# Numerical aspects of blood flow modeling for congenital heart diseases

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Grant: Leducq Foundation

# Outline

- Motivations & Context
- Clinical aspects
- Multiscale modeling
- Patients specific results
- Boundary conditions and stability

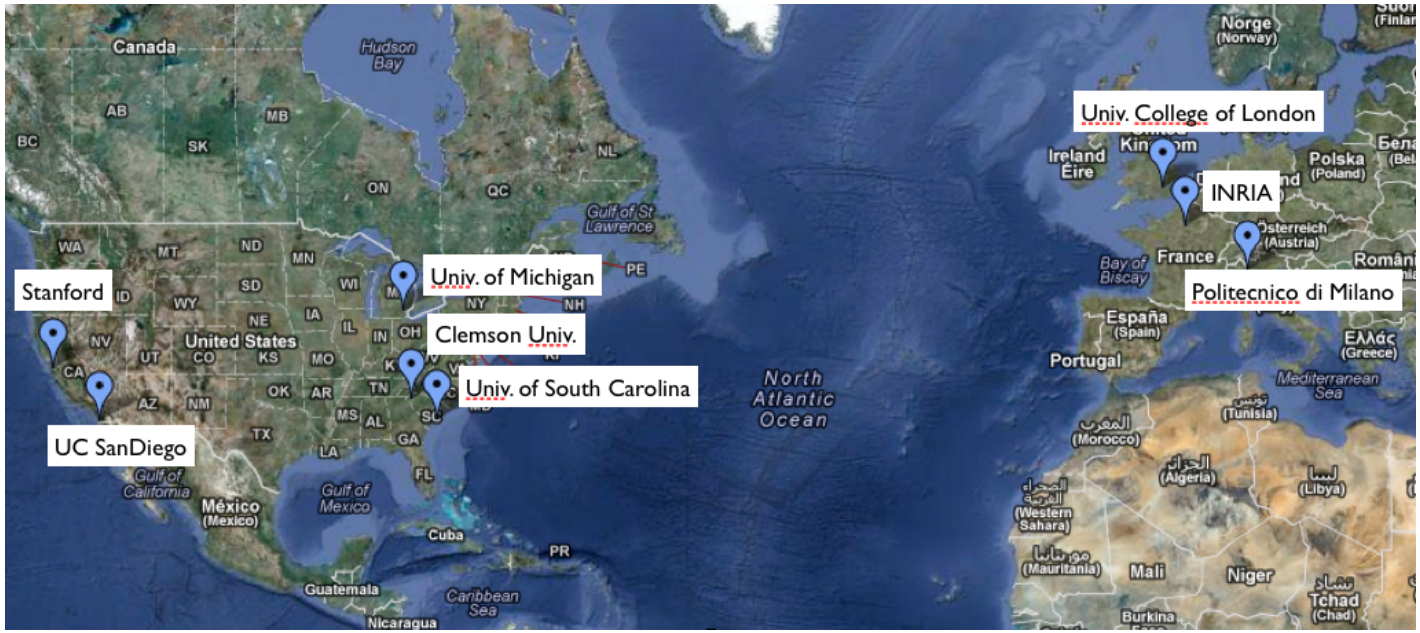
Clinical pathology and Challenges

- Structural problems with the heart present at birth
- 1% in live birth
- 3 palliative **surgeries**

Do computational methods allow to:

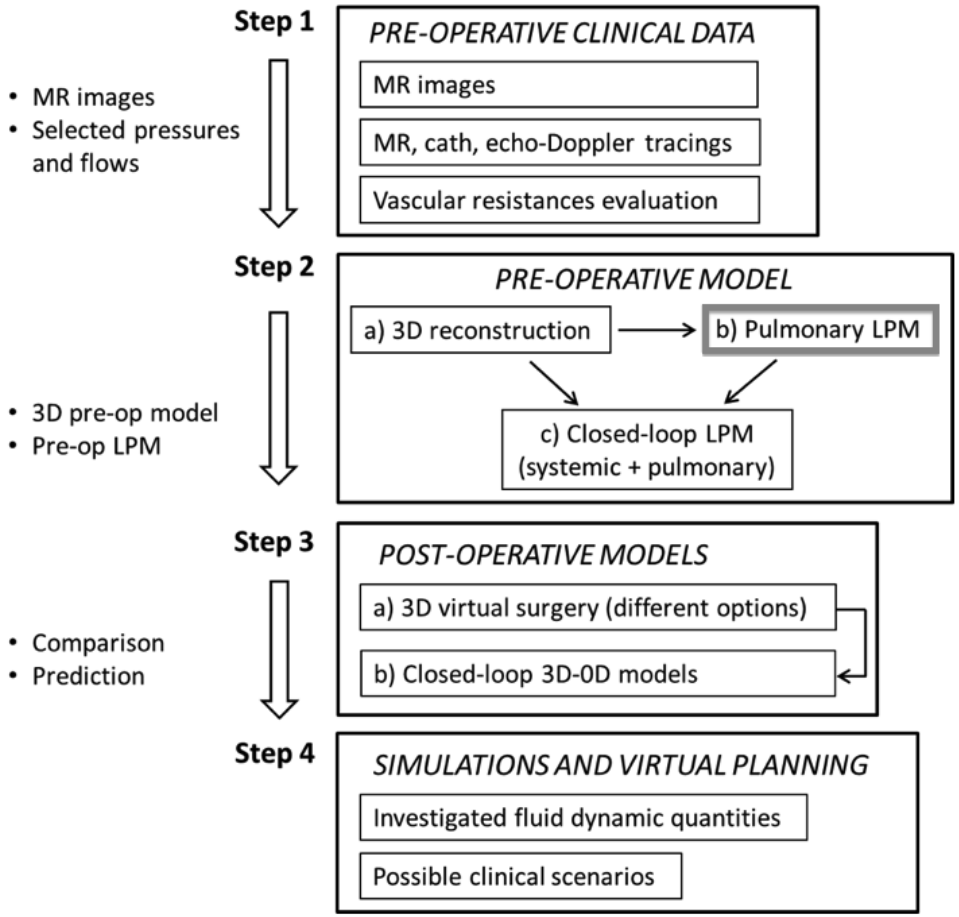
- Explore hemodynamics in patient-specific simulations ?
- Understand physiopathologies ?
- Perform virtual surgeries ?
- Help surgeons to take a decision ?

