Coupling hydrodynamics and biology to model algae growth

Anne-Céline Boulanger

LJLL,UPMC

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Outline

Outline

Biofuel

Let us begin with several definitions:

Biofuel : fuel whose energy is derived from carbon fixation.

Carbon fixation : reduction of inorganic carbon to organic compounds by living organisms. *Example* : photosynthesis

$$6CO_2 + 6H_2O + light \rightarrow C_6H_{12}O_6 + 6O_2 \tag{1}$$

Biofuel generations

First generation : biofuels made from

- fermentation of sugars derived from wheat, corn...
  Produces alcohol : bioethanol, methanol...
- oil (sunflower, colza...)
  Produces biodiesel

Problem: biofuel production to the detriment of food production.

Second generation: derived from non-food lignocellulosic crops.

$$Plants = lignin + cellulose$$

Problem: the conversion requires expensive technologies.

Third generation: algae !

Algae culture

Why algae?

- Capable of storing carbon under lipid form through photosynthesis
- Autotrophs species only need inorganic material
- Optimal conditions for the growth are easily reachable
- Natural techniques for the increase of oil production (nitrogen stress, thermal stress...)

Actual barriers

- Compromise growth/oil production
- Cost of nutrients
- Need to find a way to use the other products of an algae
- Contamination hazard for outdoor raceways
- No economical study has been done yet

depth.pdf

A raceway

Description: algae pool driven into motion by a paddlewheel

Figure: An industrial raceway (Innovalg, 85) Photo Olivier Darboux, Ifremer

Goal: optimize the biomass production by playing on the nutrients concentrations, water height, light intensity, agitation...

Outline

Shallow water approximation of Navier Stokes

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Starting point : free surface Navier Stokes equation for incompressible fluids

$$\begin{cases} div(\mathbf{u}) = 0 \\ \partial_t \mathbf{u} + (\mathbf{u}.\nabla)\mathbf{u} + \nabla p = \mathbf{g} + div\boldsymbol{\Sigma} \end{cases} \quad (2)$$

valid for $0 \leq z \leq h(t,x)$.

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Hypothesis:

- Hydrostatic hypothesis (vertical pressure variation balanced only by gravity)
- Introduction of a small parameter $\epsilon = \frac{H}{L}$
    - Rescaling in ϵ
    - Integration of NS equations along the vertical dimension
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Result : viscous Saint Venant system [Gerbeau, Perthame 2000]:

$$\begin{cases} \partial_t H + \partial_x(H\bar{u}) = 0 \\ \partial_t H\bar{u} + \partial_x(H\bar{u}^2 + \frac{gH^2}{2}) = -gH\partial_x z_b + \partial_x(4\mu\partial_x\bar{u}) + \kappa(\bar{u}) \end{cases} \tag{3}$$

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[Audusse, Bristeau, Perthame, Sainte-Marie, 2010] : a multilayer version with mass exchanges.

The Multilayer Saint-Venant system

From hydrostatic Euler equations

$$\begin{cases} \partial_x u + \partial_z w = 0 \\ \partial_t u + \partial_x u^2 + \partial_z uw + \partial_x p = 0 \\ \partial_z p = -g \end{cases} \tag{4}$$

Two main steps:

- Galerkin approximation of every variable on

$$\mathbb{P}_{0,H}^{N,t} = \left\{ \mathbb{I}_{z \in L_\alpha(x,t)}(z), \quad \alpha \in \{1, \dots, N\} \right\},$$

- Vertical integration on each layer.

The Multilayer Saint-Venant system

The Multilayer Saint Venant system writes

$$\begin{cases} \partial_t H + \sum_{\alpha=1}^{\alpha=N} \partial_x h_\alpha u_\alpha = 0 \\ \partial_t h_\alpha u_\alpha + \partial_x (h_\alpha u_\alpha^2 + \frac{g}{2} H h_\alpha) = F_{\alpha+1/2} - F_{\alpha-1/2} \end{cases} \tag{5}$$

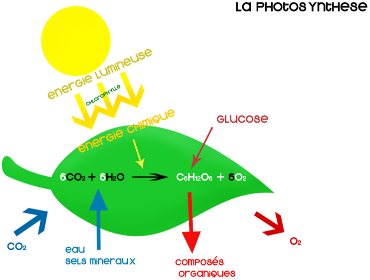
- Only one global continuity equation, $H = \sum h_\alpha$
- Exchange terms $G_{\alpha+1/2}, F_{\alpha+1/2} = u_{\alpha+1/2} G_{\alpha+1/2} + P_{\alpha+1/2}$

The advantages of this formalism compared to Navier Stokes are:

- No more z-derivative
- H is a variable of the system: once it is known, the geometry is known.
- We get a system of conservation laws with source terms.

