Smart expansion and fast calibration for jump diffusion

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1 Model specification

The log asset price is solution to

$$X_t \;=\; x_0 + \int_0^t \mu(u, X_{u-})du + \int_0^t \sigma(u, X_{u-})dW_u + J_t \tag{1}$$

where W is a standard Brownian motion and J is a compund Poisson process independent of W, $\mu(t,x) = \lambda\left(1 - e^{\eta_j + \frac{\gamma_J^2}{2}}\right) - \frac{\sigma^2(t,x)}{2}$ and $\sigma(t,x) = \nu(t)e^{(\beta(t)-1)x}$. Here ν et β are deterministic picewise functions (For more details on the practical choice of ν and β see section 4.2 in[1]). The proxy model is nothing but the Merton model given by

$$X_t^M \;=\; x_0 + \int_0^t \mu(u, x_0)du + \int_0^t \sigma(u, x_0)dW_u + J_t$$

2 Second order approximation formula for pricing European call

We use the smart expansion introduced by Benhamou, Gobet and Miri [1] to give an analytical accurate approximation of a call European price, written as the expected value under the risk neutral probability measure of the payoff function $h(x) = e^{rT}(e^{(r-q)T}e^x - K)_+$ evaluated at the maturity time T:

$$Call\,Price = \mathbb{E}\left[h\left(X_T\right)\right]$$

where r and q denote respectively the interest rate and the dividend. More precisely, for r, q and ν constants, we use the second order approximation price formula given by Theorem 2.1 of [1], to write

$$Call\,Price \simeq \mathbb{E}h(X_T^M) + \sum_{i=1}^{3} \alpha_{i,T} Greek_i^h(X_T^M) + \sum_{i=1}^{3} \beta_{i,T} \overline{Greek}_i^h(X_T^M) \tag{2}$$

where $\mathbb{E}h(X_T^M)$ is nothing but the price of an European Call asociated to the Merton model (See the formula in Remark 2.2 in [1]). Explicit forms of the greeks in formula (2) are given in Proposition 4.1 of [1].

References

[1] E.Benhamou, E. Gobet and M. Miri Smart expansion and fast calibration for jump diffusion. Finance and Stochastics 13, 4 (2009) 563-589 1