# MOdeling for Congenital Heart Alliance

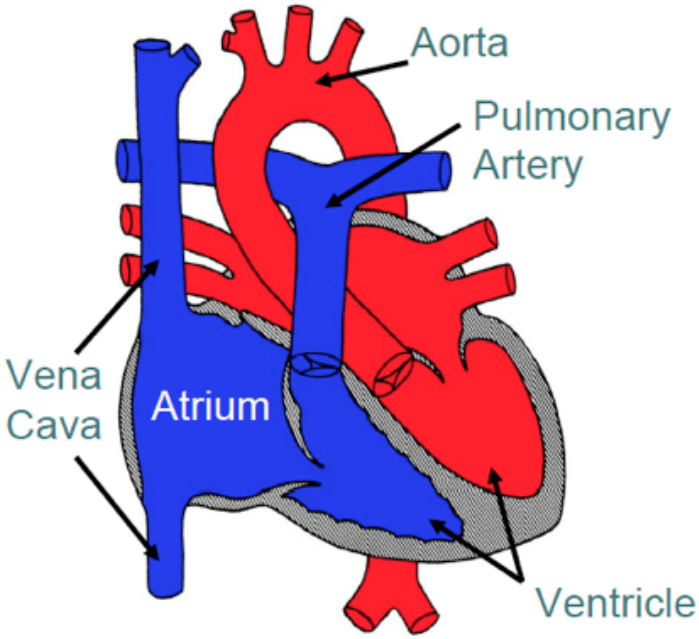



- Cardiothoracic surgery
- Pediatric cardiology
- Cardiovascular imaging and radiology
- Biomechanical and mechanical engineering
- Applied mathematics