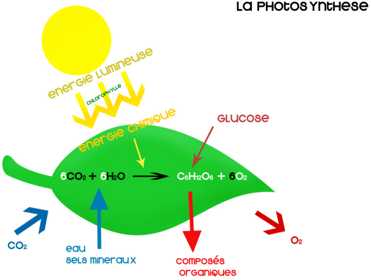
Outline

Phytoplankton environment



We want to: study the combined influence of nitrate and light on algae growth.
Nutrient: Monod, Droop, mechanistic...?
Light: photoadaptation, photoinhibition...?

Phytoplankton environment



We choose: study the evolution of 3 variables:

- The phytoplanctonic carbon, C_1
- The phytoplanctonic nitrogen, C_2
- The extracellular nutrients, C_3

We call $q = C_2/C_1$ the internal nutrient quota.

Droop model with photoadaptation

We choose: a Droop model (the growth ratio is proportional to the internal nutrient quota)

$$\mu(q) = \bar{\mu}(1 - \frac{Q_0}{q}), \tag{6}$$

with photoinhibition (the growth can be weakened by excessive light exposure)

$$\bar{\mu}(I) = \tilde{\mu}\frac{I}{I + K_{sI} + \frac{I^2}{K_{iI}}},$$

and photoadaptation (the light sensitivity is proportional to the cell history)

$$I(z) = I_0 e^{-\int_0^z (aChl(z)+b)dz}, \tag{7}$$

and Chl depends on the previous light exposure of the cell.

Derivation of the model in 2D

Add transport, diffusion, to reaction terms:

$$\begin{cases} \partial_t C_1 + \nabla \cdot (\mathbf{u} C_1) = \nu C_1 \Delta C_1 + \mu(q, I) C_1 - R C_1 \\ \partial_t C_2 + \nabla \cdot (\mathbf{u} C_2) = \nu C_2 \Delta C_2 + \lambda(C_3, q) C_1 - R C_2 \\ \partial_t C_3 + \nabla \cdot (\mathbf{u} C_3) = \nu C_3 \Delta C_3 - \lambda(C_3, q) C_1 \end{cases} \tag{8}$$

Integrate it in the multilayer model (Galerkin projection + integration):

$$\begin{cases} \dfrac{\partial h_\alpha C^1_\alpha}{\partial t} + \dfrac{\partial}{\partial x}\left(h_\alpha C^1_\alpha u_\alpha\right) = C^1_{\alpha+1/2} G_{\alpha+1/2} - C^1_{\alpha-1/2} G_{\alpha-1/2} \\ \qquad + \nu \displaystyle\int_{z_{\alpha-1/2}}^{z_{\alpha-1/2}} \Delta C_1 dz + h_\alpha(\mu(q_\alpha, I_\alpha) C^1_\alpha - R C^1_\alpha) \\ \dfrac{\partial h_\alpha C^2_\alpha}{\partial t} + \dfrac{\partial}{\partial x}\left(h_\alpha C^2_\alpha u_\alpha\right) = C^2_{\alpha+1/2} G_{\alpha+1/2} - C^2_{\alpha-1/2} G_{\alpha-1/2} \\ \qquad + \nu \displaystyle\int_{z_{\alpha-1/2}}^{z_{\alpha-1/2}} \Delta C_2 dz + h_\alpha(\lambda(C^3_\alpha, q_\alpha) C^1_\alpha - R C^2_\alpha) \\ \dfrac{\partial h_\alpha C^3_\alpha}{\partial t} + \dfrac{\partial}{\partial x}\left(h_\alpha C^3_\alpha u_\alpha\right) = C^3_{\alpha+1/2} G_{\alpha+1/2} - C^3_{\alpha-1/2} G_{\alpha-1/2} \\ \qquad + \nu \displaystyle\int_{z_{\alpha-1/2}}^{z_{\alpha-1/2}} \Delta C_3 dz - h_\alpha \lambda(C^3_\alpha, q_\alpha) C^1_\alpha \\ \qquad\qquad \alpha \in [1, \ldots, N], \end{cases}$$

