McEliece Cryptosystem in real life: security and implementation

Bhaskar Biswas and Nicolas Sendrier

SECRET - INRIA Rocq.
**Some numbers!**

<table>
<thead>
<tr>
<th>scheme</th>
<th>key gen</th>
<th>enc cycle</th>
<th>dec cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ntru</td>
<td>203983313</td>
<td>894427</td>
<td>1617090</td>
</tr>
<tr>
<td>RSA1-1024</td>
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<td>225593</td>
<td>6240622</td>
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<tr>
<td>RSA1-2048</td>
<td>1147314285</td>
<td>431452</td>
<td>30819975</td>
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<tr>
<td>RSA2-1024</td>
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<td>195172</td>
<td>6155468</td>
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<tr>
<td>RSA2-2048</td>
<td>1179499485</td>
<td>403327</td>
<td>30703080</td>
</tr>
</tbody>
</table>

**Table:** Selected parameters at a glance
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<table>
<thead>
<tr>
<th>$m$</th>
<th>$t$</th>
<th>key size (Mbits)</th>
<th>block length</th>
<th>cycles per byte</th>
<th>security</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>encr.</td>
<td>decr.</td>
</tr>
<tr>
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<td>32</td>
<td>.647</td>
<td>2048</td>
<td>516</td>
<td>5548</td>
</tr>
<tr>
<td>11</td>
<td>44</td>
<td>.860</td>
<td>2048</td>
<td>623</td>
<td>7921</td>
</tr>
<tr>
<td>12</td>
<td>21</td>
<td>.967</td>
<td>4096</td>
<td>433</td>
<td>3885</td>
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<tr>
<td>12</td>
<td>26</td>
<td>1.19</td>
<td>4096</td>
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<td>4991</td>
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<tr>
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<td>1.79</td>
<td>4096</td>
<td>577</td>
<td>7040</td>
</tr>
<tr>
<td>13</td>
<td>17</td>
<td>1.71</td>
<td>8192</td>
<td>414</td>
<td>5522</td>
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<tr>
<td>13</td>
<td>20</td>
<td>2.01</td>
<td>8192</td>
<td>445</td>
<td>6560</td>
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<tr>
<td>13</td>
<td>29</td>
<td>2.90</td>
<td>8192</td>
<td>506</td>
<td>9649</td>
</tr>
</tbody>
</table>

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Motivation

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Consider the problem and provide a careful implementation together with cryptanalysis.

Provide a reference for measuring speed and scalability.

Compare with other, number-theory based, public key schemes.
CHRONOLOGY

- Niederreiter proposed a variance! Knapsack type. 1986.

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Decoding Attacks:

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N. Courtois and M. Finiasz and N. Sendrier - Signature Scheme. 2001.
Problem statement

- Implementing a (public key) cryptosystem is a tradeoff between security and efficiency.
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- We present a slightly modified version of the scheme (which we call hybrid).
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Large public key, but the McEliece cryptosystem has a good security reduction and low complexity algorithms for encryption and decryption.

We present a slightly modified version of the scheme (which we call *hybrid*).

Two modifications, increases the information rate by putting some data in the error pattern and reduces the public key size by making use of a generator matrix in row echelon form.
System description

The McEliece encryption scheme, as any public key encryption scheme, has to be described by a triple of procedures:

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- a decryption procedure.
**System description**

We define an injective mapping $\varphi : \{0, 1\}^\ell \to W_{n,t}$. Both $\varphi$ and $\varphi^{-1}$ should be easy to compute.

- System parameters: two integers $m$ and $t$. Let $n = 2^m$ and $k = n - tm$. 