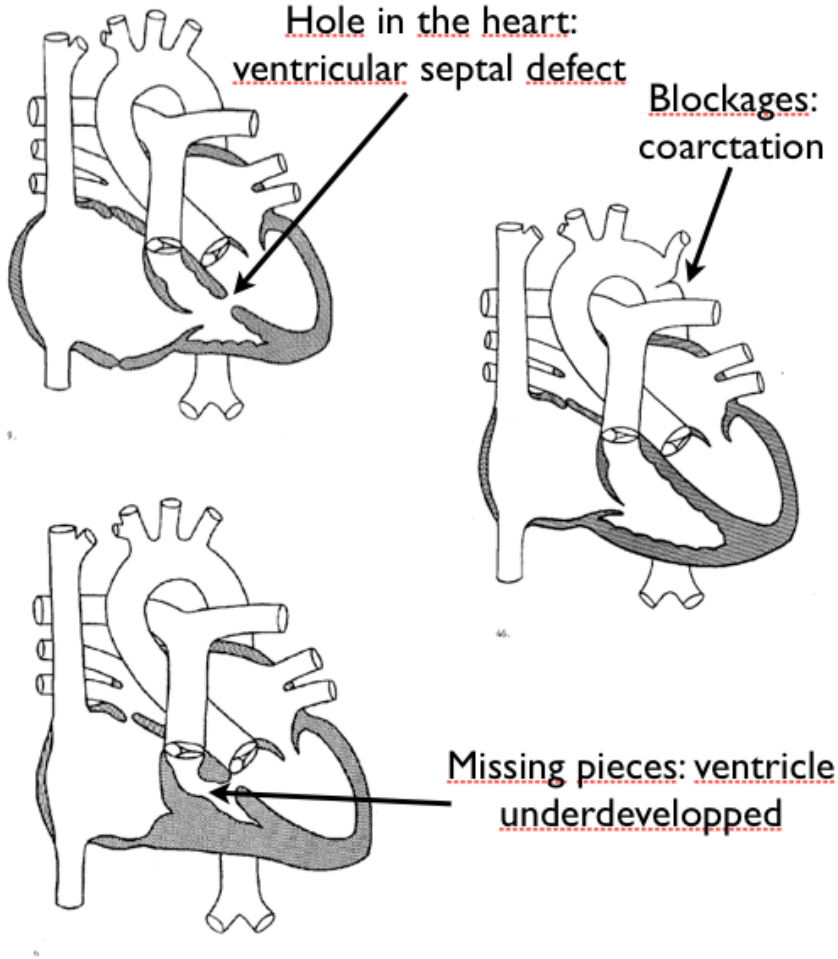
# Way of working in this collaboration



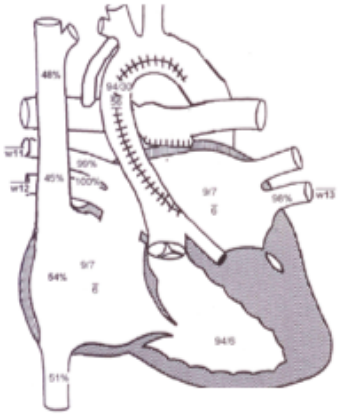
## Normal Heart



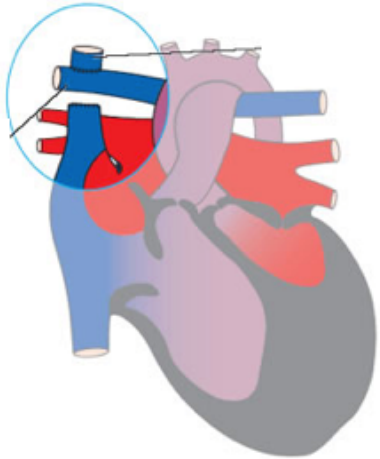

## Possible defects



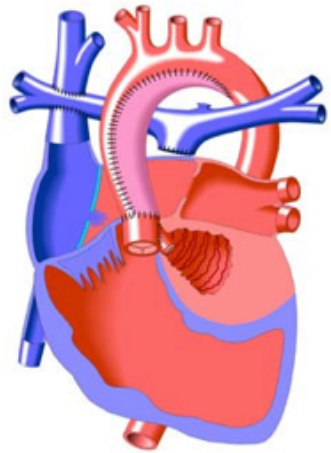
**Stage I:**
Norwood procedure



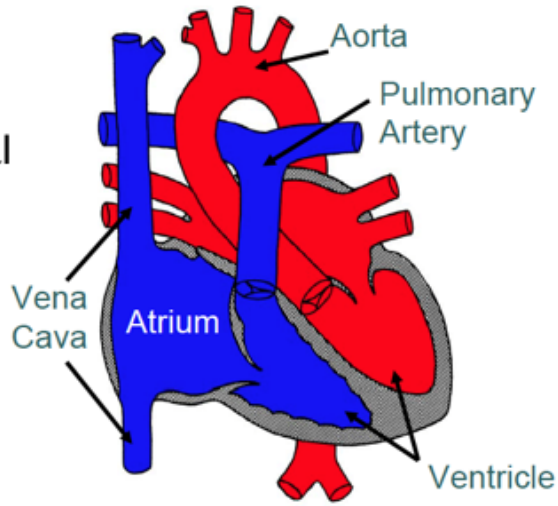
**Stage II:**
Bidirectional cavopulmonary procedure



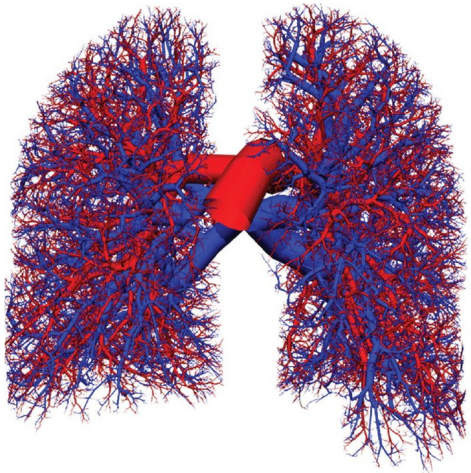
**Stage III:**
Total cavopulmonary procedure



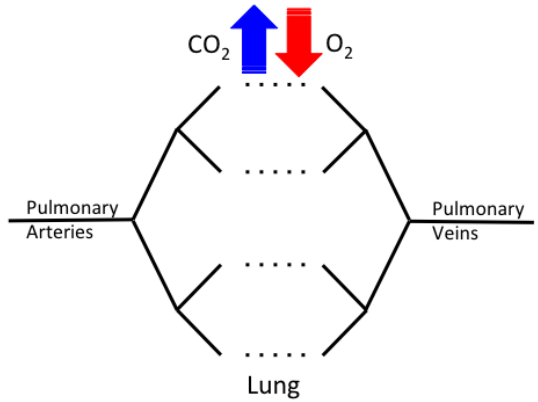
Normal heart



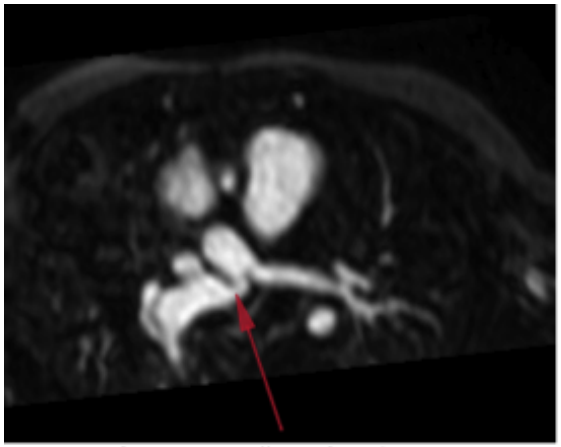
# Pulmonary circulation



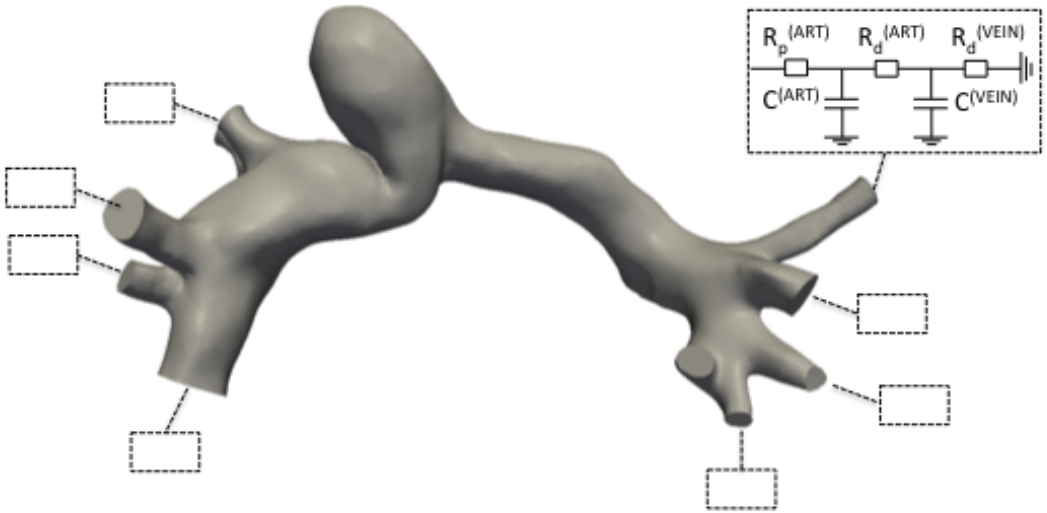
*Burrowes KS et. al.*, J Appl Physiol 2005




- Unreasonable to compute the flow in the **whole circulatory domain**.
- Motivation: given a **bounded** 3D geometry, how to model downstream domain of pulmonary arteries ?



*Courtesy of University College of London, MOCHA group*




- **Rp**: viscous effects in large vessels (arteries).
- **C**: capacitance of large vessels to store and return blood during cardiac cycle.
- **Rd**: viscous effects in small vessels (arterioles and capillaries).

