

## Help

```
#include <stdlib.h>
#include <stdio.h>
#include <string.h>
#include "pnl/pnl_random.h"
#include "pnl/pnl_vector.h"
#include "pnl/pnl_linalgsolver.h"
#include "pnl/pnl_finance.h"
#include "math/equity_pricer/implied_bs.h"
#include "pnl/pnl_mathtools.h"
#include "math/equity_pricer/gridsparse_functions.h"
#include "gridsparse_functions_varswap.h"

#define DEFAULT_VALUE_VARSWAP3D_MOD_DOMAIN_SIZE 4
#define SOLVER_PRECISION 1e-5
#define SOLVER_MAX_ITER 200
#define SOLVER_GMRES_RESTART 20

#define SPARSE_H2N(Vout,Vin,index,father) LET(Vout,index)=GET(Vin,index)+0.5*(
#define SPARSE_N2H(Vout,Vin,index,father) LET(Vout,index)=GET(Vin,index)-0.5*(
#define SPARSE_N2H_FUNC(Vin,index,father) GET(Vin,index)-0.5*( GET(Vin,(*father

////////////////////////////////////
// Stochastic Variance Swap Model Operator on Sparse Grid
////////////////////////////////////

// store the volatility on point of index i
/*
 * @param Op a SVSSparseOp contains data for abstract matrix-vector multiplicati
 * store  $\int \exp(0.5 \sum_{d=2}^{\dim} x_d^i) \int f$  where  $\int f x^i_t \int f$  is
 * Orstein Uhlhembeck process in vector Op->V_volatility
 */
void Get_Local_Volatility_init(SVSSparseOp *Op)
{
    int i, d;
    double sum = 0.0;
    LET(Op->V_volatility, 0) = 0;
    for (i = 1; i < Op->V_volatility->size; i++)
    {
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        sum = 0.0;
        for (d = 1; d < Op->G->dim; d++)
            sum += GET(Op->Model->Beta, d - 1) / GET(Op->Model->SqrtMeanReversion, d);
        LET(Op->V_volatility, i) = MIN(exp(0.5 * sum), 10);
    }
}

void Get_Local_Volatility(SVSSparseOp *Op, int i, double *vol, double *sqr_vol)
{
    *vol = GET(Op->V_volatility, i);
    *sqr_vol = (*vol) * (*vol);
}

/**
 * initilisation of the sparse operator for diffusion equation associated to
 * stochastic variance swap model
 *
 * @param pointer on SVSSparseOp
 */
void initialise_svs_sparse_operator(SVSSparseOp *Op)
{
    Op->PC = pnl_vect_create_from_zero(Op->G->size);
    Op->V_tmp0 = pnl_vect_create_from_zero(Op->G->size);
    Op->V_tmp1 = pnl_vect_create_from_zero(Op->G->size);
    Op->V_tmp2 = pnl_vect_create_from_zero(Op->G->size);
    Op->V_tmp3 = pnl_vect_create_from_zero(Op->G->size);
    Op->V_tmp4 = pnl_vect_create_from_zero(Op->G->size);
    Op->V_tmp5 = pnl_vect_create_from_zero(Op->G->size);
    Op->V_tmp6 = pnl_vect_create_from_zero(Op->G->size);
    Op->V_tmp7 = pnl_vect_create_from_zero(Op->G->size);
    Op->V_volatility = pnl_vect_create_from_zero(Op->G->size);
    Get_Local_Volatility_init(Op);
}

/**
 * Create the sparse operator for diffusion equation associated to
 * stochastic variance swap model
 *
 * @param lev int level of sparse grid

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* @param dim int dimension of model dim = n+1
with n number of factors
* @return pointer on SVSSparseOp
*/
SVSSparseOp *svs_sparse_operator_create(int lev, int N_T, VARSWAP3D_MOD *M)
{
    int d, dim;
    PnlVect *X0, *X1;
    double sum = 0.0;
    SVSSparseOp *Op = malloc(sizeof(SVSSparseOp));
    Op->Model = M;
    // Create space discretization grid
    dim = Op->Model->Nb_factor + 1;
    X0 = pnl_vect_create(dim);
