Coupling independent compartments of the cardiovascular system: fluid-fluid interaction

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Motivations
- Cardiovascular models

Application to blood flow
- Navier - Stokes equations

Simulations samples
- Coupled and reference solutions comparisons

Conclusions
- Current and further investigations
Outline

1. Motivations
   - Cardiovascular models

2. Application to blood flow
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3. Simulations samples
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4. Conclusions
   - Current and further investigations
Motivation

- **Challenge**: modelling the whole cardiovascular system.

- **Monolithic** algorithms: simultaneous solution for all compartments;

- **Partitioned** algorithms: separate solutions for each compartment.
Aim: Split cardiovascular system complexity, starting from coupling blood flow in different and independent compartments.
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   - Navier - Stokes equations

\[ \partial_t \mathbf{u}_1 - \nabla \cdot \sigma(\mathbf{u}_1, p_1) = 0 \]
\[ \nabla \cdot \mathbf{u}_1 = 0 \]

3. Simulations samples
   - Coupled and reference solutions comparisons

4. Conclusions
   - Current and further investigations
Blood models: Navier-Stokes equations

\[ \begin{align*}
\mathbf{u}_1^n &= \text{velocity} \quad p_1^n = \text{pressure} \\
\mathbf{u}_2^n &= \text{velocity} \quad p_2^n = \text{pressure}
\end{align*} \]

**Upstream Domain:** Fluid 1

\[ \begin{align*}
\rho \partial_t \mathbf{u}_1^n + \rho (\mathbf{u}_1^{n-1} \nabla) \mathbf{u}_1^n - \nabla \cdot \mathbf{\sigma}(\mathbf{u}_1^n, p_1^n) &= 0 \\
\nabla \cdot \mathbf{u}_1^n &= 0
\end{align*} \]

\[ \mathbf{u}_1^{n-1} = \mathbf{u}_1(t_{n-1}) \quad \text{previous time step} \]

\[ \mathbf{u}_1^n = \mathbf{u}_1(t_n) \quad \text{current time step} \quad p_2^n = p_2^n(t_n) \]

**Downstream Domain:** Fluid 2

\[ \begin{align*}
\rho \partial_t \mathbf{u}_2^n + \rho (\mathbf{u}_2^{n-1} \nabla) \mathbf{u}_2^n - \nabla \cdot \mathbf{\sigma}(\mathbf{u}_2^n, p_2^n) &= 0 \\
\nabla \cdot \mathbf{u}_2^n &= 0
\end{align*} \]
Blood models: Navier-Stokes equations

\[ \mathbf{u}_1^n = \text{velocity} \quad p_1^n = \text{pressure} \]

**Upstream Domain: Fluid 1**
\[
\begin{align*}
\rho \partial_t \mathbf{u}_1^n + \rho (\mathbf{u}_1^{n-1} \nabla) \mathbf{u}_1^n - \nabla \cdot (\sigma(\mathbf{u}_1^n, p_1^n)) &= 0 \\
\nabla \cdot \mathbf{u}_1^n &= 0
\end{align*}
\]

**Downstream Domain: Fluid 2**
\[
\begin{align*}
\rho \partial_t \mathbf{u}_2^n + \rho (\mathbf{u}_2^{n-1} \nabla) \mathbf{u}_2^n - \nabla \cdot (\sigma(\mathbf{u}_2^n, p_2^n)) &= 0 \\
\nabla \cdot \mathbf{u}_2^n &= 0
\end{align*}
\]

\[ \sigma(\mathbf{u}, p) \]

Stress tensor = measure of the forces acting on an inner surface in the fluid
Blood models: Navier-Stokes equations

\[ \begin{align*}
\mathbf{u}_1^n &= \text{velocity} & p_1^n &= \text{pressure} \\
\mathbf{u}_2^n &= \text{velocity} & p_2^n &= \text{pressure}
\end{align*} \]

**Upstream Domain: Fluid 1**
\[
\begin{align*}
\rho \partial_t \mathbf{u}_1^n + \rho \left( \mathbf{u}_1^{n-1} \nabla \right) \mathbf{u}_1^n - \nabla \cdot \sigma(\mathbf{u}_1^n, p_1^n) &= 0 \\
\nabla \cdot \mathbf{u}_1^n &= 0
\end{align*}
\]

**Downstream Domain: Fluid 2**
\[
\begin{align*}
\rho \partial_t \mathbf{u}_2^n + \rho \left( \mathbf{u}_2^{n-1} \nabla \right) \mathbf{u}_2^n - \nabla \cdot \sigma(\mathbf{u}_2^n, p_2^n) &= 0 \\
\nabla \cdot \mathbf{u}_2^n &= 0
\end{align*}
\]

**Transmission Conditions on \( \Sigma \)**
\[
\begin{align*}
? &= ? \\
? &= ?
\end{align*}
\]
Coupling techniques: Classical method

- **Dirichlet-Neumann** transmission conditions:

  \[
  \begin{aligned}
  u_1^n &= u_2^{n-1} \\
  \sigma(u_2^n, p_2^n) \cdot n_2 &= -\sigma(u_1^n, p_1^n) \cdot n_1
  \end{aligned}
  \]

- **Loosely coupled scheme** (explicit).