- How can we determine these parameters ?
  - Tuning of a **total equivalent resistance** at each branch
    $\Rightarrow R_{tot} = R_p^A + R_d^A + R_d^V$
  - Using empirical law to evaluate **distribution** of equivalent resistances over **arterial/venous** sides.
    $\Rightarrow R_p^A + R_d^A \approx 0.48 R_{tot}$ and $R_d^V \approx 0.52 R_{tot}$
  - Morphometric model to estimate $R_p^A$, $C^A$, $R_d^A$ in **arterial** side
  - Using empirical law to deduce equivalent venous compliance.
    $\Rightarrow C^V \approx 5.7 C^A$

# Tuning of total resistance boundary conditions

- Input data
  - **Flow split**, $fs$
  - **Transpulmonary gradient**, difference between **measured** arterial pressure and left atrial pressure, $P_m^t - P_a$
  - **Flow rate** at the inlet of interest domain $Q^{in}$
- Proportionality law between flow $Q_i^t$ and surface area $S_i$

$$\Rightarrow Q_i^t \propto S_i^\alpha$$

Then, denoting $\delta_{ir} = 1$ if outlet $i$ is on the right side,

$$\Rightarrow Q_i^t = Q^{in}(\delta_{ir} fs + (1 - \delta_{ir})(1 - fs)) \frac{S_i^\alpha}{\sum_j S_i^\alpha \delta_{ij}}$$

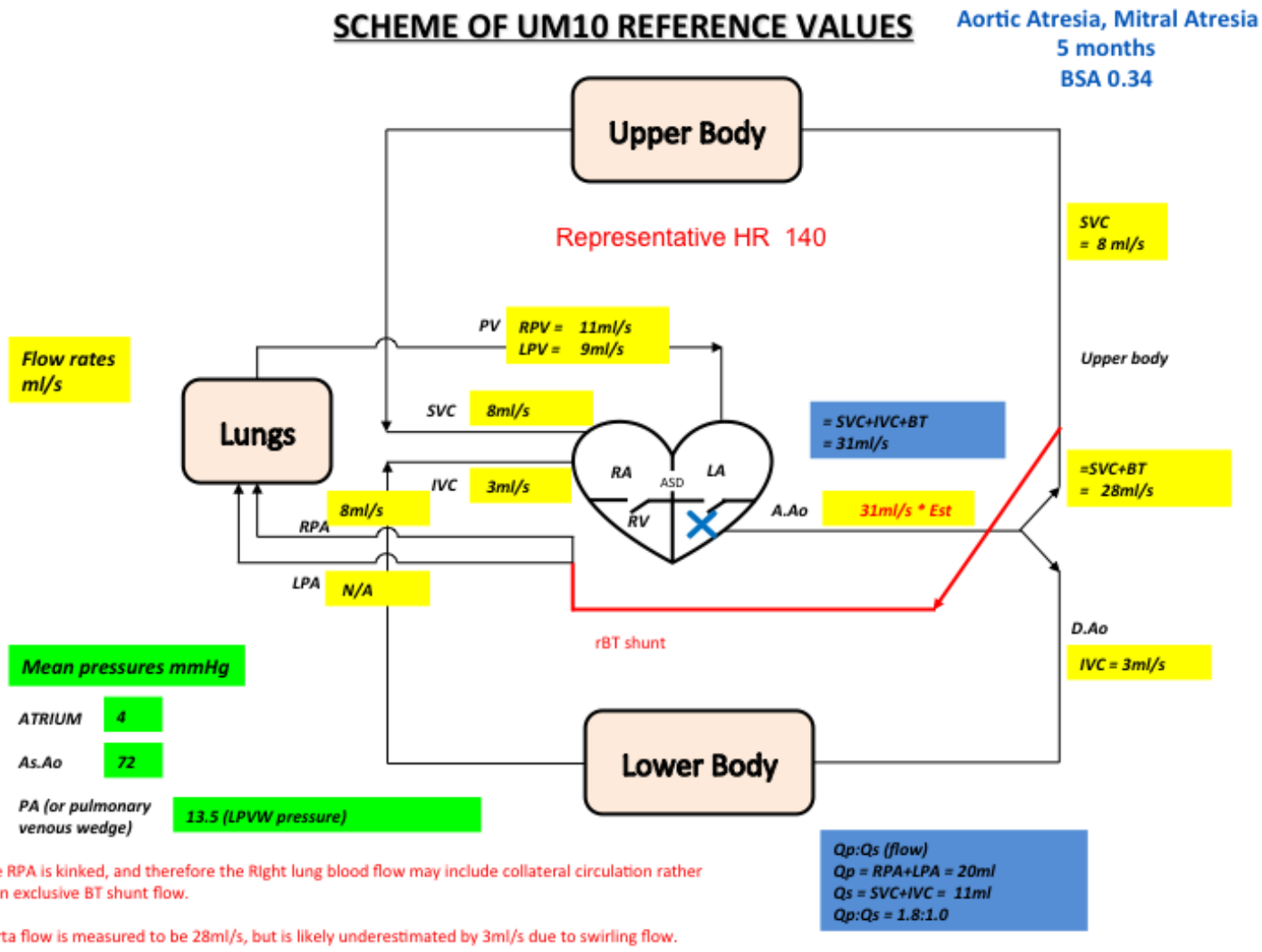
# Iterative process: tuning of total resistance

1. Initialisation of $R_i$
2. Running 3D simulation imposing resistance boundary condition
3. New $P_i^{3D}$ (then new $P_m^{3D}$), and flow $Q_i^{3D}$
4. Checking **convergence**
5. **update**

