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## cf\_callout\_kunitomoiked

Let

- $T$  = maturity date ( $T > t$ )
- $K$  = strike price
- $L$  = lower barrier
- $U$  = upper barrier
- $x$  = spot price
- $t$  = pricing date
- $\sigma$  = volatility
- $r$  = interest rate
- $\delta$  = dividend yields
- $\theta = T - t$
- $b = r - \delta$

The exact value for double barrier call/put options is given by the Ikeda-Kunitomo formula [1], which allows to compute exactly the price when the boundaries suitably depend on the time variable  $t$ . More precisely, set

$$U(s) = Ue^{\delta_1 s} \quad L(s) = Le^{\delta_2 s}$$

where the constants  $U, L, \delta_1, \delta_2$  are such that  $L(s) < U(s)$ , for every  $s \in [t, T]$ . The functions  $U(s)$  and  $L(s)$  play the role of *upper* and *lower* barrier respectively.  $\delta_1$  and  $\delta_2$  determine the curvature and the case of  $\delta_1 = 0$  and  $\delta_2 = 0$  corresponds to two flat boundaries.

*In the software, we consider only flat boundaries.*

The numerical studies suggest that in most cases it suffices to calculate the leading five terms of the series giving the price of the knock-out and knock-in double barrier call options.

Let  $\tau$  stand for the first time at which the underlying asset price  $S$  reaches at least one barrier, i.e.

$$\tau = \inf\{s > t; S_s \leq L(s) \text{ or } S_s \geq U(s)\}.$$

We define the following coefficients:

- $\mu_1 = 2 \frac{b - \delta_2 - n(\delta_1 - \delta_2)}{\sigma^2} + 1$
- $\mu_2 = 2 \frac{n(\delta_1 - \delta_2)}{\sigma^2}$
- $\mu_3 = 2 \frac{b - \delta_2 + n(\delta_1 - \delta_2)}{\sigma^2} + 1$

## Knock-Out Call Option

PAYOFF  $C_T = \begin{cases} (S_T - K)_+ & \text{if } \tau > T \\ 0 & \text{otherwise} \end{cases}$

PRICE 
$$C(t, x) = x e^{-\delta\theta} \sum_{n=-\infty}^{+\infty} \left[ \left( \frac{U^n}{L^n} \right)^{\mu_1} \left( \frac{L}{x} \right)^{\mu_2} (N(d_1^+) - N(d_2^+)) \right. \\ \left. - \left( \frac{L^{n+1}}{x U^n} \right)^{\mu_3} (N(d_3^+) - N(d_4^+)) \right] \\ - K e^{-r\theta} \sum_{n=-\infty}^{+\infty} \left[ \left( \frac{U^n}{L^n} \right)^{\mu_1-2} \left( \frac{L}{x} \right)^{\mu_2} (N(d_1^-) - N(d_2^-)) \right. \\ \left. - \left( \frac{L^{n+1}}{x U^n} \right)^{\mu_3-2} (N(d_3^-) - N(d_4^-)) \right]$$

where  $F = U e^{\delta_1\theta}$  and

$$d_1^\pm = \frac{\log(x U^{2n} / K L^{2n}) + (b \pm \frac{\sigma^2}{2}) \theta}{\sigma \sqrt{\theta}} \quad d_2^\pm = \frac{\log(x U^{2n} / F L^{2n}) + (b \pm \frac{\sigma^2}{2}) \theta}{\sigma \sqrt{\theta}} \\ d_3^\pm = \frac{\log(L^{2n+2} / K x U^{2n}) + (b \pm \frac{\sigma^2}{2}) \theta}{\sigma \sqrt{\theta}} \quad d_4^\pm = \frac{\log(L^{2n+2} / F x U^{2n}) + (b \pm \frac{\sigma^2}{2}) \theta}{\sigma \sqrt{\theta}}$$

## References

- [1] N.KUNIMOTO N.IKEDA. Pricing options with curved boundaries.  
*Mathematical finance*, 2:275–298, 1992. 1