    X1 = pnl_vect_create(dim);
    for (d = 1; d < dim; d++)
    {
        LET(X1, d) = sqrt(1 - exp(-2 * GET(Op->Model->MeanReversion, d - 1) * Op->
        LET(X0, d) = -1.0 * GET(X1, d);
        sum += GET(Op->Model->Beta, d - 1) / GET(Op->Model->SqrtMeanReversion, d -
    }
    svs_sigma_time(Op->Model, Op->Model->T);
    LET(X1, 0) = 2; //Op->Model->V0*exp(0.5*sum)*sqrt(Op->Model->T)*2;
    LET(X0, 0) = -1.0 * GET(X1, 0);
    Op->G = grid_sparse_create(X0, X1, lev);
    pnl_vect_free(&X0);
    pnl_vect_free(&X1);
    // Create time grid - Here Uniform grid
    Op->TG = premia_pde_time_homogen_grid(Op->Model->T, N_T);
    // Initialise size of temporary vectors
    initialise_svs_sparse_operator(Op);
    return Op;
}

/**
* desallocation of sparse operator for diffusion equation associated to
* stochastic variance swap model not free variance swap model
*
* @param pointer on SVSSparseOp
*/
void svs_sparse_operator_free(SVSSparseOp **Op)

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{
    premia_pde_time_grid_free(&(*Op)->TG);
    pnl_vect_free(&(*Op)->PC);
    pnl_vect_free(&(*Op)->V_tmp0);
    pnl_vect_free(&(*Op)->V_tmp1);
    pnl_vect_free(&(*Op)->V_tmp2);
    pnl_vect_free(&(*Op)->V_tmp3);
    pnl_vect_free(&(*Op)->V_tmp4);
    pnl_vect_free(&(*Op)->V_tmp5);
    pnl_vect_free(&(*Op)->V_tmp6);
    pnl_vect_free(&(*Op)->V_tmp7);
    pnl_vect_free(&(*Op)->V_volatility);
    GridSparse_free(&(*Op)->G);
    free(*Op);
    *Op = NULL;
}

/*
 * WARNING: with the convention A is matrix operator, we compute
 *  $V_{out} = a * PC V_{in} + b V_{out}$ 
 *
 * @param Op a SVSSparseOp contains data for abstract matrix-vector multiplicati
 * @param Vin a PnlVect, input parameters
 * @param a a double
 * @param b a double
 * @param Vout a PnlVect, the output
 */
void GridSparse_apply_svs_PC(SVSSparseOp *Op,
                             const PnlVect *Vin,
                             const double a,
                             const double b,
                             PnlVect *Vout)
{
    pnl_vect_clone(Vout, Vin);
    pnl_vect_mult_vect_term(Vout, Op->PC);
}

int Index[3]; //Index[0]=dir, Index[1]= Position Index[2]= left or right

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void GridSparse_add_rhs_svs(SVSSparseOp *Op,
                           PnlVect *V_rhs)
{
    double Forward, sqr_vol, vol;
    int i;
    // No optimisation,
    // Good way : compute & store term in x_1 on uniforme fine grid and
    // use it in this loop in place of evaluate premia_bs_s_square_gamma ...
    // Here we suppose r=0 - bond = 1.
    PnlVect *VT_rhs = Op->V_tmp0;
    for (i = 1; i < VT_rhs->size; i++)
    {
        Forward = Op->Model->S0 * exp(GridSparse_real_value_at_points(Op->G, 0, i))
        Get_Local_Volatility(Op, i, &vol, &sqr_vol);
        LET(VT_rhs, i) = (sqr_vol - 1.0) * pnl_bs_impli_s_square_gamma(Op->Model->
            Forward,
            Op->Model->Strike,
            premia_pde_time_grid_time(Op->TG)
            + premia_pde_time_grid_step(Op->TG));
    }
    Nodal_to_Hier(VT_rhs, Op->G);
    pnl_vect_axpby(0.5 * premia_pde_time_grid_step(Op->TG)*
        Op->Model->V0_sqr, VT_rhs, 1.0, V_rhs);
}