- **Pressure propagation in a straight vessel:**

  - **Reference solution**
  - **Classical coupling method**

- **Standard treatments:**
  - **Strongly coupled schemes** (implicit): iterate until convergence, expensive;
  - **New loosely coupled methods**: stable and computationally cheap.
Coupling techniques: New method

- Robin-Robin transmission conditions derived from Nitsche’s method:

\[
\begin{align*}
\sigma(u^n_1, p^n_1) \cdot n_1 + \alpha u^n_1 &= \alpha u^{n-1}_2 - \sigma(u^{n-1}_2, p^{n-1}_2) \cdot n_2 \\
\sigma(u^n_2, p^n_2) \cdot n_2 + \alpha u^n_2 &= \alpha u^n_1 + \sigma(u^{n-1}_2, p^{n-1}_2) \cdot n_2
\end{align*}
\]

- Nitsche’s penalty parameter: \( \alpha > 0 \)

- Relaxation method to treat interface conditions for coupled problems.

Fluid 1 - Used in Fluid - Structure problems.

[Burman, Fernández, 09]
[Hansbo, '05]
Coupling techniques: New method

- **Robin-Robin** transmission conditions derived from Nitsche’s method:

  \[
  \begin{align*}
  \sigma(u_1^n, p_1^n) \cdot n_1 + \alpha u_1^n &= \alpha u_2^{n-1} - \sigma(u_2^{n-1}, p_2^{n-1}) \cdot n_2 \\
  \sigma(u_2^n, p_2^n) \cdot n_2 + \alpha u_2^n &= \alpha u_1^n + \sigma(u_2^{n-1}, p_2^{n-1}) \cdot n_2
  \end{align*}
  \]

  - Nitsche’s penalty parameter: \( \alpha > 0 \)
    to control velocities on the interface;

- Add in the downstream Fluid 2:

  \[
  S(p_2^n, p_2^{n-1}) = \beta \int_{\Sigma} (p_2^n - p_2^{n-1}) q_2
  \]

  - Stabilization parameter: \( \beta > 0 \)
    to control pressures on interfaces.
Coupling techniques: New method

• Taking appropriate values of $\alpha$ and $\beta$ the Fluid-Fluid coupled model becomes stable.

• Pressure propagation in a straight vessel:

  new coupling method

  classical coupling method

  reference solution

• Pressure wave instabilities disappeared with Robin-Robin method.
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Haemodynamics application: aorta model

- Comparisons: Fluid-Fluid Model vs. Reference Solution in a Template of an Aorta;

- Windkessel 0D model in the outputs;

\[ i(t) = \text{blood flow} \]

\[ R_P = \text{big vessel peripheral resistance} \]

\[ C = \text{capacity of vessels to dilate} \]

\[ R_d = \text{small vessels distal resistance} \]
Haemodynamics application: aorta model

- Comparisons: Fluid-Fluid Model vs. Reference Solution in a Template of an Aorta;

- Windkessel 0D model in the outputs;

- Three surfaces representing Aortic valve;
Haemodynamics application: aorta model

- Comparisons: Fluid-Fluid Model vs. Reference Solution in a Template of an Aorta;

- Windkessel 0D model in the outputs;

- Three surfaces representing Aortic valve;

Navier-Stokes Blood flow

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \cdot \nabla u - \nabla \cdot \sigma(u, p) &= 0 \\
\nabla \cdot u &= 0
\end{align*}
\]
Haemodynamics application: aorta model

• Comparisons:
  Fluid-Fluid Model vs. Reference Solution in a Template of an Aorta;

• Windkessel 0D model in the outputs;

• Three surfaces representing Aortic valve;

• Two Solvers: FELiScE
  Finite Element for Life Science et Engineering
  [Macs & Reo Team - INRIA Saclay & Rocquencourt].
Blood velocity profile

Blood-Blood coupling

Time = 0.00000 s

Reference solution

- Time step: \( t = 1 \times 10^{-3} \);
- Inflow conditions: Waveform of cardiac output 9 liters/minute hear rate 85 beats/minute.

Smaldone, Fernández, Gerbeau, ’12
Interface Velocity and Pressure

- Error on Pressure ≈ 0.2 %
- Error on Velocity ≈ 1 %
Outputs Pressures

• Error on Pressures ≈ 2% in all outputs:
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Conclusions

- Current works:
  - Coding Software for coupling solvers;
  - Coding Solver for fluid & cardiovascular models;
  - Efficient coupled Blood-Blood model;
Conclusions

• Forthcoming works:
  - Testing with Strongly coupled schemes
  - Including Heart model;
  - Including a semplified Aortic valve model;

[M. Astorino, ’09 ]
Thanks for the attention