Some inverse scattering problems on star-shaped graphs: application to fault detection on electrical transmission line networks

Filippo Visco Comandini

Projet SISYPHE- INRIA Rocquencourt

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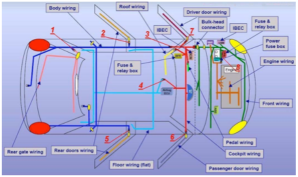
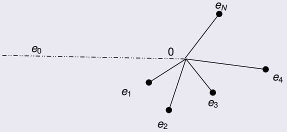
Outline

Outline

Motivation (1 min)



Source: Images CEA



Mathematics (29 min)

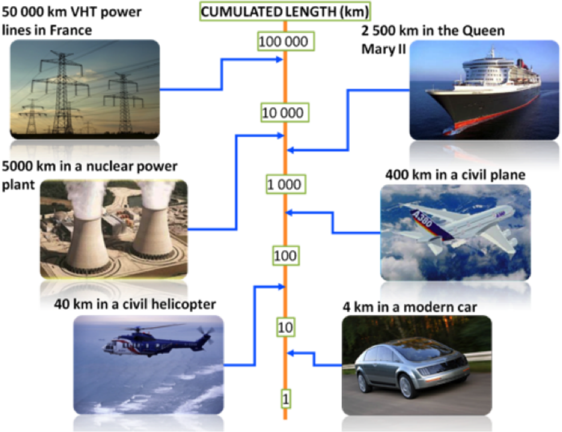
$$\begin{cases} \partial_x \nu_{1j}(k,x) = +(q_{j,d}(x) - ik)\nu_{1j}(k,x) - q_{j,+}(x)\nu_{2j}(k,x), \\ \partial_x \nu_{2j}(k,x) = -q_{j,-}(x)\nu_{1j}(k,x) - (q_{j,d}(x) - ik)\nu_{2j}(k,x), \\ \nu_{1j}(k,l_j) - \rho_j(k)\nu_{2j}(k,l_j) = 0, \quad x \in [0,l_j]. \\ \begin{pmatrix} \nu_1(x,k) \\ \nu_2(x,k) \end{pmatrix} \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ikx} + r(k) \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ikx}, \qquad x \to -\infty. \end{cases}$$

$$\nu_{10}(k,0) + \nu_{20}(k,0) = \nu_{1j}(k,0) + \nu_{2j}(k,0) \quad \forall j \in \{1,\dots,N\},$$

$$\sum_{j=1}^{N} \nu_{1j}(k,0) - \nu_{2j}(k,0) = \nu_{10}(k,0) - \nu_{20}(k,0). \qquad .$$

Outline

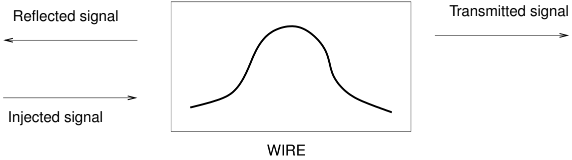
Diagnostic and fault-detection of critical networks



Frequency Domain Reflectometry

How can we find faults in a wire ?

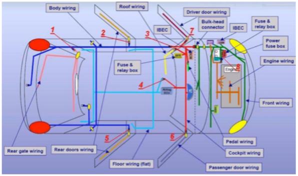
FDR is the most commonly used method : a signal is sent down a wire at some point and the signal reflected by the network is measured at the same point and analyzed in frequency for fault detection and location.



Constraint

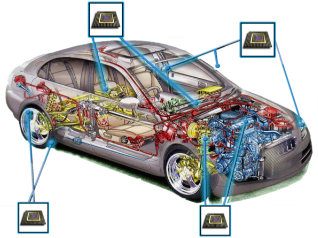
Limited number of available diagnostic port plug.

