# General approximation schemes for option prices in stochastic volatility models

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## 1 Model specification

The article proposes to work with stochastic models under the forward measure, and to take the underlying asset as a forward contract. The price of a call option with strike $K$ and maturity $T$ at time 0 is

$$\Pi_0(K) = D_{0T}\mathbb{E}(F_{TT} - K)_+$$

Where $F_{tT}$ is the price of a forward contract at time $t$, and $D_{tT}$ is the price of a zero coupon. The expectation is taken under the forward measure. More precisely, we have the following dynamics

$$\begin{cases} dF_{tT} &= \sqrt{v_{tT}}F_{tT}dW_t^1 \\ dv_t &= (a - bv_t)dt + \sigma\sqrt{v_t}dW_t^2 \end{cases}$$

where $W^1$ and $W^2$ are standard brownian motions such that $d\langle W_t^1, W_t^2\rangle = \rho dt$.

## 2 Second order approximation formula for pricing European call

We use the smart expansion introduced by Larsson [1] to give an analytical accurate approximation of the above call European price. More precisely, we use the second order approximation price formula given [1], to write

$$\Pi_0(K) \simeq \bar{\Pi}_0^0 + \left.\frac{\partial\bar{\Pi}_0^\varepsilon}{\partial\varepsilon}\right|_{\varepsilon=0} + \left.\frac{\partial^2\bar{\Pi}_0^\varepsilon}{\partial\varepsilon^2}\right|_{\varepsilon=0} \tag{1}$$

where

$$\bar{\Pi}_0^0 = D_{0T}(F_{0T}\mathcal{N}(d_1) - K\mathcal{N}(d_2)),$$

$$d_1 = \frac{1}{\sqrt{\Sigma_T}}\log\left(\frac{F_{0T}}{K}\right) + \frac{1}{2}\sqrt{\Sigma_T}, \quad d_2 = d_1 - \sqrt{\Sigma_T},$$

$$\Sigma_T = (v_0 - b)\frac{1}{b}(1 - e^{-aT}) + bT$$

Explicit forms of the greeks in formula (1) are given in details in [1].

## References

[1] Karl Larsson:General approximation schemes for option prices in stochastic volatility models. Quantitative Finance Volume 12, Issue 6, 2012

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