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mc_robbinsmoro_hw

Input parameters

- Number of iterations N
- Generator type
- Increment inc
- Confidence Value
- Volatility of volatility

Output parameters

- Price P
- Error price σ_P
- Delta δ
- Error delta σ_{delta}
- Price Confidence Interval: ICp [Inf Price,Sup Price]
- Delta Confidence Interval: ICp [Inf Delta,Sup Delta]

Description

Computation of a european or a asian option in the Hull and White stochastic volatility model.

The model is defined by

$$dS_t = (r-q)S_t dt + \sqrt{\sigma_t} S_t dW_t^1,$$
$$d\sigma_t = \nu\sigma_t dt + \zeta\sigma_t dW_t^2,$$

where W^1 and W^2 are two correlated Brownian motions with $\langle W^1, W^2\rangle_t = \rho t$. In this model, S_t has a finite mean but an infinite variance. With a linear discretization of S_t (*Euler scheme*), we see numericaly that the variance is finite but increases very quickly with the number of steps. To reduce this effect, we need to truncate this variance.
In their original paper Hull and White, [1] suggested that $\zeta = 1$ may be more realistic. However, one can use larger values for ζ in order to get a sharper contrast with the constant volatility case.

Algorithm:

/*The price*/
The objective is to compute $V_0 = \mathbb{E}[\phi(S_t, t \leq T)]$ where $(S_t)_{t\leq T}$ is the Hull and White model.
/*Simulate the discretized underlying*/
Following *Glasserman Heidelberger and Shahabuddin (1999)*[2], we consider this discretisation of the model:

$$S_{T_{i+1}} = S_{T_i}(1 + (r-q)\Delta t + \sqrt{\sigma_i \Delta t} Z_i),$$
$$\sigma_{i+1} = \min\{c, \sigma_i e^{(\nu - \frac{1}{2}\zeta^2)\Delta t + \zeta\sqrt{\Delta t}(\rho Z_i + \sqrt{1-\rho^2} Z_{m+i})}\},$$

where c is a non-negative constant. The truncation has little impact on the mean but makes the estimated variances much more stable. The constant volatility case corresponds to $\zeta = 0$.
At each iterations we generate a random gaussian vector of size $2\times N$ where N is the total number of MC iterations. Then separate this vector in two vectors af same size N to simulate both the underlying asset and the volatility.
/*Importance sampling*/
In the discretized problem we have to evaluate $\hat{V}_0 = \mathbb{E}[\hat{\phi}(Z)]$ where $Z = (Z_1, \ldots, Z_m)$ is a standard gaussian vector. Using an elementary version of Girsanov theorem leads to the following representation of $\hat{V}_0$:

$$\hat{V}_0 = \mathbb{E}[g(\mu, Z)], \tag{1}$$

with

$$g(\mu, Z) = \hat{\phi}(Z+\mu)e^{-\mu\cdot Z - \frac{1}{2}\|\mu\|^2}, \tag{2}$$

where $\|x\|$ denotes the Euclidean norm of a vector $x \in \mathbb{R}^m$ and $x \cdot y$ is the inner product of two vectors $x, y \in \mathbb{R}^m$.

/*Variance reduction*/
The idea is then to make use of a Robbins and Monro algorithm to assess the optimal μ^* that minimizes the variance of $g(\mu^*, Z)$.
/*The price computation and confidence interval*/
By rebalancing this optimal μ^* in the MC computation of the price we reduce the variance by a factor of 5 and more. Finally by the use of the central limit theorem we get a confidence interval with a length equal to the length of the MC standard confidence interval by a factor of 2.5 to 3 and even more for options that are far from the money.

References

[1] J.HULL A.WHITE. The pricing of options on assets with stochastics volatility. *J.Of Finance*, 42:281–300, 1987. 2

[2] P.GLASSERMAN P.HEIDELBERGER P.SHAHABUDDIN. Asymptotically optimal importance sampling and stratification for prcing path-dependent options. *Mathematical Finance*, 2,April:117–152, 1999. 2