega \\ \phi_i(\mathbf{x}) \cdot \mathbf{n}(\mathbf{x}) = 0 \quad \forall \mathbf{x} \in \partial\Omega \\ \langle \phi_i, \phi_k \rangle = \delta_{i,k}, \quad k \in [\![1, i]\!] \end{cases} \tag{1}$$

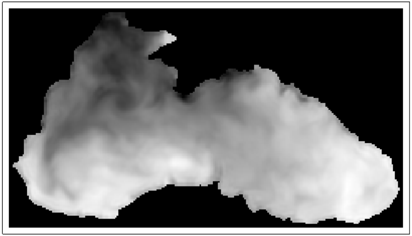
Image Basis

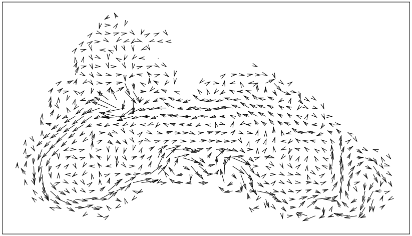
ψ_i are obtained by sequentially solving systems S_i :

$$S_i = \begin{cases} \psi_i = \min\limits_{\mathbf{f} \in L_2(\Omega)} \langle \nabla \mathbf{f}, \nabla \mathbf{f} \rangle \, d\mathbf{x} \\ \nabla \psi_i(\mathbf{x}) \cdot \mathbf{n}(\mathbf{x}) = 0 \quad \forall \mathbf{x} \in \partial\Omega \\ \langle \psi_i, \psi_k \rangle = \delta_{i,k}, \quad k \in [\![1, i]\!] \end{cases} \tag{2}$$

Black Sea motion estimation

Results of Assimilation in the reduced model:





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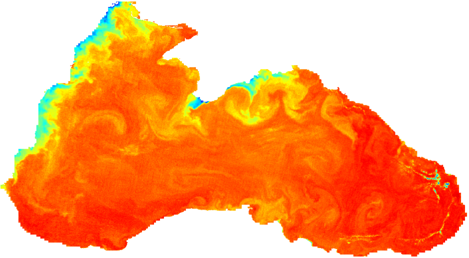

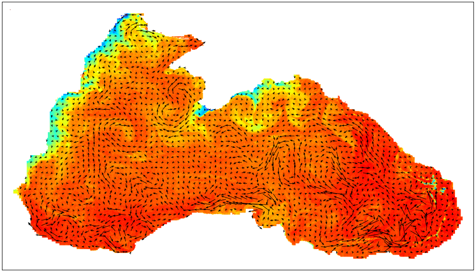

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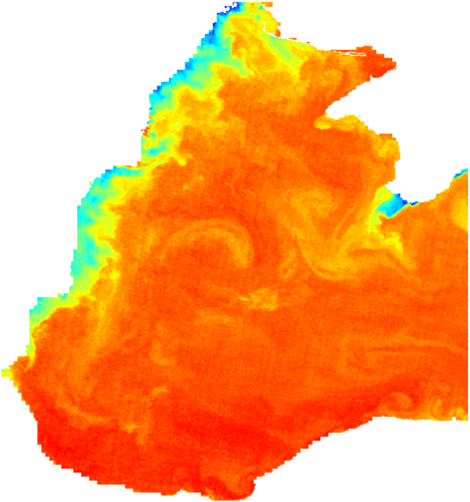

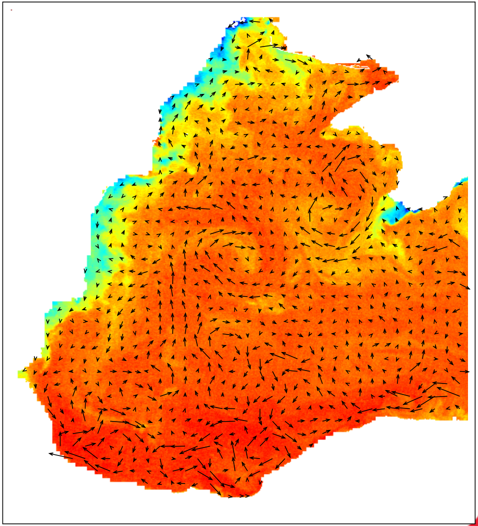

Prospective

Methods

- Optimal basis for reduced models
- Non linear observation operators, linked to image structures
- Characterization of model errors
- Comparison of 4D-Var and ensemble methods

Objectives

- Motion modeling of geophysical flows
- Short-term tracking and forecast of clouds
- Forecast of ocean currents from image data

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