

[Help](#)

```
#include <cmath>
#include <iostream>
#include "config.h"

#include "numerics.h"

const double SQ2P = sqrt(2.0 * M_PI);
const double FPMIN = 1.0e-100;
const double EPS = 0.00000001;
const double ITMAX = 100;

double gammp(double a, double x)
{
    //Returns P(a,x)
    double gamser, gammcf, gln;
    if (x < 0.0 || a <= 0.0) myerror("Invalid arguments in routine gammp");
    if (x < (a + 1.0))
    {
        gser(&gamser, a, x, &gln);
        return gamser;
    }
    else
    {
        gcf(&gammcf, a, x, &gln);
        return 1.0 - gammcf;
    }
}

double gammq(double a, double x)
{
    //Returns Q(a,x)
    double gamser, gammcf, gln;
    if (x < 0.0 || a <= 0.0) myerror("Invalid arguments in routine gammq");
    if (x < (a + 1.0))
    {
        gser(&gamser, a, x, &gln);
        return 1.0 - gamser;
    }
    else
```

```
    {
        gcf(&gammcf, a, x, &gln);
        return gammcf;
    }
}
```

```
void gser(double *gamser, double a, double x, double *gln)
//low-level routine for calculating Q(a; x)
{
    int n;
    double sum, del, ap;

    *gln = pnl_lgamma(a);
    if (x <= 0.0)
    {
        if (x < 0.0) myerror("x less than 0 in routine gser");
        *gamser = 0.0;
        return;
    }
    else
    {
        ap = a;
        del = sum = 1.0 / a;
        for (n = 1; n <= ITMAX; n++)
        {
            ++ap;
            del *= x / ap;
            sum += del;
            if (fabs(del) < fabs(sum)*EPS)
            {
                *gamser = sum * exp(-x + a * log(x) - (*gln));
                return;
            }
        }
        myerror("a too large, ITMAX too small in routine gser");
        return;
    }
}
```

```
void gcf(double *gammcf, double a, double x, double *gln)
{
    //low-level routine for calculating Q(a; x)
    double an, b, c, d, del, h;
    *gln = pnl_lgamma(a);
    b = x + 1.0 - a;
    c = 1.0 / FPMIN;
    d = 1.0 / b;
    h = d;
    int i;
    for (i = 1; i <= ITMAX; i++)
    {
        an = -i * (i - a);
        b += 2.0;
        d = an * d + b;
        if (std::fabs(d) < FPMIN) d = FPMIN;
        c = b + an / c;
        if (std::fabs(c) < FPMIN) c = FPMIN;
        d = 1.0 / d;
        del = d * c;
        h *= del;
        if (std::fabs(del - 1.0) < EPS) break;
    }
    if (i > ITMAX) myerror("a too large, ITMAX too small in routine gcf");
    *gammcf = std::exp(-x + a * std::log(x) - (*gln)) * h;
}

double normPDF(double x)
{
    return std::exp(-x * x / 2) / SQ2P;
}

double normCDF(double x)
{
    if (x > 0) return 1.0 - normCDF(-x);
    if (x * x < 3.0) return 0.5 * (1 - gammp(0.5, 0.5 * x * x));
    else return 0.5 * gammq(0.5, 0.5 * x * x);
}
```

```

double expint(int n, double x)
{
    int nm1;
    double a, b, c, d, del, fact, h, psi, ans = 0;

    nm1 = n - 1;
    if (n < 0 || x < 0.0 || (x == 0.0 && (n == 0 || n == 1)))
        myerror("bad argument in expint");
    else
    {
        if (n == 0) ans = exp(-x) / x; //special case
        else
        {
            if (x == 0.0) ans = 1.0 / nm1; //another special case
            else
            {
                if (x > 1.0)
                {
                    b = x + n;
                    c = 1.0 / FPMIN;
                    d = 1.0 / b;
                    h = d;
                    for (int i = 1; i < ITMAX; i++)
                    {
                        a = -i * (nm1 + i);
                        b += 2.0;
                        d = 1.0 / (a * d + b);
                        c = b + a / c;
                        del = c * d;
                        h *= del;
                        if (fabs(del - 1.0) < EPS)
                        {
                            ans = h * exp(-x);
                            return ans;
                        }
                    }
                    myerror("continued fraction failed in exprint");
                }
            }
        }
        else
        {
            ans = (nm1 != 0 ? 1.0 / nm1 : -log(x) - EULER);
        }
    }
}

