An asymptotic model for computing the scattering of waves by small heterogeneities in the time domain

Simon Marmorat

EPI POems INRIA Saclay / CEA List

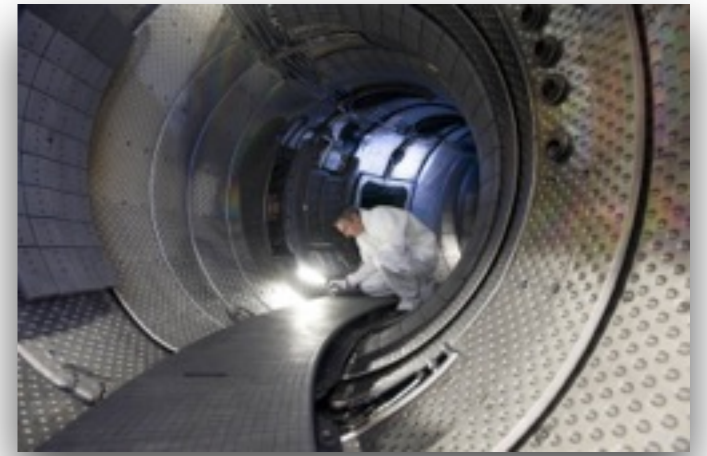
Joint work with Patrick Joly (EPI POems INRIA/ENSTA/CNRS)
Xavier Claeys (LJLL, Paris)

Motivation and context

« *Non-destructive testing* (*NDT*) *is a wide group of analysis techniques used in science and industry to evaluate the properties of a material, component or system without causing damage*. » Wikipedia

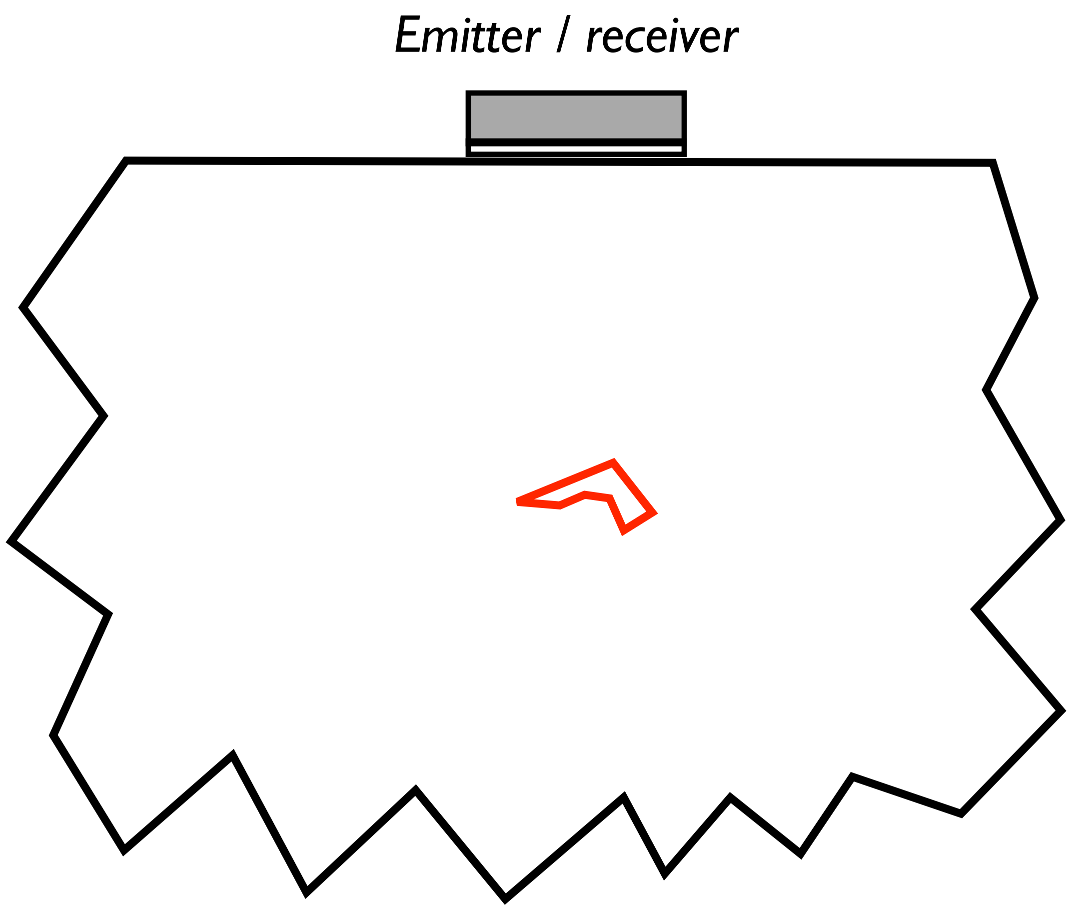
Applications:

- Medicine (ultrasonography, EEG, …)
- Geophysics (ground exploration)
- Industrial maintenance (planes, nuclear plants, …)



Non destructive testing

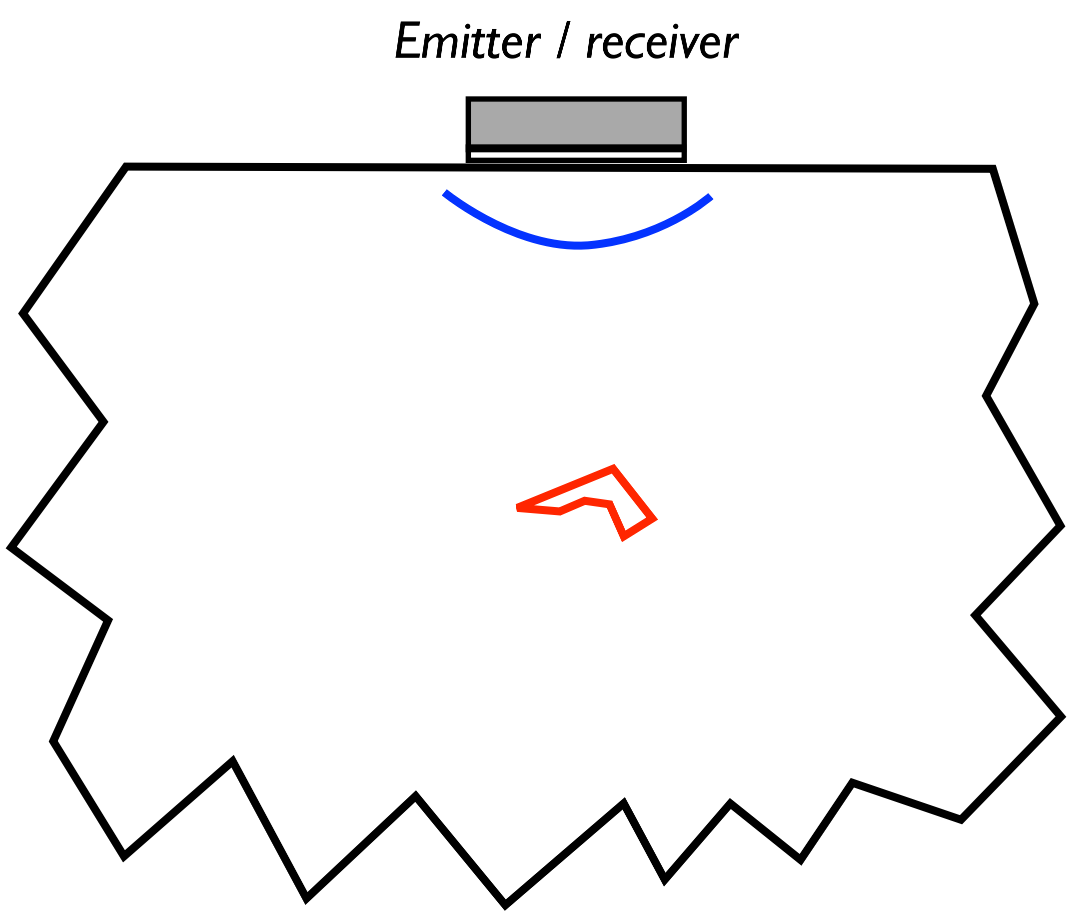
Study ultrasonic wave propagation with applications to Non-Destructive Testing.



