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```
/*
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 *
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 * Or it might be one day....
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 * Software Foundation, 59 Temple Place - Suite 330, Boston, MA 02111-1307, USA
 */

/*
 * Put together by John Smith john at arrows dot demon dot co dot uk,
 * using ideas by others.
 *
 * Calculate erf(z) for complex z.
 * Three methods are implemented; which one is used depends on z.
 *
 * The code includes some hard coded constants that are intended to
 * give about 14 decimal places of accuracy. This is appropriate for
 * 64-bit floating point numbers.
 *
 * Oct 1999: Fixed a typo that in
 *      const double complex cerf_continued_fraction( const double complex z )
 * that caused erroneous answers for erf(z) where real(z) negative
 */
```

```

/*
 * Put in Premia by Jérôme Lelong
 * used by Parisian Options
 */

#include <cmath>
#include <complex>

using namespace std;

typedef complex<double> complex_double ;

#ifndef M_SQRT2
#define M_SQRT2      1.41421356237309504880168872420969808    /* sqrt(2) */
#endif

#ifndef M_PI
#define M_PI          3.14159265358979323846264338327950288    /* pi */
#endif

/*#include <octave/oct.h>*/

/*#include "f77-fcn.h"*/

/*
 * Abramowitz and Stegun: (eqn: 7.1.14) gives this continued
 * fraction for erfc(z)
 *
 *
 * 
$$\text{erfc}(z) = \sqrt{\pi} \cdot \exp(-z^2) \cdot \cfrac{1}{z + \cfrac{1/2}{z + \cfrac{1}{z + \cfrac{3/2}{z + \cfrac{2}{z + \cfrac{5/2}{\dots}}}}}}$$

 *
 * This is evaluated using Lentz's method, as described in the narative
 * of Numerical Recipes in C.
 *
 * The continued fraction is true providing real(z)>0. In practice we
 * like real(z) to be significantly greater than 0, say greater than 0.5.
 */

static complex_double cerfc_continued_fraction(complex_double z)

```

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{
    double tiny = 1e-20 ;      /* a small number, large enough to calculate 1/tiny
    double eps = 1e-15 ;      /* large enough so that 1.0+eps > 1.0, when using *
    /* the floating point arithmetic */
    /*
    * first calculate  $z + \frac{1}{2} \frac{1}{z + \frac{1}{z + \dots}}$ 
    *
    *
    */
    complex_double f = z ;
    complex_double C = f ;
    complex_double D = 0.0 ;
    complex_double delta ;
    double a ;

    a = 0.0 ;
    do
    {
        a = a + 0.5 ;
        D = z + a * D ;
        C = z + a / C ;

        if (D.real() == 0.0 && D.imag() == 0.0)
            D = tiny ;

        D = 1.0 / D ;

        delta = (C * D) ;

        f = f * delta ;

    }
    while (abs(1.0 - delta) > eps) ;

    /*
    * Do the first term of the continued fraction
    */
    f = 1.0 / f ;

    /*
    * and do the final scaling

```

```

    */
    f = f * exp(-z * z) / sqrt(M_PI) ;

    return f ;
}

static complex_double cerf_continued_fraction(complex_double z)
{
    if (z.real() > 0)
        return 1.0 - cerfc_continued_fraction(z) ;
    else
        return -1.0 + cerfc_continued_fraction(-z) ;
}

/*
 * Abramowitz and Stegun, Eqn. 7.1.5 gives a series for erf(z)
 * good for all z, but converges faster for smallish abs(z), say abs(z)<2.
 */

static complex_double cerf_series(complex_double z)
{
    double tiny = 1e-20 ;          /* a small number compared with 1.*/
    /* warning("cerf_series:");*/
    complex_double sum = 0.0 ;
    complex_double term = z ;
    complex_double z2 = z * z ;
    int n;
    for (n = 0; n < 3 || abs(term) > abs(sum)*tiny; n++)
    {
        sum = sum + term / (2.0 * n + 1.0) ;
        term = -term * z2 / (n + 1.0) ;
    }

    return sum * 2.0 / sqrt(M_PI) ;
}

/*
 * Numerical Recipes quotes a formula due to Rybicki for evaluating
 * Dawson's Integral:
 *
 *  $\exp(-x^2) \int_0^x \exp(t^2) dt = \frac{1}{\sqrt{\pi}} \lim_{n \rightarrow \infty} \sum_{h=0}^n \frac{\exp(-(z-n.h)^2)}{n}$ 

```

```

*           0 to x                               h->0 n odd
*
* This can be adapted to erf(z).
*/
static complex_double cerf_rybicki(complex_double z)
{
    /* warning("cerf_rybicki:"); */
    double h = 0.2 ;           /* numerical experiment suggests this is small enough
    complex_double I = complex_double(0.0, 1.0);
    /*
    * choose an even n0, and then shift z->z-n0.h and n->n-h.
    * n0 is chosen so that real((z-n0.h)^2) is as small as possible.
    */

    int n0 = 2 * (int)(floor(z.imag() / (2.0 * h) + 0.5)) ;

    complex_double z0 = I * (double)n0 * h;
    complex_double zp = z - z0 ;
    complex_double sum = 0.0;
    /*
    * limits of sum chosen so that the end sums of the sum are
    * fairly small. In this case exp(-(35.h)^2)=5e-22
    *
    */
    int np;
    for (np = -35; np <= 35; np += 2)
    {
        complex_double t = zp - np * h * I ;
        complex_double b = exp(t * t) / (double)(np + n0);
        sum += b ;
    }

    sum = sum * 2.0 * exp(-z * z) / M_PI ;

    return (I * sum);
}

static complex_double cerf(complex_double z)
{
    /* Use the method appropriate to size of z -
    * there probably ought to be an extra option for NaN z, or infinite z

```

```
    *
    */
    if (abs(z) < 2.0)
        return cerf_series(z) ;
    else if (fabs(z.real()) < 0.5)
        return cerf_rybicki(z) ;
    else
        return cerf_continued_fraction(z) ;
}

complex_double normal_cerf(complex_double z)
{
    return (0.5 * (cerf(z / M_SQRT2) + 1.0));
}
```