

## Help

```

#ifndef __time_change_levy__
#define __time_change_levy__

#include "pnl/pnl_vector.h"
#include "pnl/pnl_band_matrix.h"
#include "pnl/pnl_tridiag_matrix.h"
#include "pde_tools.h"

extern dcomplex Ctgamma_log(dcomplex z);

/*
The two following class of model come from :

@article {MR1995283,
  AUTHOR = {Carr, Peter and Geman, H{\ 'e}lyette and Madan, Dilip B. and
            Yor, Marc},
  TITLE = {Stochastic volatility for {L}\ 'evy processes},
  JOURNAL = {Math. Finance},
  FJOURNAL = {Mathematical Finance. An International Journal of Mathematics,
              Statistics and Financial Economics},
  VOLUME = {13},
  YEAR = {2003},
  NUMBER = {3},
  PAGES = {345--382},
  ISSN = {0960-1627},
  MRCLASS = {91B28 (60G51)},
  MRNUMBER = {MR1995283 (2005a:91054)},
}

CIR stochastic clock

the CIR process can be use as rate of time change. it follows the SDE


$$dy_t = \kappa \left( \eta - y_t \right) dt + \lambda \sqrt{y_t} dW_t$$


*/
typedef struct _CIR_diffusion CIR_diffusion;

```

```

struct _CIR_diffusion
{
    double Kappa;
    double Eta;
    double Lambda;
    double Drift; // proportional to Drift correction
    double y0;
    double Kappa_sqr;
    double Lambda_sqr;
    double Kappa_sqr_eta_div_lambda_sqr;
    double Two_kappa_eta_div_lambda_sqr;

    double time;
    double Jump_drift;
    double Jump_drift_psi;
    void *Levy;
    dcomplex(*characteristic_exponent)(dcomplex u, void *mod);
};

```

```

extern CIR_diffusion *CIR_diffusion_create(double Kappa, double Eta, double Lambda,
    void *Levy_,
    dcomplex(*characteristic_exponent_)(dcomplex, void *),
    double *jump_drift);
extern dcomplex CIR_diffusion_characteristic_exponent(dcomplex u, double t, void *mod);
extern dcomplex CIR_diffusion_ln_characteristic_function(dcomplex u, double t, void *mod);
extern double CIR_diffusion_get_sigma_square(CIR_diffusion *Process);
extern void CIR_diffusion_fourier_stiffness(CIR_diffusion *mod, double hx, double htau);
extern void CIR_diffusion_update(CIR_diffusion *process, double t);
/*

```

Gamma- OU stochastic clock

the rate of time change is now solution of the SDE

$$dy_t = -\lambda y_t dt + dz_{\{\lambda t\}}.$$

Choice  $z_t$  as a compound poisson process,

$$z_t = \sum_{n=1}^{N_t} x_n$$

where  $N_t$  is a Poisson process with intensity parameter  $\alpha$

and each  $x_n$  follows an exponential law with mean  $\frac{1}{\beta}$ .

\*/

```

typedef struct _GammaOU_diffusion GammaOU_diffusion;

struct _GammaOU_diffusion
{
    double Lambda;
    double Alpha;
    double Beta;
    double Drift; // proportional to Drift correction
    double y0;

    double Lambda_a;
    double Lambda_b;
    double y0_one_m_el_div_lambda;
    double y0_el;
    double one_m_el_div_lambda;
    double beta_el;

    double time;
    double Jump_drift;
    double Jump_drift_psi;
    void *Levy;
    dcomplex(*characteristic_exponent)(dcomplex u, void *mod);
};

extern GammaOU_diffusion *GammaOU_diffusion_create(double Lambda, double Alpha,
    void *Levy_,
    dcomplex(*characteristic_exponent_)(dcomplex, void *),
    double *jump_drift);
extern dcomplex GammaOU_diffusion_characteristic_exponent(dcomplex u, double t,
extern dcomplex GammaOU_diffusion_ln_characteristic_function(dcomplex u, double t,
extern double GammaOU_diffusion_get_sigma_square(GammaOU_diffusion *Process);
extern void GammaOU_diffusion_fourier_stiffness(GammaOU_diffusion *mod, double h,
extern void GammaOU_diffusion_update(GammaOU_diffusion *process, double t);

extern void test_CIR_diffusion(void);
extern void test_GammaOU_diffusion(void);

typedef struct _Heston_diffusion Heston_diffusion;