$$R_i = \frac{(P_m^t - P_a) - (P_m^{3D} - P_i^{3D})}{Q_i^t}$$

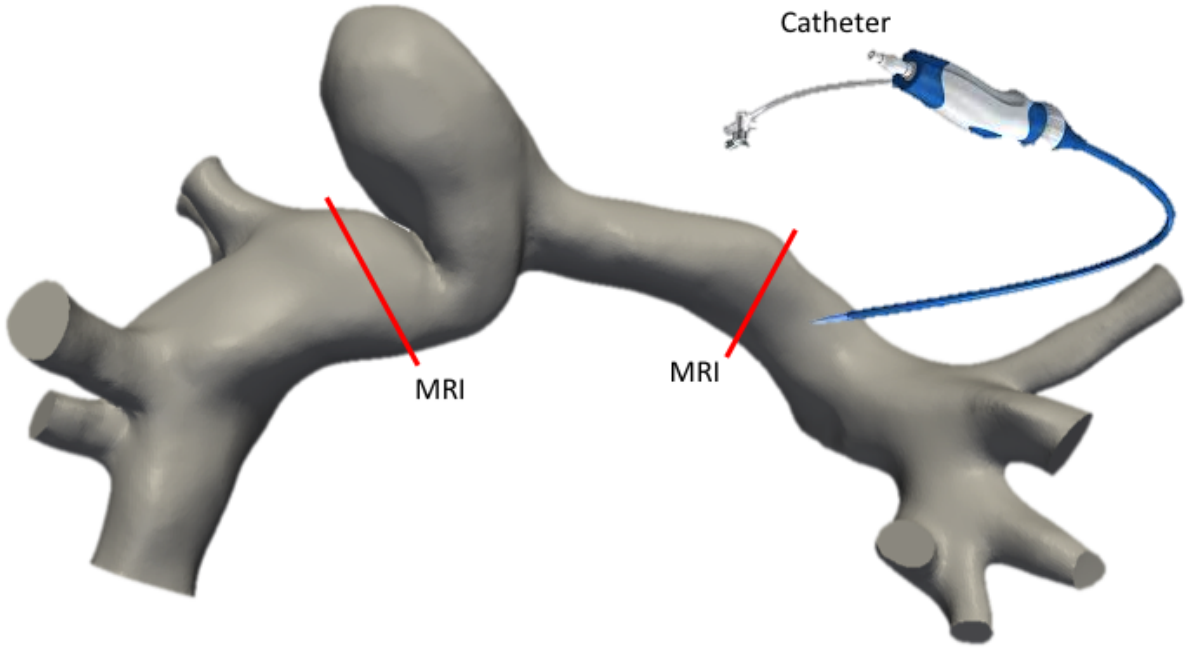
6. Go to 2 until convergence.

$\Rightarrow$ Convergence with less than 10 iterations.

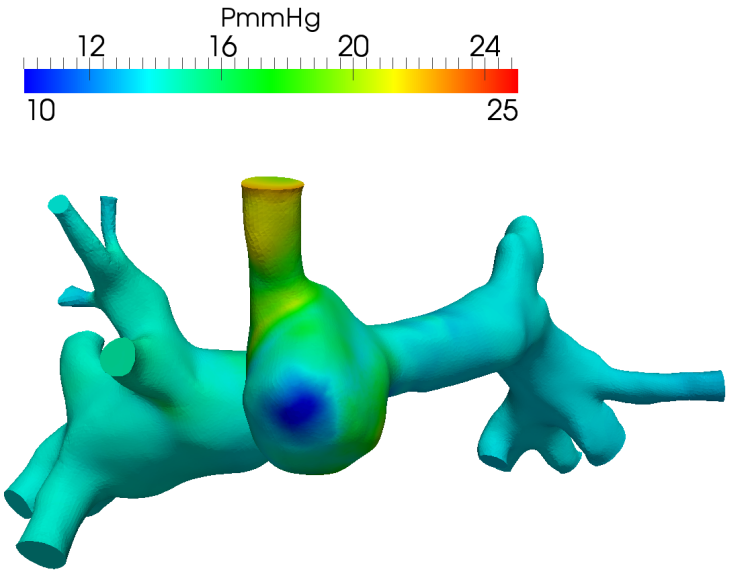


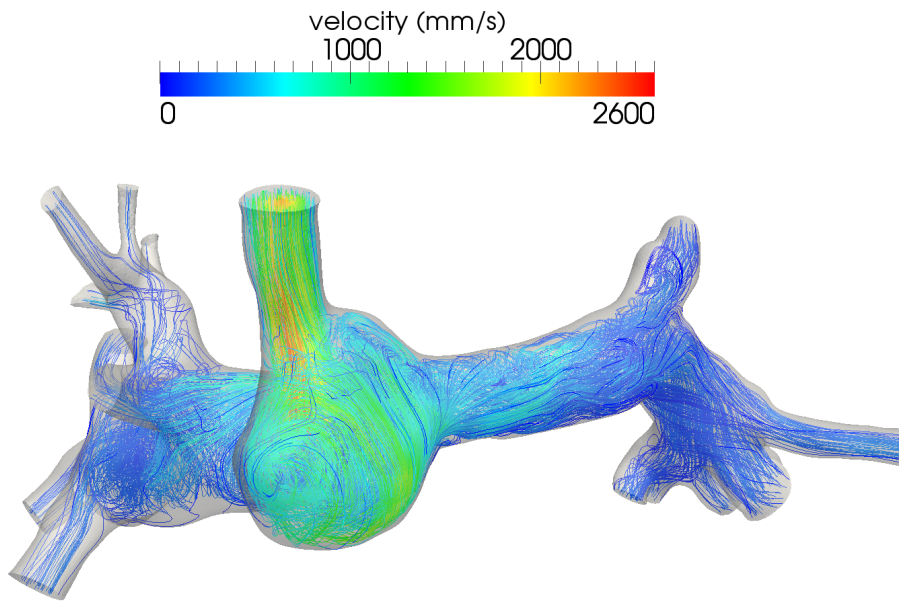

The RPA is kinked, and therefore the RIght lung blood flow may include collateral circulation rather than exclusive BT shunt flow.

Aorta flow is measured to be 28ml/s, but is likely underestimated by 3ml/s due to swirling flow.

