Credit Default Swaps with Correlated CIR++ Intensity and Interest Rate

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Abstract

Consider a Credit Default Swap (CDS) as defined in Brigo and Alfonsi (2004), and suppose that the interest rates and default intensity are modeled by correlated CIR++ stochastic processes. Our goal is to compute numerically the CDS rate R_f.

Contents

1 Credit Default Swaps

A credit default swap (CDS) is a contract ensuring protection against default. This contract is specified by a number of parameters. Let us start by assigning a maturity T.

Consider two companies “A” and “B” who agree on following: if a third reference company “C” defaults at time $\tau < T$, “B” pays to “A” a ceratin cash amount Z, supposed to be deterministic in our model, at the default time τ itself. This cash amount is a “protection” for “A” in case “C” defaults. In exchange for this protection, company “A” agrees to pay periodically to “B” a fixed amount “R_f”, referred as the CDS rate. Payments occur at times $\mathcal{T} = \{T_1, ..., T_n\}$, $\alpha_i = T_i - T_{i-1}$, $T_0 = 0$, $T_n = T$, fixed in advance at time $t = 0$ up to default time τ if this occurs before maturity T, or until maturity T if no defaults occurs. With these notations we may write the CDS discounted value seen from “B” at time t as

$$\mathbf{1}_{\tau>t}\left[D(t,\tau)(\tau - T_{\beta(\tau)-1})R_f\mathbf{1}_{\tau<T} + \sum_{i=\beta(t)}^{n} D(t,T_i)\alpha_i R_f \mathbf{1}_{\tau>T_i} - D(t,\tau)Z\mathbf{1}_{\tau<T}\right], \tag{1}$$

where for any $s \in [0,T]$, $\beta(s) \in \{1, ..., n\}$ and $T_{\beta(s)}$ is the first date of $T_1, ..., T_n$ following t. The stochastic discount factor at time t for maturity T is denoted by $D(t,T) = B(t)/B(T)$, where $B_t = \exp\left(\int_0^t r_u du\right)$ denotes the bank-account numeraire, r being the instantaneous short interest rate.

We denote by $\mathrm{CDS}(t, \mathcal{T}, T, R_f, Z)$ the price at time t of the above CDS, and by $\mathcal{P}_t$ the random variable in (1). We will compute the CDS price at time t, according to risk-neutral valuation (Bielecki and Rutkowski (2002)):

$$\mathrm{CDS}(t, \mathcal{T}, T, R_f, Z) = \mathbf{1}_{\tau>t}\mathbf{E}\left\{\mathcal{P}_t | \mathcal{F}_t \vee \sigma\left(\{\tau < u\}, u \leq t\right)\right\}, \tag{2}$$

where $\mathcal{F}_t$ is the basic filtration without default, typically representing the information flow of interest rate, intensities and possibly other default-free quantities, and $\mathbf{E}$ denotes the risk-neutral expectation in the enlarged probability space supporting τ.

As Z is given, R_f must be set to the value that makes the CDS fair at time t, i.e. R_f is such that $\mathrm{CDS}(t, \mathcal{T}, T, R_f, Z) = 0$. This explains why R_f is called "the CDS rate."

1.1 CIR++ model

Consider the Cox-Ingersoll-Ross (1985) process

$$dx_t^\alpha = k(\theta - x_t^\alpha)dt + \sigma\sqrt{x_t^\alpha}dW_t, \tag{3}$$

where the parameter vector is $\alpha = (k, \theta, \sigma, x_0^\alpha)$, with $k, \theta, \sigma, x_0^\alpha$ positive deterministic constants. The condition

$$2k\theta > \sigma^2$$

ensures that the origin is inaccesible to the process x^α, so that the process x^α remains positive. We can now define the CIR++ model, consisting of the following extension of (3)

$$r_t = x_t^\alpha + \varphi(t;\alpha), \;\; t \geq 0, \tag{4}$$

with

$$\varphi(t;\alpha) = f(0,t) - f^{\text{CIR}}(0,t;\alpha),$$

where $f(0,t)$, which is a function integrable on closed intevals, will be defined later, and

$$\begin{aligned} f^{\text{CIR}}(0,t;\alpha) \;\; &= \;\; 2k\,\theta\frac{\exp\{th\} - 1}{2h + (k+h)(\exp\{th\} - 1)} \\ &\quad + x_0\,\frac{4h^2\exp\{th\}}{[2h + (k+h)(\exp\{th\} - 1)]^2}, \end{aligned}$$

with $h = \sqrt{k^2 + 2\sigma^2}$.

