Coupling From The Past with Oracle Skipping

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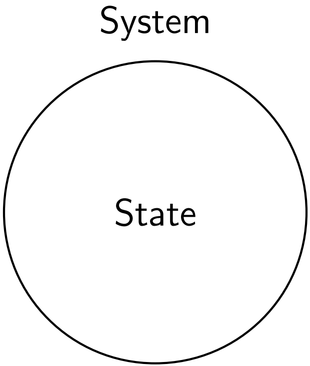
Supervisors:

Ana Bušić
Anne Bouillard

Dyogene
(Inria — ENS)

Markov Chains

Markov Chains



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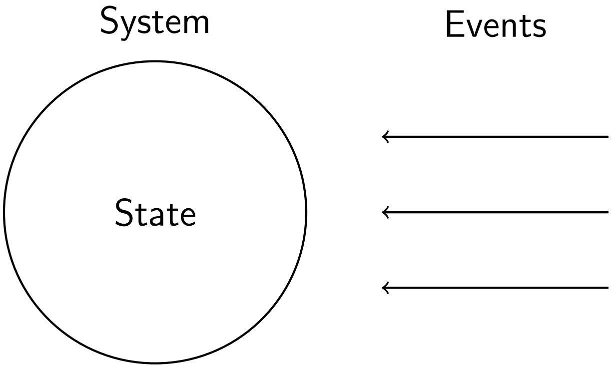

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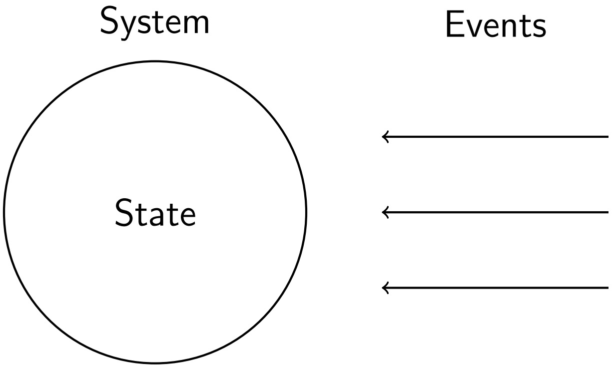


Events are independent

Markov Chains

System

Events

State: 6

Arrival

Departure

Events are independent

Markov Chains

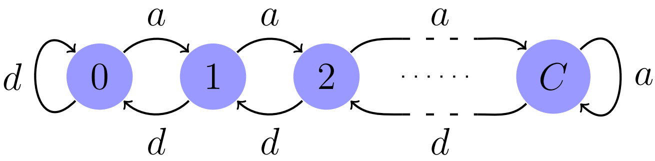
System

State: 6

Events

Arrival

Departure



Markov Chains

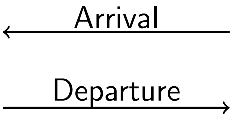
Arrival

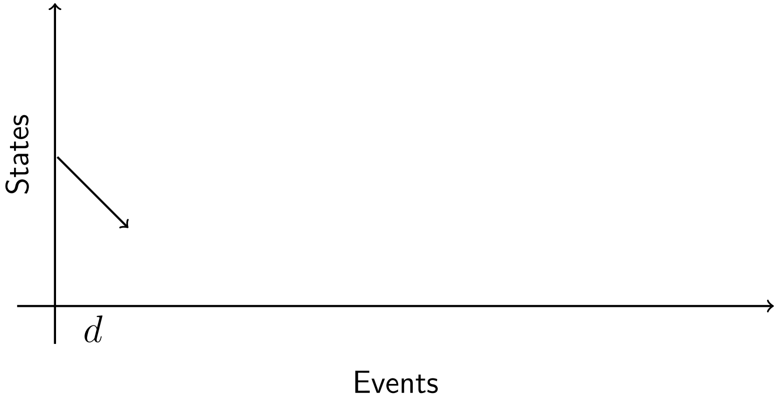
Departure

States

Events

Markov Chains

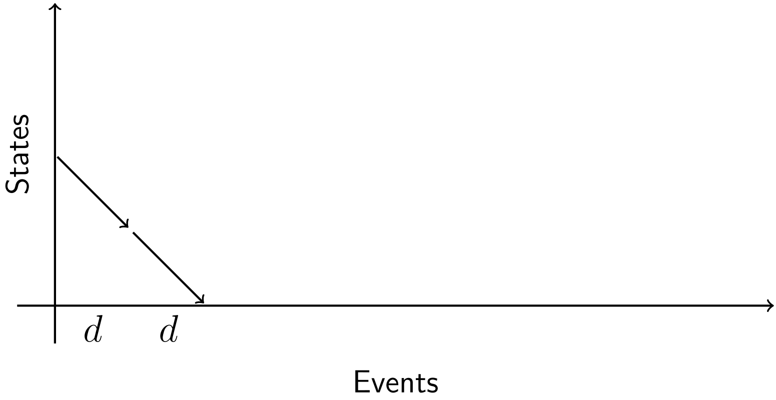




Markov Chains

Arrival

Departure



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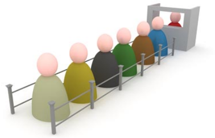

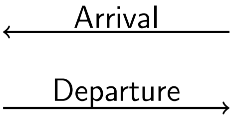

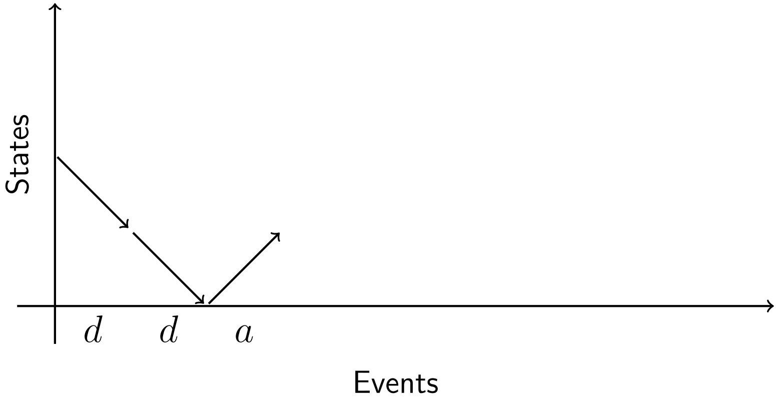

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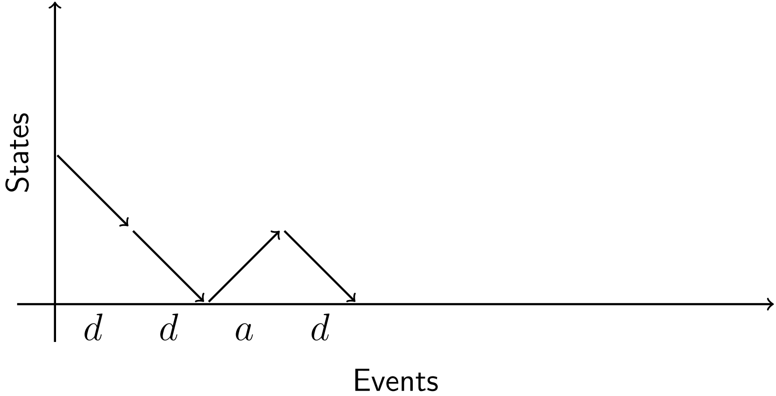

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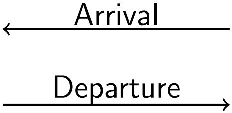