From the knowledge of the incident and the scattered waves, recover information (shape, size, physical properties, …) on the object of interest.

Non destructive testing

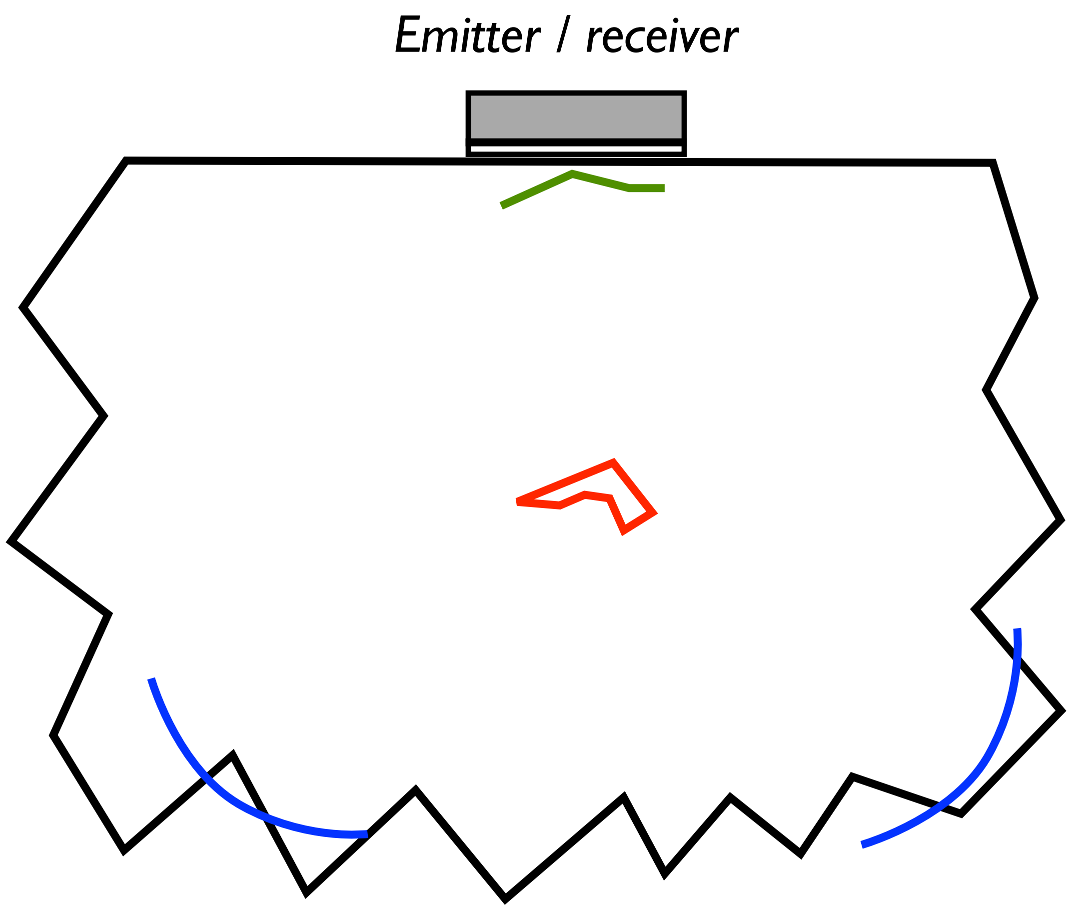
Study ultrasonic wave propagation with applications to Non-Destructive Testing.



From the knowledge of the incident and the scattered waves, recover information (shape, size, physical properties, …) on the object of interest.

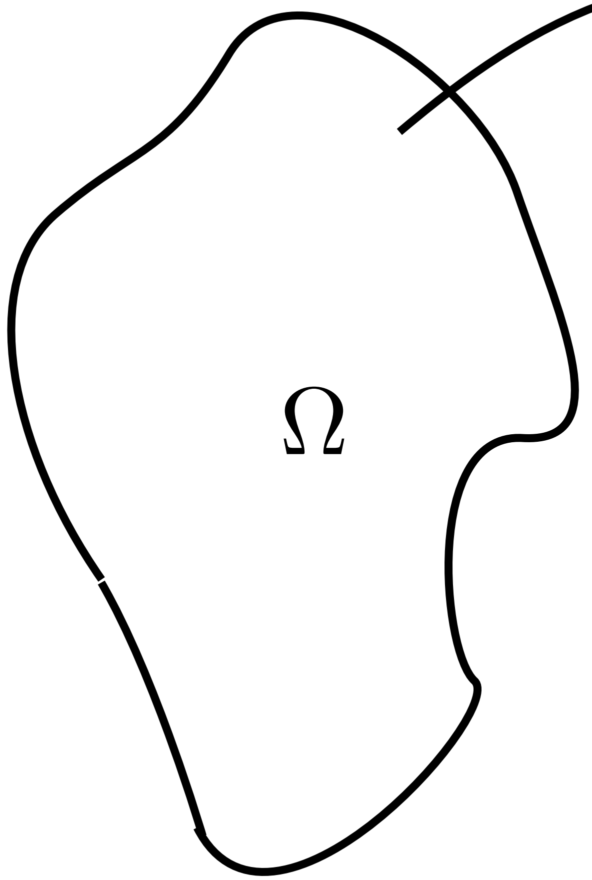
Non destructive testing

Study ultrasonic wave propagation with applications to
Non-Destructive Testing.



