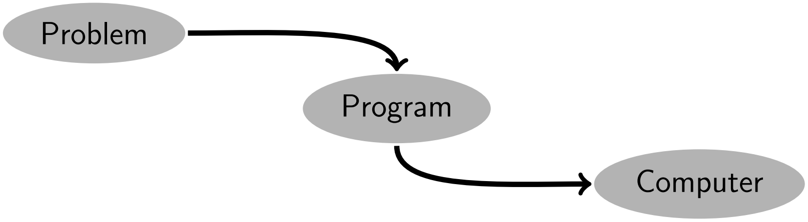
Lazy computing

Thibaut Balabonski
GALLIUM

November 20, 2012

Solving problems with computers



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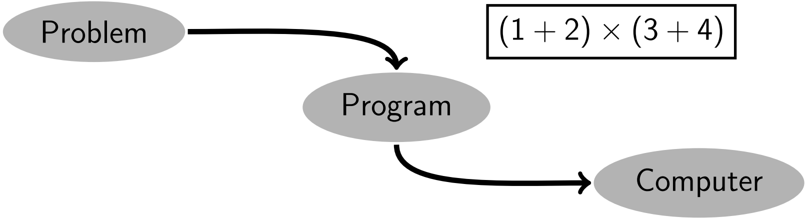

$$\boxed{(1+2)\times(3+4)}$$

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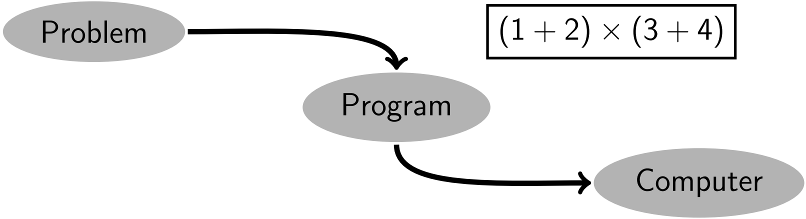

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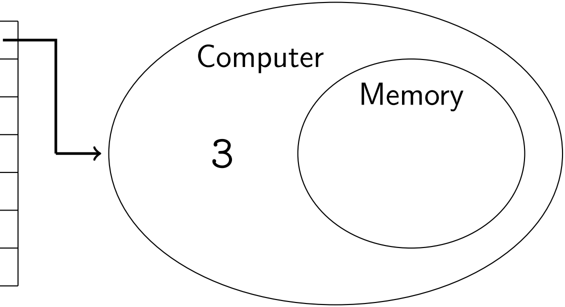

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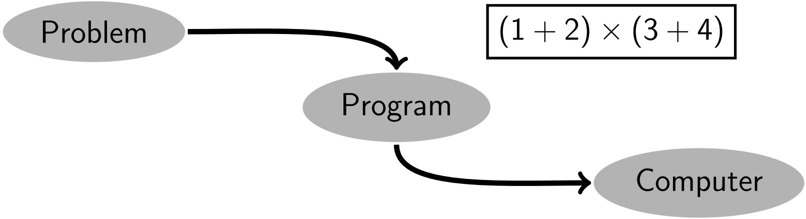

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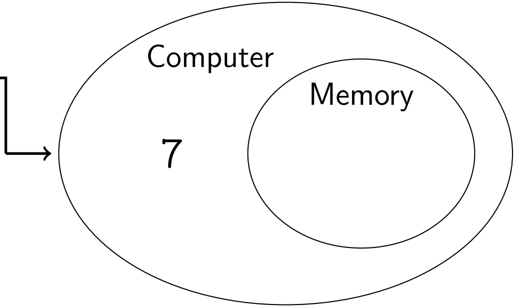

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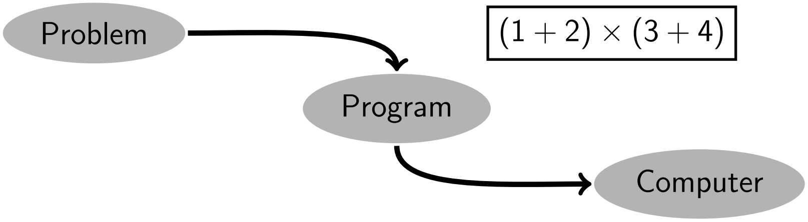

