Multilevel Monte Carlo for Pricing European Call Option in Scott's Stochastic Volatility Model

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1 Model specification

We consider the following stochastic volatility model with a general form given by

$$\begin{cases} S_t &= s_0 + \int_0^t rS_u du + \int_0^t f(Y_u) S_u(\rho dW_u + \sqrt{1-\rho^2} dB_t) \\ Y_t &= y_0 + \int_0^t b(Y_u) du + \int_0^t \sigma(Y_t) dW_t, \end{cases}$$

where $(S_t)_{t\in[0,T]}$ is the asset price, r the instantaneous interest rate, $(B_t)_{t\in[0,T]}$ and $(W_t)_{t\in[0,T]}$ are independent standard one-dimensional Brownian motions, $\rho \in [-1,1]$ is the correlation between the Brownian motions respectively driving the asset price and the process $(Y_t)_{t\in[0,T]}$. The volatility process is $(f(Y_t))_{t\in[0,T]}$ where the transformation function f is usually taken positive and strictly monotonic in order to ensure that the effective correlation between the stock price and the volatility keeps the same sign. In the particular case where $f(y) = \sqrt{y}$, $b(y) = \mu y$ and $\sigma(y) = \xi Y$, we recover the Hull & White model see [2]. In this work, we focus on the Scott's stochastic volatility model given by

$$\begin{cases} S_t &= s_0 + \int_0^t rS_u du + \int_0^t \sigma_0 e^{Y_u} S_u(\rho dW_u + \sqrt{1-\rho^2} dB_t) \\ Y_t &= y_0 + \int_0^t \kappa(\theta - Y_u) du + \int_0^t \nu dW_t, \end{cases} \tag{1}$$

which corresponds to choose $f(y) = \sigma_0 e^y$, $b(y) = \kappa(\theta - y)$ and $\sigma(y) = \nu$.

2 European Call Price

Our aim, consists in implementing the paper by Jourdain & Sbai [?] in order to compute the European call price $\mathbb{E}\left(e^{-rT}(S_T - K)_+\right)$, where the stock price process $(S_t)_{t\in[0,T]}$ follows the Scott's stochastic volatility model (1). A common way to compute this quantity is to use a Monte Carlo method given by

$$\frac{1}{M}\sum_{i=1}^{M} \mathrm{BS}_T\left(s_0 e^{\rho\left(F(\bar{Y}_T^{N,i}) - F(y_0)\right) + \bar{m}_T^{N,i} + \left(\frac{(1-\rho^2)\bar{v}_T^{N,i}}{2T} - r\right)T}, \frac{(1-\rho^2)\bar{v}_T^{N,i}}{T}\right),$$

where $\mathrm{BS}_T(s,v)$ stands for the price of a European Call option with maturity T in the Black & Scholes model with initial stock price s, volatility v and constant interest rate r. The number M is the total number of Monte Carlo samples and the index i refers to independent draws. The quantities $\bar{Y}_T^N$, $\bar{m}_T^{N,i}$ and $\bar{v}_T^N$ are respectively discretization schemes of the quantities

$$Y_T, \quad m_T = \int_0^T r - \frac{\sigma_0^2 e^{2Y_s}}{2} - \rho\sigma_0 e^{Y_s}\left(\frac{\kappa}{\nu}(\theta - Y_s) + \frac{\nu}{2}\right) dt \text{ and } v_T = \int_0^T \sigma_0^2 e^{2Y_s} ds.$$

The function F is given by $F(y) = \frac{\sigma_0 e^u}{\nu} - 1$. In the Premia code we use the Euler scheme to approximate Y_T and Riemann schemes to approximate both m_T and v_T. In their paper, Jourdain & Sbai [?] use a Multilevel Monte Carlo method instead of the classical Monte Carlo one (described above) to compute the European Call price. The multi-level Monte Carlo method is introduced by Giles [1] as an extended method of the statistical Romberg one of Kebaier [4]. When approximating the expected value of a function of a stochastic differential equation solution, these methods improve efficiently the computational complexity of a standard Monte Carlo algorithm.

References

[1] M.B. Giles. Multilevel Monte Carlo path simulation Operations Research, 2008 56:607–617 1

[2] J. Hull and A. White. The pricing of options on assets with stochastic volatilities. The Journal of Finance, 42(2) 281–300, 1987. 1

[3] as:05 B. Jourdain, M. Sbai. High order discretization schemes for stochastic volatility models, accepted for publication in Journal of Computational Finance

[4] A. Kebaier. Romberg Extrapolation: A New Variance Reduction Method and Applications to Option Pricing. Ann. Appl. Probab. 15 (2005), no. 4, 2681–2705

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