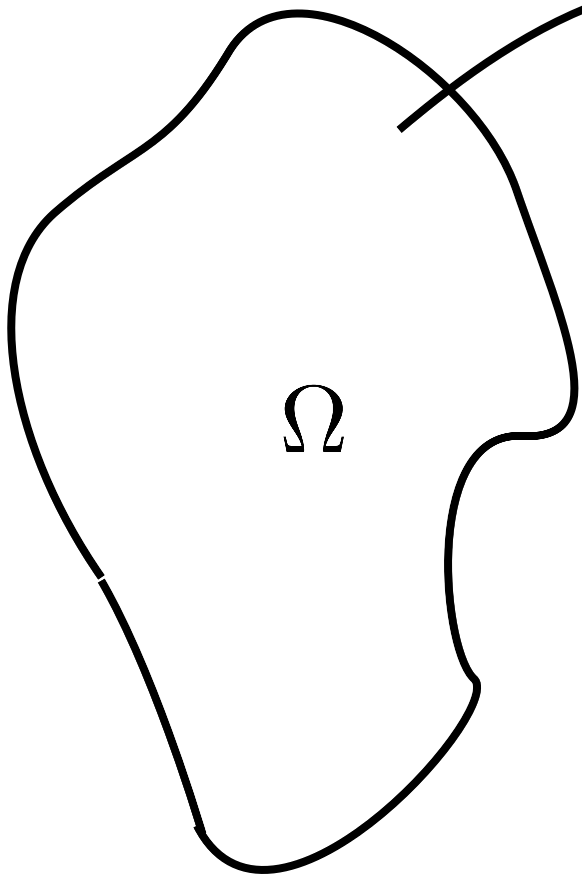
From the knowledge of the incident and the scattered waves, recover information (shape, size, physical properties, ...) on the object of interest.

What is Numerical Analysis ?



Equation modeling some physical phenomenon occurring on Ω, with unknown $u(x,t)$

What is Numerical Analysis ?

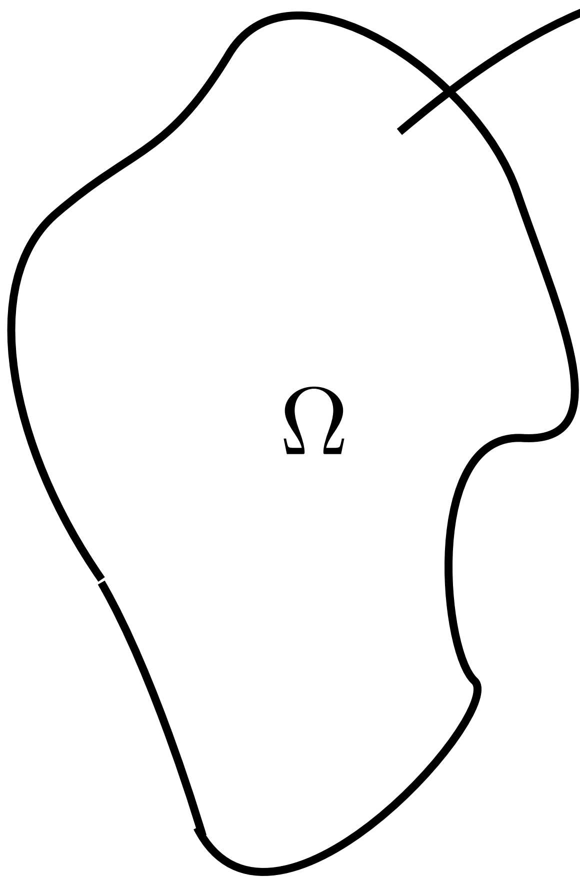


Equation modeling some physical phenomenon occurring on Ω, with unknown $u(x,t)$

Prove the equation has a unique solution

Equation is said to be **well-posed**

What is Numerical Analysis ?



Equation modeling some physical phenomenon occurring on Ω, with unknown $u(x,t)$

↓

Prove the equation has a unique solution

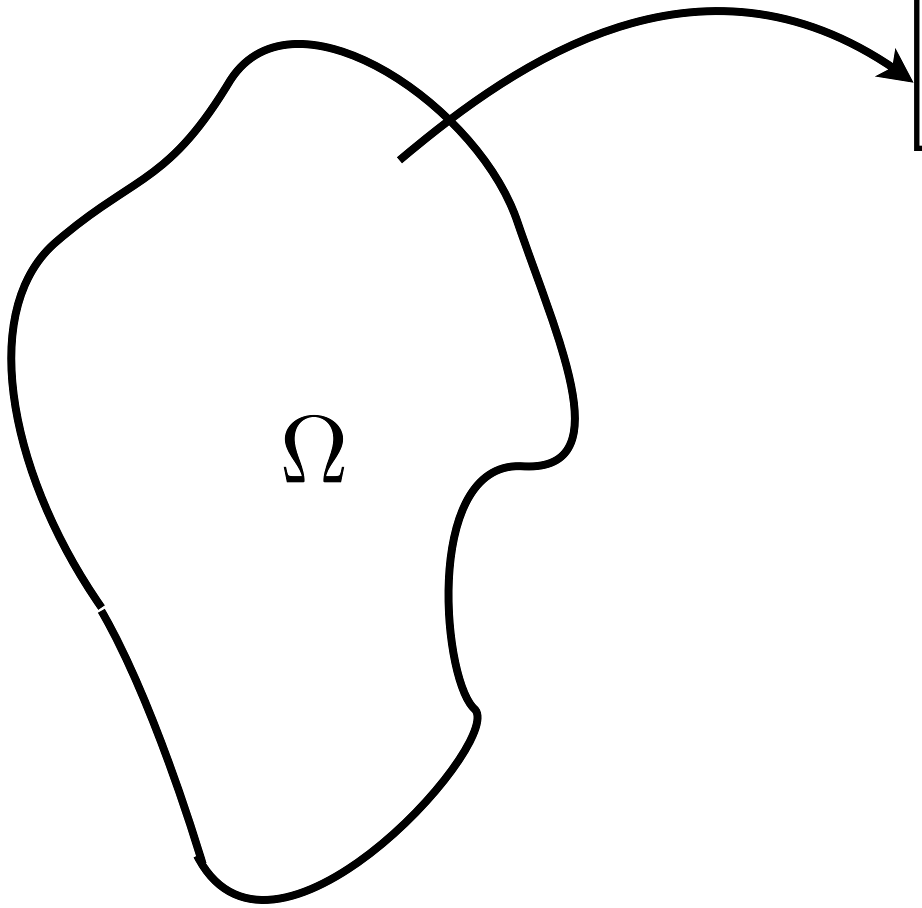
Equation is said to be **well-posed**

↓

Usually no analytical (*no nice formula*) solution

When the equation or the geometry are not simplistic

What is Numerical Analysis ?