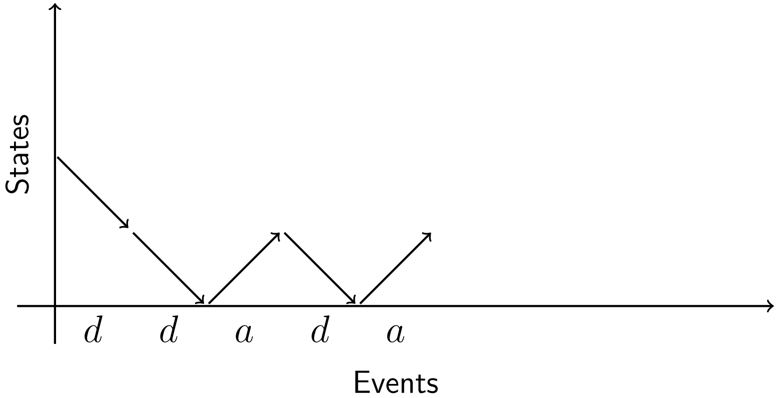

Markov Chains

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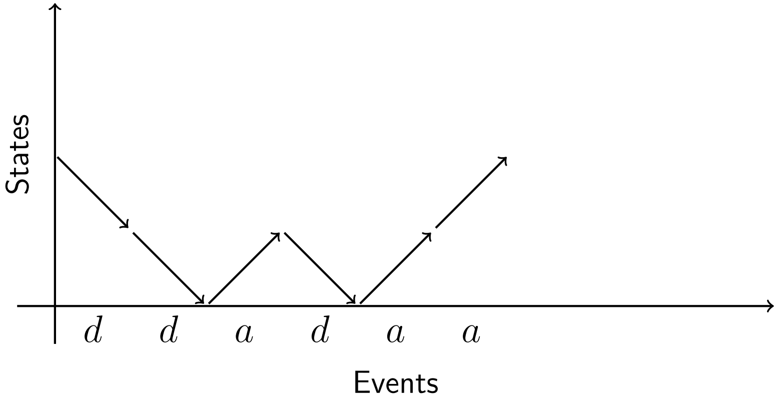

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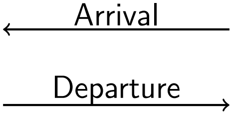

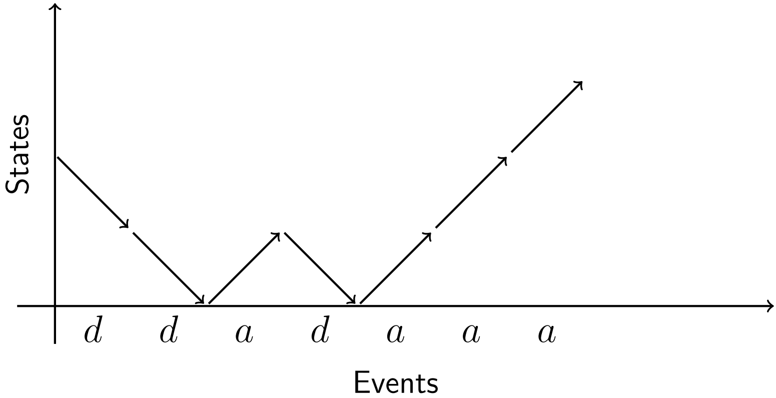

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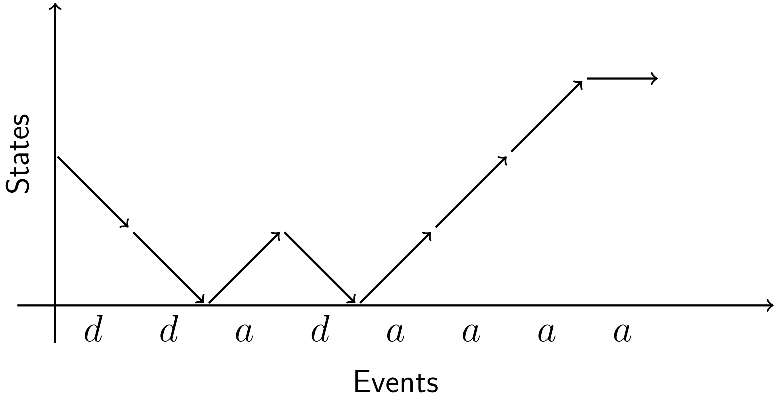

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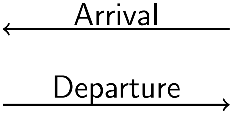
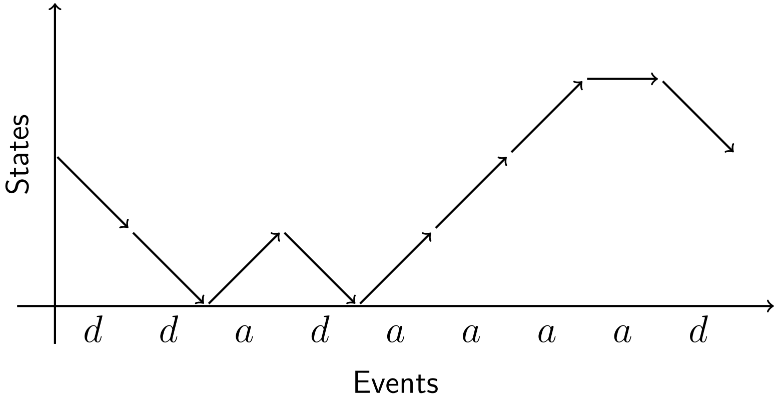
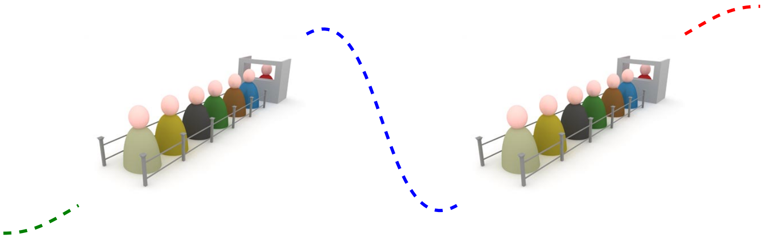