1.2 CIR++ short-rate model

We suppose that the short-rate r_t is defined by Eq. (4). Denote by f instantaneous forward rates, i.e. $f(t,T) = -\partial \ln P(t,T)/\partial T$.

Denote by $P^M(0,T)$ the price of the T-maturity market zero-coupon; in what follows the superscript "M" stands for "market". The initial market zero-coupon interest-rate curve $T \to P^M(0,T)$ is automatically calibrated by our model if we set $\varphi(t;\alpha) = \varphi^{\text{CIR}}(t;\alpha)$, where

$$\varphi^{\text{CIR}}(t;\alpha) = f^M(0,t) - f^{\text{CIR}}(0,t;\alpha).$$

Consider a given discrete set of zero-coupon prices

$$P(0,0) = 1, \; P(0,t_1), \; ..., \; P(0,t_n),$$

with $t_n = T$. The function $t \to f^M(0,t)$, defined on $[0,T]$, is piecewise constant and such that

$$e^{-\int_0^{t_i} f^M(0,u)du} = P(0,t_i), \;\; 1 \leq i \leq n.$$

Thus, for $1 \leq i \leq n$

$$f^M(0,t) = const. = f_i, \;\; t \in]t_{i-1}, t_i].$$

1.3 CIR++ intensity model

For the default intensity model of a default time τ, consider a similar approach and set:

$$\lambda_t = y_t^\beta + \psi(t;\beta), \;\; t \geq 0, \tag{5}$$

where

$$dy_t^\beta = \kappa(\mu - y_t^\beta)dt + \nu\sqrt{y_t^\beta}dZ_t, \tag{6}$$

$\beta = (\kappa, \mu, \nu, y_0^\beta)$. The short-interest rate and intensity processes are allowed to be correlated, by assuming that the driving Brownian motions W and Z are instantaneously correlated according to

$$dW_t\, dZ_t = \rho\, dt.$$

Here again the function $\psi(t;\beta) = \psi^{\text{CIR}}(t;\beta)$, with

$$\psi^{\text{CIR}}(t;\beta) = \gamma^M(0,t) - g^{\text{CIR}}(0,t;\beta),$$

where $g^{\text{CIR}}(0,t;\beta)$ is defined as in Section 1.1. Given an initial discrete set of CDS rates

$$R_f(0,t_1),\; ...,\; R_f(0,t_m),$$

with $t_m = T$, we can compute the function $t \to \gamma^M(0,t)$, which is piecewise linear, and such that, if we denote by $R_f^{\gamma^M}(0,t)$ the CDS rate (correspondint to maturity t) computed using the intensity γ^M, we have

$$R_f^{\gamma^M}(0,t_i) = R_f(0,t_i), \;\; 1 \leq i \leq n.$$

With this choice for λ, in the credit derivatives world we have formulae that are analogous to the ones for the interest-rate derivatives products. Thus, the risk-neutral survival probability is

$$\mathbf{E}(\mathbf{1}_{\tau > t}) = \mathbf{E}(e^{-\int_0^t \lambda(u)du}) \tag{7}$$

2 Pricing CDS

2.1 Pricing CDS when r and τ are independent, in a deterministic intensity model

Suppose that the default time τ admits the (deterministic) intensity function γ (also called hazard rate), i.e. the hazard function Γ is given by $\Gamma(T) = \int_0^T \gamma(u)du$.

When interest rates and default time are independent Eq. (2) writes (see Brigo and Alfonsi (2004)):

$$\begin{aligned} \text{CDS}(t,\mathcal{T},T,R_f,Z) \;=\; & R_f \int_t^T P(t,u)(u - T_{\beta(u)-1})\gamma(u)e^{-\int_t^u \gamma(s)ds}du & (8)\\ & + R_f \sum_{i=\beta(t)}^{n} P(t,T_i)\alpha_i e^{-\int_t^{T_i}\gamma(s)ds} & (9)\\ & - Z\int_t^T P(t,u)\gamma(u)e^{-\int_t^u \gamma(s)ds}du. & (10)\end{aligned}$$

where $P(t,u)$ denotes the price of the u-maturity zero-coupon starting from t.