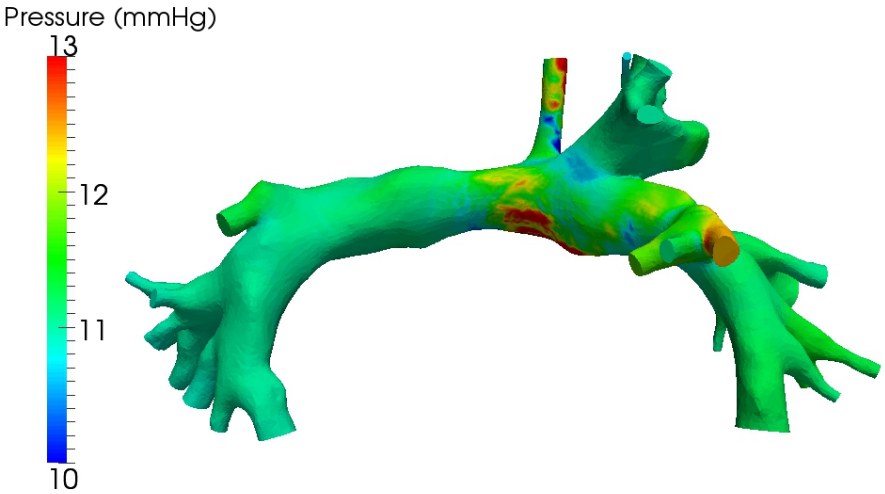


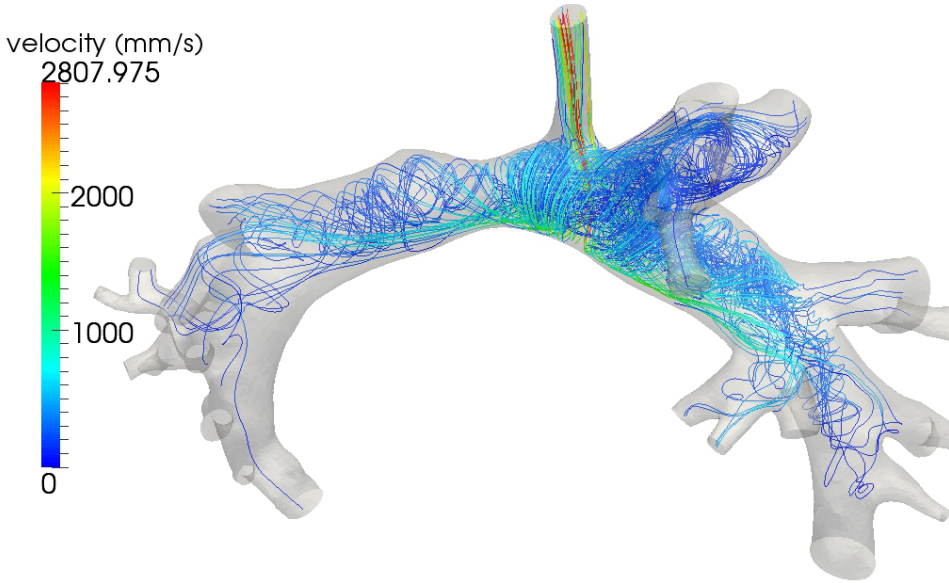

- **kink** of the RPA $\Rightarrow$ discussion with **clinicians**: **pressure drop** in the RPA is expected.



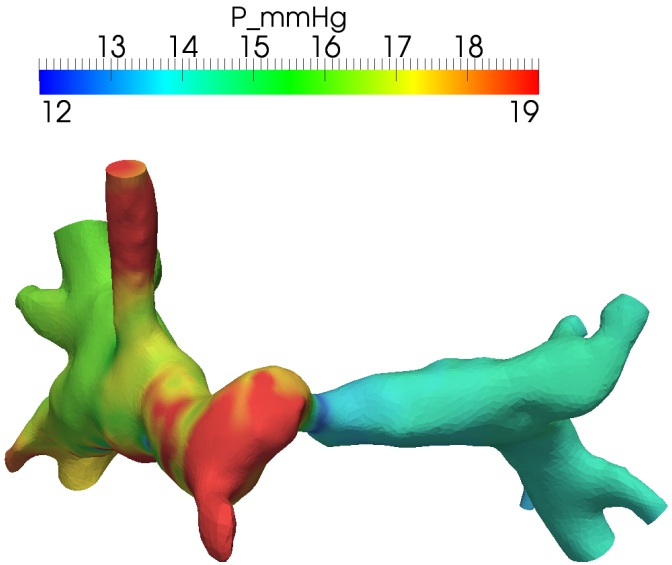



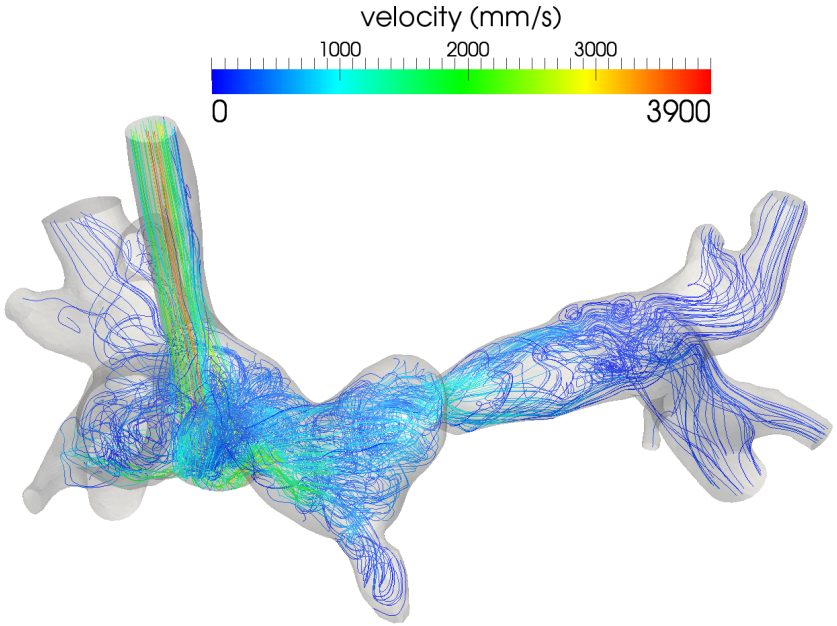

| | clinical data | numerical results |
|---|---|---|
| LPA average pressure (mmHg) | 13.5 | 13.4 |
| RPA average pressure (mmHg) | - | 13.9 |
| Pressure difference (L/R) (mmHg) | - | 0.5 |
| flow split | 0.55 | 0.55 |