Markov Chains

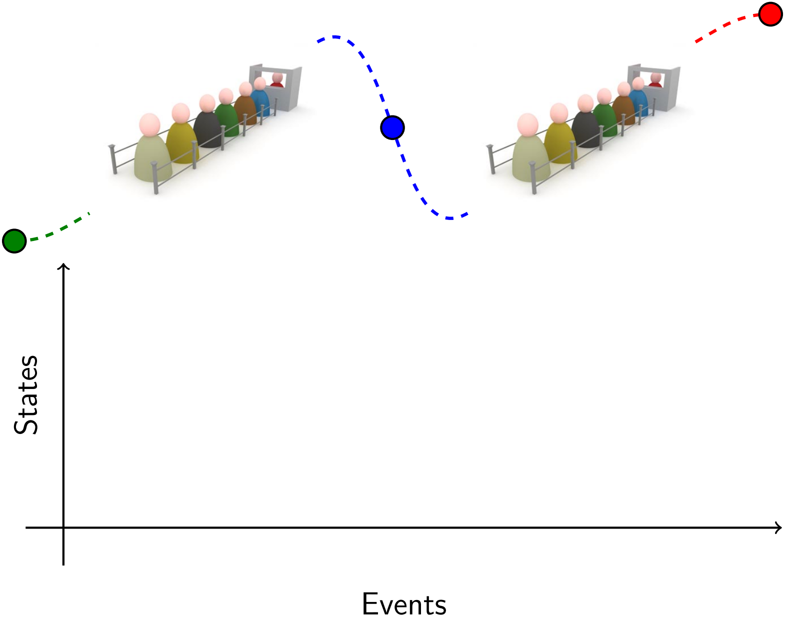
System



Events

Arrival Transfer Departure

Markov Chains



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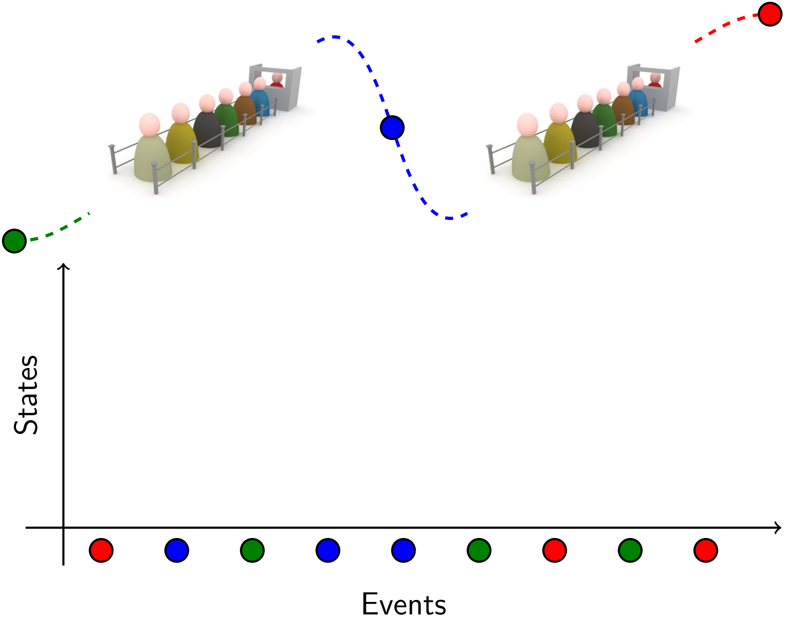

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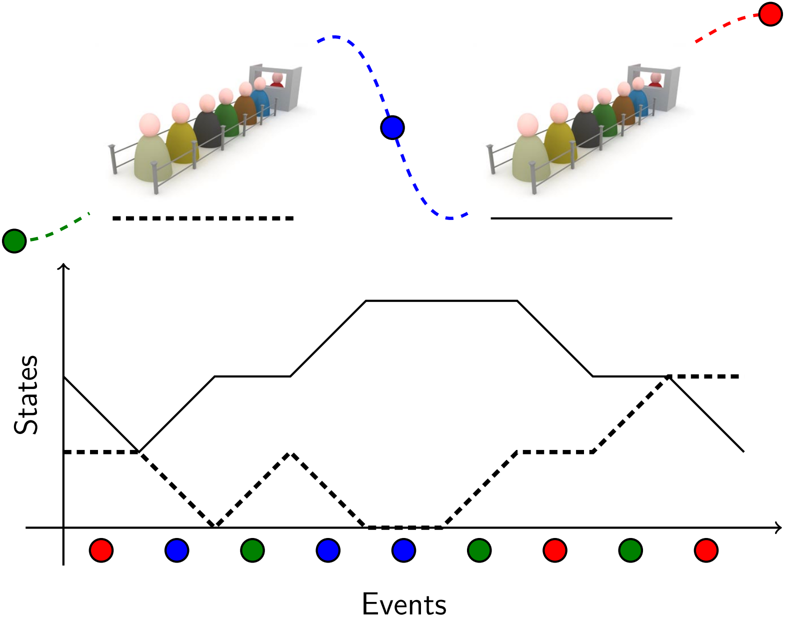
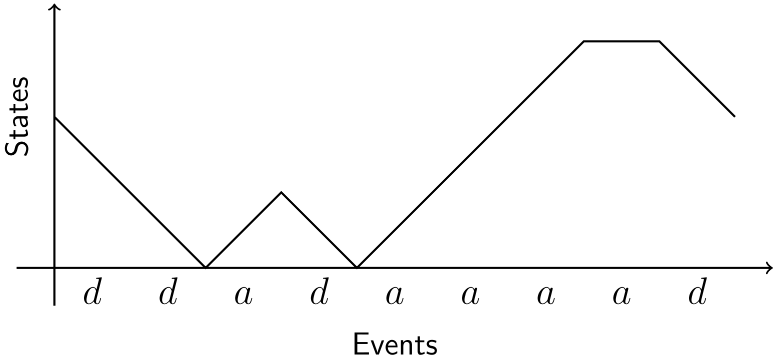


Applications

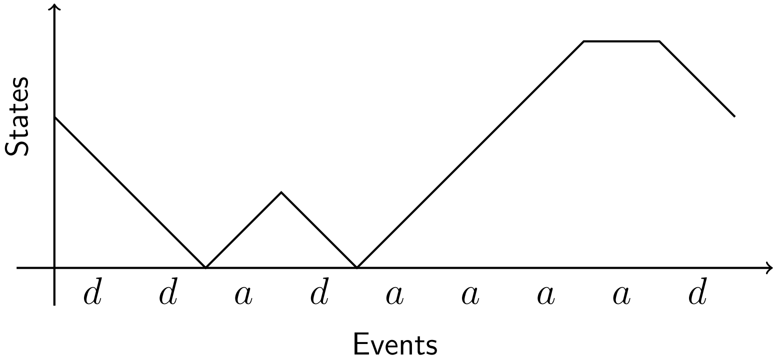
- Network Analysis
  - Delay estimation
  - Congestion estimation
  - Loss estimation
- Biology
  - Population evolution
  - Epidemics
- Physics
  - Microscopic gas models

Stationary Distribution



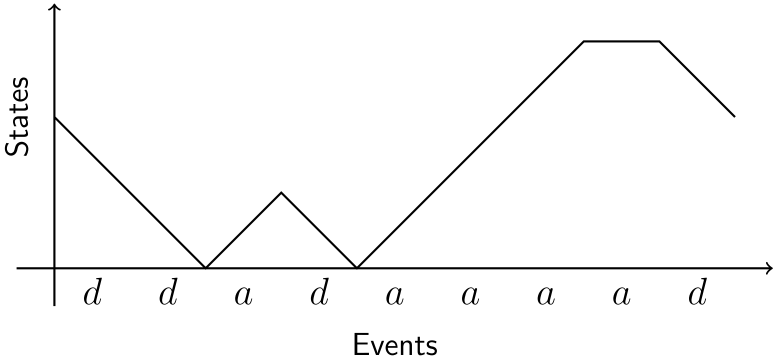
Stationary Distribution

- System is in equilibrium



Stationary Distribution

- System is in equilibrium
- Bias due to initial state "forgotten"



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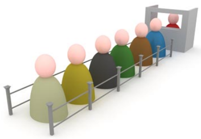


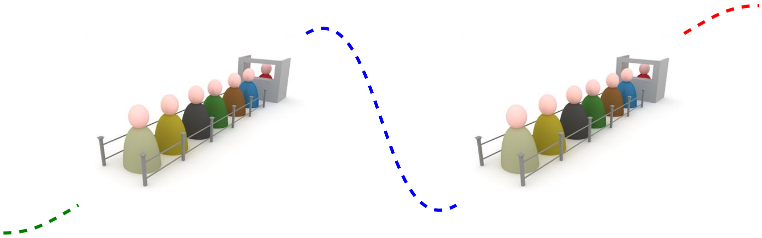
- System is in equilibrium
- Bias due to initial state "forgotten"

$$\mathbf{P}\left(\text{arrival}\right) = \mathbf{P}\left(\text{departure}\right) = \tfrac{1}{2}$$

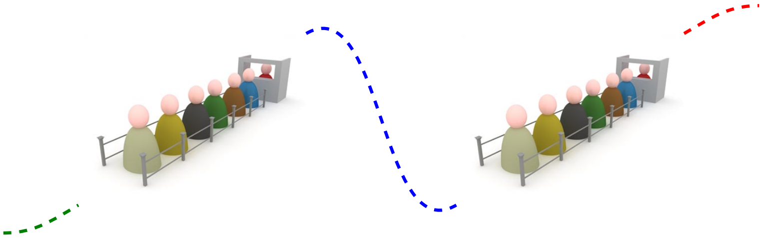
$$\forall k,\ p_t\left(k\right) = \tfrac{1}{C+1} \ \Rightarrow\ \forall k,\ p_{t+1}\left(k\right) = \tfrac{1}{C+1}$$

