Source | Model | Option
| Model_Option | Help on mc methods | Archived Tests

mc_standard2d

Input parameters:

- StepNumber N
- Generator_Type
- Confidence Value

Output parameters:

- Price P
- Error Price σ_P
- Deltas δ_1, δ_2
- Errors delta $\sigma_{\delta_1}, \sigma_{\delta_2}$
- Price Confidence Interval: IC_P =[Inf Price, Sup Price]
- Delta Confidence Intervals: IC_{δ_j} =[Inf Delta, Sup Delta]

Description:

Computation for a Call on Maximum - Put on Minimum - Exchange or Bestof European Option of its Price and its Delta with the Standard Monte Carlo or Quasi-Monte Carlo simulation. In the case of Monte Carlo simulation, this method also provides an estimation for the integration error and a confidence interval.

- The underlying asset prices evolve according to the two-dimensional Black and Scholes model, that is:

$$\begin{cases} dS_u^1 = S_u^1((r-d_1)du + \sigma_1 dB_u^1), \;\; S_{T-t}^1 = s^1 \\ dS_u^2 = S_u^2((r-d_2)du + \sigma_2 dB_u^2), \;\; S_{T-t}^2 = s^2 \end{cases}$$

where S_T^j denotes the spot at maturity T, s^j is the initial spot and $(B_u^1, u \geq 0)$ and $(B_u^2, u \geq 0)$ denote two real-valued Brownian motions with instantaneous correlation ρ. A description for correlated brownian motions and their simulation is given in the part about random variable simulation.
Then we have:

$$\begin{cases} S_T^1 = s^1 \exp((r - d_1 - \frac{\sigma_1^2}{2})t) \exp(\sigma_{11} B_t^1) \\ S_T^2 = s^2 \exp((r - d_2 - \frac{\sigma_2^2}{2})t) \exp(\sigma_{21} B_t^1 + \sigma_{22} B_t^2) \end{cases}$$

where the parameters $\sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22}$ are given in the following matrix A:

$$\begin{vmatrix} \sigma_{1,1} & \sigma_{1,2} \\ \sigma_{2,1} & \sigma_{2,2} \end{vmatrix} = \begin{vmatrix} \sigma_1 & 0 \\ \rho\sigma_2 & \sqrt{1-\rho}\sigma_2 \end{vmatrix}$$

such that $AA^t = \Gamma$ where Γ is the covariance matrix expressed by:

$$\begin{vmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{vmatrix}$$

- The price of an option is

$$P = E\left[\exp(-rt) f(K, S_T^1, S_T^2)\right]$$

where f denotes the payoff of the option, K the strike and t time to maturity.
The Deltas are given by:

$$\begin{aligned} \delta_1 &= \tfrac{\partial}{\partial s^1} E[\exp(-rt) f(K, S_T^1, S_T^2)] \\ \delta_2 &= \tfrac{\partial}{\partial s^2} E[\exp(-rt) f(K, S_T^1, S_T^2)] \end{aligned}$$

- Estimators are expressed as:

$$\widetilde{P} = \frac{1}{N} \exp(-rt) \sum_{i=1}^{N} P(i)$$

$$\widetilde{\delta}_j = \frac{1}{N} \exp(-rt) \sum_{i=1}^{N} \frac{\partial}{\partial s^j} P(i) = \frac{1}{N} \exp(-rt) \sum_{i=1}^{N} \delta_j(i)$$

The values for $P(i)$ and $\delta_j(i)$ are detailed for each option.

- **Put on the Minimum**: The payoff is $(K - \min(S_1, S_2))^+$.

$$P(i) = \left[K - \min(S_T^1(i), S_T^2(i))\right]^+$$

If $P(i) > 0$ then:

$$\delta_1(i)$$

$$\delta_2(i) = \begin{cases} -\frac{\partial S_T^2(i)}{\partial s^2} = -\frac{S_T^2(i)}{s^2} & \text{if} \quad S_T^2(i) \leq S_T^1(i) \\ 0 & \text{otherwise} \end{cases}$$

- **Call on the Maximum**: The payoff is $(\max(S_1, S_2) - K)^+$.

$$P(i) = \Big[\max(S_T^1(i), S_T^2(i)) - K\Big]^+$$

If $P(i) > 0$ then:

$$\delta_1(i) = \begin{cases} \frac{\partial S_T^1(i)}{\partial s^1} = \frac{S_T^1(i)}{s^1} & \text{if} \quad S_T^1(i) \geq S_T^2(i) \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_2(i) = \begin{cases} \frac{\partial S_T^2(i)}{\partial s^2} = \frac{S_T^2(i)}{s^2} & \text{if} \quad S_T^1(i) \geq S_T^2(i) \\ 0 & \text{otherwise} \end{cases}$$

- **Exchange Option**: The payoff is $(S_1 - ratio \times S_2)^+$.

$$P(i) = \Big(S_T^1(i) - ratio \times S_T^2(i)\Big)^+$$

$$\delta_1(i) = \begin{cases} \frac{S_T^1(i)}{s^1} & \text{if} \quad P(i) > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_2(i) = \begin{cases} -ratio \times \frac{S_T^2(i)}{s^2} & \text{if} \quad P(i) > 0 \\ 0 & \text{otherwise} \end{cases}$$