Equation modeling some physical phenomenon occurring on Ω, with unknown $u(x,t)$

↓

Prove the equation has a unique solution

Equation is said to be **well-posed**

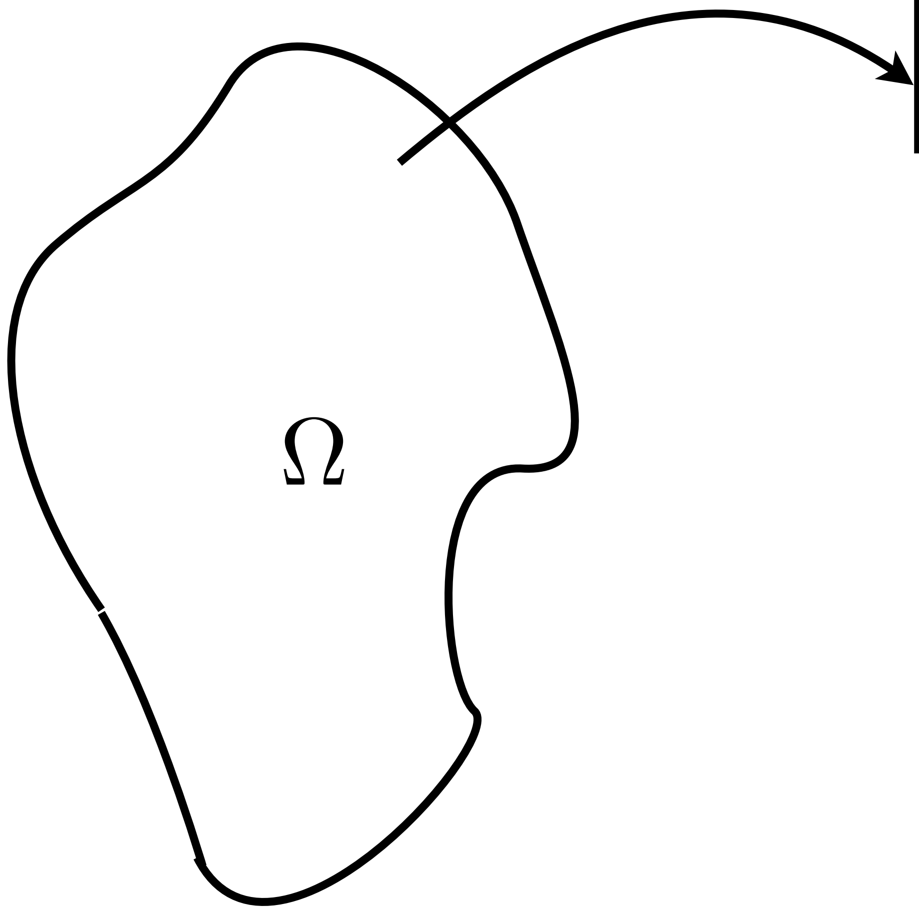
↓

Usually no analytical (*no nice formula*) solution

When the equation or the geometry are not simplistic

How to represent the phenomenon anyway?
Thanks to simulation!

What is Numerical Analysis ?



Equation modeling some physical phenomenon occurring on Ω, with unknown $u(x,t)$

Prove the equation has a unique solution

Equation is said to be **well-posed**

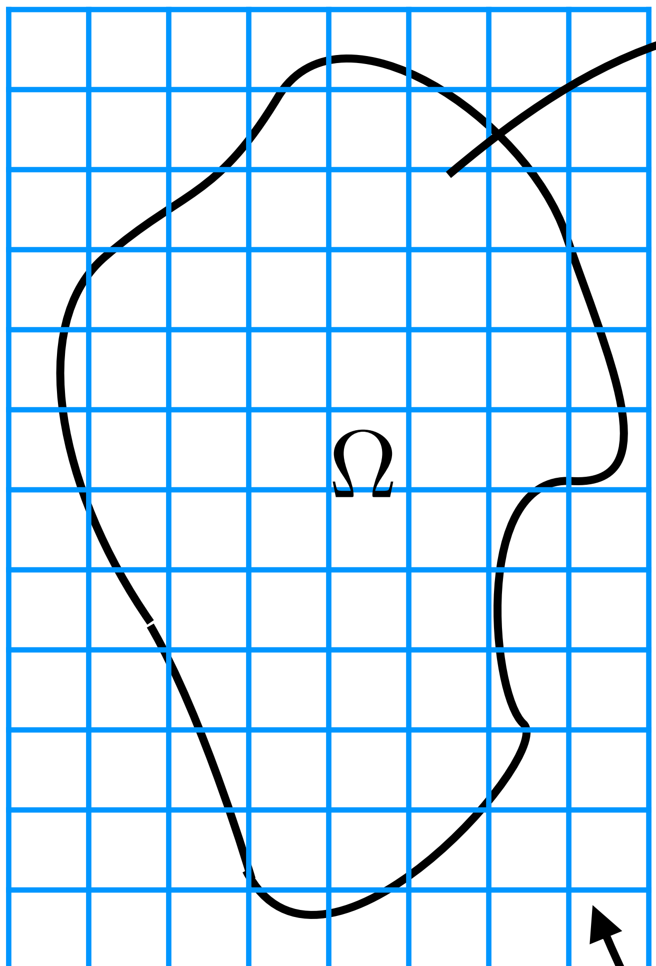
Usually no analytical (*no nice formula*) solution

When the equation or the geometry are not simplistic

How to represent the phenomenon anyway? Thanks to simulation!

Represent Ω by a finite number of points, compute an approximation of the unknown on each points.

What is Numerical Analysis ?



Mesh

Equation modeling some physical phenomenon occurring on Ω, with unknown $u(x,t)$

Prove the equation has a unique solution

Equation is said to be **well-posed**

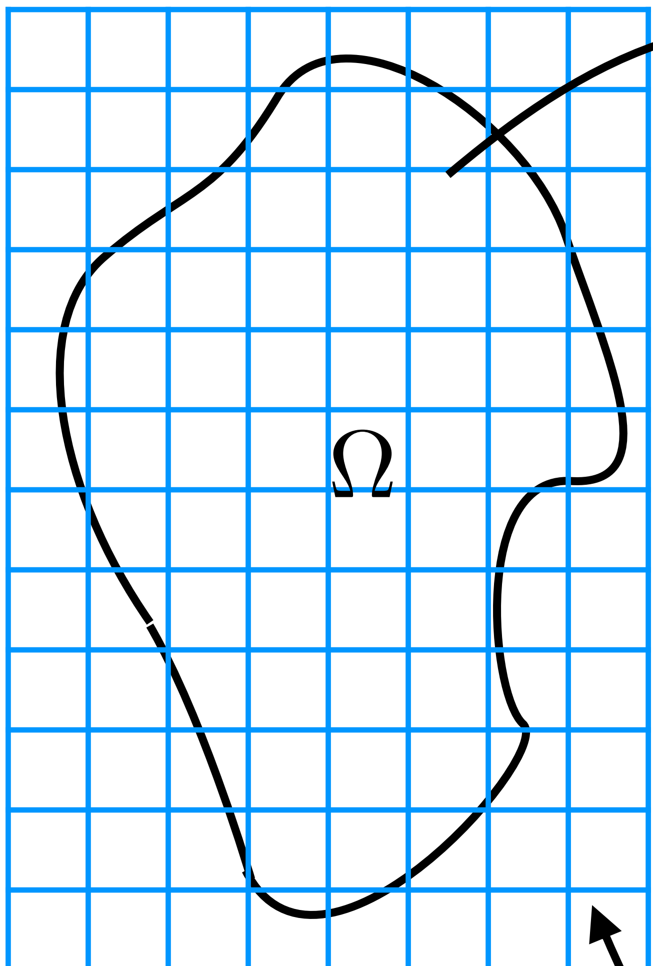
Usually no analytical (*no nice formula*) solution

When the equation or the geometry are not simplistic

How to represent the phenomenon anyway? Thanks to simulation!

Represent Ω by a finite number of points, compute an approximation of the unknown on each points.

What is Numerical Analysis ?