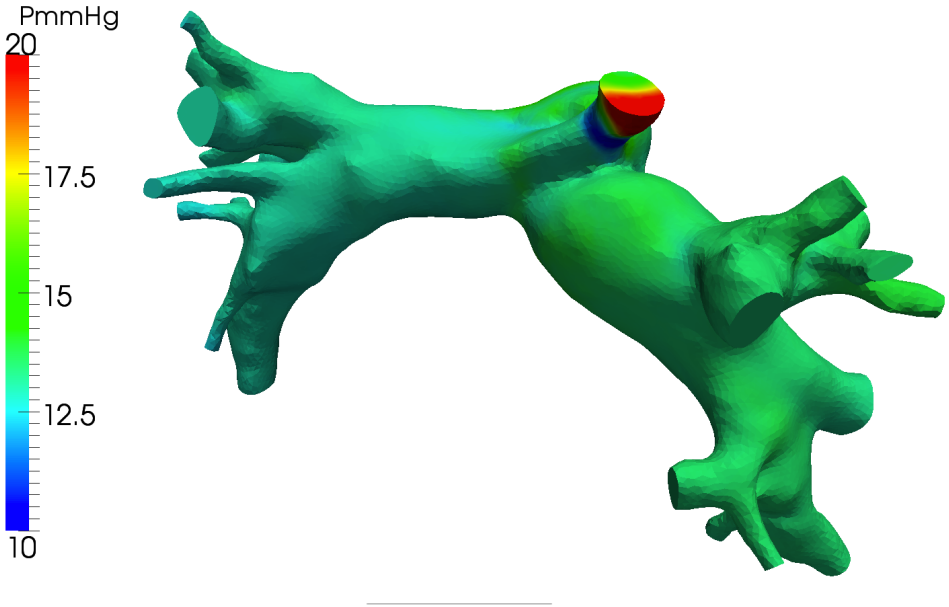



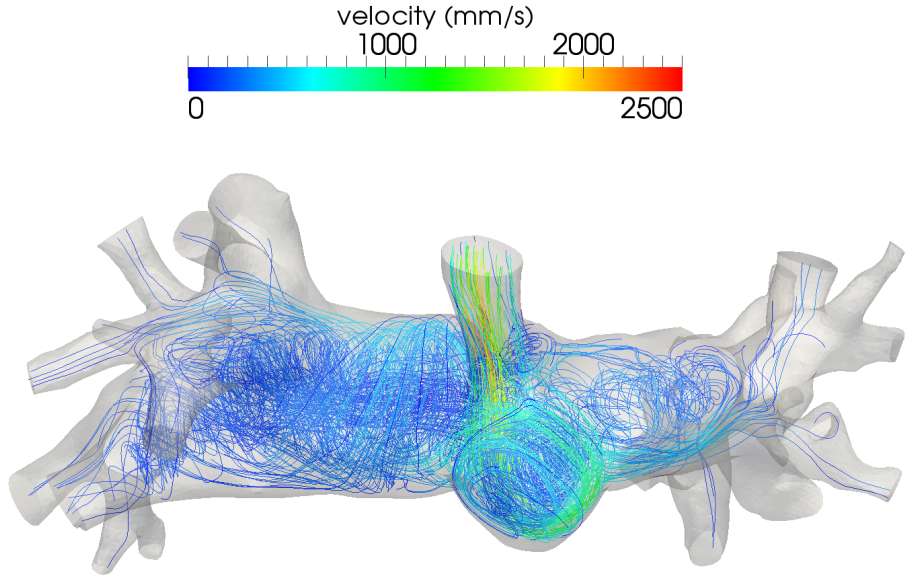

| | clinical data | numerical results |
|---|---|---|
| LPA average pressure (mmHg) | 11 | 11 |
| RPA average pressure (mmHg) | 11 | 11.2 |
| Pressure difference L/R (mmHg) | 0 | 0.2 |
| Flow split | 0.52 | 0.52 |




| | clinical data | numerical results |
|---|---|---|
| LPA average pressure (mmHg) | 14 | 14 |
| RPA average pressure (mmHg) | 17 | 17 |
| Pressure difference (L/R) (mmHg) | 3 | 3 |
| flow split | 0.66 | 0.66 |



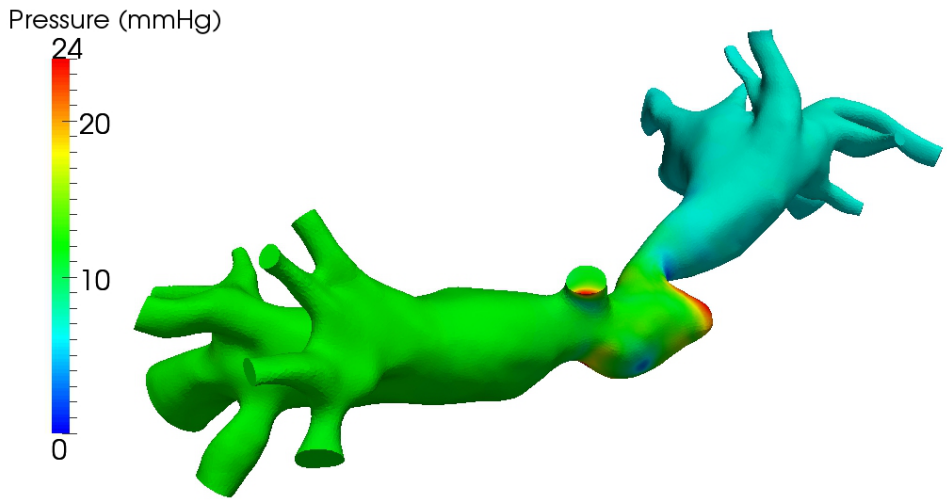




| | clinical data | numerical results |
|---|---|---|
| LPA average pressure (mmHg) | 12.7 | 12.7 |
| RPA average pressure (mmHg) | - | 13.4 |
| Pressure difference (L/R) (mmHg) | - | 0.7 |
| flow split | 0.46 | 0.46 |

# Importance of accuracy in clinical measurements

Patient with measured values such as:

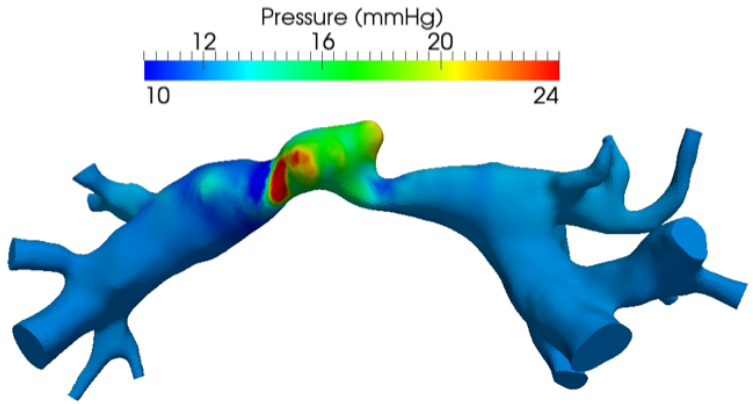
- $fs = 0.46$
- $P^t = 12$ mmHg




$\Rightarrow$ Discussion with clinicians: pressure is not coherent with at most 3mmHg difference between left and right pulmonary veins wedge pressures.