Outline

1. Motivations
2. The hydrodynamical model
3. The biological model
4. Numerical scheme
5. Kinetic interpretation
6. Raceway modeling

Finite volume scheme

Finally, we can summarize the hydro-bio system in:

$$\partial_t X + \partial_x F(X) = S_e + S_v + S_p + S_{bio} \tag{9}$$

Time discretization (splitting)

$$\begin{cases} \dfrac{\tilde{X}^{n+1} - X^n}{\Delta t} + \partial_x F(X^n) = S_e(X^n, \tilde{X}^{n+1}) + S_p(X^n) + S_{bio}(X^n) \\ \dfrac{X^{n+1} - \tilde{X}^{n+1}}{\Delta t} - S_v(X^n, X^{n+1}) = 0 \end{cases} \tag{10}$$

Space discretization

- Division of the X-domain in N cells C_i
- Integration of the system over $[t_i, t_{i+1}] \times [x_i, x_{i+1}]$

$$\boxed{\tilde{X}_i^{n+1} - X_i^n + \sigma_i^n[F_{i+1/2}^n - F_{i-1/2}^n] = \Delta t S_{e,i}^n + \Delta t S_{p,i}^n + \Delta t S_{bio,i}^n} \tag{11}$$

Outline

Kinetic interpretation

Particularity of the problem

$$\left.\begin{array}{l}\text{No proof of MSV system being hyperbolic} \\ \text{No analytical computation of the eigenvalues}\end{array}\right\} \text{Riemann problem not possible}$$

Use of kinetic schemes

$$\boxed{\text{MSV+Bio system} \Leftrightarrow \begin{cases} \partial_t M_\alpha + \xi \partial_x M_\alpha + N_{\alpha-\frac{1}{2}} - N_{\alpha+\frac{1}{2}} = Q_\alpha \\ \partial_t C^i M_\alpha + \xi \partial_x C^i M_\alpha + C^i N_{\alpha-\frac{1}{2}} - C^i N_{\alpha+\frac{1}{2}} = R_\alpha \end{cases} \quad (12)}$$

- $(M_\alpha, C^i M_\alpha)$ particle density, (Q_α, R_α) collision term ($= 0$ a.e.)
- $\int_R \xi^p M d\xi$, $\int_R \xi^p U d\xi$ give the macroscopic variables
- linear transport equation $\Rightarrow$ upwind scheme

Idea

$$F_{i+1/2} = \int_R \xi M_{i+1/2} d\xi$$

Analytical validation

Analytical solution of Euler system with Biology [B., Sainte-Marie 2012]

- 2D and 3D hydro solutions for any topography $z_b(x)$
- in 2D, stationary continuous solutions (u, H, T) satisfy

$$u(x,z) = \alpha\beta\frac{cos(\beta(z-z_b))}{sin(\beta H)}, \quad \left(g(H+z_b) + \frac{\alpha^2\beta^2}{2sin^2(\beta H)}\right)_x = 0$$

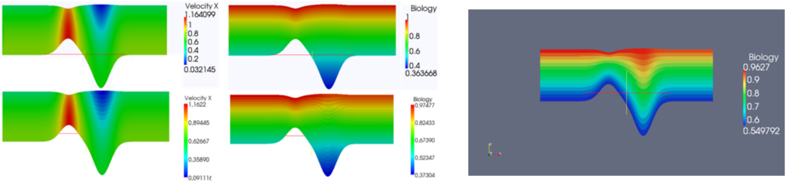
$$T(x,z) = e^{-(H-(z-z_b))}$$

where T is solution of the simple biological system

$$\partial_t T + \partial_x uT + \partial_z wT = f(x,z)T(x,z) \tag{13}$$

and

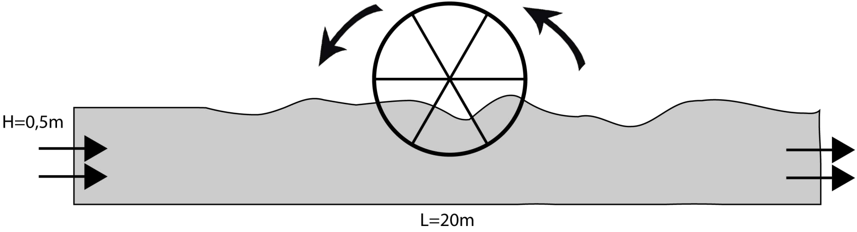
$$f(x,z) = \alpha\beta\frac{cos(\beta(z-z_b))}{sin(\beta H)}\left(\frac{tan(\beta(z-z_b))}{tan(\beta H)} - 1\right)\frac{\partial H}{\partial x}$$