2.2 Pricing CDS when r and τ are correlated

When interest rates and default intensity are correlated, the first ideea is to simulate trajectories of the correlated process r and λ, and to use then Monte Carlo techniques to compute the expectation in Eq. (2). We implement also a second method, based on an analytical approximation for some credit derivatives terms involving correlated CIR processes.

2.3 Simulating λ and r

Explicit scheme for simulating a CIR process For a discretization $t_0 = 0 < t_1 < ... < t_n = T$, of the interval $[0,T]$, consider the scheme (called Explicit(0), see Alfonsi (2005))

$$\hat{x}^{\alpha}_{t_{i+1}} = \left[\left(1 - \frac{k}{2}(t_{i+1} - t_i)\right)\sqrt{\hat{x}^{\alpha}_{t_i}} - \frac{\sigma(W_{t_{i+1}} - W_{t_i})}{2(1 - \frac{k}{2}(t_{i+1} - t_i))}\right]^2 + \left(k\theta - \frac{\sigma^2}{4}\right)(t_{i+1} - t_i). \quad (11)$$

Simulating correlated CIR processes We write Z as $Z_t = \rho W_t + \sqrt{1-\rho^2}W'_t$, where W'_t is a Brownian motion independent of W, and we obtain the increments of (W,Z) between t_i and t_{i+1} through simulation of the increments of W, and W'; these increments are independent, centered Gaussian variables with variance $\Delta_i = t_{i+1} - t_i$.

2.4 Gaussian dependence mapping

For $u \in [t,T]$, denote by

$$\begin{aligned} M_1^{\mathrm{CIR}++}(u) &\triangleq \mathbf{E}\left[\lambda_u \exp\left\{-\int_t^u (r_s + \lambda_s)ds\right\}\right] \\ M_2^{\mathrm{CIR}++}(u) &\triangleq \mathbf{E}\left[\exp\left\{-\int_t^u (r_s + \lambda_s)ds\right\}\right]. \end{aligned}$$

When λ and r are independent, the value of the CDS at time t can be written as (see [BrAlf]):

$$\begin{aligned} \mathrm{CDS}(t,\mathcal{T},T,R_f,Z) = \quad & R_f \int_t^T M_1^{\mathrm{CIR}++}(u)(u - T_{\beta(u)-1})du & (12) \\ + \quad & R_f \sum_{i=\beta(t)}^{n} \alpha_i M_2^{\mathrm{CIR}++}(T_i) & (13) \\ - \quad & Z \int_t^T M_1^{\mathrm{CIR}++}(u)du. & (14) \end{aligned}$$

If, for $t \le u \le T$, we know to compute $M_1^{\text{CIR++}}(u)$ and $M_2^{\text{CIR++}}(u)$, then using (12), (13) and (14) we can compute the CDS value. Indeed, knowing $M_1^{\text{CIR++}}(u)$, for $t \le u \le T$, we can compute numerically the integrals in (12) and (14).

How about the computation of $M_1^{\text{CIR++}}(u)$ and $M_2^{\text{CIR++}}(u)$? Looking closely at this problem one find that these expectations can be trustworthy approximated not only when λ and r are independent, but also when λ and r are correlated. *Thus, in the general case $\rho \neq 0$ we will approximate the* CDS *value using* (12)-(14).

We present now the approximation formulae to compute $M_1^{\text{CIR++}}(u)$ and $M_2^{\text{CIR++}}(u)$.

2.4.1 Preliminaries

Because $r_s = x_s^\alpha + \varphi(s;\alpha)$ and $\lambda_s = y_s^\beta + \psi(s;\beta)$ we have

$$\begin{aligned} M_1^{\text{CIR++}}(u) &= e^{-\int_t^u (\varphi(s;\alpha)+\psi(s;\beta))ds} \Big(\psi(u;\beta)\mathbf{E}\left[\exp\left\{-\int_t^u (x_s^\alpha + y_s^\beta)ds\right\}\right] \\ &\qquad + \mathbf{E}\left[y_u^\beta \exp\left\{-\int_t^u (x_s^\alpha + y_s^\beta)ds\right\}\right]\Big), \\ M_2^{\text{CIR++}}(u) &= e^{-\int_t^u (\varphi(s;\alpha)+\psi(s;\beta))ds}\mathbf{E}\left[\exp\left\{-\int_t^u (x_s^\alpha + y_s^\beta)ds\right\}\right]. \end{aligned} \tag{15}$$