```

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void GridSparse_preconditioner_svs_init(SVSSparseOp *Op)
{
    int i, d;
    double vol, sqr_vol, jacobi;
    i = 1;
    do
    {
        jacobi = 0;
        for (d = 1; d < Op->G->dim; d++)
        {
            jacobi += //0.5*GET(Op->Model->Beta,d-1)*GET(Op->Model->Beta,d-1)*
                GET(Op->Model->MeanReversion, d - 1) * (2 << ((log2int(pnl_mat_int_g
            }
        Get_Local_Volatility(Op, i, &vol, &sqr_vol);
    }
}

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        jacobi += 0.5 * sqr_vol * Op->Model->V0_sqr *
            (2 << ((log2int(pnl_mat_int_get(Op->G->Points, i, 0)) + 1)));
        LET(Op->PC, i) = 1.0 / sqrt(1.0 + premia_pde_time_grid_step(Op->TG) * jacobi);
        i++;
    }
    while (i < Op->G->size);
}

int Operator_Initialisation_Log1F(const PnlVect *Vin, SVSSparseOp *Op)
{
    // compute : = u + theta L(u) with L :=BS operator
    PnlVect *Drift_Price, * Vol_Price, * Drift_Price_T, * Vol_Price_T;
    int Index[3]; //Index[0]=dir, Index[1]= Position Index[2]= left or right
    int *father;
    double vol, sqr_vol;
    Drift_Price = Op->V_tmp0;
    Vol_Price = Op->V_tmp1;
    Drift_Price_T = Op->V_tmp2;
    Vol_Price_T = Op->V_tmp3;
    Index[2] = 0;
    Index[0] = 1;
    //>> on y_1 = x_2
    Index[1] = 1;
    father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
    do
    {
        SPARSE_H2N(Drift_Price, Vin, Index[1], father);
        Index[1]++;
        father += 2;
    }
    while (Index[1] < Op->G->size);
    Index[1] = 1;
    father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
    do
    {
        Get_Local_Volatility(Op, Index[1], &vol, &sqr_vol);
        LET(Vol_Price_T, Index[1]) = sqr_vol * GET(Drift_Price, Index[1]);
        LET(Drift_Price_T, Index[1]) = vol * GET(Drift_Price, Index[1]);
        SPARSE_N2H(Drift_Price, Drift_Price_T, Index[1], father);
        SPARSE_N2H(Vol_Price, Vol_Price_T, Index[1], father);
        Index[1]++;
    }
}

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        father += 2;
    }
    while (Index[1] < Op->G->size);
    return OK;
}

/*
 * Compute \ cL u ... the discret operator
 */
int Operator_SVS_X1F(const PnlVect *Vin, PnlVect *Vout, SVSSparseOp *Op, const d
{
    PnlVect *Drift_Price, * Vol_Price, *Correl_Price, *dYY, *dSY, *dS, *Price_Dir;
    int Index[3];
    //>> Index[0]=dir, Index[1]= Position Index[2]= left or right
    int *father;
    int *neig;
    double coeff, alpha, beta_sqr;
    double sigma = 0.5 * Op->Model->V0_sqr;
    double coeff_2 = Op->Model->Rho * Op->Model->V0_time;
    alpha = a * Op->theta_time_scheme * premia_pde_time_grid_step(Op->TG);
    if (Op->G->dim == 2)
        Operator_Initialisation_Log1F(Vin, Op);
    else
        return WRONG;
    Drift_Price = Op->V_tmp0;
    Vol_Price = Op->V_tmp1;
    Correl_Price = Op->V_tmp2;
    dYY = Op->V_tmp4;
    dSY = Op->V_tmp5;
    dS = Op->V_tmp6;
    Price_Dir = Op->V_tmp7;
    pnl_vect_set_zero(Price_Dir);
    pnl_vect_set_zero(dSY);
    pnl_vect_set_zero(dYY);
    pnl_vect_set_zero(dS);
    //>>compute nodale representation in dimension Dim
    Index[2] = 0;
    //>> Log spot variables
    Index[0] = 0;
    //>> Left father
    if (Index[0] > Op->G->dim)