The stationary distribution is the uniform distribution

Stationary Distribution

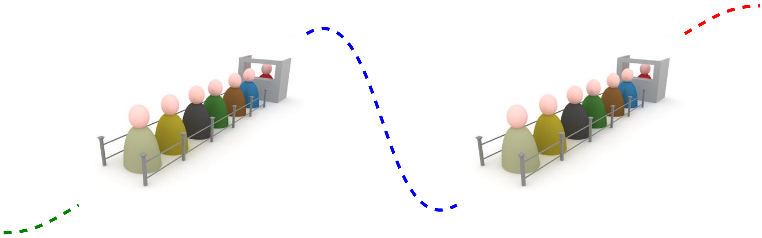


Stationary Distribution



The stationary distribution is very hard to compute

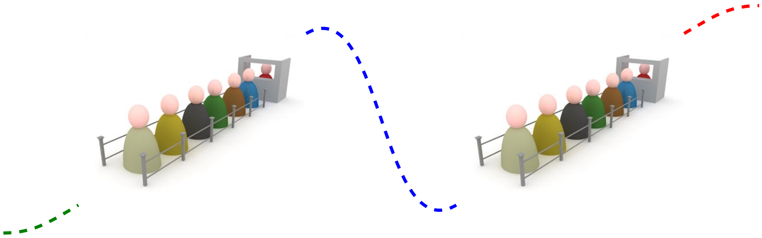
Stationary Distribution



The stationary distribution is very hard to compute

$$\mathbf{E}\left[\text{number of clients}\right] = ?$$
$$\mathbf{E}\left[\text{end-to-end delay}\right] = ?$$
$$\mathbf{P}\left\{\text{refusing a client}\right\} = ?$$

Stationary Distribution



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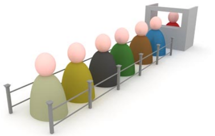
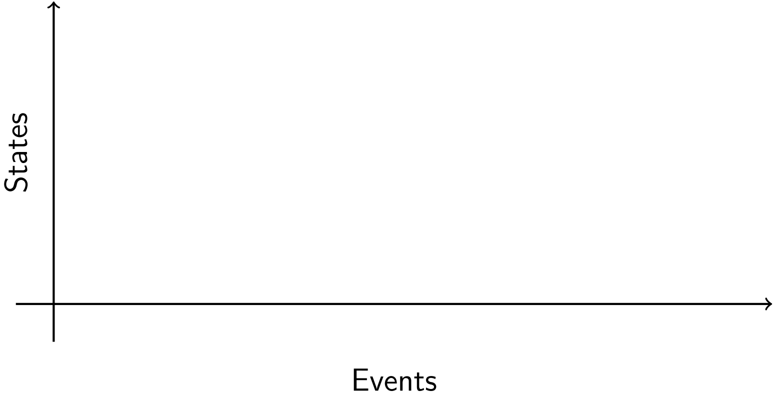
It is enough to be able to generate random samples

Monte Carlo Markov Chains





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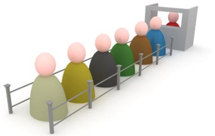
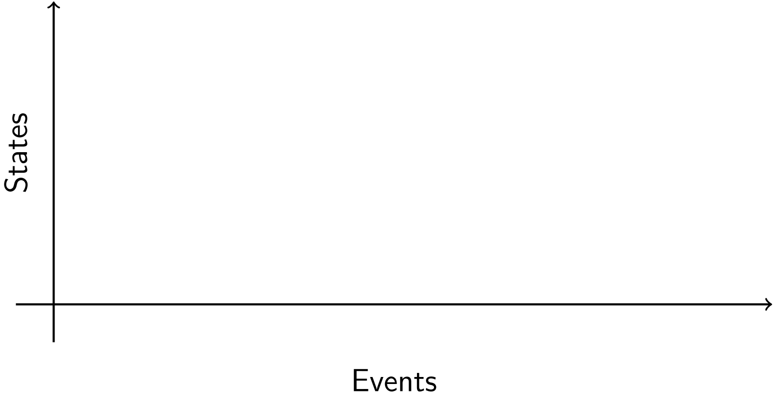


- Markov chains converge towards their stationary distribution



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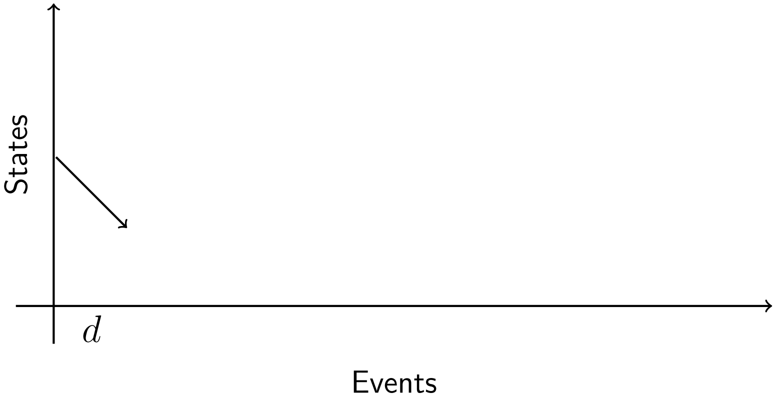

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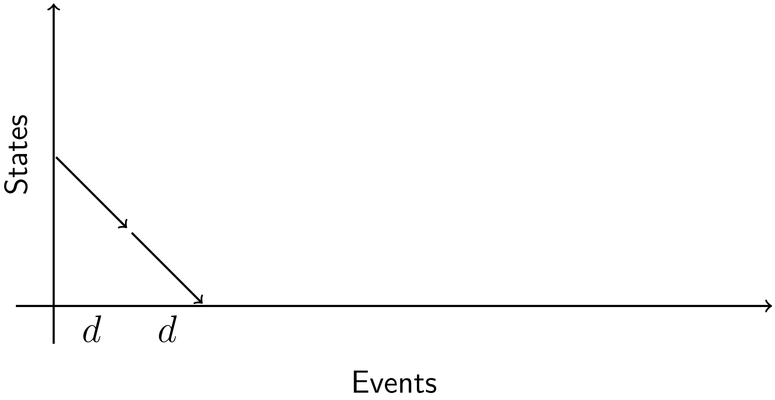

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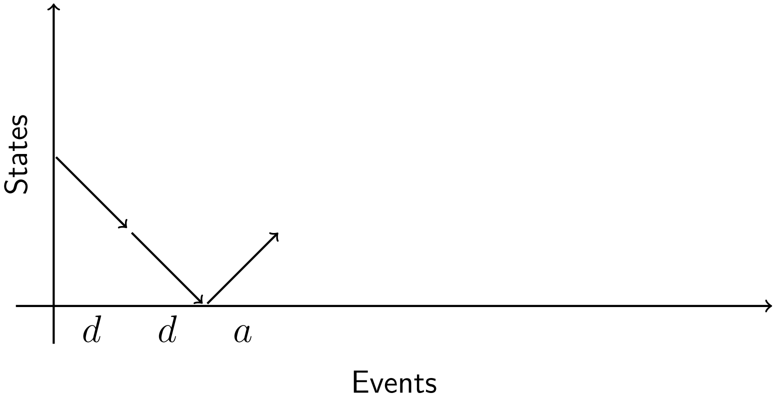