Mesh

Equation modeling some physical phenomenon occurring on Ω, with unknown $u(x,t)$

Prove the equation has a unique solution

Equation is said to be **well-posed**

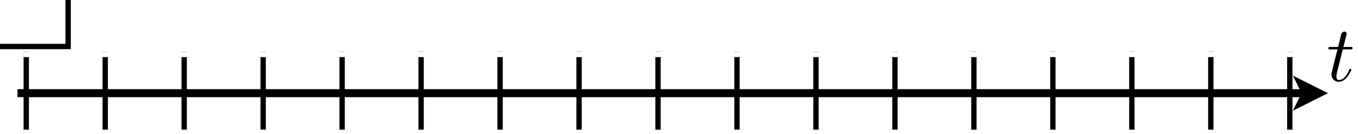
Usually no analytical (*no nice formula*) solution

When the equation or the geometry are not simplistic

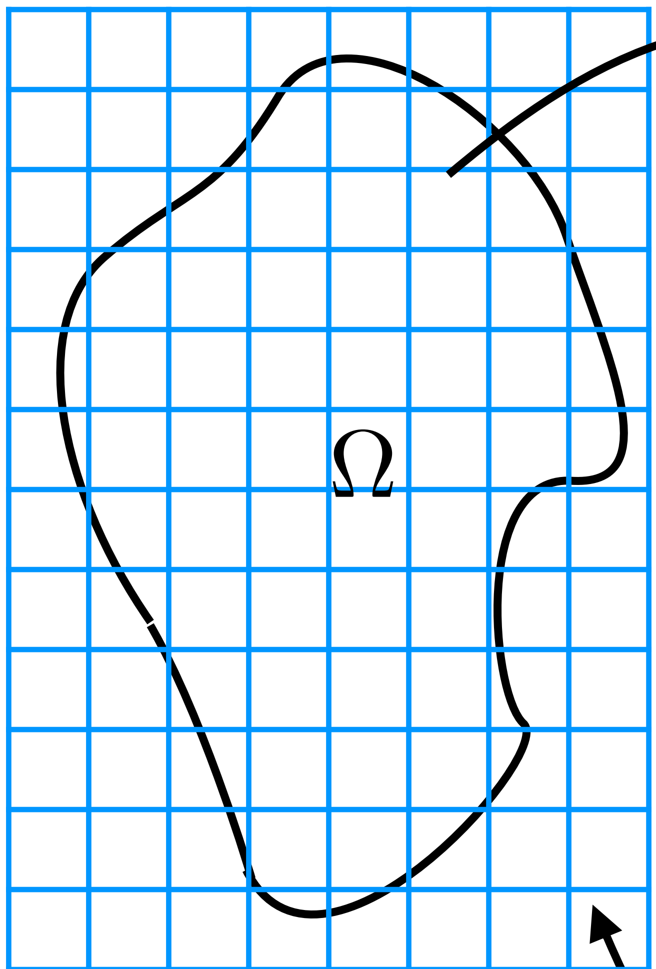
How to represent the phenomenon anyway? Thanks to simulation!

Represent Ω by a finite number of points, compute an approximation of the unknown on each points.

Since equation time dependent, we also discretize time



What is Numerical Analysis ?



Mesh

Equation modeling some physical phenomenon occurring on Ω, with unknown $u(x,t)$

Prove the equation has a unique solution

Equation is said to be **well-posed**

Usually no analytical (*no nice formula*) solution

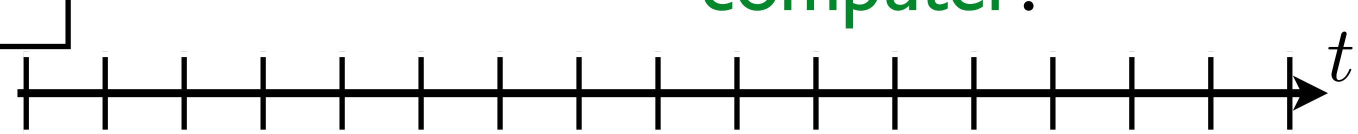
When the equation or the geometry are not simplistic

How to represent the phenomenon anyway? Thanks to simulation!

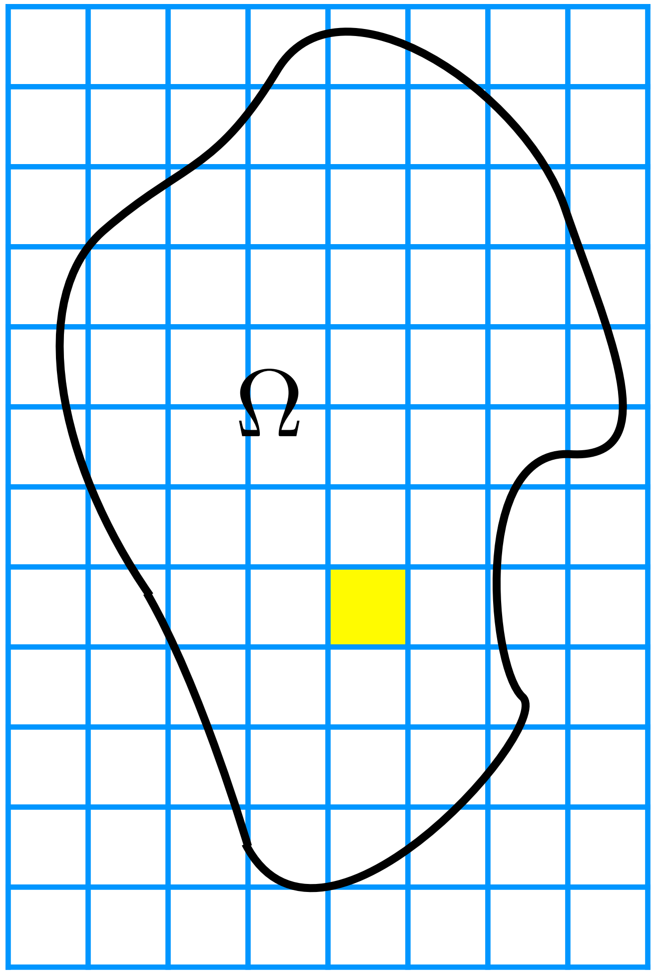
Represent Ω by a finite number of points, compute an approximation of the unknown on each points.

Since equation time dependent, we also discretize time

Leads to a matrix system: solvable by a computer!



What is Numerical Analysis ?

