Inverse analysis of fluid-structure systems. Application to the estimation of arterial stiffness

CRISTÓBAL BERTOGLIO

Seminaire des doctorants INRIA-Rocquencourt

Joint work

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- Dominique Chapelle, Philippe Moireau (MACS team)

euHeart consortium

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Outline

1. Why do doctors want to know the arterial stiffness?

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2. Modeling of fluid-structure interaction

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3. Numerical examples

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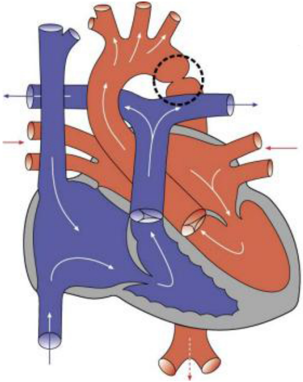
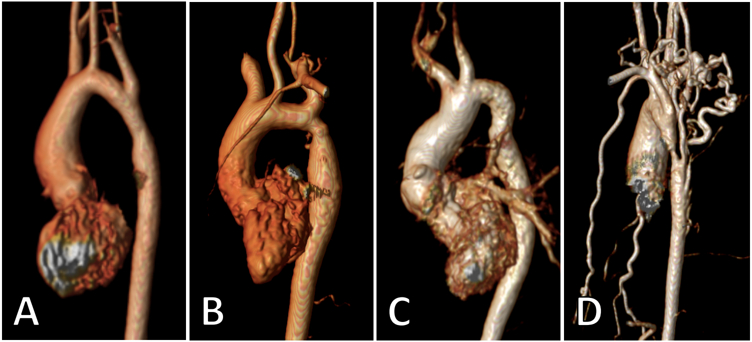
Aortic coarctation

- **Aortic coarctation**: narrowing of aorta after arch



- Pre-coarctation: higher pressure
  $\Rightarrow$ enlarged heart
- Post-coarctation: lower pressure
  $\Rightarrow$ bad perfusion to distal organs

- Types of coarctation: great variability

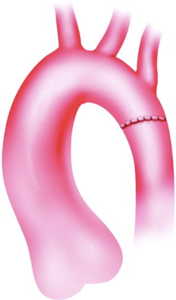


Mild *Moderate* *Tortuous* *Severe*

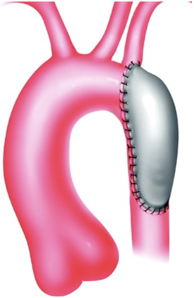
Blood pressure is the main indicator of severity

Aortic coarctation

- Main treatment: surgical repair



End-to-end anastomosis



Patch augmentation

Aortic coarctation

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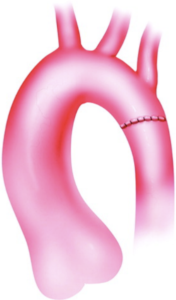

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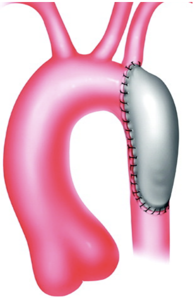

Patch augmentation

Hypertension persits after repair ... increase of arterial stiffness ??

Aortic coarctation

- Fluid-structure interaction phenomena

Medical image *Extracted surfaces*

Outline

Fluid-structure interaction

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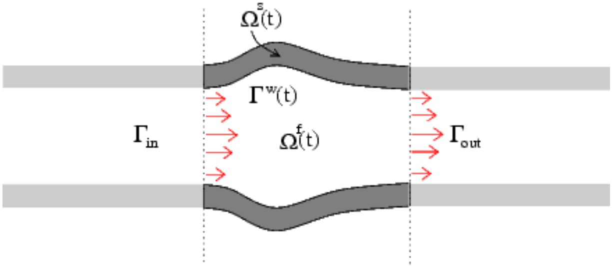
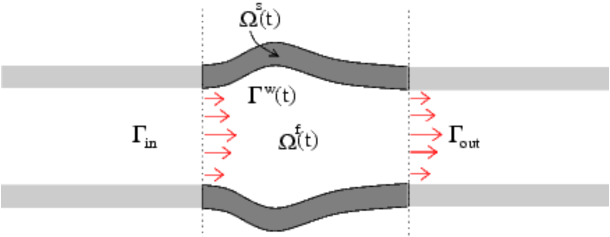
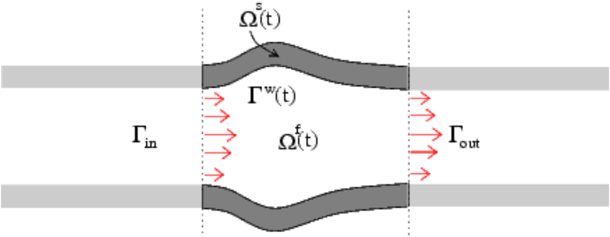