```

```

        fact = 1.0;
        for (int i = 1; i <= ITMAX; i++)
        {
            fact *= -x / i;
            if (i != nm1) del = -fact / (i - nm1);
            else
            {
                psi = -EULER;
                for (int ii = 1; ii <= nm1; ii++) psi += 1.0 / ii;
                del = fact * (-log(x) + psi);
            }
            ans += del;
            if (fabs(del) < fabs(ans)*EPS) return ans;
        }
        myerror("series failed in exprint");
    }
}

return ans;
}

double bessil(double x)
{
    double ax, ans;
    double y; //Accumulate polynomials in double precision

    if ((ax = fabs(x)) < 3.75) //Polynomial fit
    {
        y = x / 3.75;
        y *= y;
        ans = ax * (0.5 + y * (0.87890594 + y * (0.51498869 + y * (0.15084934 + y *
            + y * (0.301532e-2 + y * 0.32411e-3))))));
    }
    else
    {
        y = 3.75 / ax;
        ans = 0.2282967e-1 + y * (-0.2895312e-1 + y * (0.1787654e-1 - y * 0.420059
        ans = 0.39894228 + y * (-0.3988024e-1 + y * (-0.362018e-2 + y * (0.163801e
            + y * (-0.1031555e-1 + y * ans))));
        ans *= (exp(ax) / sqrt(ax));
    }
}

```

```

    }
    return x < 0.0 ? -ans : ans;
}

double bessk1(double x)
{
    double y, ans; //Accumulate polynomials in double precision

    if (x <= 2.0) //Polynomial fit
    {
        y = x * x / 4;
        ans = (log(x / 2) * bessi1(x)) + (1. / x) * (1 + y * (0.15443144 + y * (-0.
            + y * (-0.18156897 + y * (-0.1919402e-1 + y * (-0.110404e-2 + y * (-
        }
    else
    {
        y = 2. / x;
        ans = (exp(-x) / sqrt(x)) * (1.25331414 + y * (0.23498619 + y * (-0.365562
            + y * (-0.780353e-2 + y * (0.325614e-2 + y *
        }
    }
    return ans;
}

//-----

void polint(double xa[], double ya[], int n, double x, double *y, double *dy)
//Given arrays xa[1..n] and ya[1..n], and given a value x, this routine returns
//an error estimate dy. If P(x) is the polynomial of degree N ? 1 such that P(xa
//1, . . . , n, then the returned value y = P(x).
{
    int ns = 1;
    double den, dif, dift, ho, hp, w;
    double *c = new double[n];
    double *d = new double[n];
    c--; //for c and range between 1 and n
    d--; //instead of 0 and n-1
    dif = fabs(x - xa[1]);
    for (int i = 1; i <= n; i++) //Here we find the index ns of the closest table e
    {
        if ((dift = fabs(x - xa[i])) < dif)

```

```

        {
            ns = i;
            dif = dift;
        }
        c[i] = ya[i]; //and initialize the tableau of c's and d's.
        d[i] = ya[i];
    }
    *y = ya[ns--]; //This is the initial approximation to y.
    for (int m = 1; m < n; m++) //For each column of the tableau,
    {
        for (int i = 1; i <= n - m; i++) //we loop over the current c's and d's and
        {
            ho = xa[i] - x;
            hp = xa[i + m] - x;
            w = c[i + 1] - d[i];
            if ((den = ho - hp) == 0.0) myerror("Error in routine polint");
            //This error can occur only if two input xa's are (to within roundoff)
            den = w / den;
            d[i] = hp * den; //Here the c's and d's are updated.
            c[i] = ho * den;
        }
        *y += (*dy = (2 * ns < (n - m) ? c[ns + 1] : d[ns--]));
        //After each column in the tableau is completed, we decide which correction
        //we want to add to our accumulating value of y, i.e., which path to take
        //tableau-forking up or down. We do this in such a way as to take the most
        //line" route through the tableau to its apex, updating ns accordingly to
        //where we are. This route keeps the partial approximations centered (insofar
        //on the target x. The last dy added is thus the error indication.
    }
    c++;
    d++;
    delete[] d;
    delete[] c;
}

```