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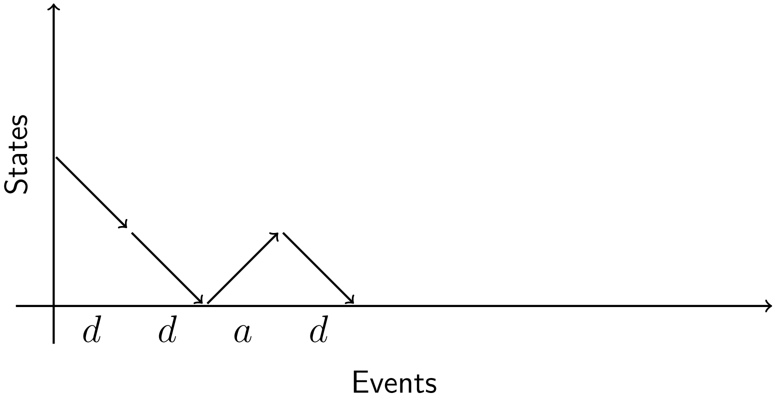

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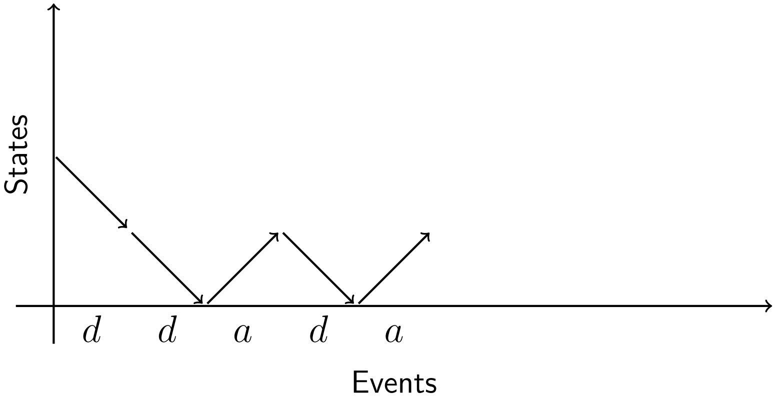

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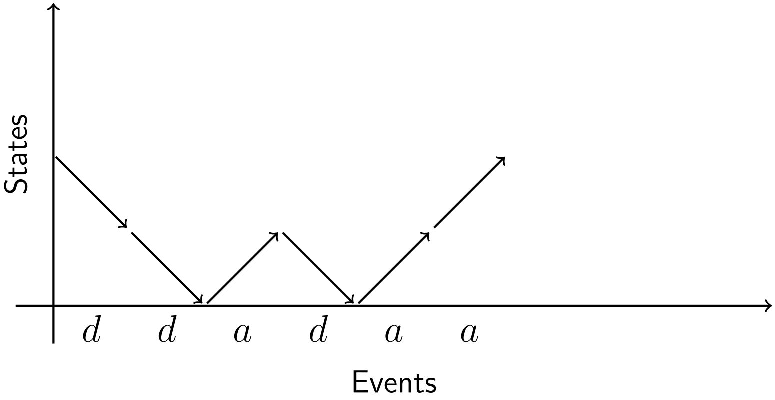

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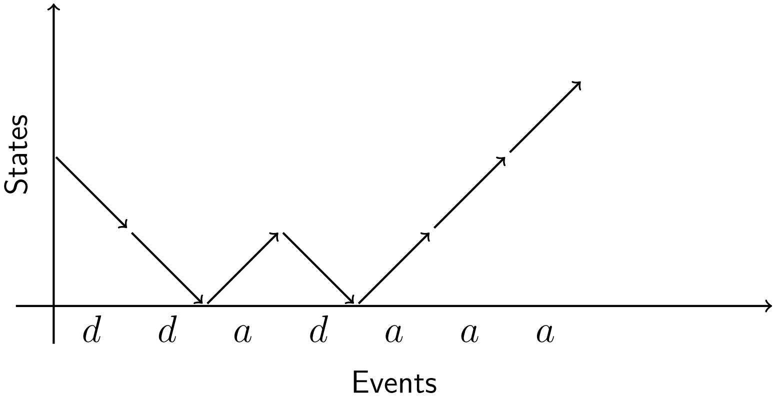

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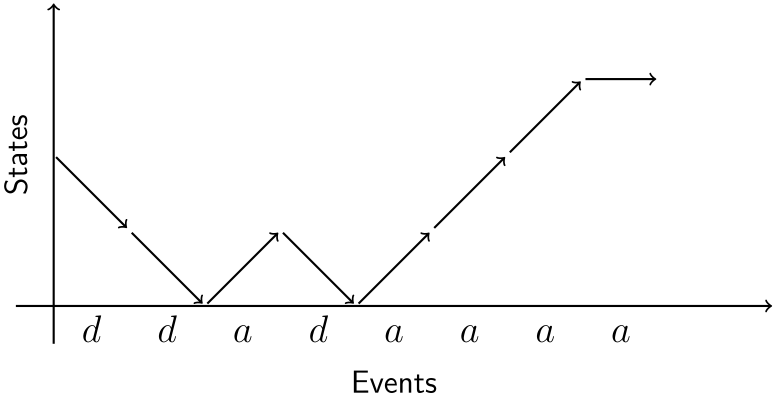

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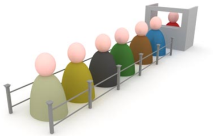

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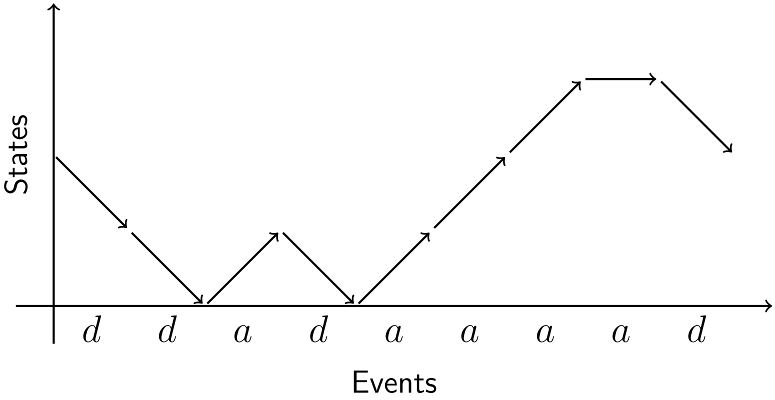
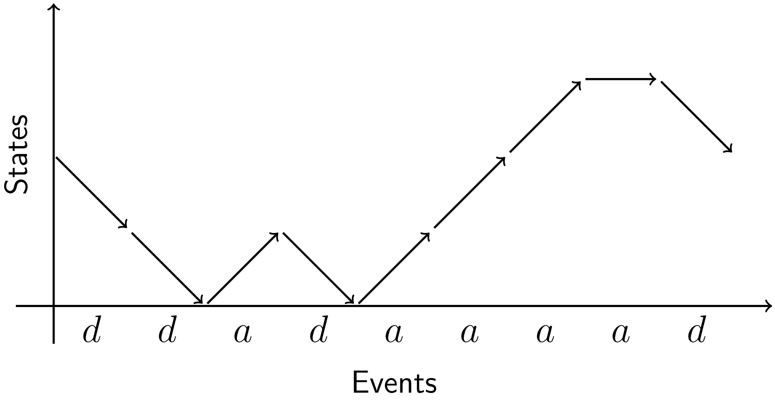


Monte Carlo Markov Chains

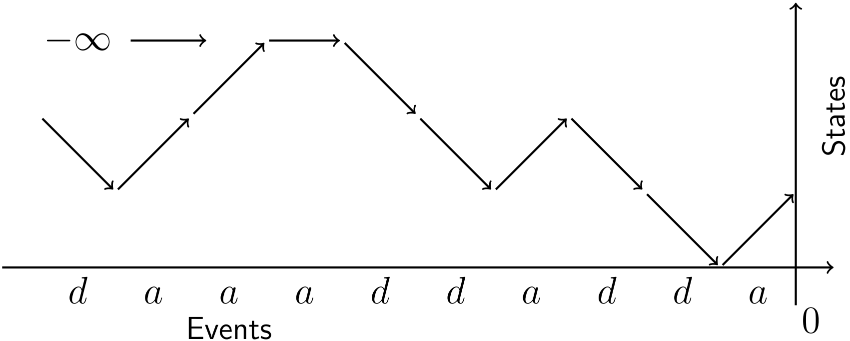
- Markov chains converge towards their stationary distribution
- How long should we wait ?



