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mc_merton

Input parameters:

- Number of iterations N
- Generator_Type
- Confidence Value

Output parameters:

- Price P
- Error Price σ_P
- Delta δ
- Error delta σ_δ
- Price Confidence Interval: $IC_P = [\text{Inf Price}, \text{Sup Price}]$
- Delta Confidence Interval: $IC_\delta = [\text{Inf Delta}, \text{Sup Delta}]$

Description:

Computation for a Call - Put - CallSpread or Digit European Option of its Price and its Delta via the Malliavin method with the [Standard Monte Carlo](#) or [Quasi-Monte Carlo simulation](#). In the case of Monte Carlo simulation, the method also provides an estimation for the integration error and a confidence interval.

The underlying asset price evolves according to the Merton model, that is:

$$\begin{cases} S_{T-t} = s \\ \frac{dS_u}{S_u} = (r - \lambda \mathbb{E}U_1 - d)du + \sigma dB_u + d(\sum_{j=1}^{N_u} U_j), \end{cases} \quad (1)$$

where $(B_u)_{t \geq 0}$ is a Brownian motion, $(N_u)_{u \geq 0}$ is a Poisson process with deterministic jump intensity λ , $(U_u)_{j \geq 1}$ is a sequence of positive, independent stochastic variables and σ is a constant, such that $\sigma > 0$. We suppose that r is a deterministic risk-free interest rate. Then, we have

$$S_T = s \left(\prod_{j=1}^{N_T} (U_j + 1) \right) e^{(r - \lambda \mathbb{E} U_1 - d - \frac{\sigma^2}{2})T + \sigma B_T}. \quad (2)$$

Where S_T denotes the spot at maturity T , s is the initial spot, t is the time to maturity.

In this context we suppose that the jump variables U are log-normal distributed with constant mean μ and variance γ .

The Price of an option at $T - t$ is:

$$P = E [\exp(-rt) f(K, S_T, R)]$$

where f denotes the payoff of the option, K the strike and R the rebate (for Digit option only).

Using the Malliavin integration by parts formula the Delta is given by:

$$\delta = E \left[\frac{P B_T}{s \sigma T} \right]$$

Estimators are expressed as:

$$\tilde{P} = \frac{1}{N} \exp(-rt) \sum_{i=1}^N P(i)$$

where $P(i) = f(S_T(i), K)$

$$\tilde{\delta} = \frac{1}{N} \exp(-rt) \sum_{i=1}^N \delta(i)$$

where

$$\delta(i) = \frac{P(i) B_T(i)}{s \sigma T}.$$

The values for $P(i)$ are detailed for each option.

- **Put:** The payoff is $(K - S_T)^+$. We have:

$$P(i) = (K - S_T(i))^+$$

- **Call:** The payoff is $(S_T - K)^+$.

The *Call-Put Parity* relations for price and delta are expressed by:

$$C = P + s \exp(-dt) - K \exp(-rt)$$

$$\delta_C = \delta_P + \exp(-dt)$$

where C and P respectively denotes the Call and the Put prices.

They will be used for a Call simulation (which corresponds to a method of control variate and leads generally to a reduced variance for the estimator).

- **CallSpread:** The payoff is $(S_T - K_1)^+ - (S_T - K_2)^+$.

We have:

$$P(i) = [(S_T(i) - K_1)^+ - (S_T(i) - K_2)^+]$$

- **Digit:** The payoff is $R1_{\{S_T - K \geq 0\}}$.

We have:

$$P(i) = R1_{\{S_T(i) - K \geq 0\}}$$

For digital options we use the Malliavin techniques.

Algorithm:

```
/* Value to construct the confidence interval */
For example if the confidence value is equal to 95% then the value  $z_\alpha$  used
to construct the confidence interval is 1.96. This parameter is taken into
account only for MC simulation and not for QMC simulation.
/*Initialization*/
/*Call-Spread*/
Strike  $K_1$  and  $K_2$  used for a Call-Spread option.
/*Median forward stock and delta values*/
Computation of intermediate values we use several times in the program.
/* Change a Call into a Put to apply the Call-Put parity */
In case of Call, we modify parameters of the option; they will be reinitialized
at the end of the simulation program. Simulation will be done as for a put.
```

- /*MC sampling*/

Initialization of the simulation: generator type, dimension, size N of the sample.

```
/* Test after initialization for the generator */
```

Test if the dimension of the simulation is compatible with the selected generator. (See remarks on QMC simulation, especially on dimension of low-discrepancy sequences).

Definition of a parameter which exprimes if we realize a MC or QMC simulation. Some differences then appear in the algorithm for simulation of a gaussian variable and in results in the simulation.

/* Begin N iterations */

- /* Simulation of a gaussian variable according to the generator type, that is Monte Carlo or Quasi Monte Carlo. */

Call to the appropriate function to generate a standard gaussian variable. See the part about simulation of random variables for explanations on this point. We just recall that for a MC simulation, we use the Gauss-Abramovitz algorithm, and for a QMC simulation we use an inverse method.

- /* Simulation of a poisson variable N_T with parameter λT */

- /*Price*/

At the iteration i , we obtain

$$S_T(i) = s \exp \left[\left(r - \lambda \mathbb{E}U_1 - d - \frac{\sigma^2}{2} \right) T \right] \exp(\sigma B_T(i)) \left(\prod_{j=1}^{N_T} \exp(\mu + \gamma g_j) \right)$$

$$P(i) = \text{Payoff}(S_T(i), K)$$

from a simulation of $B_t(i)$ with the selected generator as $\sqrt{t}g_i$ where g_i is a standard Gaussian variable.

Payoff functions are given for each option in the previous section.

- /*Delta*/

Calculation of Delta δ_i with formula previously detailed for each option.

/*Digit*/

/*CallSpread*/

/*Call-Put*/

/*Sum*/

Computation of the sums $\sum P_i$ and $\sum \delta_i$ for the mean price and the mean delta.

/*Sum of squares*/

Computation of the sums $\sum P_i^2$ and $\sum \delta_i^2$ necessary for the variance price and the variance delta estimations. (finally only used for MC estimation)

/* End N iterations */

• /*Price*/

The price estimator is:

$$P = \frac{1}{N} \exp(-rt) \sum_{i=1}^N P(i)$$

The error estimator is σ_P with :

$$\sigma_P^2 = \frac{1}{N-1} \left(\frac{1}{N} \exp(-2rt) \sum_{i=1}^N P(i)^2 - P^2 \right)$$

• /*Delta*/

$$\delta = \frac{1}{N} \exp(-rt) \sum_{i=1}^N \delta(i)$$

The error estimator is σ_δ with:

$$\sigma_\delta^2 = \frac{1}{N-1} \left(\frac{1}{N} \exp(-2rt) \sum_{i=1}^N \delta(i)^2 - \delta^2 \right)$$

• /* Call Price and Delta with the Call Put Parity */

We now compute the price and the delta in case of a call, because call was considered as a put until now.

Parameters of the option are reinitialized. This step is necessary: if you want to begin an other call simulation just after a first one with the standard method, the parameters must have been modified to specify that we really consider a Call and not a Put.

• /* Price Confidence Interval */

The confidence interval is given as:

$$IC_P = [P - z_\alpha \sigma_P; P + z_\alpha \sigma_P]$$

with z_α computed from the confidence value.

• /* Delta Confidence Interval */

The confidence interval is given as:

$$IC_\delta = [\delta - z_\alpha \sigma_\delta; \delta + z_\alpha \sigma_\delta]$$

with z_α computed from the confidence value.

Confidence intervals are always computed, but for a QMC simulation they don't work, thus they don't appear in the results.

References