Outline

Raceway modeling

Geometry: a 2D rectangular pool along x and z axis (H = 50 cm and L = 20m)



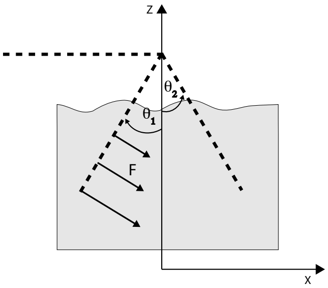
Raceway modeling

Agitation:

$$F_x(x,z,t) = F\left(\sqrt{(x-x_{wheel})^2+(z-z_{wheel})^2}\omega\right)^2 \cos\theta \qquad (14)$$

$$F_z(x,z,t) = F\left(\sqrt{(x-x_{wheel})^2+(z-z_{wheel})^2}\omega\right)^2 \sin\theta \qquad (15)$$

where F is a constant, θ is the angle between the blade and the vertical direction, $\omega = \dot{\theta}$.



Hydrodynamics

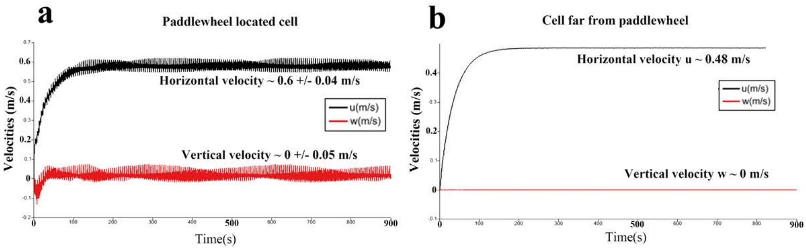


Figure: (a): Velocities along vertical and horizontal axis in a cell located near from the wheel rotating at angular speed $\omega = 0.8 rad/s$. The flow is very turbulent. (b): Velocities along vertical and horizontal axis in a cell located far from the wheel. An asymptotic value of $0.48 m.s^{-1}$ is reached.

Hydrodynamics

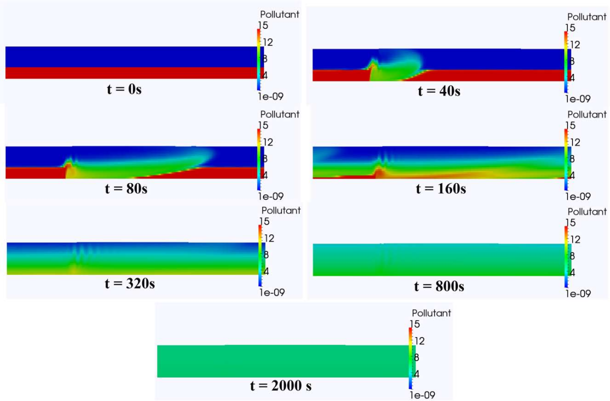


Figure: Snapshots of the tracer (pollutant) concentration in a raceway set into motion by a paddlwheel of angular velocity $\omega = 0.85 rad/s$. It is clear that after several minutes, the raceway is totally homogeneous. Therefore, the paddlewheel has indeed the required effect on the mixing.

Lagrangian trajectories

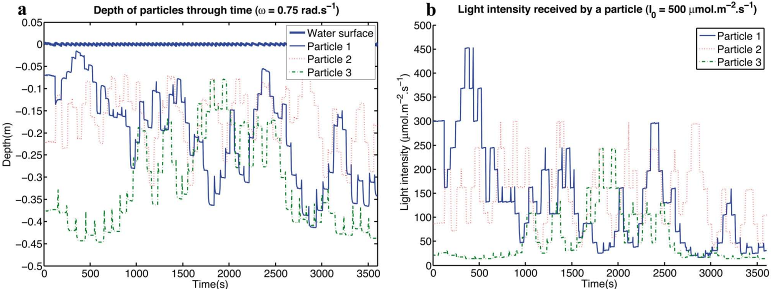


Figure: (a):Trajectories of three particles during the simulations. The large curve represents the water surface at the middle of the pool. The other plot is the height of a given particle through time. The algae undergo sudden changes of depth every time it meets the wheel.(b): Perceived light from the microalgae. Particles are subject to even greater irradiance changes since the light is exponentially decaying.