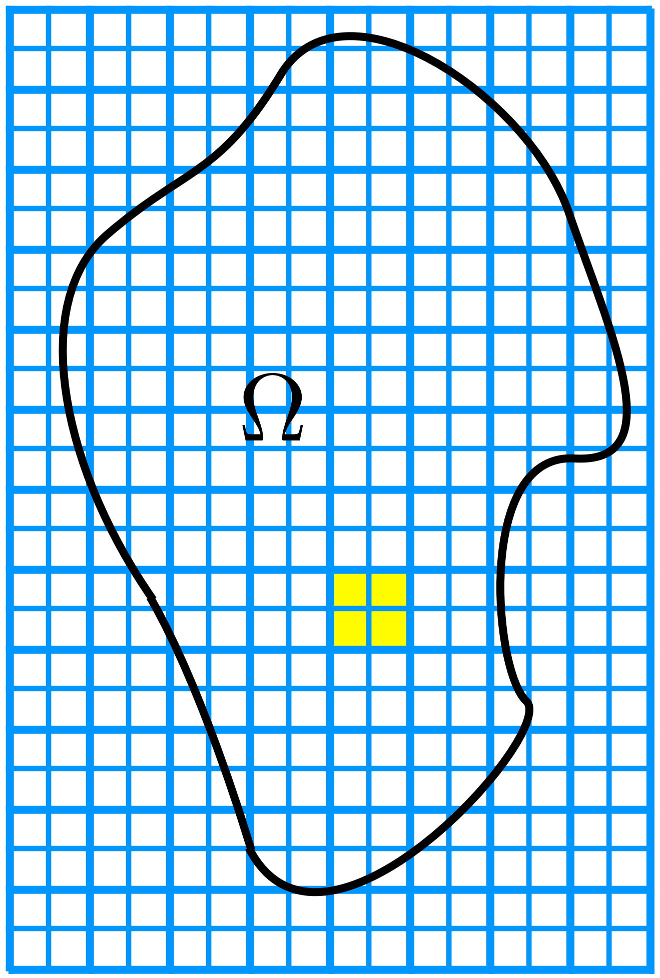


Mesh

Key parameter: the number of points on the mesh n

number of degrees of freedom

What is Numerical Analysis ?



Mesh

Key parameter: the number of points on the mesh n

number of degrees of freedom

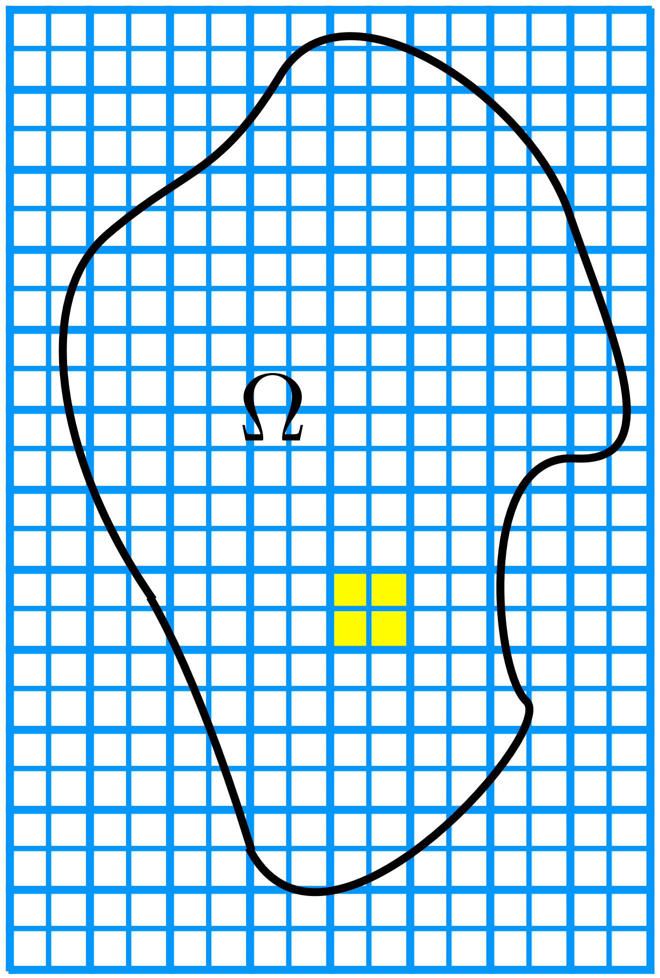
More degrees of freedom

Better approximation

Higher computational cost

inverting a matrix of size n
has complexity $O(n^3)$

What is Numerical Analysis ?



Mesh

Key parameter: the number of points on the mesh n

number of degrees of freedom

More degrees of freedom

Better approximation

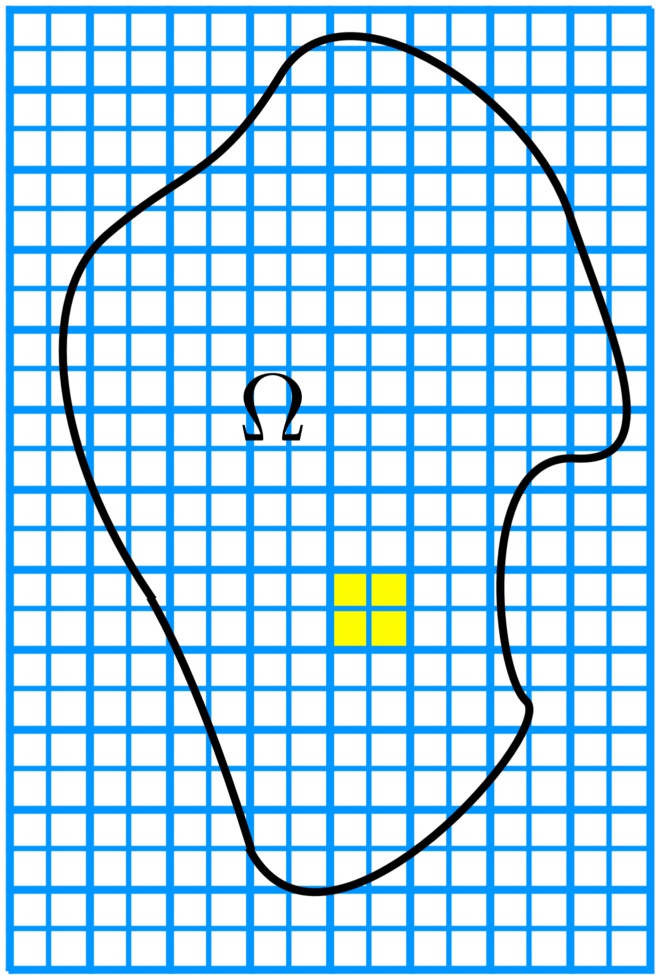
Higher computational cost

inverting a matrix of size n
has complexity $O(n^3)$

We note Δx the maximum diameter of mesh cells

and Δt the time step.

What is Numerical Analysis ?



Mesh

Key parameter: the number of points on the mesh n

number of degrees of freedom

More degrees of freedom

Better approximation

Higher computational cost

inverting a matrix of size n *has complexity* $O(n^3)$

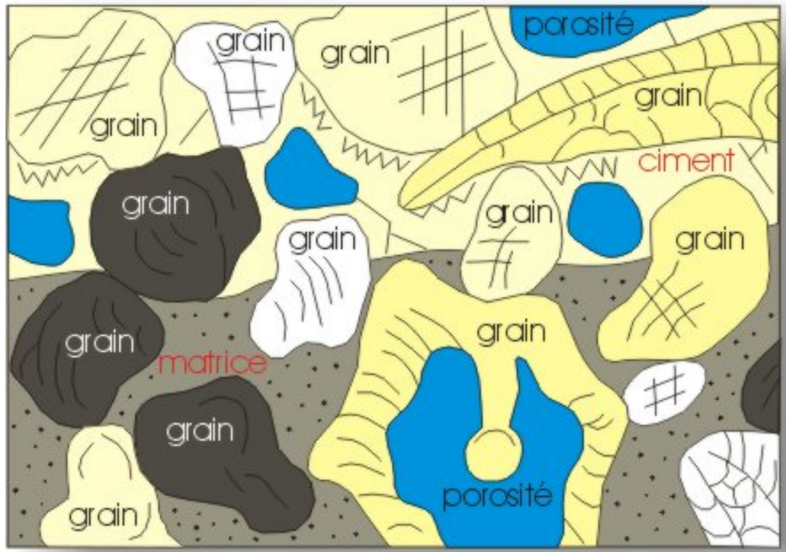
We note Δx the maximum diameter of mesh cells