```

```

struct _Heston_diffusion
{
    double Eta;
    double Kappa;
    double Rho;
    double Theta;
    double Sigma;
    double sigma_sqr;
    double theta_sqr;
    double sigma_sqr_d_eta_kappa;
    double etakappathetam2;
    double rho_theta;
    double Drift;
};

extern Heston_diffusion *Heston_diffusion_create(double Eta_, double Kappa_, double
    double Theta_, double Sigma_,
    double *jump_drift);

extern dcomplex Heston_diffusion_characteristic_exponent(dcomplex u, double t, v
extern dcomplex Heston_diffusion_ln_characteristic_function(dcomplex u, double t

/*

    dS_t = (r-q-\ lambda_y \ mu) S_t dt + \ sqrt{V_t} S_t dW_t^1 + J_y S_t dq_y(t)
    dV_t = \ kappa_{nu} \ left( \ eta_{nu} + V_t \ right) + \ theta_{nu} \ sqrt{V_
    dW^1 dW^2 = \ rho dt

    (1+J_y) is a lognormally distributed with mean $\ mu_y$ and variance
    $\ sigma_y^2$

    $q_{y}$ is an indenpendent Poisson process with arrival rate
    $\ lambda_{y}$
    $\ mu= \ left( \ exp{\ mu_y+\ sigma_y^2/2}-1 \ right)$.

*/

typedef struct _Bates_diffusion Bates_diffusion;

struct _Bates_diffusion

```

```

{
    double Eta;
    double Kappa;
    double Theta;
    double Rho;
    double Sigma;

    double mu_J;
    double Sigma_J;
    double Lambda_J;

    double sigma_sqr;
    double theta_sqr;
    double sigma_sqr_d_eta_kappa;
    double etakappathetam2;
    double rho_theta;
    double lnonepmuj;
    double sigmaj_sqr_demi;
    double Drift;
};

extern Bates_diffusion *Bates_diffusion_create(double Eta_, double Kappa_, double
    double Theta_, double Sigma_,
    double mu_J_,
    double Sigma_J_, double Lambda_J_, double *jump_drift);
extern dcomplex Bates_diffusion_characteristic_exponent(dcomplex u, double t, vo
extern dcomplex Bates_diffusion_ln_characteristic_function(dcomplex u, double t,
/*
@article {MR1841412,
    AUTHOR = {Barndorff-Nielsen, Ole E. and Shephard, Neil},
    TITLE = {Non-{G}aussian {0}rnstein-{U}hlenbeck-based models and some of
        their uses in financial economics},
    JOURNAL = {J. R. Stat. Soc. Ser. B Stat. Methodol.},
    FJOURNAL = {Journal of the Royal Statistical Society. Series B.
        Statistical Methodology},
    VOLUME = {63},
    YEAR = {2001},
    NUMBER = {2},
    PAGES = {167--241},
    ISSN = {1369-7412},

```

```

MRCLASS = {62M07 (62M09 62M10 62P20)},
MRNUMBER = {MR1841412 (2002c:62127)},
}

```

The square volatility follows the SDE of the form :

$$d\sigma_t^2 = -\lambda \sigma_t^2 dt + dZ_{\lambda t}$$

where  $\lambda > 0$  and  $Z$  is a subordinator.

The risk neutral dynamic of the log price  $x_t = \log S_t$  are given by

$$dW_t = (r - q - \lambda k(-\rho) - \sigma_t^2/2) dt + \sigma_t dW_t + \rho dZ_t$$

$$\quad x_0 = \log(S_0).$$

where  $k(u) = \log\{\mathbb{E}[\exp(-u Z_1)]\}$ .

Choose  $Z_t$  as a compound poisson process,

$$Z_t = \sum_{n=1}^{N_t} x_n$$

where  $N_t$  is a Poisson process with intensity parameter  $\alpha$

and each  $x_n$  follows an exponential law with mean  $\frac{1}{\beta}$ .

