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## mc\_antithetic

### Input parameters:

- Number of iterations  $N$
- Generator\_Type
- Increment  $inc$
- Confidence Value

### Output parameters:

- Price  $P$
- Error Price  $\sigma_P$
- Delta  $\delta$
- Error delta  $\sigma_\delta$
- Price Confidence Interval:  $IC_P = [\text{Inf Price}, \text{Sup Price}]$
- Delta Confidence Interval:  $IC_\delta = [\text{Inf Delta}, \text{Sup Delta}]$

### Description:

Computation for a Call - Put - CallSpread or Digit European Option of its Price and its Delta with the [Antithetic Variables](#) method with [Monte Carlo](#) or [Quasi-Monte Carlo](#) simulation. In the case of Monte Carlo simulation, the method also provides an estimation for the integration error and a confidence interval.

The underlying asset price evolves according to the Black and Scholes model, that is:

$$dS_u = S_u((r - d)du + \sigma dB_u), \quad S_{T-t} = s$$

then

$$S_T = s \exp \left( \left( r - d - \frac{\sigma^2}{2} \right) t \right) \exp(\sigma B_t)$$

where  $S_T$  denotes the spot at maturity  $T$ ,  $s$  is the initial spot,  $t$  is the time to maturity.

The Price of an option at  $T - t$  is:

$$P = E [\exp(-rt) f(K, S_T, R)]$$

where  $f$  denotes the payoff of the option,  $K$  the strike and  $R$  the rebate (for Digit option only).

The Delta is given by:

$$\delta = \frac{\partial}{\partial s} E [\exp(-rt) f(K, S_T, R)]$$

Estimators with antithetic variables are expressed as:

$$\tilde{P} = \frac{1}{2N} \exp(-rt) \sum_{i=1}^N (P(i) + P'(i))$$

$$\tilde{\delta} = \frac{1}{N} \exp(-rt) \sum_{i=1}^N \frac{\partial}{\partial s} P(i) = \frac{1}{2N} \exp(-rt) \sum_{i=1}^N (\delta(i) + \delta'(i))$$

$P(i)$  and  $P'(i)$  are antithetic payoffs obtained from  $S_T(i)$  and  $S'_T(i)$  given by:

$$\begin{cases} S_T(i) = s \exp \left[ \left( r - d - \frac{\sigma^2}{2} \right) t \right] \exp(\sigma B_t(i)) \\ S'_T(i) = s \exp \left[ \left( r - d - \frac{\sigma^2}{2} \right) t \right] \exp(\sigma B'_t(i)) \end{cases}$$

where  $B_t(i) = \sqrt{t}g_i$  and  $B'_t(i) = -\sqrt{t}g_i$  are antithetic brownian motions.  $g_i$  is a standard gaussian variable.

Values for  $P(i)$  and  $\delta(i)$  are detailed for each option in the following points.

Values for  $P'(i)$  and  $\delta'(i)$  are obtained in the same way.

- **Put:** The payoff is  $(K - S_T)^+$ . We have:

$$P(i) = (K - S_T(i))^+$$

$$\delta(i) = \begin{cases} -\frac{\partial S_T(i)}{\partial s} = -\frac{S_T(i)}{s} & \text{if } P(i) > 0 \\ 0 & \text{otherwise} \end{cases}$$

- **Call:** The payoff is  $(S_T - K)^+$ .

The *Call-Put Parity* relations for price and delta are expressed by:

$$C = P + s \exp(-dt) - K \exp(-rt)$$

$$\delta_C = \delta_P + \exp(-dt)$$

where  $C$  and  $P$  respectively denotes the Call and the Put prices.

They will be used for a Call simulation (which corresponds to a method of control variate and leads generally to a reduced variance for the estimator).

- **CallSpread:** The payoff is  $(S_T - K_1)^+ - (S_T - K_2)^+$ .

We have:

$$P(i) = [(S_T(i) - K_1)^+ - (S_T(i) - K_2)^+]$$

$$\delta(i) = \begin{cases} \frac{\partial S_T(i)}{\partial s} = \frac{S_T(i)}{s} & \text{if } S_T(i) > K_1 \text{ and } S_T(i) < K_2 \\ -\frac{\partial S_T(i)}{\partial s} = -\frac{S_T(i)}{s} & \text{if } S_T(i) > K_2 \text{ and } S_T(i) < K_1 \\ 0 & \text{otherwise} \end{cases}$$

- **Digit:** The payoff is  $R1_{\{S_T - K \geq 0\}}$ .

We have:

$$P(i) = R1_{\{S_T(i) - K \geq 0\}}$$

To have an estimation of the Delta in the case of a Digit option, we need to use the increment value *inc* at each iteration  $i$  as:

$$\delta_i = \begin{cases} \frac{R}{2s \cdot inc} & \text{if } S_T(i)(s(1 + inc)) > K \text{ and } S_T(i)(s(1 - inc)) < K \\ 0 & \text{otherwise} \end{cases}$$

$S_T(i)(s(1 + inc))$  is the spot value at  $T$  with initial value  $s(1 + inc)$ .