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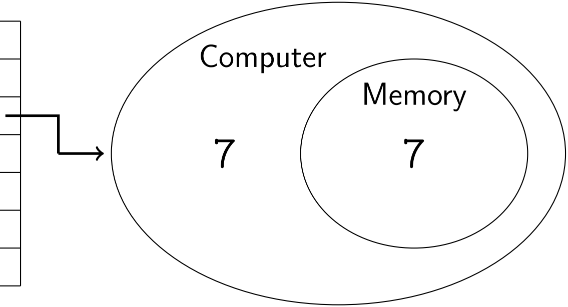

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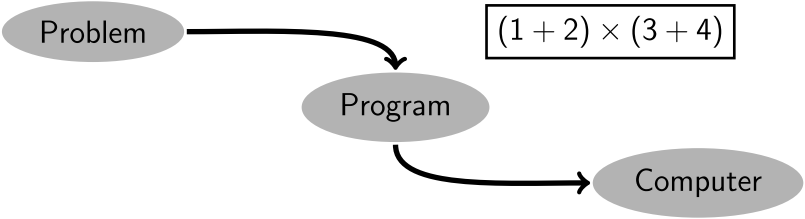

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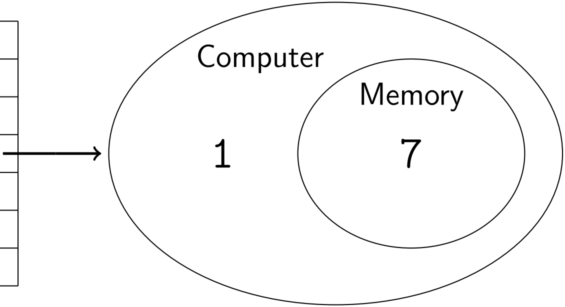

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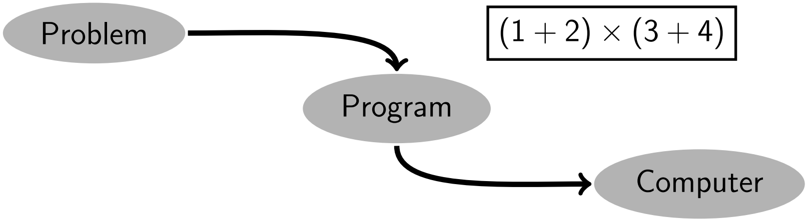


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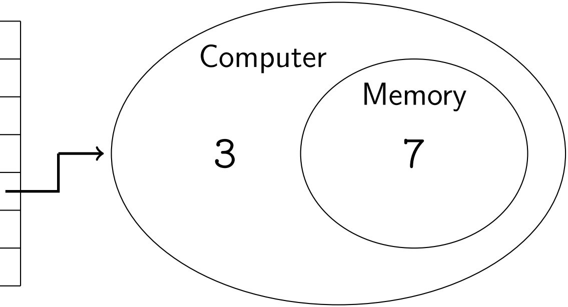

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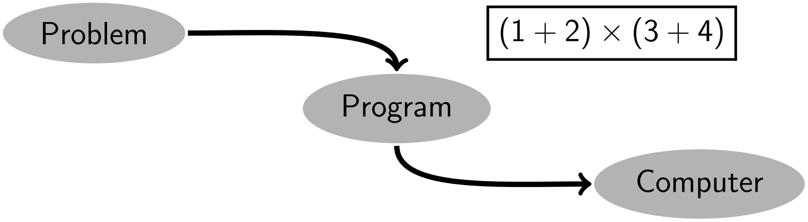

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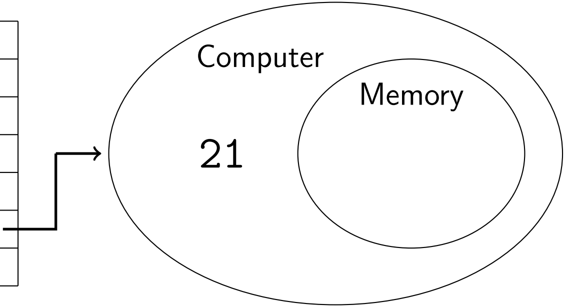

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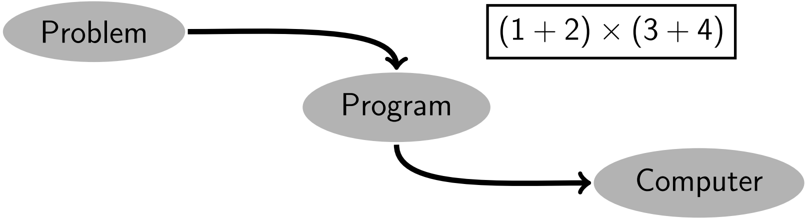

