fd_howard2d

Input parameters:

- SpaceStepNumber N
- TimeStepNumber M
- Epsilon

Output parameters:

- Price
- Delta1
- Delta2

The algorithm of Howard has been introduced by Howard in [1].

/*Memory Allocation*/

/*Covariance Matrix*/

/*Space localisation/*
Define the integration domain $D = [-l, l]^2$ using probabilistic estimation.

/*Space Step/*
Define the space step $h = \dfrac{2l}{N}$.

/*Time Step*/
Define the time step $k = \dfrac{T}{M}$.

/*Terminal Values/*
Put the value of the payoff into a vector P.

/*Homegenous Dirichlet Conditions/*

/*Factors of scheme*/
Initialize the matrix M^h issued from the discretization of the operator A in the case of Dirichlet Boundary conditions.

/*Finite difference Cycle/*
At any time step, we have to solve the linear complementarity problem.

> **/*Init Control*/**
> We initializate the control pp and the second member R.
>
> **/*Howard cycle*/**
>
> We solve the linear complementarity problem using the Howard algorithm, which consists in constructing a convergent sequence u^p whose limit is u.
> Let epsilon>0 be given.

Step 1 Let u^k be given, we compute $(i,j) \to pp^k[i][j] = argmin(M^{pp}u^k(i,j) - f^{pp}[i][j])$ where $pp = 0$ or 1 (the domain is divided into 2 regions: the continuation region and the exercice region), M^0 is the matrix M^h issued from the discretization of the operator A, $M^1 = Id$, $f^0 = R$, $f^1 = Obst$.

Step 2 We solve the linear system $M^{pp^k}u = G^{pp^k}$ by the Gauss factorization. It gives u^{k+1}.

The stopping criteria is

$$\|u^{k+1} - u^k\|_\infty < epsilon. \tag{1}$$

/*Price*/

/*Delta*/

/*Memory Desallocation*/

References

[1] Howard, R.A.: Dynamic Programming and Markov Process. (MIT Press. 1960) 1