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{
    printf("error in dimension");
    abort();
}
Index[1] = 1; //First point on map
father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
do
{
    // SPARSE_H2N(Price_Dir,Vin,Index[1],father);
    // Not need without interest rate
    SPARSE_H2N(dSY, Drift_Price, Index[1], father);
    SPARSE_H2N(dYY, Vol_Price, Index[1], father);
    Index[1]++;
    father += 2;
}
while (Index[1] < Op->G->size);
Index[1] = 1;
father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
neig = pnl_hmat_int_lget(Op->G->Ind_Neigh, Index);
coeff = 0.5 * Op->Model->Rho * Op->Model->V0_time * Op->Model->Sum_Beta;
do
{
    // >> Term :  $\sigma^2/2 \frac{d^2 P}{dx^2}$ 
    LET(Vol_Price, Index[1]) = sigma * FD_Lap_Stencil_Center(Index[1], Index[0],

    // >> Term :  $-\sigma^2/2 \frac{d P}{dx}$ 
    // >> Term :  $-\sigma / 2 \rho \text{ Sum\_Beta } \frac{dP}{dx}$ 

    //>> Centered Scheme
    //LET(Vol_Price,Index[1])-=sigma*FD_Conv_Stencil_Center(Index[1],Index[0],
    //LET(Vol_Price,Index[1])-=coeff*FD_Conv_Stencil_Center(Index[1],Index[0],

    //>> Decentered Scheme
    LET(Vol_Price, Index[1]) -= sigma * FD_Conv_Stencil_DeCenter(Index[1], Ind
    LET(Vol_Price, Index[1]) -= coeff * ((coeff > 0) ? FD_Conv_Stencil_DeCente
        (-1.) * FD_Conv_Stencil_DeCenter(Inde

    // >> correlation Term  $\sigma^2/2 \frac{d P}{dx}$ 
    LET(dS, Index[1]) = coeff_2 * FD_Conv_Stencil_Center(Index[1], Index[0], d
    SPARSE_N2H(Correl_Price, dS, Index[1], father);
    LET(Vout, Index[1]) += alpha * (SPARSE_N2H_FUNC(Vol_Price, Index[1], fathe

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        Index[1]++;
        father += 2;
        neig += 2;
    }
    while (Index[1] < Op->G->size);

    Index[0]++;
    // variable x_2 = y_1
    if (Index[0] > Op->G->dim)
    {
        printf("error in dimension");
        abort();
    }
    Index[1] = 1;
    father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
    do
    {
        SPARSE_H2N(Price_Dir, Vin, Index[1], father);
        SPARSE_H2N(dSY, Correl_Price, Index[1], father);
        Index[1]++;
        father += 2;
    }
    while (Index[1] < Op->G->size);
    Hier_to_Nodal_in_dir(Index[0], Vin, Price_Dir, Op->G);
    Index[1] = 1;
    father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
    neig = pnl_hmat_int_lget(Op->G->Ind_Neigh, Index);
    beta_sqr = GET(Op->Model->MeanReversion, Index[0] - 1);
    do
    {
        coeff = GridSparse_real_value_at_points(Op->G, Index[0], Index[1]);
        LET(Vol_Price, Index[1]) = beta_sqr * FD_Lap_Stencil_Center(Index[1], Inde
        LET(Vol_Price, Index[1]) -= GET(Op->Model->MeanReversion, Index[0] - 1) *
                                ((coeff > 0) ? FD_Conv_Stencil_DeCenter(Index[
                                (-1.0) * FD_Conv_Stencil_DeCenter(Index[1], I

        //FD_Conv_Stencil_Center(Index[1],Index[0],Price_Dir,Op->G,*neig,*neig+1)

        LET(Vol_Price, Index[1]) += GET(Op->Model->SqrtMeanReversion, Index[0] - 1
                                FD_Conv_Stencil_Center(Index[1], Index[0], dSY

        //>> Correl x y_1 term

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        //>> Back to hierarchical representation
        LET(Vout, Index[1]) += alpha * (SPARSE_N2H_FUNC(Vol_Price, Index[1], father));
        Index[1]++;
        father += 2;
        neig += 2;
    }
    while (Index[1] < Op->G->size);
    return OK;
}

```