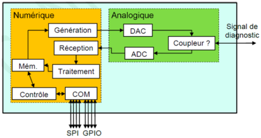
Choice of subnetwork to monitor



Source: Images CEA

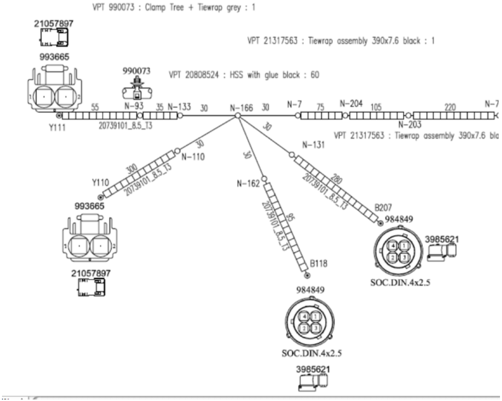
Global architecture



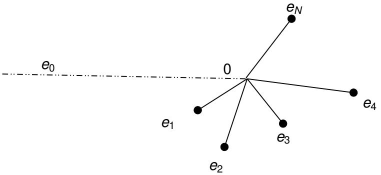


Functions of fault detection modules

Choice of subnetwork to monitor



Choice of critical sub-networks



Algorithm's design

Outline

Impedance Matrix

- V_1 and V_2 are the *Voltages*
- I_1 and I_2 are the *Intensities of the current*

Impedance Matrix **Z**

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

In matrix form :

$$\mathbf{V} = \mathbf{Z}\,\mathbf{I}$$

Matrix representation

- Impedance Matrix **Z** : perfect for locating electrical faults.
- Scattering Matrix **S** : perfect for measures of diagnostic devices.

Characteristic impedance

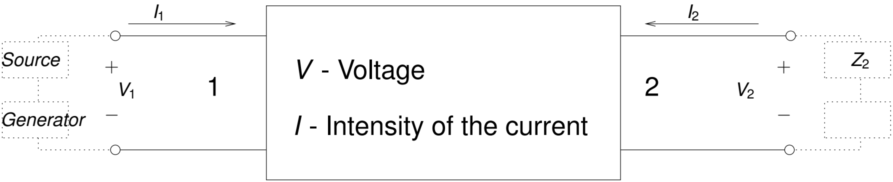


Image impedance

- Impose I_1,
- Measure $V_1(Z_2)$,
- Compute $\mathcal{Z}_1(Z_2) = \frac{I_1}{V_1}$.

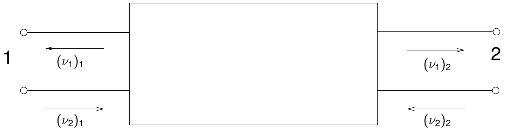
Characteristic impedance

Characteristic impedance $Z_{c,2}$ of port 1 : it is the fixed point of the application

$$Z_{c,2} = \mathcal{Z}_1(Z_{c,2}).$$

Outline

Scattering representation



ν_2 and ν_1 are the direct and reflected power waves

Change of variables

$$(\nu_1)_i = \frac{V_i - Z_{c,i} I_i}{2\sqrt{\Re Z_{c,i}}}, \quad (\nu_2)_i = \frac{V_i + Z_{c,i} I_i}{2\sqrt{\Re Z_{c,i}}}. \quad i = 1, 2.$$

Scattering Matrix

$$\begin{pmatrix} (\nu_1)_1 \\ (\nu_1)_2 \end{pmatrix} = \begin{pmatrix} r_1 & t_{12} \\ t_{21} & r_2 \end{pmatrix} \begin{pmatrix} (\nu_2)_1 \\ (\nu_2)_2 \end{pmatrix}$$

In matrix form ;

$$\boldsymbol{\nu}_1 = \mathbf{S} \boldsymbol{\nu}_2$$

Scattering problems

Direct Scattering Problem

Useful for simulations

$$\mathbf{Z} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix}$$

describes the conditions of the network

$$\mathbf{S} = \begin{pmatrix} r_1 & t_{12} \\ t_{21} & r_2 \end{pmatrix}$$

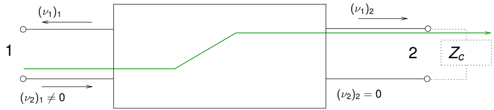
describes the measures of the reflectometer