Denoting by

$$\begin{aligned} M_1^{\text{CIR}}(u) &\triangleq \mathbf{E}\left[y_u^\beta \exp\left\{-\int_t^u (x_s^\alpha + y_s^\beta)ds\right\}\right] \\ M_2^{\text{CIR}}(u) &\triangleq \mathbf{E}\left[\exp\left\{-\int_t^u (x_s^\alpha + y_s^\beta)ds\right\}\right], \end{aligned}$$

we have that

$$\begin{aligned} M_1^{\text{CIR++}}(u) &= e^{-\int_t^u (\varphi(s;\alpha)+\psi(s;\beta))ds}\left[\psi(u;\beta)M_2^{\text{CIR}}(u) + M_1^{\text{CIR}}(u)\right] \\ M_2^{\text{CIR++}}(u) &= e^{-\int_t^u (\varphi(s;\alpha)+\psi(s;\beta))ds} M_2^{\text{CIR}}(u). \end{aligned}$$

The ideea is to "map" the two-dimensional CIR dynamics in an analogous tractable two-dimensional Gaussian dynamics that preserve as much as possible of the original CIR structure, and then do calculations in the Gaussian model. Recall that the CIR process and the Vasicek process for interest rate give both affine models. The first one is more convenient because it ensures positive values while the second one is more analytically tractable. Indeed, in our two-dimensional CIR model we have no formula for $M_1^{\text{CIR}}(u)$ and $M_2^{\text{CIR}}(u)$ for $\rho \neq 0$, while in the two-dimensional Vasicek case one can easily derive such formulae (see below).

2.4.2 Mapping a CIR dynamic in a Vasicek one

Let x^α be the CIR process defined in (3)

$$dx_s^\alpha = k(\theta - x_s^\alpha)ds + \sigma\sqrt{x_s^\alpha}dW_s, \qquad x_t^\alpha = x_0, \quad t \le s \le T,$$

and let $x^{\alpha_T,V}$ be the following Vasicek process

$$dx_s^{\alpha_T,V} = k(\theta - x_s^{\alpha_T,V})ds + \sigma^{\alpha_T,V}dW_s, \qquad x_t^{\alpha_T,V} = x_0, \quad t \le s \le T,$$

where $\sigma^{\alpha_T,V}$, the volatility of the Vasicek process, is computed solving the equation

$$\mathbf{E}\left[\exp\left\{-\int_t^T x_s^{\alpha_T,V}ds\right\}\right] = \mathbf{E}\left[\exp\left\{-\int_t^T x_s^{\alpha}ds\right\}\right].$$

In the above equation, expectations on both sides are analytically known, being bond price formulae for Vasicek and CIR models respectively. Then

$$\sigma^{\alpha_T,V} = k\sqrt{2\,\frac{\log(P^{\text{CIR}}(t,T;x^\alpha)) + \theta(T-t) - (\theta - x_t)g(k,T-t)}{(T-t) - 2g(k,T-t) + g(2k,T-t)}},$$

with $g(a,s) = (1-e^{-as})/a$.

2.4.3 Gaussian dependence mapping approximation

Recall our two CIR processes x^α and y^β

$$\begin{aligned} dx_s^\alpha &= k(\theta - x_s^\alpha)ds + \sigma\sqrt{x_s^\alpha}dW_s, \\ dy_s^\beta &= \kappa(\mu - y_s^\beta)ds + \nu\sqrt{y_s^\beta}dZ_s, \end{aligned}$$

with $dW_s\,dZ_s = \rho\,ds$, and consider the corresponding Vasicek processes $x^{\alpha_T,V}$ and $y^{\beta_T,V}$

$$\begin{aligned} dx_s^{\alpha_T,V} &= k(\theta - x_s^{\alpha_T,V})ds + \sigma^{\alpha_T,V}dW_s, \\ dy_s^{\beta_T,V} &= \kappa(\mu - y_s^{\beta_T,V})ds + \nu^{\beta_T,V}dZ_s, \end{aligned}$$

constructed as in Section 2.4.2.