```

int Operator_Initialisation_Log3F(const PnlVect *Vin, SVSSparseOp *Op)
{
    // Pointwise multiplication
    // Use for computation of u + theta L(u)
    PnlVect *Drift_Price, * Vol_Price, * Drift_Price_T, * Vol_Price_T;
    // >> Index[0]=dir, Index[1]= Position Index[2]= left or right
    int Index[3];
    int *father;
    double vol, sqr_vol;
    Drift_Price = Op->V_tmp0;
    Vol_Price = Op->V_tmp1;
    Drift_Price_T = Op->V_tmp2;
    Vol_Price_T = Op->V_tmp3;
    // >> on y_1 = x_2
    Index[2] = 0;
    Index[0] = 1;
    Index[1] = 1;
    father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
    do
    {
        SPARSE_H2N(Drift_Price, Vin, Index[1], father);
        Index[1]++;
        father += 2;
    }
    while (Index[1] < Op->G->size);
    // >> on y_2 = x_3
    Index[0]++;
    Index[1] = 1;
    father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
    do
    {

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        SPARSE_H2N(Drift_Price_T, Drift_Price, Index[1], father);
        Index[1]++;
        father += 2;
    }
    while (Index[1] < Op->G->size);
    // >> on y_3 = x_4
    Index[0]++;
    Index[1] = 1;
    father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
    do
    {
        SPARSE_H2N(Drift_Price, Drift_Price_T, Index[1], father);
        Index[1]++;
        father += 2;
    }
    while (Index[1] < Op->G->size);
    Index[1] = 1;
    father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
    do
    {
        Get_Local_Volatility(Op, Index[1], &vol, &sqr_vol);
        LET(Vol_Price_T, Index[1]) = sqr_vol * GET(Drift_Price, Index[1]);
        LET(Drift_Price_T, Index[1]) = vol * GET(Drift_Price, Index[1]);
        SPARSE_N2H(Drift_Price, Drift_Price_T, Index[1], father);
        SPARSE_N2H(Vol_Price, Vol_Price_T, Index[1], father);
        Index[1]++;
        father += 2;
    }
    while (Index[1] < Op->G->size);
    // >> Come back to Hierarchic representation
    Index[0]--;
    Index[1] = 1;
    father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
    do
    {
        SPARSE_N2H(Drift_Price_T, Drift_Price, Index[1], father);
        SPARSE_N2H(Vol_Price_T, Vol_Price, Index[1], father);
        Index[1]++;
        father += 2;
    }
    while (Index[1] < Op->G->size);

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    Index[0]--;
    Index[1] = 1;
    father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
    do
    {
        SPARSE_N2H(Drift_Price, Drift_Price_T, Index[1], father);
        SPARSE_N2H(Vol_Price, Vol_Price_T, Index[1], father);
        Index[1]++;
        father += 2;
    }
    while (Index[1] < Op->G->size);
    return OK;
}

/*
 * Compute \ cL u ... the discret operator
 */
int Operator_SVS_X3F(const PnlVect *Vin, PnlVect *Vout, SVSSparseOp *Op, const d
{
    PnlVect *Drift_Price, * Vol_Price, *Correl_Price, *dYY, *dSY, *dS, *Price_Dir;
    int Index[3];
    //Index[0]=dir, Index[1]= Position Index[2]= left or right
    int *father, * neig;
    double coeff, alpha, sigma, coeff_2, beta_sqr;
    alpha = a * Op->theta_time_scheme * premia_pde_time_grid_step(Op->TG);
    if (Op->G->dim == 4)
        Operator_Initialisation_Log3F(Vin, Op);
    else
        return WRONG;
    // Rename data vectors of SVSSparseOp use for store computation
    Drift_Price = Op->V_tmp0;
    Vol_Price = Op->V_tmp1;
    Correl_Price = Op->V_tmp2;
    dYY = Op->V_tmp4;
    dSY = Op->V_tmp5;
    dS = Op->V_tmp6;
    Price_Dir = Op->V_tmp7;
    // >>compute nodale representation in dimension Dim :
    Index[2] = 0; // >> Left father
    Index[0] = 0; // >> First dimension, here Log spot variables
    if (Index[0] > Op->G->dim)