Industrial problems

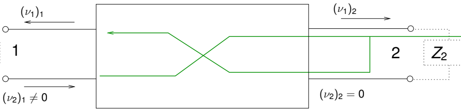
Inverse Scattering Problems

Hard and Soft faults

HEALTHY NETWORKS

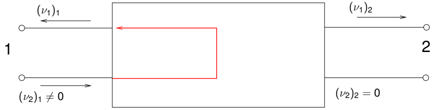


No wire fault, matched load

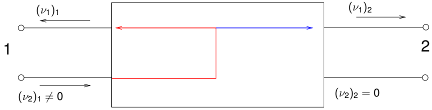


No wire fault, unmatched load

CORRUPTED NETWORKS



Hard fault

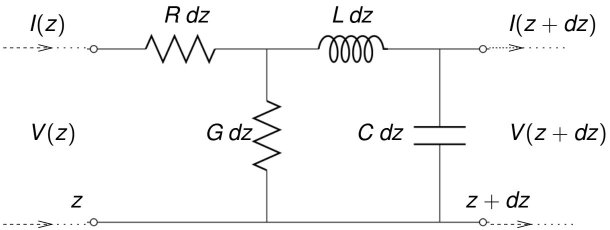


Soft fault

Outline

Telegrapher's Model

Telegrapher's equations in harmonic regime



- $L(z)$ is the inductance ;
- $C(z)$ capacitance ;
- $R(z)$ series resistance ;
- $G(z)$ shunt conductance.

Transmission line equations

$$\begin{cases} \frac{d}{dz} I(k,z) = +(ikC(z) + G(z))V(k,z) \\ \frac{d}{dz} V(k,z) = -(ikL(z) + R(z))I(k,z) \end{cases} \qquad + B.C.$$

where $I(k,z)$ and $V(k,z)$ are, respectively, the intensity of the current and the voltage at position z and frequency k.

From telegrapher model to Zakharov-Shabat equation

Characteristic impedance

$$Z_c^\infty(z) = \sqrt{\frac{L(z)}{C(z)}}$$

Power waves are used instead of I and V

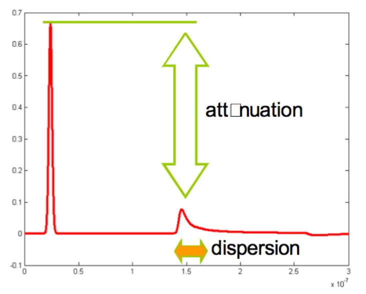
$$\begin{cases} \nu_1(k,x) = \frac{1}{\sqrt{2}}\left[(Z_c^\infty)^{-1/2}(x)V(k,x) - (Z_c^\infty)^{1/2}(x)I(k,x)\right], \\ \nu_2(k,x) = \frac{1}{\sqrt{2}}\left[(Z_c^\infty)^{-1/2}(x)V(k,x) + (Z_c^\infty)^{1/2}(x)I(k,x)\right]. \end{cases}$$

Zakharov-Shabat equations

Zakharov-Shabat equations with a source on the left ($\nu_r = 0$).

$$\begin{aligned}
\partial_x \nu_1(k,x) + ik\nu_1(k,x) &= +q_d(x)\nu_1(k,x) - q_+(x)\nu_2(k,x),\\
\partial_x \nu_2(k,x) - ik\nu_2(k,x) &= -q_-(x)\nu_1(k,x) - q_d(x)\nu_2(k,x),\\
\nu_2(k,x_l) - \rho_l(k)\nu_1(k,x_l) &= (1-\rho_l(k))\nu_l(k),\\
\nu_1(k,x_r) - \rho_r(k)\nu_2(k,x_r) &= 0.
\end{aligned}$$

Potentials

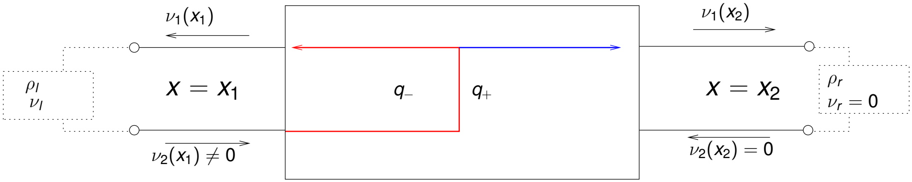


Dissipation : $q_d(x)$

Dispertion : $q_\pm(x)$

Zakharov-Shabat equations

$$\frac{d}{dx}\begin{pmatrix} \nu_1(x,k) \\ \nu_2(x,k) \end{pmatrix} = \begin{pmatrix} q_d(x) - ik & -q_+(x) \\ -q_-(x) & -(q_d(x) - ik) \end{pmatrix} \begin{pmatrix} \nu_1(x,k) \\ \nu_2(x,k) \end{pmatrix}.$$