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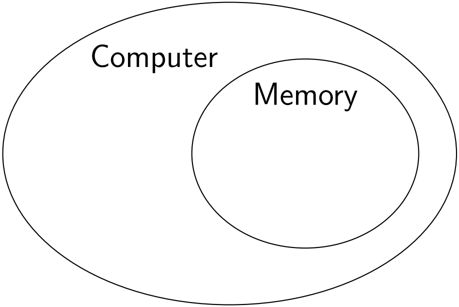

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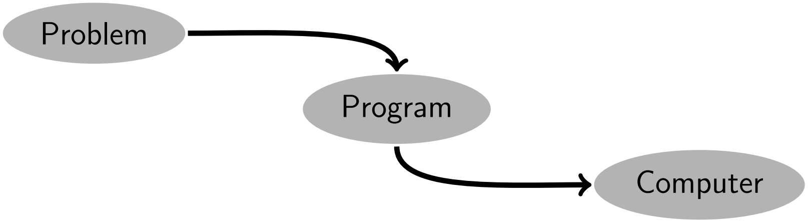
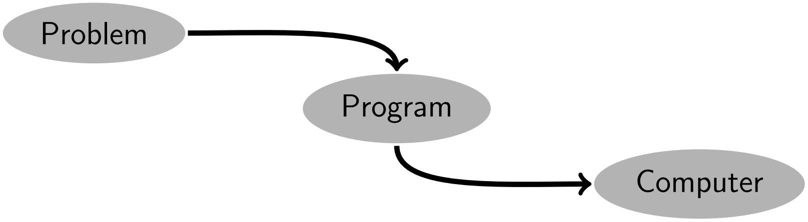


Gallium-related issues:

- Programming languages and compilation
- Emphasis on safety

Solving problems with computers



Gallium-related issues:

- Programming languages and compilation
- Emphasis on safety

In this talk:

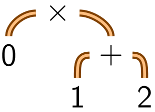
- Emphasis on **efficiency**

Chosing a path

$$
\begin{tikzcd}
& (1+2)\times(3+4) \arrow[dl] \arrow[dr] & \\
3\times(3+4) \arrow[dr] & & (1+2)\times 7 \arrow[dl] \\
& 3\times 7 \arrow[d] & \\
& 21 &
\end{tikzcd}
$$

Freedom implies responsibility

- Outermost prevents unneeded computations.
- Innermost prevents duplication of subprograms.



$$\begin{tikzcd}
0 \times (1+2) \arrow[rr] \arrow[dr] & & 0 \\
& 0 \times 3 \arrow[ur] &
\end{tikzcd}$$

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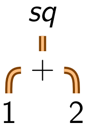
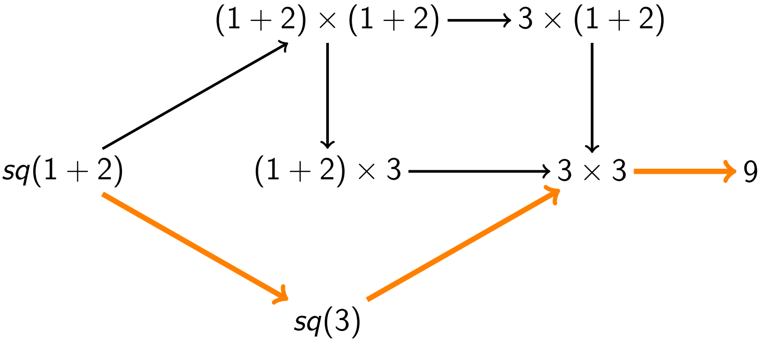
$$sq(e) \;=\; e \times e$$





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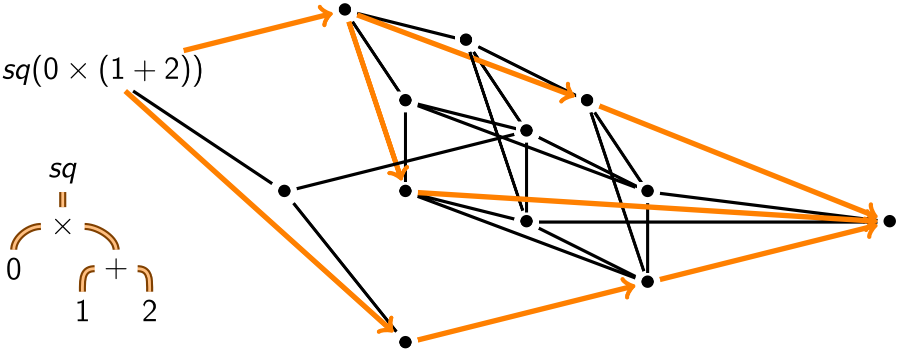

Evaluation strategies

Goal

Minimize the number of rewriting steps.