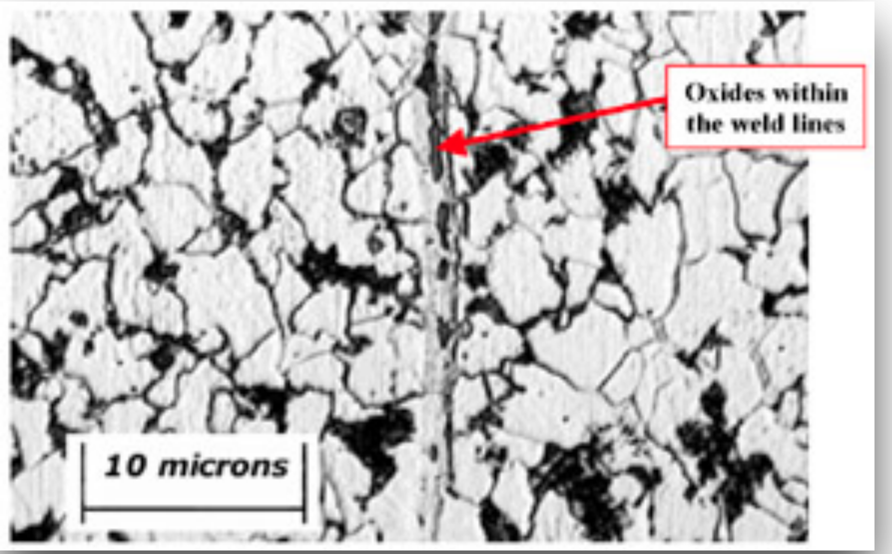
and Δt the time step.

Remark: $n \simeq \dfrac{1}{\Delta x}$

PhD motivations

- Non destructive testing for ultrasonic waves in media containing small heterogeneities (for instance concrete with small aggregate in it).



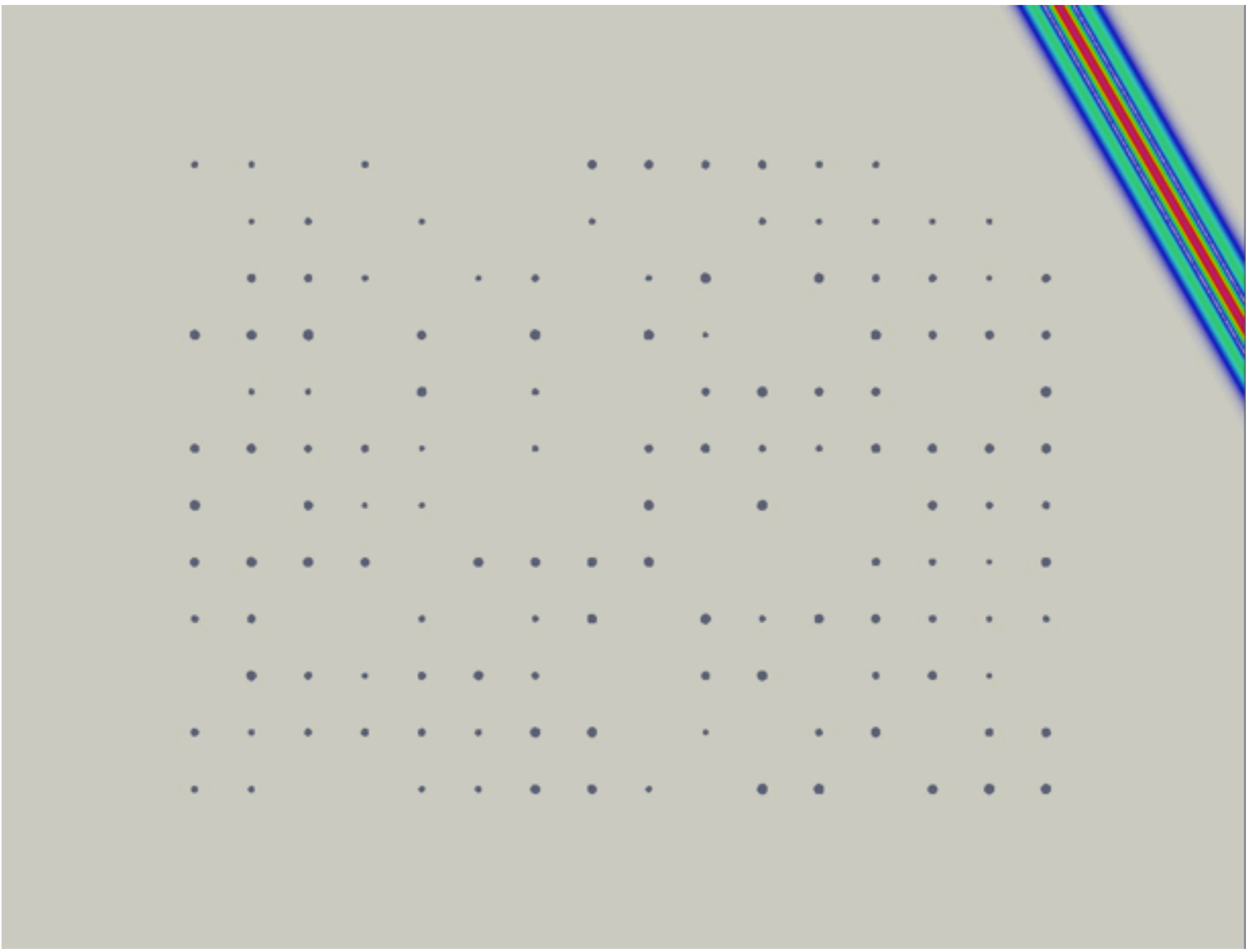


- Propagation of waves inside smooth media disturbed by small defects with respect to the wavelength.

- Classical simulation tools struggles to deal with multi scale problems.

PhD motivations

Simulation of an elastic wave propagating inside a media disturbed by small inclusions