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    {
        printf("error in dimension");
        abort();
    }
Index[1] = 1; // >> First point on map
father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
do
{
    SPARSE_H2N(dSY, Drift_Price, Index[1], father);
    SPARSE_H2N(dYY, Vol_Price, Index[1], father);
    Index[1]++;
    father += 2;
}
while (Index[1] < Op->G->size);
Index[1] = 1;
father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
neig = pnl_hmat_int_lget(Op->G->Ind_Neigh, Index);
sigma = 0.5 * Op->Model->V0_sqr;
coeff = 0.5 * Op->Model->Rho * Op->Model->V0_time * Op->Model->Sum_Beta;
coeff_2 = Op->Model->Rho * Op->Model->V0_time;
do
{
    // >> Term :  $\sigma^2/2 \frac{d^2 P}{dx^2}$ 
    LET(Vol_Price, Index[1]) = sigma * FD_Lap_Stencil_Center(Index[1], Index[0],

    // >> Term :  $-\sigma^2/2 \frac{d P}{dx}$ 
    // >> Term :  $-\sigma / 2 \rho \text{ Sum\_Beta } \frac{dP}{dx}$ 

    //>> Centered Scheme
    //LET(Vol_Price, Index[1]) -= sigma * FD_Conv_Stencil_Center(Index[1], Index[0],
    //LET(Vol_Price, Index[1]) -= coeff * FD_Conv_Stencil_Center(Index[1], Index[0],

    //>> Decentered Scheme
    LET(Vol_Price, Index[1]) -= sigma * FD_Conv_Stencil_DeCenter(Index[1], Index[0],
    LET(Vol_Price, Index[1]) -= coeff * ((coeff > 0) ? FD_Conv_Stencil_DeCenter(Index[1], Index[0],
                                                (-1.) * FD_Conv_Stencil_DeCenter(Index[1], Index[0],

    // >> correlation Term  $\sigma^2/2 \frac{d P}{dx}$ 
    LET(dS, Index[1]) = coeff_2 * FD_Conv_Stencil_Center(Index[1], Index[0], dS, Index[0],
    // Without centered scheme
    //LET(dS, Index[1]) = coeff_2 * ((coeff_2 < 0) ? FD_Conv_Stencil_DeCenter(Index[1], Index[0],

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//          (-1.0)*FD_Conv_Stencil_DeCenter(Index[1],Index[0],dSY,Op
SPARSE_N2H(Correl_Price, dS, Index[1], father);
LET(Vout, Index[1]) += alpha * (SPARSE_N2H_FUNC(Vol_Price, Index[1], fathe
Index[1]++;
father += 2;
neig += 2;
}
while (Index[1] < Op->G->size);
Index[0]++; // variable x_2 = y_1
if (Index[0] > Op->G->dim)
{
    printf("error in dimension");
    abort();
}
Index[1] = 1;
father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
do
{
    SPARSE_H2N(Price_Dir, Vin, Index[1], father);
    SPARSE_H2N(dSY, Correl_Price, Index[1], father);
    Index[1]++;
    father += 2;
}
while (Index[1] < Op->G->size);
Hier_to_Nodal_in_dir(Index[0], Vin, Price_Dir, Op->G);
Index[1] = 1;
father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
neig = pnl_hmat_int_lget(Op->G->Ind_Neigh, Index);
beta_sqr = GET(Op->Model->MeanReversion, Index[0] - 1);
do
{
    coeff = GridSparse_real_value_at_points(Op->G, Index[0], Index[1]);
    LET(Vol_Price, Index[1]) = beta_sqr * FD_Lap_Stencil_Center(Index[1], Inde
    LET(Vol_Price, Index[1]) -= GET(Op->Model->MeanReversion, Index[0] - 1) *
        ((coeff > 0) ? FD_Conv_Stencil_DeCenter(Index[1], Index[0], dSY, Op
        (-1.0) * FD_Conv_Stencil_DeCenter(Index[1], Index[0], dSY, Op
    // spot vol correlation term
    LET(Vol_Price, Index[1]) += GET(Op->Model->SqrtMeanReversion, Index[0] - 1)
        FD_Conv_Stencil_Center(Index[1], Index[0], dSY, Op->G);
    //>> vol vol correlation term, for next correlation term ...
    LET(Drift_Price, Index[1]) = GET(Op->Model->SqrtMeanReversion, Index[0] -

```