- **BestOf Option**: The payoff is $[\max(S_1 - K_1, S_2 - K_2)]^+$.

$$P(i) = \Big[\max(S_T^1(i) - K_1, S_T^2(i) - K_2)\Big]^+$$

If $P(i) > 0$ then:

$$\delta_1(i) = \begin{cases} \frac{S_T^1(i)}{s^1} & \text{if} \quad S_T^1(i) - K_1 \geq S_T^2(i) - K_2 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_2(i) = \begin{cases} \frac{S_T^2(i)}{s^2} & \text{if} \quad S_T^1(i) - K_1 \geq S_T^2(i) - K_2 \\ 0 & \text{otherwise} \end{cases}$$

Algorithm:

/* Value to construct the confidence interval */
For example if the confidence value is equal to 95% then the value z_α used to construct the confidence interval is 1.96. This parameter is taken into account only for MC simulation and not for QMC simulation.
/*Initialization*/
/* Covariance Matrix */
/* Coefficients of the matrix A such that $AA^t = \Gamma$ */ This covariance matrix allows to generate the correlated two-dimensional brownian motions.
/*Median forward stock and delta values*/
Computation of intermediate values we use several times in the program.

- /*MC sampling*/

Initialization of the simulation: generator type, dimension, size N of the sample
/* Test after initialization for the generator */
Test if the dimension of the simulation is compatible with the selected generator. (See remarks on QMC simulation, especially on dimension of low-discrepancy sequences). For standard Monte Carlo in the two-dimensional Black and Scholes model, we never have any problem with the dimension, fixed to 2 at the beginning of the programm.
Definition of a parameter which exprimes if we realize a MC or QMC simulation. Some differences then appear in the algorithm for simulation of a gaussian variable and in results in the simulation.

/* Begin N iterations */

- /*Gaussian Random Variables*/

Generation of 2 gaussian variables g_1 and g_2 used for the Brownian motions as $\sqrt{t}g_j$.
Simulation of independent gaussian variables according to the generator type, that is Monte Carlo or Quasi Monte Carlo.
Call to the appropriate function to generate a standard gaussian variable. See the part about simulation of random variables for explanations on this point. We just recall that for a MC simulation, we use the Gauss-Abramovitz algorithm, and for a QMC simulation we use an inverse method and a two-dimensional low-discrepancy sequence.

- /*Price*/

At the iteration i, we obtain

$$P(i) = \text{payoff}(K, S_T^1(i), S_T^2(i))$$

- /*Delta*/

Calculation of Delta $\delta_1(i)$ and $\delta_2(i)$ for the different cases with formula given previously.
/*Call on the Maximum*/
/*Put on the Minimum*/
/*Best of*/
/*Exchange*/
Formula were previously described.

/*Sum*/
Computation of the sums $\sum P(i)$ and $\sum \delta_j(i)$ for the mean price and the means delta.

/*Sum of squares*/
Computation of the sums $\sum P(i)^2$ and $\sum(\delta_j(i))^2$ necessary for the variance price and the variances delta estimations. (finally only used for MC estimation)

/* End N iterations */

- /*Price*/

The price estimator is:

$$P = \frac{1}{N} \exp(-rt) \sum_{i=1}^{N} P(i)$$

The error estimator is σ_P with :

$$\sigma_P^2 = \frac{1}{N-1} \left(\frac{1}{N} \exp(-2rt) \sum_{i=1}^{N} P(i)^2 - P^2 \right)$$

The confidence interval is

$$IC_P = [P - z_\alpha \sigma_P; P + z_\alpha \sigma_P]$$

with z_α computed from the confidence value.

- /*Delta*/

- /* Delta1 estimator */
The delta estimator is:

$$\delta_1 = \frac{1}{N} \exp(-rt) \sum_{i=1}^{N} \delta_1(i)$$

The error estimator is σ_{δ_1} with:

$$\sigma_{\delta_1}^2 = \frac{1}{N-1} \left(\frac{1}{N} \exp(-2rt) \sum_{i=1}^{N} \delta_1^2(i) - \delta_1^2 \right)$$

The confidence interval is given as:

$$IC_{\delta_1} = [\delta_1 - z_\alpha \sigma_{\delta_1}; \delta_1 + z_\alpha \sigma_{\delta_1}]$$

with z_α computed from the confidence value.

- /* Delta2 estimator */
The delta estimator is:

$$\delta_2 = \frac{1}{N} \exp(-rt) \sum_{i=1}^{N} \delta_2(i)$$

The error estimator is σ_{δ_2} with:

$$\sigma_{\delta_2}^2 = \frac{1}{N-1} \left(\frac{1}{N} \exp(-2rt) \sum_{i=1}^{N} \delta_2^2(i) - \delta_2^2 \right)$$

The confidence interval is given as:

$$IC_{\delta_2} = [\delta_2 - z_\alpha \sigma_{\delta_2}; \delta_2 + z_\alpha \sigma_{\delta_2}]$$

with z_α computed from the confidence value.

References