$S_T(i)(s(1 + inc))$  and  $S_T(i)(s(1 - inc))$  are computed with the same brownian motion for each iteration. Thus we always have  $S_T(i)(s(1 + inc)) > S_T(i)(s(1 - inc))$  and there is only one case for which  $\delta_i > 0$ .

## Algorithm:

/\* Value to construct the confidence interval \*/

For example if the confidence value is equal to 95% then the value  $z_\alpha$  used to construct the confidence interval is 1.96. This parameter is taken into account only for MC simulation and not for QMC simulation.

/\*Initialization\*/

/\*Call-Spread\*/

Strike  $K_1$  and  $K_2$  used for a Call-Spread option.

/\*Median forward stock and delta values\*/

Computation of intermediate values we use several times in the program.

/\* Change a Call into a Put to apply the Call-Put parity \*/

In case of Call, we modify parameters of the option; they will be reinitialized at the end of the simulation program. Simulation will be done as for a put.

• /\*MC sampling\*/

Initialization of the simulation: generator type, dimension, size  $N$  of the sample

/\* Test after initialization for the generator \*/

Test if the dimension of the simulation is compatible with the selected generator. (See remarks on QMC simulation, especially on dimension of low-discrepancy sequences). For standard Monte Carlo in the one-dimensional Black and Scholes model, we never have any problem with the dimension, fixed to 1 at the beginning of the program.

Definition of a parameter which exprimes if we realize a MC or QMC simulation. Some differences then appear in the algorithm: for simulation of a gaussian variable and in results in the simulation.

/\* Begin N iterations \*/

- /\* Simulation of a gaussian variable according to the generator type, that is Monte Carlo or Quasi Monte Carlo. \*/

Call to the appropriate function to generate a standard gaussian variable. See the part about simulation of random variables for explanations on this point. We just recall that for a MC simulation, we use the Gauss-Abramovitz algorithm, and for a QMC simulation we use an inverse method.

- /\*Price\*/

At the iteration  $i$ , we obtain  $S_T(i)$  and  $S'_T(i)$  from a simulation according to the selected generator of antithetics brownian motions  $B_t(i)$  as  $\sqrt{t}g_i$  and  $B'_t(i)$  as  $-\sqrt{t}g_i$  where  $g_i$  is a standard gaussian variable.

$$P(i) = \text{Payoff}(S_T(i), K)$$

$$P'(i) = \text{Payoff}(S'_T(i), K)$$

- /\*Delta\*/

Calculation of Delta  $\delta_i$  and  $\delta'_i$  with formula previously detailed for each option.

/\*Digit\*/

/\*CallSpread\*/

/\*Call-Put\*/

/\*Sum\*/

Computation of the sums  $\sum \frac{1}{2}(P(i) + P'(i))$  and  $\sum \frac{1}{2}(\delta(i) + \delta'(i))$  for the mean price and the mean delta.

/\*Sum of squares\*/

Computation of the sums  $\sum (P(i) + P'(i))^2$  and  $\sum (\delta(i) + \delta'(i))^2$  necessary for the variance price and the variance delta estimations. (finally only used for MC estimation)

/\* End N iterations \*/

• /\*Price\*/

The price estimator is:

$$P = \frac{1}{N} \exp(-rt) \sum_{i=1}^N \frac{1}{2} (P(i) + P'(i))$$

The error estimator is  $\sigma_P$  with :

$$\sigma_P^2 = \frac{1}{N-1} \left( \frac{1}{N} \exp(-2rt) \sum_{i=1}^N \frac{1}{4} (P(i) + P'(i))^2 - P^2 \right)$$

• /\*Delta\*/

$$\delta = \frac{1}{N} \exp(-rt) \sum_{i=1}^N \frac{1}{2} (\delta(i) + \delta'(i))$$

The error estimator is  $\sigma_\delta$  with:

$$\sigma_\delta^2 = \frac{1}{N-1} \left( \frac{1}{N} \exp(-2rt) \sum_{i=1}^N \frac{1}{4} (\delta(i) + \delta'(i))^2 - \delta^2 \right)$$

• /\* Call Price and Delta with the Call Put Parity \*/

We now compute the price and the delta in case of a call, because call was considered as a put until now.

Parameters of the option are reinitialized. This step is necessary: if you want to begin an other call simulation just after a first one with the antithetic

method, the parameters must have been modified to specify that we really consider a Call and not a Put.

- /\* Price Confidence Interval \*/

The confidence interval is given as:

$$IC_P = [P - z_\alpha \sigma_P; P + z_\alpha \sigma_P]$$

with  $z_\alpha$  computed from the confidence value.

- /\* Delta Confidence Interval \*/

The confidence interval is given as:

$$IC_\delta = [\delta - z_\alpha \sigma_\delta; \delta + z_\alpha \sigma_\delta]$$

with  $z_\alpha$  computed from the confidence value.

Confidence intervals are always computed, but for a QMC simulation they don't work, thus they don't appear in the results.

## References