# Importance of accuracy in clinical measurements

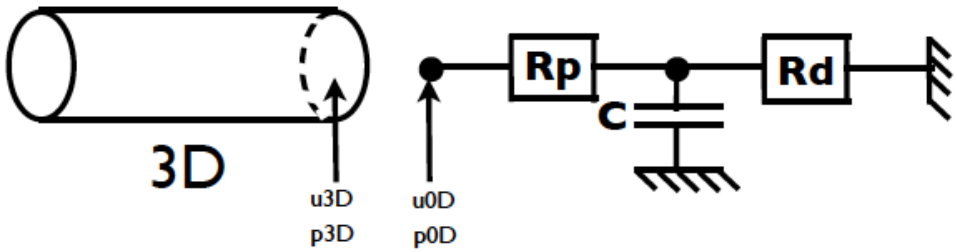
- Problem with clinical data.
- 3D simulation with 12 mmHg imposed at both sides.
- New flow split found, $fs = 0.54$ (instead of 0.46)




$\Rightarrow$ pressure distribution according to clinical point of view

RCRext

## Motivation:




$$\int_{\Sigma} u_{3D} \cdot n = q_{0D}$$

## Different ways to couple the pressure:

i) $\forall x \in \Sigma, \quad p_{3D}(x,t) = p_{0D}(t)$

ii) $\forall x \in \Sigma, \quad \sigma \cdot n(x,t) = p_{0D}(t)$

iii) $\forall x \in \Sigma, \quad \sigma \cdot n(x,t) + \frac{1}{2}\rho u^2(x,t) = p_{0D}(t)$

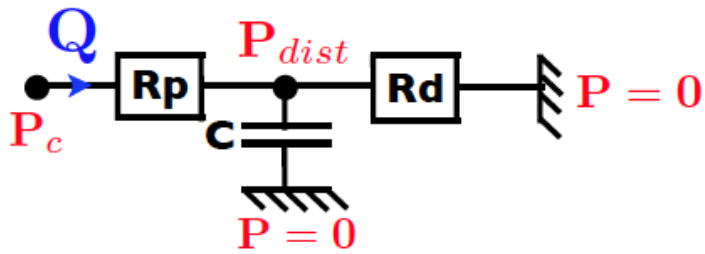
➡ Poiseuille flow: violation of this condition

# 3D-0D coupling case with divergence

RCRext

## Energy of Windkessel model:

$$\left| \begin{array}{r} \mathbf{P}_c = \mathbf{P}_{dist} + R_p \mathbf{Q} \\ c\frac{d}{dt}\mathbf{P}_{dist} = \mathbf{Q} - \frac{\mathbf{P}_{dist}}{R_d} \end{array} \right.$$




$$\boxed{\frac{c}{2}\frac{d}{dt}\mathbf{P}_{dist}^2 + \frac{1}{R_d}\mathbf{P}_{dist}^2 + R_p\mathbf{Q}^2 = P_c\mathbf{Q}}$$

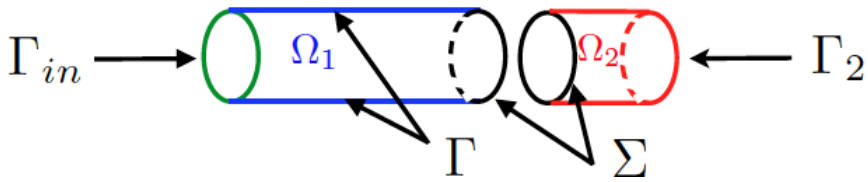
## Coupling 3D-0D:

$$\frac{d}{dt}E_c + P_{visc} = -\rho\int_\Sigma \mathbf{u}\frac{\mathbf{u}^2}{2}\cdot n + \int_\Sigma \sigma\mathbf{u}\cdot n$$

$$\frac{d}{dt}E_c + P_{visc} = P_{in} + \rho\int_{\Sigma_{out}} \frac{\mathbf{u}^2}{2}\mathbf{u}\cdot n - \int_{\Sigma_{out}} \sigma\mathbf{u}\cdot n$$


$$\boxed{\frac{d}{dt}E_c + P_{visc} + R_pQ^2 + \frac{c}{2}\frac{d}{dt}\mathbf{P}_{dist}^2 + \frac{1}{R_d}\mathbf{P}_{dist}^2 = P_{in}}$$

- Idea: adding a small 3D domain with modified Navier-Stokes model.
- Aim: get a global energy similar to Navier-Stokes + Windkessel.




- In the added 3D part

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho\mathbf{u}\nabla\mathbf{u} + \rho\frac{\mathbf{u}}{2}div(\mathbf{u}) - div(2\mu D(\mathbf{u})) + \nabla\mathbf{p} + \gamma\mathbf{u} = \mathbf{0}$$
$$\alpha\frac{\partial \mathbf{p}}{\partial t} + \beta\mathbf{p} + div(\mathbf{u}) = \mathbf{0}$$
$$\mathbf{u}_{|\Gamma_2} = \mathbf{0}$$

## 3D-0D Windkessel coupling

$$\frac{d}{dt}E_c + P_{visc} + R_p Q^2 + \frac{c}{2}\frac{d}{dt}\mathbf{P}^2_{dist} + \frac{1}{R_d}\mathbf{P}^2_{dist} = P_{in}$$

## 3D-3D modified NS coupling

$$\frac{d}{dt}E_c + P_{visc} + \gamma\int_{\Omega_2}\mathbf{u}^2 + \frac{\alpha}{2}\frac{d}{dt}\int_{\Omega_2}\mathbf{p}^2 + \beta\int_{\Omega_2}\mathbf{p}^2 = P_{in}$$

# NSM

Thank you