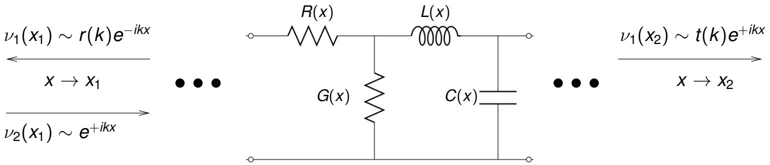


Identification of potentials

$$\{q_+(x), q_-(x), q_d(x),\} \Leftrightarrow \left\{ \frac{d}{dx} \log \frac{L(x)}{C(x)}, \frac{R(x)}{L(x)}, \frac{G(x)}{C(x)} \right\}$$

Inverse scattering problem

Reflectometry experiment



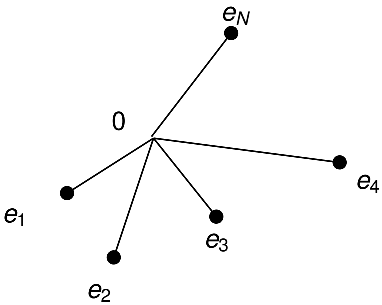
Reflectometry problem

What are the identifiable line parameters, when $r(k)$ is known ?

Outline

Star shaped network Γ

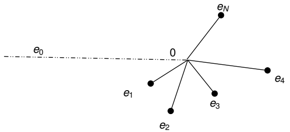
- $\{e_1, \ldots, e_N\}$ finite lines,
- each branch e_j is parametrized by $[0, l_j]$



- $\{R_j(x), L_j(x), C_j(x), G_j(x)\}_{j=1}^N$ are the line parameters.

Zakharov-Shabat eqs on a star-shaped network

Lossy case



On each branch e_j, for $j = 1, \ldots, N$

$$\begin{cases} \partial_x \nu_{1j}(k,x) = +(q_{j,d}(x) - ik)\nu_{1j}(k,x) - q_{j,+}(x)\nu_{2j}(k,x), \\ \partial_x \nu_{2j}(k,x) = -q_{j,-}(x)\nu_{1j}(k,x) - (q_{j,d}(x) - ik)\nu_{2j}(k,x), \\ \nu_{1j}(k,l_j) - \rho_j(k)\nu_{2j}(k,l_j) = 0, \quad x \in [0,l_j]. \\ \begin{pmatrix} \nu_1(x,k) \\ \nu_2(x,k) \end{pmatrix} \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ikx} + r(k) \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ikx}, \qquad x \to -\infty. \end{cases}$$

Boundary conditions at central node - Kirchhoff rules

$$\nu_{10}(k,0) + \nu_{20}(k,0) = \nu_{1j}(k,0) + \nu_{2j}(k,0) \quad \forall j \in \{1, \ldots, N\},$$

$$\sum_{j=1}^{N} \nu_{1j}(k,0) - \nu_{2j}(k,0) = \nu_{10}(k,0) - \nu_{20}(k,0).$$

And now ?

We have shown where the equations come from. But we need to prove

Direct scattering problem

the direct scattering problem is well-posed, i.e. for q_+, q_-, q_d in certain class, there exists unique a reflection coefficient $r(k)$.

Inverse scattering problems

- Identifiability results.
- Algorithm for retrieving potentials from the reflection coefficients.

THANK YOU !

Main (Simplified) results - Lossy case

Identifiability results

Theorems

If $l_j \neq l_i$ for $i \neq j$ $r_{\mathcal{N}}(k) = r'_{\mathcal{N}}(k)$ implies for $j = 1, \dots, N$:

T4 (electrical distances) $l_j = l'_j$.

T5 (line loss factors) $\int_0^{l_j} q_{j,d}(s)ds = \int_0^{l_j} q'_{j,d}(s)ds$

$$\text{where } \int_0^{l_j} q_{j,d}(s)ds = \int_0^{l_j} \left(\frac{R_j}{L_j}(s) + \frac{G_j}{C_j}(s) \right) ds$$

and $\int_0^{l_j} \cosh(\int_0^x (q_{j,d}(s) - q'_{j,d}(s))ds) \Big(q'_{j,+}(x) q'_{j-}(x) - q_{j,-}(x) q_{j,+}(x) \Big) dx = 0$.

T6 $\frac{G_j}{C_j} = \frac{G'_j}{C'_j}$ and $\frac{R_j}{L_j} = \frac{R'_j}{L'_j}$ if line parameters R_j, L_j, C_j, G_j are constant.