Coupling from the Past

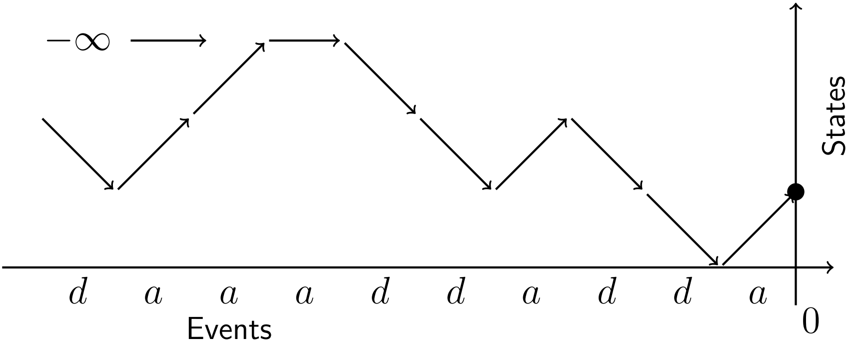
Coupling from the Past

- Run a Markov chain from $-\infty$



Coupling from the Past

- Run a Markov chain from $-\infty$
- The state at time 0 has the right distribution



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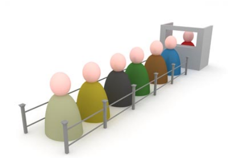

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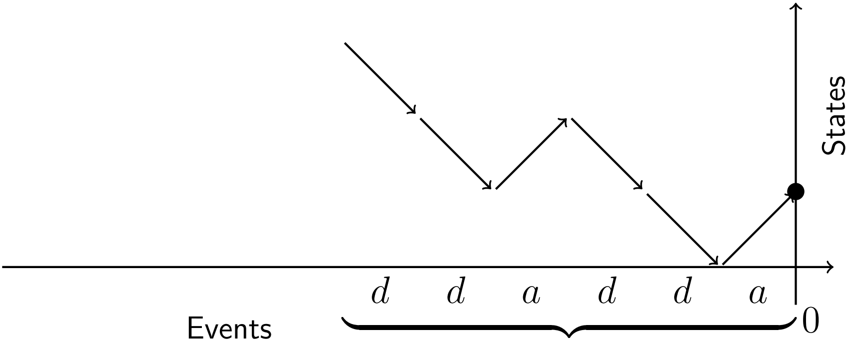

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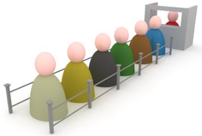

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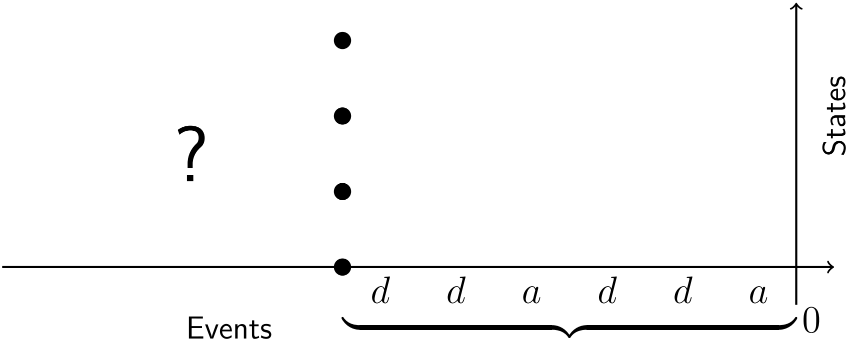

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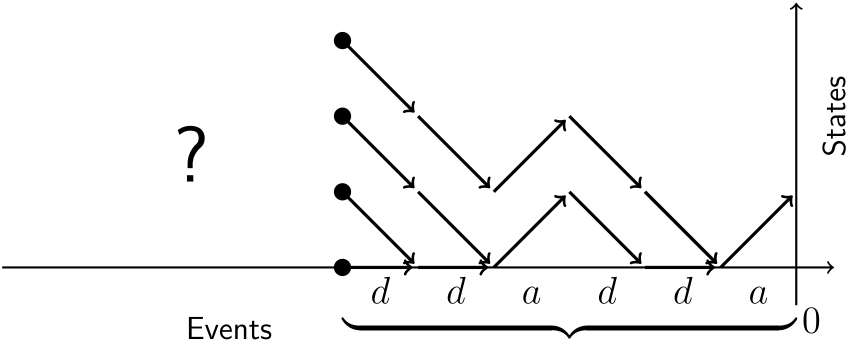

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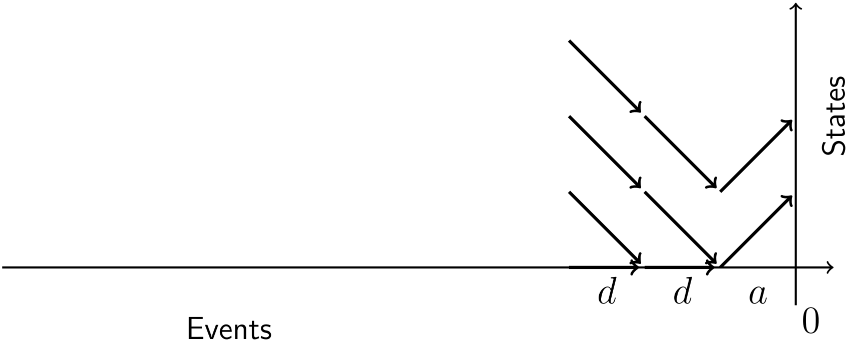

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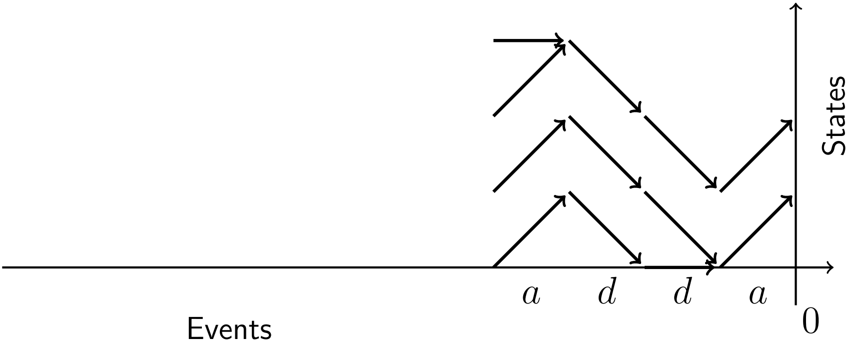

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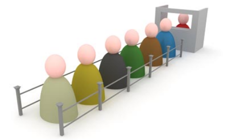