```

        * FD_Conv_Stencil_Center(Index[1], Index[0],
//>> Back to hierarchical representation
LET(dS, Index[1]) = SPARSE_N2H_FUNC(Drift_Price, Index[1], father);
//>> Back to hierarchical representation
LET(Vout, Index[1]) += alpha * (SPARSE_N2H_FUNC(Vol_Price, Index[1], father),
Index[1]++;
father += 2;
neig += 2;
}
while (Index[1] < Op->G->size);

Index[0]++; //>> Variable x_3=y_2
if (Index[0] > Op->G->dim)
{
    printf("error in dimension");
    abort();
}
Index[1] = 1;
father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
do
{
    SPARSE_H2N(Price_Dir, Vin, Index[1], father);
    SPARSE_H2N(dSY, Correl_Price, Index[1], father);
    SPARSE_H2N(dYY, dS, Index[1], father);
    Index[1]++;
    father += 2;
}
while (Index[1] < Op->G->size);
Index[1] = 1;
father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
neig = pnl_hmat_int_lget(Op->G->Ind_Neigh, Index);
beta_sqr = GET(Op->Model->MeanReversion, Index[0] - 1);
do
{
    coeff = GridSparse_real_value_at_points(Op->G, Index[0], Index[1]);
    LET(Vol_Price, Index[1]) = beta_sqr * FD_Lap_Stencil_Center(Index[1], Index[0],
    LET(Vol_Price, Index[1]) -= GET(Op->Model->MeanReversion, Index[0] - 1) *
        ((coeff > 0) ? FD_Conv_Stencil_DeCenter(Index[1], Index[0],
        (-1.0) * FD_Conv_Stencil_DeCenter(Index[1], Index[0],
// spot vol correlation term

```

```

    LET(Vol_Price, Index[1]) += GET(Op->Model->SqrtMeanReversion, Index[0] - 1
                                FD_Conv_Stencil_Center(Index[1], Index[0], dSY
//>> Correl y_1 y_2 Term
    LET(Vol_Price, Index[1]) += GET(Op->Model->SqrtMeanReversion, Index[0] - 1
                                FD_Conv_Stencil_Center(Index[1], Index[0], dYY
//>> vol vol correlation term, for next correlation term ...
    LET(Drift_Price, Index[1]) = GET(Op->Model->SqrtMeanReversion, Index[0] -
                                * FD_Conv_Stencil_Center(Index[1], Index[0],
//>> Back to hierarchical representation
    LET(dS, Index[1]) += SPARSE_N2H_FUNC(Drift_Price, Index[1], father);
    LET(Vout, Index[1]) += alpha * (SPARSE_N2H_FUNC(Vol_Price, Index[1], fathe
    Index[1]++;
    father += 2;
    neig += 2;
}
while (Index[1] < Op->G->size);
Index[0]++;//>>Variable x_4=y_3
if (Index[0] > Op->G->dim)
{
    printf("error in dimension");
    abort();
}
Index[1] = 1;
father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
do
{
    SPARSE_H2N(Price_Dir, Vin, Index[1], father);
    SPARSE_H2N(dSY, Correl_Price, Index[1], father);
    SPARSE_H2N(dYY, dS, Index[1], father);
    Index[1]++;
    father += 2;
}
while (Index[1] < Op->G->size);
Index[1] = 1;
father = pnl_hmat_int_lget(Op->G->Ind_Father, Index);
neig = pnl_hmat_int_lget(Op->G->Ind_Neigh, Index);
beta_sqr = GET(Op->Model->MeanReversion, Index[0] - 1);
do
{
    coeff = GridSparse_real_value_at_points(Op->G, Index[0], Index[1]);
    LET(Vol_Price, Index[1]) = beta_sqr * FD_Lap_Stencil_Center(Index[1], Inde

```



```

    LET(Vol_Price, Index[1]) -= GET(Op->Model->MeanReversion, Index[0] - 1) *
                                ((coeff > 0) ? FD_Conv_Stencil_DeCenter(Index[1], Index[0], dSY
                                (-1.0) * FD_Conv_Stencil_DeCenter(Index[1], Index[0], dSY
    // spot vol correlation term
    LET(Vol_Price, Index[1]) += GET(Op->Model->SqrtMeanReversion, Index[0] - 1) *
                                FD_Conv_Stencil_Center(Index[1], Index[0], dSY
    //>> Correl y_3 y_2 + y_1 Term
    LET(Vol_Price, Index[1]) += GET(Op->Model->SqrtMeanReversion, Index[0] - 1) *
                                FD_Conv_Stencil_Center(Index[1], Index[0], dSY
    //>> Back to hierarchical representation
    LET(Vout, Index[1]) += alpha * (SPARSE_N2H_FUNC(Vol_Price, Index[1], father
    Index[1]++;
    father += 2;
    neig += 2;
}
while (Index[1] < Op->G->size);
return OK;
}