We adopt the following approximations:

$$\begin{aligned} M_2^{\text{CIR}}(u) &\triangleq \mathbf{E}\left[\exp\left\{-\int_t^u (x_s^\alpha + y_s^\beta)ds\right\}\right] && (16) \\ &\approx \mathbf{E}\left[\exp\left\{-\int_t^u (x_s^{\alpha_u,V} + y_s^{\beta_u,V})ds\right\}\right] && (17) \end{aligned}$$

$$\begin{aligned} M_1^{\text{CIR}}(u) &\triangleq \mathbf{E}\left[y_u^\beta \exp\left\{-\int_t^u (x_s^\alpha + y_s^\beta)ds\right\}\right] && (18) \\ &\approx \mathbf{E}\left[y_u^{\beta_u,V} \exp\left\{-\int_t^u (x_s^{\alpha_u,V} + y_s^{\beta_u,V})ds\right\}\right] + \Delta_u && (19) \end{aligned}$$

where

$$\begin{aligned} \Delta_u &\triangleq \mathbf{E}\left[\exp\left\{-\int_t^u x_s^\alpha ds\right\}\right]\mathbf{E}\left[y_u^\beta \exp\left\{-\int_t^u y_s^\beta ds\right\}\right] \\ &\quad - \mathbf{E}\left[\exp\left\{-\int_t^u x_s^{\alpha_u,V} ds\right\}\right]\mathbf{E}\left[y_u^{\beta_u,V} \exp\left\{-\int_t^u y_s^{\beta_u,V} ds\right\}\right] \\ &= P^{\text{CIR}}(t,u;x^\alpha)\left[\left(-\frac{\partial P^{\text{CIR}}}{\partial u}(t,u;y^\beta)\right) - M_3^{\text{V}}(u)\right], && (20) \end{aligned}$$

where $M_3^{\mathrm{V}}(u)$ is defined in (23).

Put

$$
\begin{aligned}
M_1^{\mathrm{V}}(u) &\triangleq \mathbf{E}\left[y_u^{\beta_u,V}\exp\left\{-\int_t^u (x_s^{\alpha_u,V}+y_s^{\beta_u,V})ds\right\}\right] && (21)\\
M_2^{\mathrm{V}}(u) &\triangleq \mathbf{E}\left[\exp\left\{-\int_t^u (x_s^{\alpha_u,V}+y_s^{\beta_u,V})ds\right\}\right] && (22)\\
M_3^{\mathrm{V}}(u) &\triangleq \mathbf{E}\left[y_u^{\beta_u,V}\exp\left\{-\int_t^u y_s^{\beta_u,V}ds\right\}\right] && (23)
\end{aligned}
$$

Then, from (18)-(19) and (16)-(17), we have

$$
\begin{aligned}
M_1^{\mathrm{CIR}}(u) &\approx M_1^{\mathrm{V}}(u)+\Delta_u && (24)\\
M_2^{\mathrm{CIR}}(u) &\approx M_2^{\mathrm{V}}(u). && (25)
\end{aligned}
$$

At this point we can say that the work is completely done, because, for $0\le u\le T$, $M_1^{\mathrm{V}}(u)$, $M_2^{\mathrm{V}}(u)$ and $M_3^{\mathrm{V}}(u)$ have known analytical expressions, as indicated in what follows.

2.4.4 Computing $M_1^{\mathrm{V}}(u)$, $M_2^{\mathrm{V}}(u)$ and $M_3^{\mathrm{V}}(u)$

Let $x^{\alpha_T,V}$ and $y^{\beta_T,V}$ be two Vasicek processes as follows

$$
\begin{aligned}
dx_s^{\alpha_T,V} &= k(\theta-x_s^{\alpha_T,V})ds+\sigma_v dW_s,\\
dy_s^{\beta_T,V} &= \kappa(\mu-y_s^{\beta_T,V})ds+\nu_v dZ_s,
\end{aligned}
$$

defined for $t\le u\le T$, with $dW_s\,dZ_s=\rho\,ds$. Lemmas 3.1 and 3.2 from [BrAlf] imply the following formulae.