Question

In **which order** should we perform the steps?

Richer programming languages

- Functions
- Data structures

```
map(f, []     )  =  []
map(f, x : xs)  =  f(x) : map(f, xs)

> map(sq, 1 : 2 : 3 : [])
1 : 4 : 9 : []
```

Richer programming languages

- Functions
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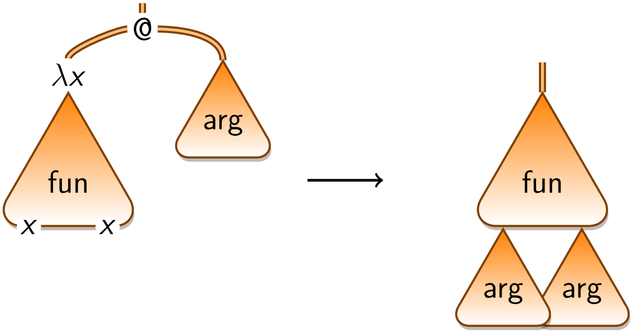
```

Second-order rewriting

Lambda-calculus: computing with functions

Church, 1936



$(\lambda x.f)a$ $\longrightarrow$ $f^{\{x:=a\}}$

$(\lambda x.xx)((\lambda y.y)a)$ $\quad$ $((\lambda y.y)a)((\lambda y.y)a)$

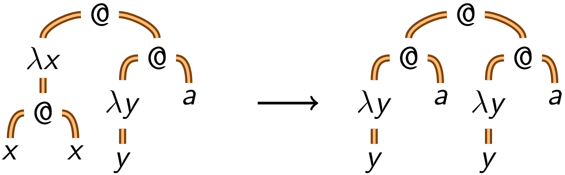
Shortest simple path

$$\begin{tikzcd}
 & ((\lambda y.y)a)((\lambda y.y)a) \arrow[r] \arrow[d] & a((\lambda y.y)a) \arrow[d] \\
(\lambda x.xx)((\lambda y.y)a) \arrow[ur] \arrow[dr] & ((\lambda y.y)a)a \arrow[r] & aa \\
 & (\lambda x.xx)a \arrow[ur] &
\end{tikzcd}$$

Theorem: uncomputability (Barendregt et al. 1976)

Optimal strategies for the λ-calculus
cannot be computable.

Wadsworth's call-by-need (1971)



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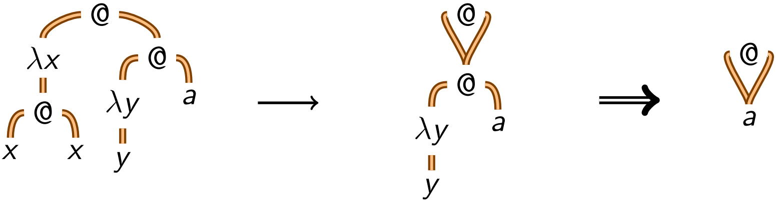
