New approximations in local volatility models

Ahmed KEBAIER

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Premia 18

1 Model specification

This paper presents a new approximation formula of European options in local volatility model with stochastic interest rates. More precisely, we consider the CEV diffusion $(S_t)_{t\geq 0}$ with a time-dependent level $(\nu_t)_{0\leq t\leq T}$ and time-dependent skew $(\beta_t)_{0\leq t\leq T}$

$$S_t \;=\; S_0 + \int_0^t (r_u - q_u) S_u du + \int_0^t \nu_u S_u^{\beta} dW_u \tag{1}$$

where $(r_t)_{0\leq t\leq T}$ and $(q_t)_{0\leq t\leq T}$ are respectively the time-dependent interest rate and the time-dependent dividend processes.

2 Second order approximation formula for pricing European call based on both the at the Money (ATM) and at strike local volatility

We use the smart expansion introduced by Gobet and Suleiman [1] to give an analytical accurate approximation of a call European price, written as the expected value under the risk neutral probability measure of the payoff function $h(x) = (x - K)_+$ evaluated at the maturity time T:

$$Call\ Price = \mathbb{E}\left[e^{-\int_0^T r_s ds} h\left(S_T\right)\right].$$

More precisely, for r, q and ν constants, we use the second order approximation price formula based on the ATM local volatility and given by Theorem 2 of [1], to write

$$Call\ Price \simeq Call^{BS}(0, S_0; T, K) + \alpha_{1,T}\left(\frac{3}{2} S_0^2 \partial_S^2 Call^{BS}(0, S_0; T, K) + S_0^3 \partial_S^3 Call^{BS}(0, S_0; T, K)\right) \tag{2}$$

where

$$Call^{BS}(t, S; T, K) = S e^{-q(T-t)} \mathcal{N}(d_1) - K e^{-r(T-t)} \mathcal{N}(d_2),$$

$$d_1 = \frac{1}{\sigma\sqrt{T-t}} \log\left(\frac{Se^{-q(T-t)}}{Ke^{-r(T-t)}}\right) + \frac{1}{2}\sigma\sqrt{T-t},$$
$$d_2 = d_1 - \sigma\sqrt{T-t},$$
$$\alpha_{1,T} = (\beta-1)\nu^4 S_0^{4(\beta-1)} \frac{\frac{1}{2}\exp(2(r-q)T) - \exp((r-q)T) + \frac{1}{2}}{(r-q)^2}.$$

Explicit forms of the greeks in formula (2) are given in section C of [1].

We also use the second order approximation price formula based on the local volatility at strike and given by Theorem 3 of [1], to write

$$Call\ Price \simeq Call^{BS}(0, S_0; T, K) + \tilde{\alpha}_{1,T}\left(\frac{3}{2}K2\partial_K^2 Call^{BS}(0, S_0; T, K) + K_0^3\partial_K^3 Call^{BS}(0, S_0; T, K)\right) \quad (3)$$

where

$$\tilde{\alpha}_{1,T} = (\beta-1)\nu^4 K^{4(\beta-1)} \frac{\frac{1}{2}\exp(2(q-r)T) - \exp((q-r)T) + \frac{1}{2}}{(r-q)^2};$$

Also the explicit forms of the greeks in formula (3) are given in section C of [1].

Finally, the two Call prices, given by approximation formulas (2) and (3), are computed by the Premia code source function.

References

[1] Gobet, E.; Suleiman, A. New Approximations in Local Volatility Models. Preprint (2010) 1, 2