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## tr\_rogersstapleton\_downout

Output parameters:

- Price
- Delta

**Description:** Rogers and Stapelton [1] propose to approximate the logarithm of the stock price ( $X_t = X_0 + \sigma W_t + (r - \delta - \sigma^2/2)t$ ) by the random walk  $(\xi_n)_n$  where for some fixed  $\Delta x > 0$ ,

$$\begin{cases} \xi_n = X_{\tau_n} \\ \tau_0 = 0, \tau_{n+1} = \inf\{t > \tau_n, |X_t - X_{\tau_n}| > \Delta x\} \end{cases}$$

The price of the European option with payoff function  $\varphi$ , maturity  $T$ , up and out barrier  $b^*$  (resp. down and out barrier  $b_*$ ) and rebate  $R$  is approximated by

$$\sum_{n \geq 0} \mathbb{P}(\nu = n) \mathbb{E} \left( e^{-r\eta T/n} R 1_{\{\eta < n\}} + e^{-rT} \varphi(\xi_n) 1_{\{\eta \geq n\}} \middle| \nu = n \right) \quad (1)$$

where  $\nu = \sup\{n : \tau_n \leq T\}$  and  $\eta = \inf\{n : \xi_n \geq \log(b^*)\}$  (resp  $\inf\{n : \xi_n \leq \log(b_*)\}$ ).

/\*Up and Down probabilities\*/

Computes the probabilities of an up step  $\mathbb{P}(\xi_n = x + \Delta x | \xi_n = x)$  and a down step  $\mathbb{P}(\xi_n = x - \Delta x | \xi_n = x)$ . For  $x = x^*$  the grid point immediately below the up-barrier  $b^*$  or  $x = x_*$  the grid point immediately above the down-barrier  $b_*$ , modified probabilities are calculated.

/\*moments of tau1\*/

Computes the three first moments of the random variable  $\tau_1$ .

/\*Initialization\*/

Initializes the variables before the

/\*recursion on the number of time-steps\*/

The price is computed according to (1) thanks to a recursion on the number  $n$  of time-steps.

/\*computation of the probability of  $\nu=n$ \*/

Computation of the probability  $\mathbb{P}(\nu = n)$  thanks to a refinement of the Central Limit Theorem. When this probability which appears as a multiplicative coefficient in (1) is smaller than 0.000005, the conditional expectation  $\mathbb{E} \left( e^{-r\eta T/n} R 1_{\{\eta < n\}} + e^{-rT} \varphi(\xi_n) 1_{\{\eta \geq n\}} \middle| \nu = n \right)$  is not calculated. Otherwise,

/\*contribution for a fixed number of time-steps\*/

it is computed thanks to a backward resolution on a tree with  $n$  time steps. Two cases are distinguished to construct the tree. If  $\log(b^*) \leq X_0 + n\Delta x$  (resp.  $\log(b_*) \geq X_0 - n\Delta x$ ) the barrier can be hit (/\*Barrier hit\*/). Otherwise it cannot and the construction is simpler. (/\*Barrier not hit\*/).

/\*end of the recursion\*/

The recursion on the number  $n$  on time-steps ends when the cumulative probability  $Q = \sum_{m=0}^n \mathbb{P}(\nu = m)$  becomes greater than 0.99999.

## References

- [1] L.C.G.ROGERS E.J.STAPLETON. Fast accurate binomial pricing. *preprint*, 1997. 1