One can show that the process  $\sigma_t^2$  is a stationary process with a

marginal law that follows a Gamma distribution with mean  $\alpha$  and

variance  $\frac{\alpha}{\beta}$ . In this case,

$$k(u) = \frac{-\alpha}{\beta + u}.$$

\*/

```

typedef struct _BNS_diffusion BNS_diffusion;

```

```

struct _BNS_diffusion

```

```

{

```

```

    double Lambda;

```

```

    double Rho;

```

```

    double Beta;

```

```

    double Alpha;

```

```

    double Sigma0;

```

```

    double Sigma0_sqr ;

```

```

    double Lambda_m1;

```

```

    double Drift; // proportional to Drift correction

```

```

};

```

```

extern BNS_diffusion *BNS_diffusion_create(double Lambda_, double Rho_,
    double Beta_, double Alpha_,
    double Sigma0_, double *jump_drift);

```

```
extern dcomplex BNS_diffusion_characteristic_exponent(dcomplex u, double t, void
extern dcomplex BNS_diffusion_ln_characteristic_function(dcomplex u, double t, v
```

```
/*
```

```
@article {MR1793362,
  AUTHOR = {Duffie, Darrell and Pan, Jun and Singleton, Kenneth},
  TITLE = {Transform analysis and asset pricing for affine
           jump-diffusions},
  JOURNAL = {Econometrica},
  FJOURNAL = {Econometrica. Journal of the Econometric Society},
  VOLUME = {68},
  YEAR = {2000},
  NUMBER = {6},
  PAGES = {1343--1376},
  ISSN = {0012-9682},
  CODEN = {ECMTA7},
  MRCLASS = {91B28 (60J60)},
  MRNUMBER = {MR1793362 (2001m:91081)},
}
```

```
}
```

```
dS_t = (r-q-\ lambda_y \ mu) S_t dt + \ sqrt{V_t} S_t dW_t^1 + J_y S_t dq_y(t) \
dV_t = \ kappa_{\nu} \ left( \ eta_{\nu} + V_t \ right) + \ theta_{\nu} \ sqrt{V_t} \
dW_t^2 + J_V dq_{\ \nu}(t)
```

```
dW^1 dW^2 = \ rho dt
```

$(1+J_y)$  is a lognormally distributed with mean  $\mu_y$  and variance  $\sigma_y^2$

$J_V$  has an exponential distribution with mean  $\mu_{\nu}$

$q_y$  and  $q_{\nu}$  are independent Poisson process with arrivals rates  $\lambda_y$  and  $\lambda_{\nu}$

$\mu = \left( \exp\{\mu_y + \sigma_y^2/2\} - 1 \right)$ .

```
*/
```

```
typedef struct _DPS_diffusion DPS_diffusion;
```

```
struct _DPS_diffusion
{
    double Eta;
    double Kappa;
    double Rho;
    double Theta;
    double Sigma;

    double mu_y;
    double Sigma_y_sqr_demi;
    double Lambda_y;

    double mu_v;
    double Lambda_v;

    double sigma_cy_sqr_demi;
    double mu_cy;
    double mu_cv;
    double Lambda_c;
    double rho_j;

    double s_lambda;

    double sigma_sqr;
    double theta_sqr;
    double sigma_sqr_d_eta_kappa;
    double etakappathetam2;
    double rho_theta;

    double Drift;
};

extern DPS_diffusion *DPS_diffusion_create(double Eta_, double Kappa_,
    double Rho_, double Theta_,
    double Sigma_, double mu_y_,
    double Sigma_y_, double Lambda_y_,
    double mu_v_, double Lambda_v_,
    double mu_cy_, double Sigma_cy_,
```



```

    double mu_cv_, double Lambda_c_,
    double rho_j_, double *jump_drift);
extern dcomplex DPS_diffusion_characteristic_exponent(dcomplex u, double t, void
extern dcomplex DPS_diffusion_ln_characteristic_function(dcomplex u, double t, v

```

```

typedef struct _Levy_diffusion Levy_diffusion;

```

```

struct _Levy_diffusion
{
    void *process;
    dcomplex(*characteristic_exponent)(dcomplex u, double t, void *mod);
    dcomplex(*ln_characteristic_function)(dcomplex u, double t, void *mod);
    // Artificial volatility term to come back to parabolic problem
    double vol_square;
};

```

```

extern Levy_diffusion *Levy_diffusion_create(void *process_, dcomplex(*characret
    dcomplex(*ln_characrteristic_function_)(dcomplex u, double t, void *mod));
extern dcomplex Levy_diffusion_characteristic_exponent(dcomplex u, double t, Lev
extern double Levy_diffusion_get_sigma_square(Levy_diffusion *Levy);
extern void Levy_diffusion_fourier_stiffness(Levy_diffusion *mod, double t, doub
extern dcomplex Levy_diffusion_ln_characteristic_function(dcomplex u, double t,
extern dcomplex Levy_diffusion_characteristic_function(dcomplex u, double t, Lev

```

```

#endif

```