Fluid-structure interaction



- Solid: elastodynamics (d, v) in moving $\Omega_s(t)$

Fluid-structure interaction



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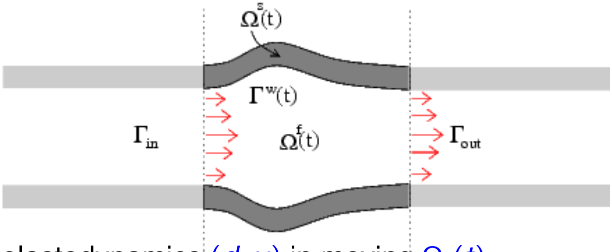
- Fluid: Navier-Stokes (u, p) in moving $\Omega_f(t)$

Fluid-structure interaction



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- Coupling: velocity $(u = v)$ and stresses at $\Omega_f(t) \cap \Omega_s(t)$

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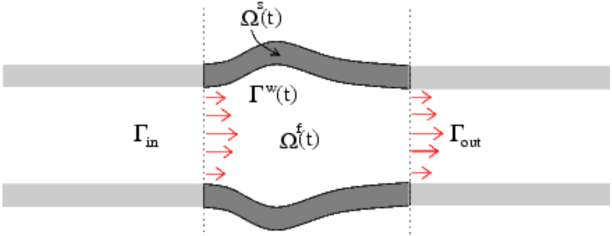

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- Discretization: finite elements (space) & finite differences (time)

Forward problem

- State: $X_n = [d_n \; v_n \; u_n \; p_n]$ at time t_n
- Given X_0 (initial condition) and θ (parameters) compute

$$X_{n+1} = A_{n+1}(X_n, \theta) \; , n \geq 0$$

Fluid-structure interaction

FSI simulation *Medical image*

The inverse problem

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- Observations $Z_n = H(X_n) + \chi_n$ with dim(Z_n)<<dim(X_n)

  $H(\cdot)$: observation operator, χ_n: measurement noise

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Least squares

Find $\hat{\theta}$ which minimizes

$$J(\hat{\theta}) = \sum_n \|Z_n - H(X_n)\|^2_{W_n^{-1}} + \|\hat{\theta} - \hat{\theta}_0\|^2_{P_0^{-1}}$$

s.t. $X_{n+1} = A_{n+1}(\hat{X}_n, \theta)$, $X_0 = \hat{X}_0$.

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Filtering

- Sequential fitting of Z_n & X_n

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- No tangent operator

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Filtering

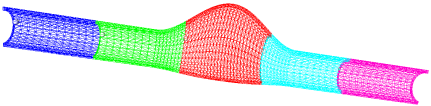
- Sequential fitting of Z_n & X_n
- No tangent operator
- Large full matrices