Biological results

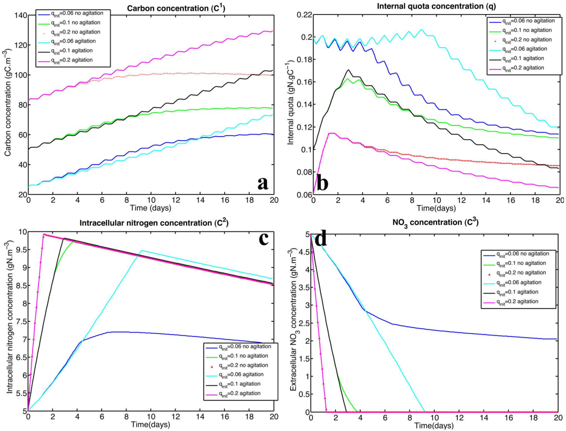


Figure: (a): Carbon concentration; (b): Internal quota q; (c): nitrogen concentration; (d): substrate concentration(NO_3). Those plots illustrate the average concentrations in the raceway for 6 simulations. Three were carried out without agitation, and the other three had the agitation term. In each situation, agitation, leading to homogenization leads to a better productivity. However, for certain initial conditions, the improvement is quite slow (after several days), since the biological variables do not evolve as quickly as hydrodynamics does.

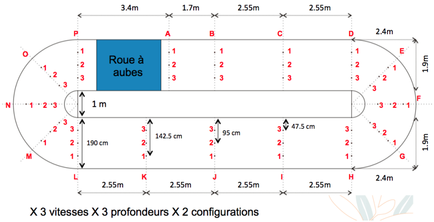
3D investigation

Simulation of a passive tracer in the raceway.

Passive tracer

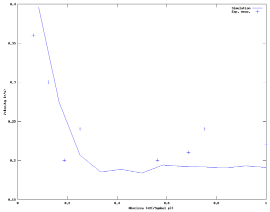
Experimental validation

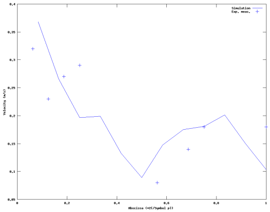
Measurement points



X 3 vitesses X 3 profondeurs X 2 configurations

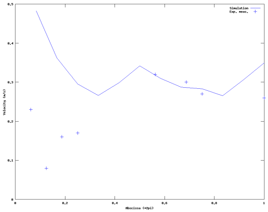
Comparison (25 Hz, aligned blades, paths 2 and 3, H = 30 cm)



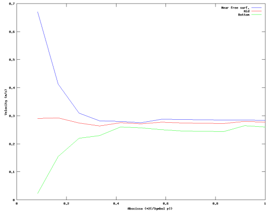


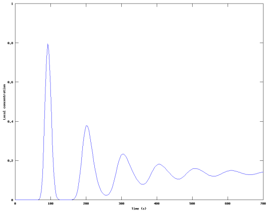
Experimental validation

Comparison (25 Hz, aligned blades, paths 1 and 3, H = 20 cm)



Comparison (25 Hz, aligned blades, path 3, H = 30 cm)





Lagrangian trajectories

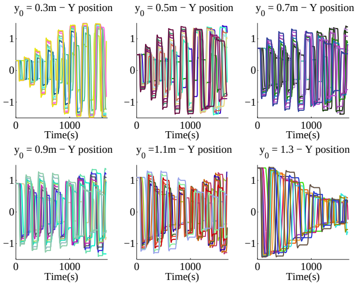


Figure: Y position of the particles

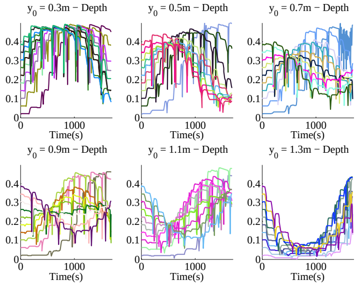


Figure: Depth of the particles

Animation

3D Trajectories

Conclusion

- First order results in 2D are encouraging
- Job in 3D is in progress (hydrodynamics OK)
- Future work : data assimilation

Thank you for your attention

Any questions ?