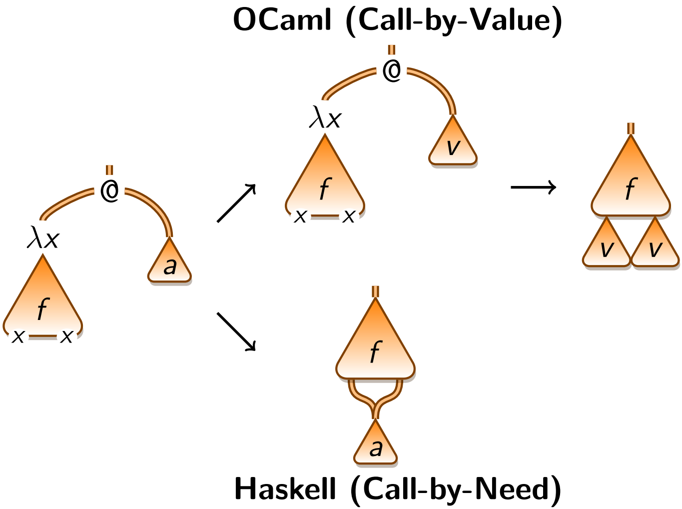


$$
\begin{tikzcd}
& ((\lambda y.y)a)((\lambda y.y)a) \arrow[dr, Rightarrow] & \\
(\lambda x.xx)((\lambda y.y)a) \arrow[ur] \arrow[dr] & & aa \\
& (\lambda x.xx)a \arrow[ur] &
\end{tikzcd}
$$

Real compilers use weak reduction

Restriction on evaluation: not inside functions.



New features of weak reduction (my work)

- The optimal strategy is still **uncomputable**.
- Call-by-need is **as good as** the optimal strategy.

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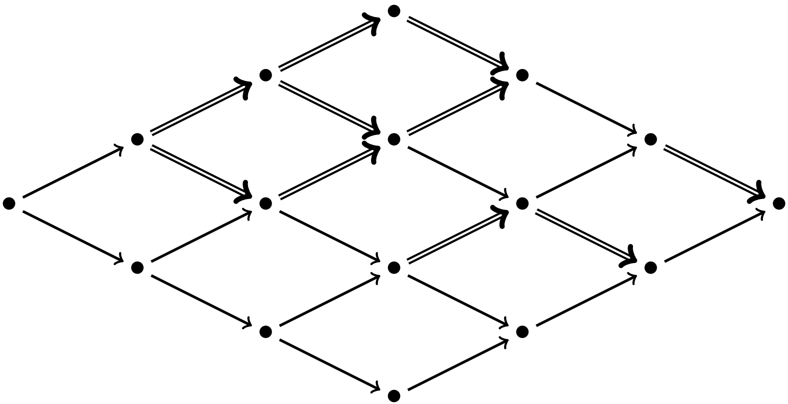
Shared needed evaluation

- Use shared evaluation.
- Consider only needed steps.



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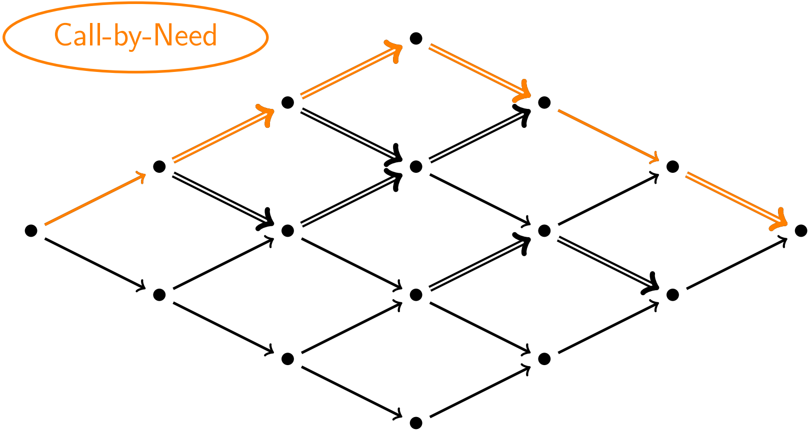

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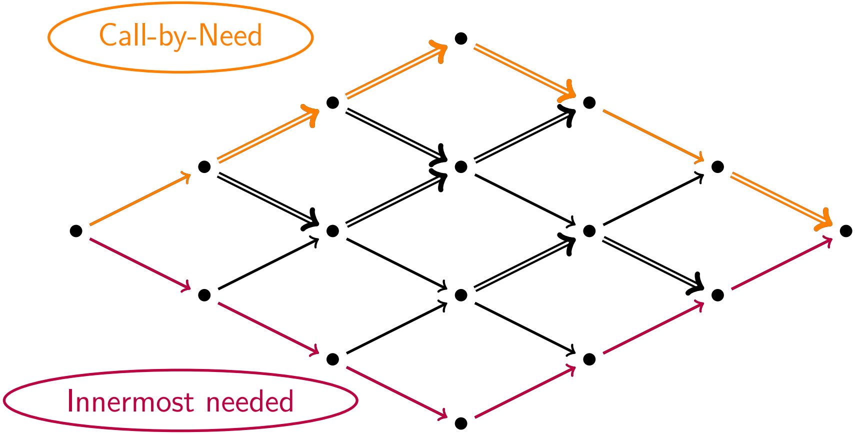

Conclusion

- Optimal strategies are not computable.
- Sharing adds shortcuts to the reduction space.
- Shared evaluation is as good as an optimal strategy
  and is computable.

Question: what is the cost of sharing?