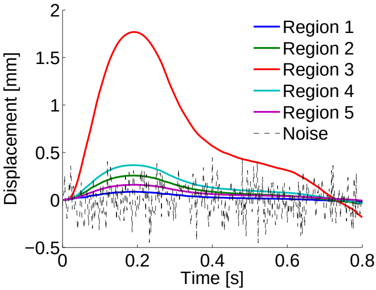
Outline

Academic example

- Example: estimation of stiffness distribution

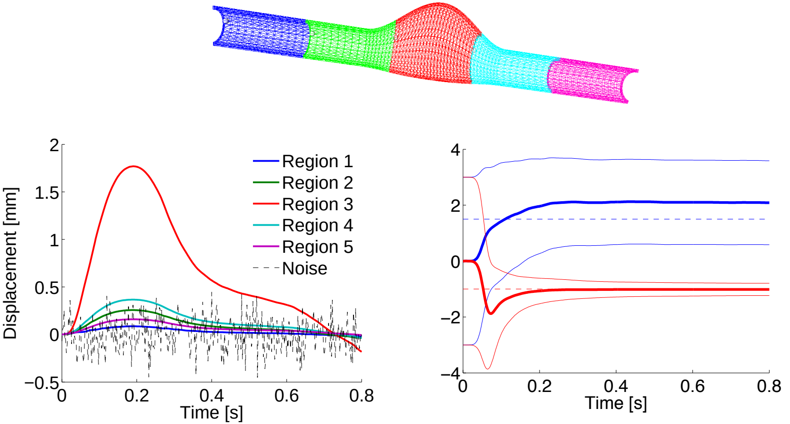
Academic example





Measurements

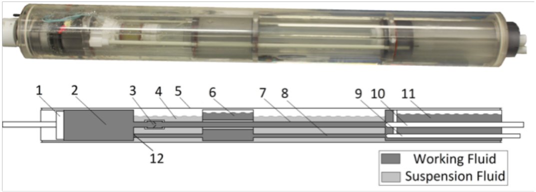
Academic example



Measurements *Estimation of* $\hat{\theta}_n$ & Std. dev.

Example with real data

- Silicon rubber aortic phantom (joint with TUe-KCL-U. Sheffield)



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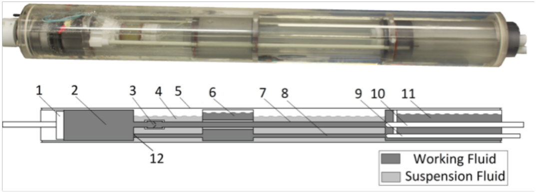
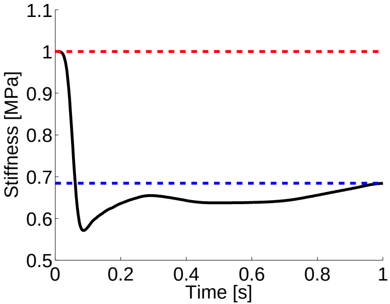


FSI simulation

Medical images

Example with real data

- Stiffness estimation using segmented surfaces



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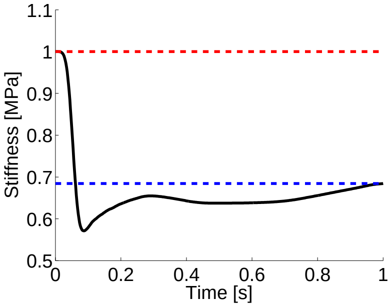

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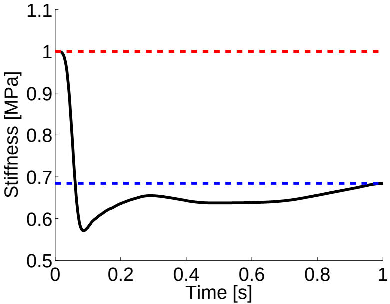
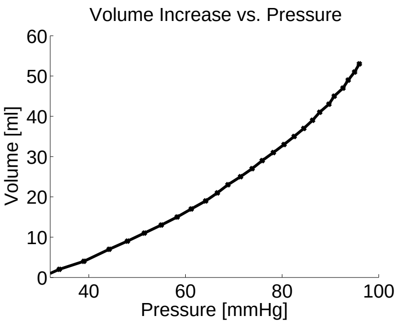


Example with real data

- Comparison with mechanical tests



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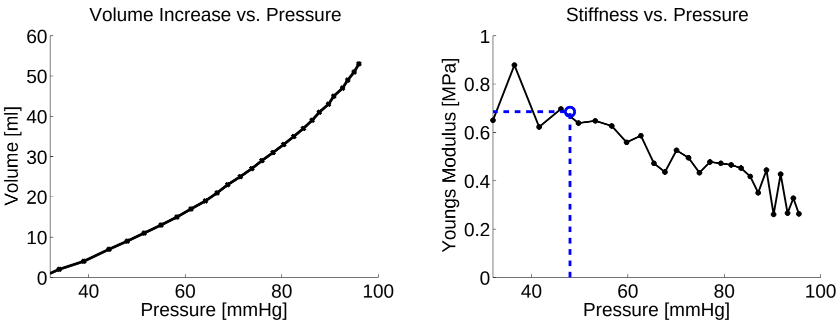

Merci!