$$
\begin{aligned}
M_1^{\mathrm{V}}(T) &= m_B\exp\left\{-m_A+\frac{1}{2}\sigma_A^2\right\}-\bar\rho\sigma_A\sigma_B\exp\left\{-m_A+\frac{1-\bar\rho^2}{2}\sigma_A^2\right\},\\
M_2^{\mathrm{V}}(T) &= \exp\left\{-m_A+\frac{1}{2}\sigma_A^2\right\},\\
M_3^{\mathrm{V}}(T) &= m_B\exp\left\{-m_A^{deg}+\frac{1}{2}(\sigma_A^{deg})^2\right\}-\bar\rho^{deg}\sigma_A^{deg}\sigma_B\exp\left\{-m_A^{deg}+\frac{1-(\bar\rho^{deg})^2}{2}(\sigma_A^{deg})^2\right\},
\end{aligned}
$$

with

$$\begin{aligned}
m_A &= (\mu+\theta)(T-t) - [(\theta - x_t)g(k,T-t) + (\mu - y_t)g(\kappa,T-t)],\\
m_B &= \mu - (\mu - y_t)e^{-\kappa(T-t)},\\
m_A^{deg} &= \mu(T-t) - (\mu - y_t)g(\kappa,T-t),\\
\sigma_A^2 &= \left(\frac{\nu_v}{\kappa}\right)^2 (T-t-2g(\kappa,T-t)+g(2\kappa,T-t))\\
&\quad + \left(\frac{\sigma_v}{k}\right)^2 (T-t-2g(k,T-t)+g(2k,T-t))\\
&\quad + \frac{2\rho\sigma_v\nu_v}{k\kappa}(T-t-g(\kappa,T-t)-g(k,T-t)+g(\kappa+k,T-t)),\\
\sigma_B^2 &= \nu_v^2 g(2\kappa,T-t),\\
(\sigma_A^{deg})^2 &= \left(\frac{\nu_v}{\kappa}\right)^2 (T-t-2g(\kappa,T-t)+g(2\kappa,T-t)),\\
\bar\rho &= \frac{1}{\sigma_A\sigma_B}\left[\frac{\nu_v^2}{\kappa}(g(\kappa,T-t)-g(2\kappa,T-t))\right.\\
&\quad \left. + \frac{\rho\sigma_v\nu_v}{k}(g(\kappa,T-t)-g(\kappa+k,T-t))\right],\\
\bar\rho^{deg} &= \frac{1}{\sigma_A^{deg}\sigma_B}\left[\frac{\nu_v^2}{\kappa}(g(\kappa,T-t)-g(2\kappa,T-t))\right],\\
g(a,s) &= (1-e^{-as})/a.
\end{aligned}$$

See the computation tree in Figure 2.4.4.

3 Available numerical routines

3.1 Pricing defaultable bonds

We have to compute $\mathbf{E}(\mathbf{1}_{\tau>T}P(0,T))$.

3.1.1 Closed form formula ($\rho = 0$)

When r and λ are uncorrelated, we can use the closed form formulae for the zero-coupon bond and the survival probabilies.