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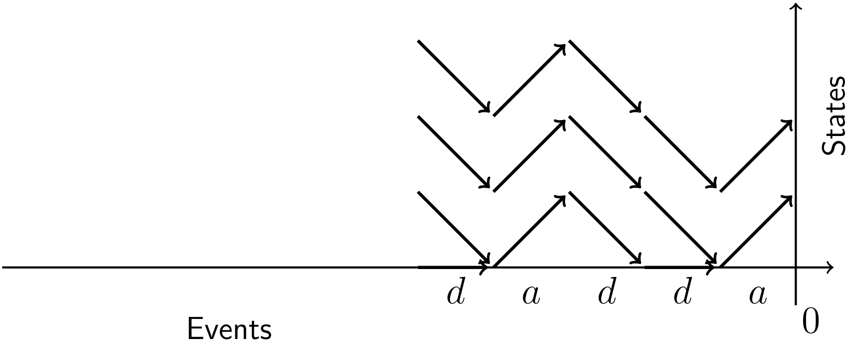

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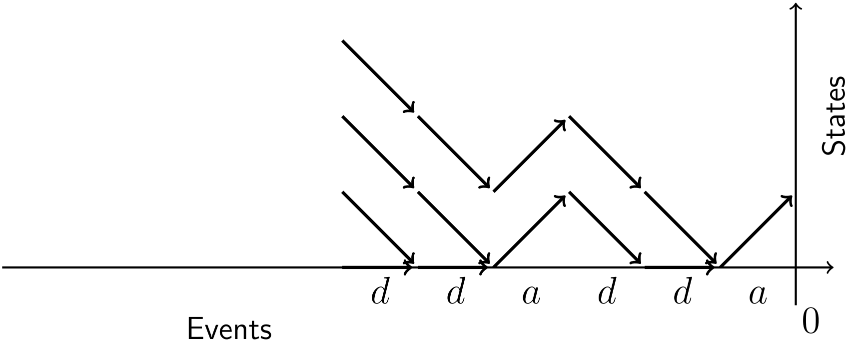

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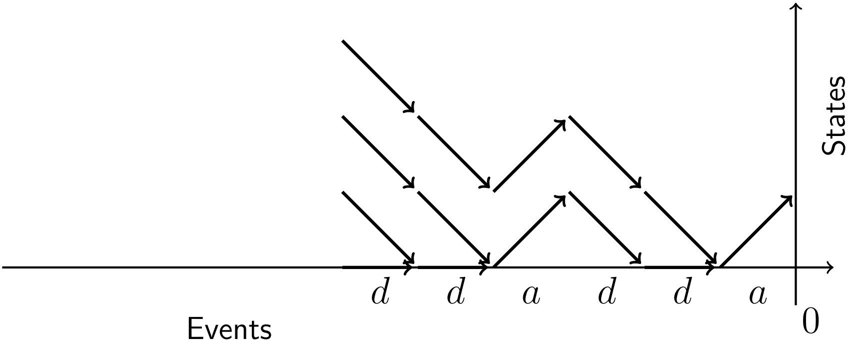
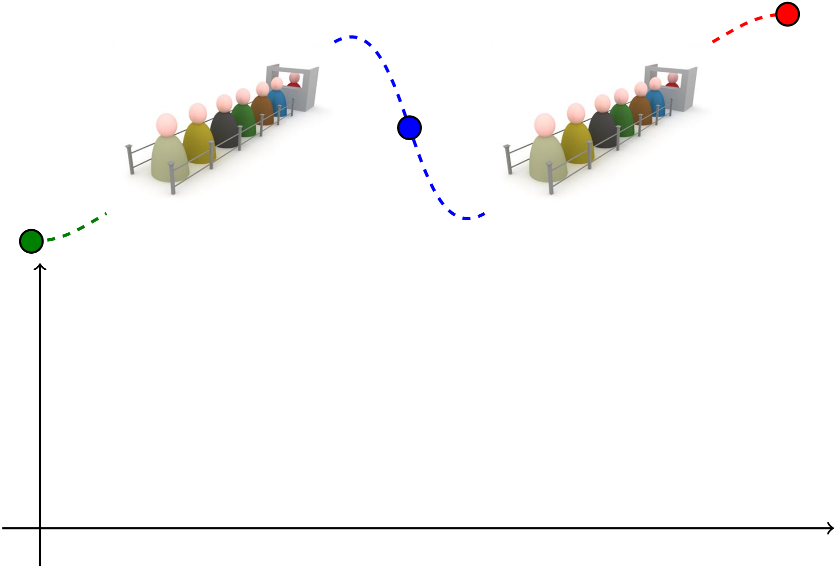


Propp and Wilson (1996)

Contribution



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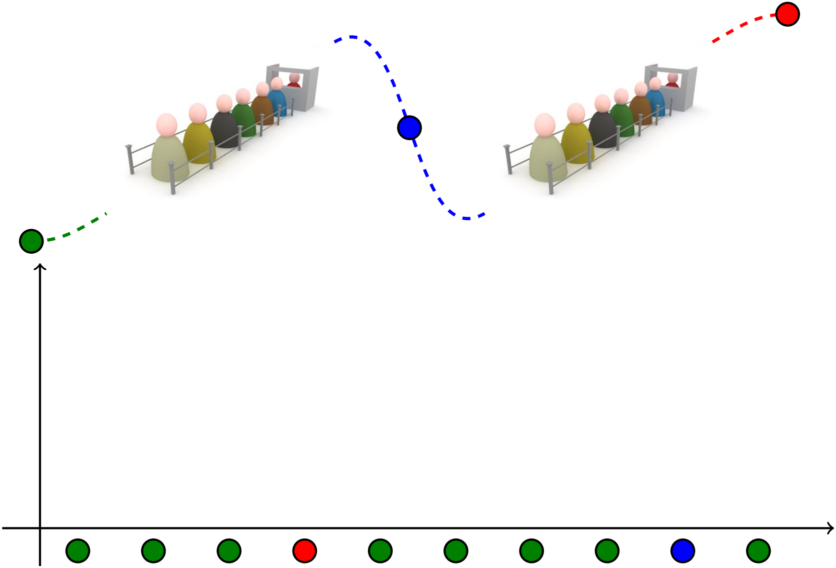

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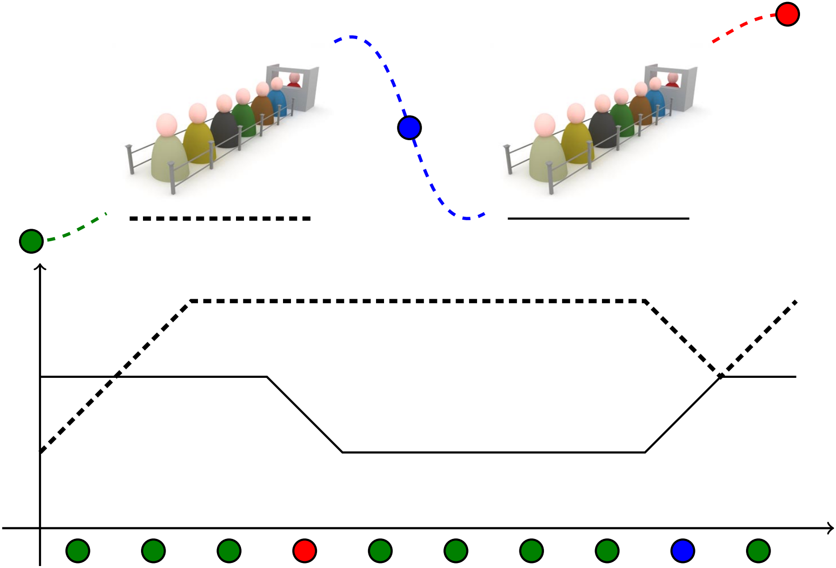