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- **System parameters**: two integers $m$ and $t$. Let $n = 2^m$ and $k = n - tm$.
- **Key generation**:
  - generate a support $L = (\alpha_1, \ldots, \alpha_n)$ of $n$ distinct elements of $\mathbb{F}_{2^m}$,
  - generate a monic irreducible generator polynomial $g(z) \in \mathbb{F}_{2^m}[z]$ of degree $t$.
- The secret key is the pair $(L, g)$ (i.e. the Goppa code $\Gamma(L, g)$ and its decoder)
- The public key is a binary $k \times (n - k)$ matrix $R$ where $G = (Id | R)$ is a generator matrix of $\Gamma(L, g)$ in row echelon form.
Encryption: the plaintext is in $\{0, 1\}^k \times \{0, 1\}^\ell$ and the ciphertext in $\{0, 1\}^n$

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\begin{align*}
\{0, 1\}^k \times \{0, 1\}^\ell &\longrightarrow \{0, 1\}^n \\
(x, x') &\longmapsto E_G(x, \varphi(x'))
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- Decryption: the ciphertext has the form $y = xG + e$, with $e = \varphi(x')$ of Hamming weight $\leq t$. Applying the decoder of $\Gamma(L, g)$ on $y$ will provide $x$ and $x' = \varphi^{-1}(e)$. 
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- We use the error to encode information bits.
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- We use the error to encode information bits.
- We use a public key in row echelon form.

Those changes improve credentiality and as we will see, have no real impact on security.
**One way encryption scheme**

**Definition (OWE)**

A public key encryption scheme is a *One Way Encryption* scheme if the probability of success of any of its adversary running in polynomial time is negligible.
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In practice, one needs more than just an OWE scheme.

- McE, though it is OWE, is vulnerable to many attacks.
- Given perfect hash functions exists, there are generic conversions, which, starting from an OWE scheme, provide a scheme resistant against adaptative chosen ciphertext attack.
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- Given perfect hash functions exists, there are generic conversions, which, starting from an OWE scheme, provide a scheme resistant against adaptative chosen ciphertext attack.

We can prove, given the two following assumptions, the hybrid McEliece encryption scheme is OWE.
Security assumptions

- Hardness of decoding in the average case: The success probability
  \[
  \Pr_{\Omega}(A(xG + e, G) = (x, e)) | \Omega = \{0, 1\}^k \times W_{n,t} \times M_{k \times n}
  \]
  of any adversary $A$ running in polynomial time is negligible.

- Pseudo-randomness of binary Goppa Codes: there exists no efficient distinguisher for Goppa codes. In other words, the generator matrix of a Goppa code looks random.
The hybrid McEliece scheme is one-way.

Proof omitted.
Recall, we generate the support $L$ and generator polynomial $g$.

- The pair $\{L, g\}$ is the private key(s).
- The public keys is $R$, a binary $k \times (n - k)$ matrix, where $G = (Id \mid R)$ is a generator matrix of $\Gamma(L, g)$ in row echelon form.

Recall again, encryption is computed as,

$$(x, x') \mapsto E_G(x, \varphi(x'))$$
Prof. Nicolas Sendrier is going to deliver a talk on this part. Il vaut mieux prier Dieu que ses saints!!!
Decryption consist of 2 distinct stages.

- Decoding of Goppa code to generate the error positions. Next 2 slides!

- Retrieval of plain text.
  This is the inverse function applied to retrieve the message.
  Where, $x' = \varphi^{-1}(e)$. 
Goppa code decoding

The 3 stages involved in the decoding process, are,

- The syndrome computation.
- Solving the key equatoin.
- Computation of roots.

We precompute all the $f_j(z) = (z - \alpha_j)^{-1} \mod g(z)$. The syndrome of the word $a = (a_1, \ldots, a_n)$ is the sum

$$R_a(z) = \sum_{j=1}^{n} f_j(z) = \sum_{j=1}^{n} \frac{a_j}{z - \alpha_j} \mod g(z)$$
Patterson algorithm and Computation of roots

- The key equation is solved by Patterson algorithm. We chose to precompute the square roots modulo $g(z)$ (that is the $s_i(z)$ such that $s_i(z)^2 = z^i \mod g(z), 0 \leq i < t$).
- The roots of the locator polynomial are found by an exhaustive search on the field elements.
**Simulation results**

**Figure:** Encryption cost vs binary work factor for different extension degrees

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**Simulation results**

**Figure:** Decryption cost vs binary work factor for different extension degrees

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We presented here a new modified version of McEliece cryptosystem and its full implementation. We have shown that code-based public key encryption scheme compares favorably with optimized implementation of number theory based schemes. We plan to explore possibilities to improve the implementation, as well as widening the scope of parameters in our simulations.
Thank you! Questions?

Questions and remarks....shoot please!