3.1.2 Monte Carlo technique

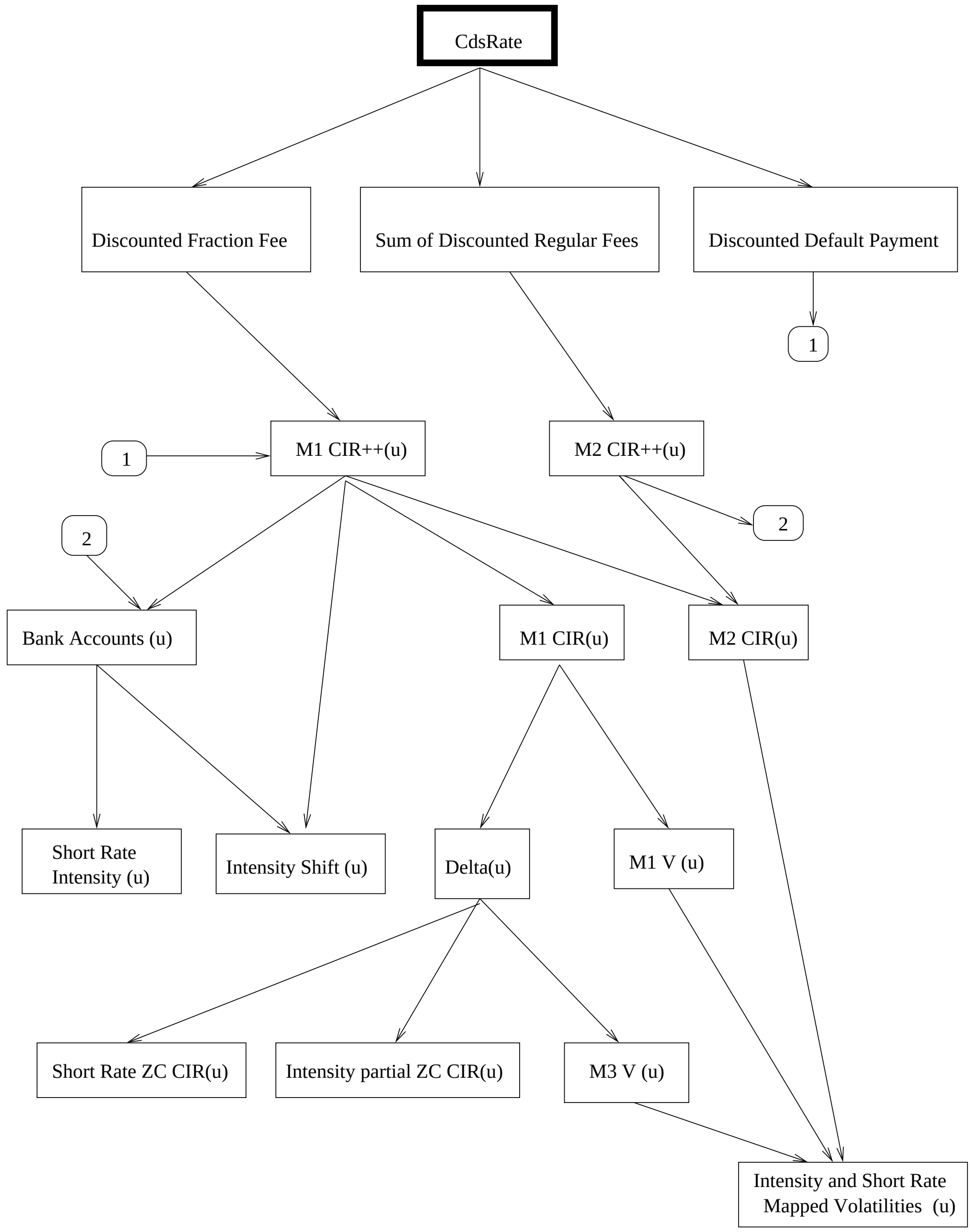
We simulate the correlated couple $(r_t,\ \lambda_t)$, over $[0,T]$, and compute the desired expectation.

3.2 Computing the CDS rate

Recall Eqs. (1) and (2). We have to compute

$$\mathrm{CDS}(t,\mathcal{T},T,R_f,Z) = \mathbf{E}_t(\mathcal{P}_t)$$

Computing ths CDS Rate using Gaussian Mapping Approximation



as a function of R_f, and then to choose the R_f that makes the CDS fair at time t, i.e.

$$\mathrm{CDS}(t, \mathcal{T}, T, R_f, Z) = 0.$$

3.2.1 Closed form formula ($\rho = 0$)

When r and λ are uncorrelated, the CDS price is written down in (8)-(10). Our model is such constructed that it exactely match the zero-coupon prices and survival probabilities. Thus in these equations we can directely use the *market* zero-coupon prices and survival probabilities. Applying numerical integration we compute the integrals in (8) and (10), and then completely evaluate the CDS rate R_f, which is such that $\mathrm{CDS}(t, \mathcal{T}, T, R_f, Z) = 0$.

3.2.2 Gaussian mapping analytical approximation

We use analytical formulae to approximate the expectations in (12)-(14) and apply numerical integration to compute integrals in (12) and (14). Then R_f is chosed such that $\mathrm{CDS}(0, \mathcal{T}, T, R_f, Z) = 0$.

3.2.3 Monte Carlo technique

To compute $\mathbf{E}(\mathcal{P}_0)$, we simulate the correlated couple $(r_t,\ \lambda_t)$, over $[0, T]$, and apply the control variate technique to reduce the variance; the control variate we used is $\mathbf{1}_{\tau > T}$.

4 Implementation

The implementation is done in the C++ language. Some remarks concerning the numerical algorithms are in order.

The numerical integrations are computed using Riemann summations and also Simpson's Rule.

CIR++ processes are simulated using the Explicit(0) scheme from Alfonsi (2005).

We also use the C++ random number generator library writen by Robert Davies. This library is freely available at the adress: http://www.robertnz.net/

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