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ap_out_laplace

Output parameters:

- Price
- Delta

Fixed Double Limit options are priced with Laplace Transform method of [1]

/*Computation of Laplace transform*/

$$\mathcal{L}(f(x)) = F(\lambda) = \int_0^{\infty} \exp(-\lambda x) f(x) dx$$

$\mu = \sqrt{2\lambda + \nu^2}$, $\nu = \frac{2y}{\sigma^2} - 1$, $U = \text{Upper_limit}$, $L = \text{Lower_limit}$,
 $K = \text{Strike}$, $x = S(0)$, $x^+ = S(0) * (1 + INC)$

$$h = \frac{K}{x}, \quad INC = 10^{-8}$$

/*Call Case*/

We have (from [1])

$$\begin{aligned}
 F(\lambda) = \int_0^{\infty} \exp(-\lambda x) f(x) dx = & \frac{\left(1 - \left(\frac{x}{U}\right)^{2\mu}\right)}{\left(1 - \left(\frac{L}{U}\right)^{2\mu}\right)} * \left\{ \left(\frac{L^2}{xK}\right)^{\mu} * \left(\frac{K}{x}\right)^{\nu+1} * \frac{1}{\mu(\mu-\nu)(\mu-\nu-1)} \right\} \\
 & + \frac{\left(1 - \left(\frac{L}{x}\right)^{2\mu}\right)}{\left(1 - \left(\frac{L}{U}\right)^{2\mu}\right)} * \left\{ 2 \left(\frac{x}{U}\right)^{\mu-\nu-1} \left[\frac{1}{\mu^2 - (\nu+1)^2} - \frac{\frac{K}{U}}{\mu^2 - \nu^2} \right] + \left(\frac{xK}{U^2}\right)^{\mu} * \frac{\left(\frac{K}{x}\right)^{\nu+1}}{\mu(\mu+\nu)(\mu+\nu+1)} \right\}
 \end{aligned}$$

/*Inversion parameters*/

According to the algorithm due to [2]

$$A = 19.1, \quad N = 15, \quad M = 11,$$

/* INVERSION */

We should remind that the inversion is made throw h .

We compute

$$sum = \frac{h}{e^{\frac{A}{2}}} * x = \frac{F_x(\frac{A}{2h})}{2} \quad \text{and} \quad sum1 = \frac{h}{e^{\frac{A}{2}}} * x^+ = \frac{F_{x^+}(\frac{A}{2h})}{2}$$

/* Computation of $S[1] = s(N)$ and $Q[1] = s_{INC}(N)$ which is the first approximation of $f(t)$ */

$$S[1] = \frac{h}{e^{\frac{A}{2}}} * s(N) = \frac{F_x(\frac{A}{2h})}{2} + \sum_{k=1}^{k=N} (-1)^k Re \left(F_x \left(\frac{A+2ik\pi}{2h} \right) \right)$$

$$Q[1] = \frac{h}{e^{\frac{A}{2}}} * s_{INC}(N) = \frac{F_{x^+}(\frac{A}{2h})}{2} + \sum_{k=1}^{k=N} (-1)^k Re \left(F_{x^+} \left(\frac{A+2ik\pi}{2h} \right) \right)$$

/* Computation of $s(N+j)$, $s_{INC}(N+j)$ $j \leq M+1$ for Euler approximations */

$$S[j] = S[j-1] + (-1)^{N+j} * Re \left(F_x \left(\frac{A+2(N+j)k\pi}{2h} \right) \right);$$

$$Q[j] = Q[j-1] + (-1)^{N+j} * Re \left(F_{x^+} \left(\frac{A+2(N+j)k\pi}{2h} \right) \right);$$

/* Computation of Euler approximations */

$$Avg = Avg + Cnp(M, i) * s(N+i);$$

$$Avg2 = Avg2 + Cnp(M, i) * s_{INC}(N+i);$$

/* f(h) value */

$$Fun = \frac{e^{\frac{A}{2}}}{h} * 2^{-M} * Avg;$$

$$Fun2 = \frac{e^{\frac{A}{2}}}{h} * 2^{-M} * Avg2;$$

/*Black-Sholes price for call option*/

From this inversion, we can compute the Double-Limit call option, with the

help of the Black-Scholes price of a European call option

```
dummy = Call_BlackScholes_73(1., h, t, r, divid, sigma, &price, &delta);
```

```
dummy = Call_BlackScholes_73(1., h2, t, r, divid, sigma, &price2, &delta2);
```

```
/* Call Price */
```

```
CTtK = x * price - x * exp(-r * t) * Fun;
```

where the variable price is from

```
Call_BlackScholes_73(1., h, t, r, divid, sigma, &price, &delta)
```

```
/*Delta for call option*/
```

```
 $\Delta_C = (CTtK - (price2 - price)/(h2 - h) * K) / (x - exp(-r * t) * (Fun2 - Fun)) / INC;$ 
```

```
/*Price*/
```

```
/*Delta */
```

References

- [1] H.GEMAN M.YOR. Pricing and hedging double barrier options: a probabilistic approach. *Mathematical finance*, 6:365–378, 1996. [1](#)
- [2] J.ABATE W.WHITT. Numerical inversion of laplace transform of probability distribution. *ORSA Journal of Computing*, 7(1), Winter 1995. [2](#)