/*
 * WARNING: with the convention A is matrix operator, we compute
 * V_out = a * A V_in + b Vout
 * Here A = (Mass + sign Delta_t SVS_FD_Operator )
 *
 * @param Op a SVSSparseOp contains data for abstract matrix-vector multiplicati
 * @param Vin a PnlVec, input parameters
 * @param a a double
 * @param b a double
 * @param Vout a PnlVec, the output
 */
void GridSparse_apply_svs(SVSSparseOp *Op,
                          const PnlVect *Vin,
                          const double a,
                          const double b,
                          PnlVect *Vout)
{
    //>> Do V_out = a Mass V_in + b Vout
    pnl_vect_axpby(a, Vin, b, Vout);
    //>> Do V_out += a Rigidity V_in
    if (Op->G->dim <= 2)

```

```

    Operator_SVS_X1F(Vin, Vout, Op, a);
else
    Operator_SVS_X3F(Vin, Vout, Op, a);
}

/*
 * Solve FD discret Sparse version of for diffusion equation associated to
 * stochastic variance swap model with theta-scheme in time
 * Vout - theta delta_t SVS_Operator Vout
 * = Vin +(1- theta) delta_t SVS_Operator Vin + delta_t Source_term ,
 *
 * @param Op a SVSSparseOp contains data for abstract matrix-vector multiplicati
 * @param Vin a PnlVec, the RHS
 * @param Vout a PnlVec, the output
 */
void GridSparse_Solve_svs(SVSSparseOp *Op, PnlVect *Vres)
{
    int *neig;
    PnlVect *V_rhs;
    PnlBicgSolver *Solver;
    //PnlGmresSolver* Solver;
    double theta = 0.0; // Euler implicit
    int Index[3] = {0, 1, 0};
    V_rhs = pnl_vect_create_from_zero(Vres->size);
    Op->theta_time_scheme = -1.0 + theta;
    GridSparse_preconditioner_svs_init(Op);
    Solver = pnl_bicg_solver_create(Vres->size, SOLVER_MAX_ITER, SOLVER_PRECISION)
    //Solver=pnl_gmres_solver_create(Vin->size,SOLVER_MAX_ITER,SOLVER_GMRES_RESTAR
    premia_pde_time_start(Op->TG);
    svs_sigma_time(Op->Model, 0.0);
    do
    {
        Op->theta_time_scheme = theta;
        if (theta != 0.0)
            GridSparse_apply_svs(Op, Vres, 1.0, 0.0, V_rhs);
        else
            pnl_vect_clone(V_rhs, Vres);

        GridSparse_add_rhs_svs(Op, V_rhs);
    }
}

```

```

    Op->theta_time_scheme = -1.0 + theta;
    //pnl_gmres_solver_solve((void*)GridSparse_apply_svs,
    pnl_bicg_solver_solve((void *)GridSparse_apply_svs,
                          Op,
                          (void *)GridSparse_apply_svs_PC,
                          Op,
                          Vres,
                          V_rhs,
                          Solver);
    svcs_sigma_time(Op->Model, premia_pde_time_grid_time(Op->TG));
}
while (premia_pde_time_grid_increase(Op->TG));
pnl_bicg_solver_free(&Solver);
//pnl_gmres_solver_free(&Solver);
pnl_vect_mult_double(Vres, Op->Model->Bond);
pnl_vect_free(&V_rhs);
Hier_to_Nodal(Vres, Op->G);
// Here we put delta on Vres[0] ...
neig = pnl_hmat_int_lget(Op->G->Ind_Neigh, Index);
LET(Vres, 0) = FD_Conv_Stencil_Center(Index[1], Index[0], Vres, Op->G, *(neig)
}

```