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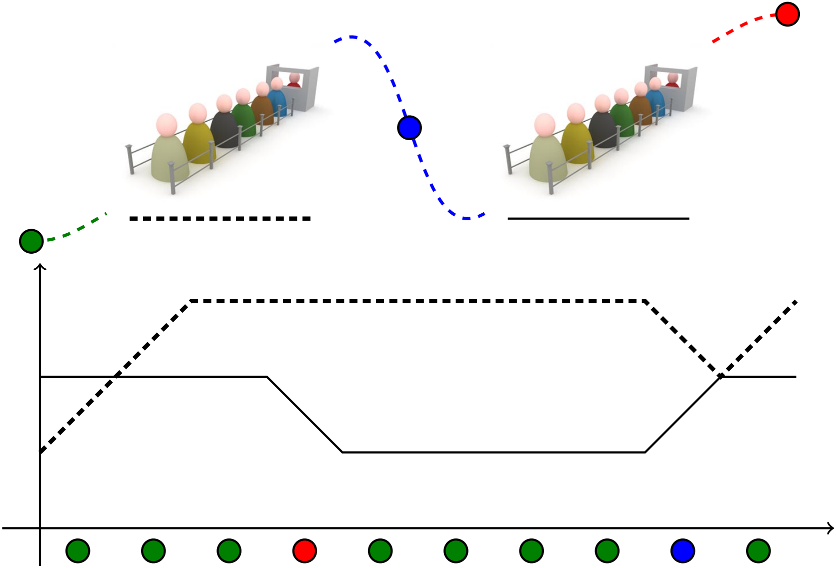
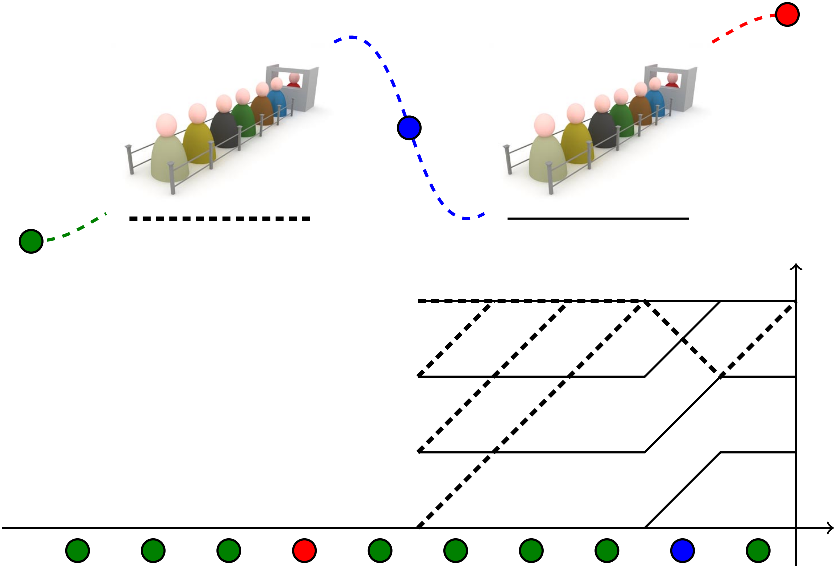


Skipping events changes the stationary distribution

Contribution



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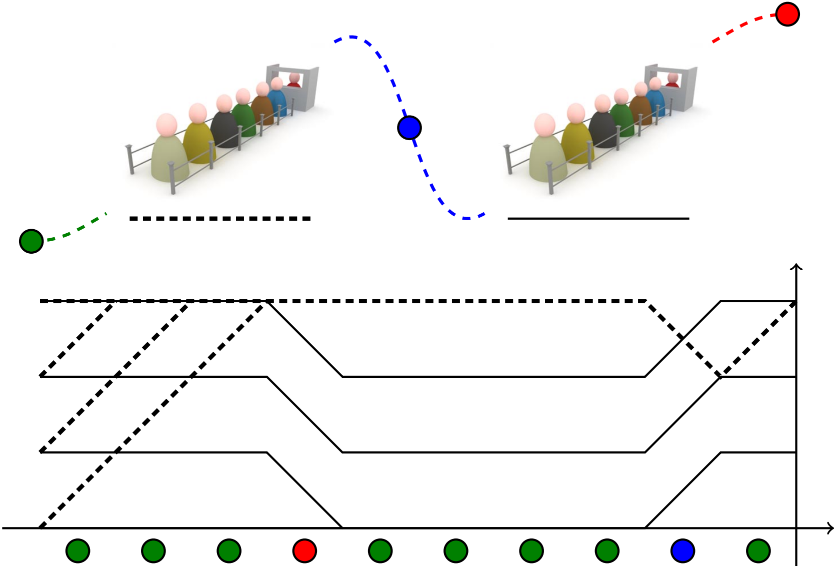
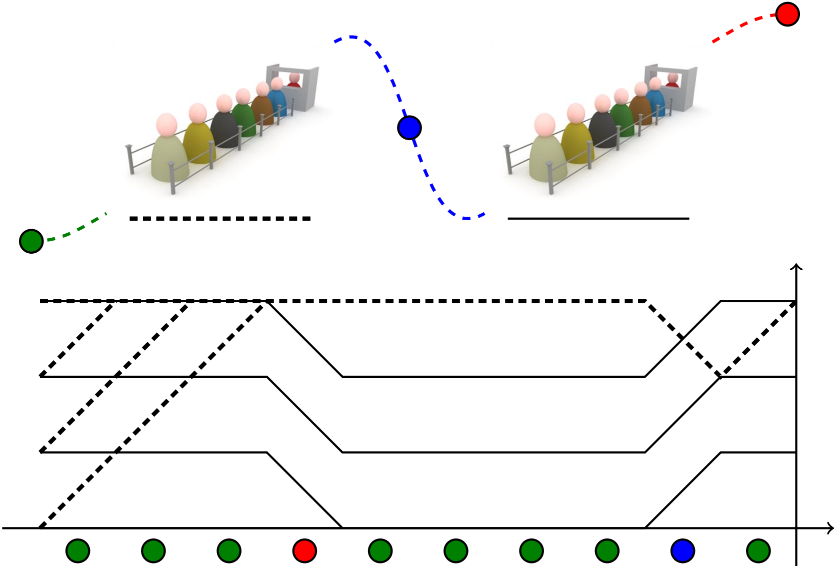
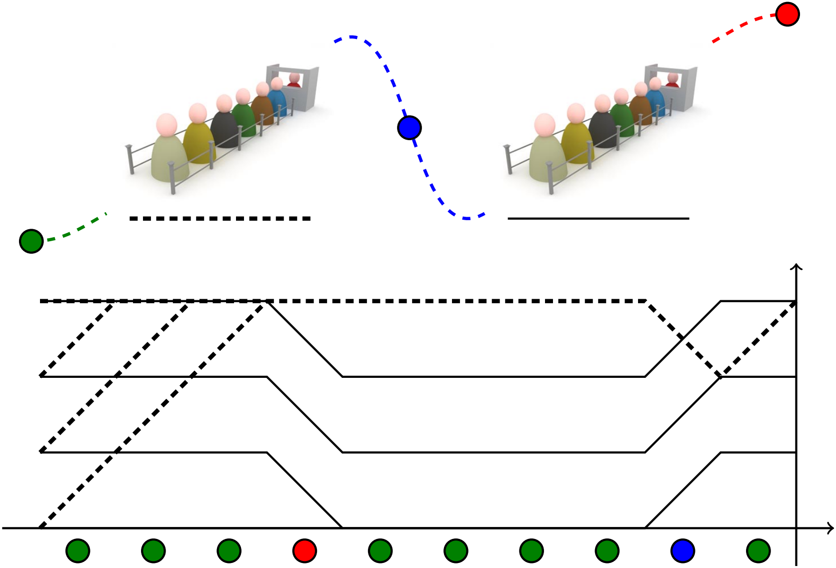


Contribution



Events must be dynamically removed

Contribution



Some deleted events must be reinserted

Questions?