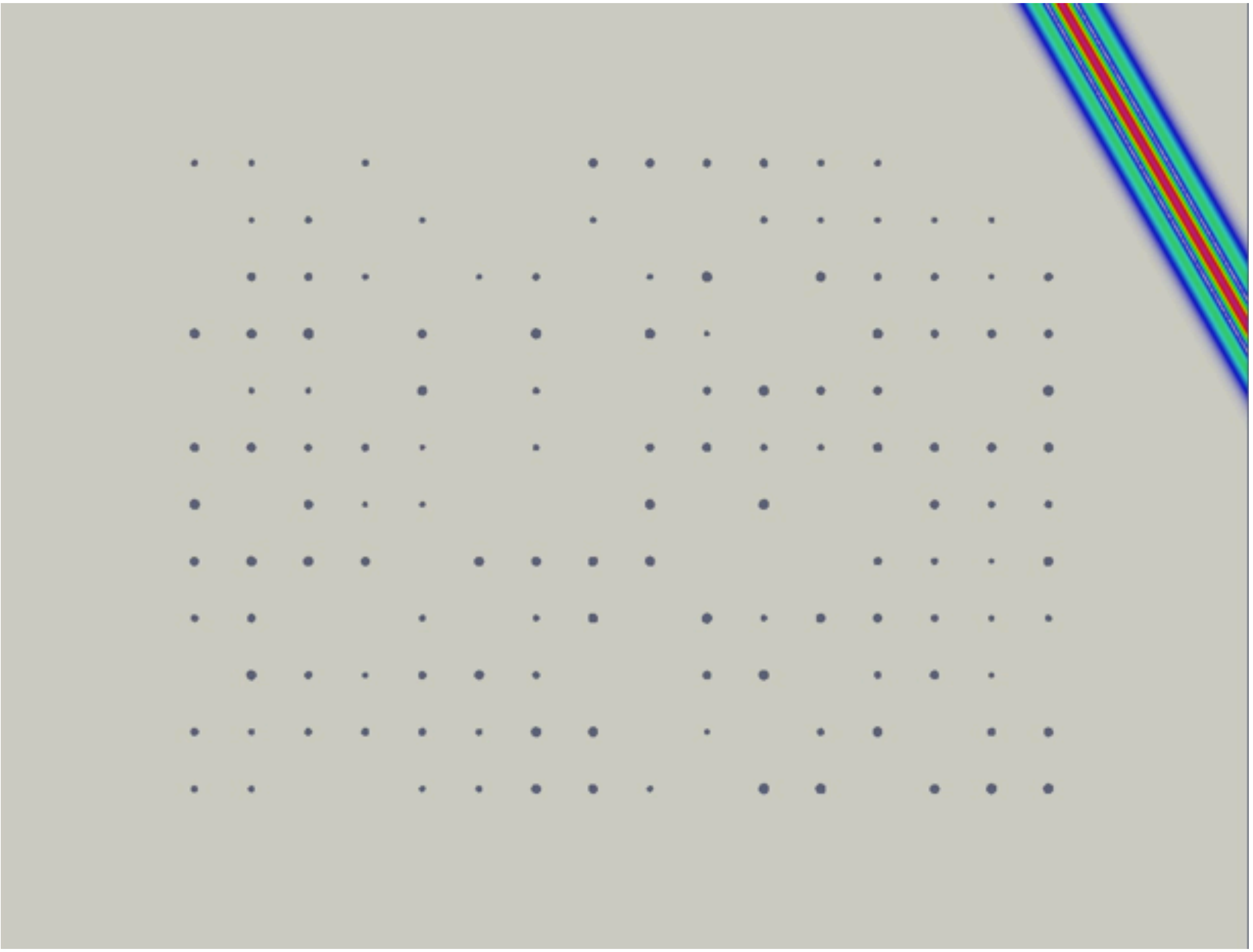


Finite element method

Simulation time (μs)	35
CPU time (s)	3527
Degrees of freedom	465132
$\Delta t \quad (\mu s)$	0.0013

PhD motivations

Simulation of an elastic wave propagating inside a media disturbed by small inclusions



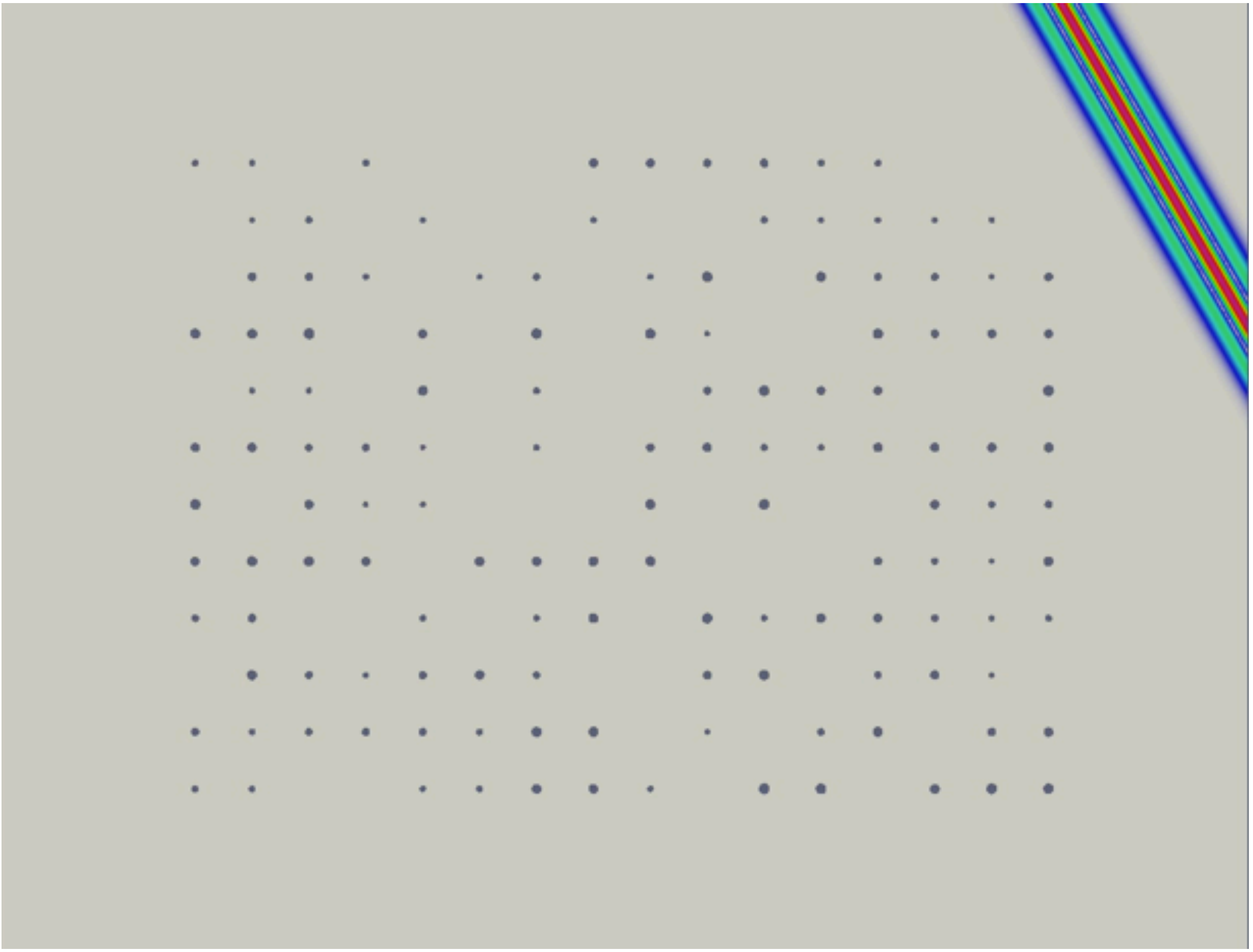
Finite element method

Simulation time (μs)	35
CPU time (s)	3527
Degrees of freedom	465132
$\Delta t \quad (\mu s)$	0.0013

CPU time for simulation in a medium of same size without defects is about 5 minutes!!!

PhD motivations

Simulation of an elastic wave propagating inside a media disturbed by small inclusions



Finite element method

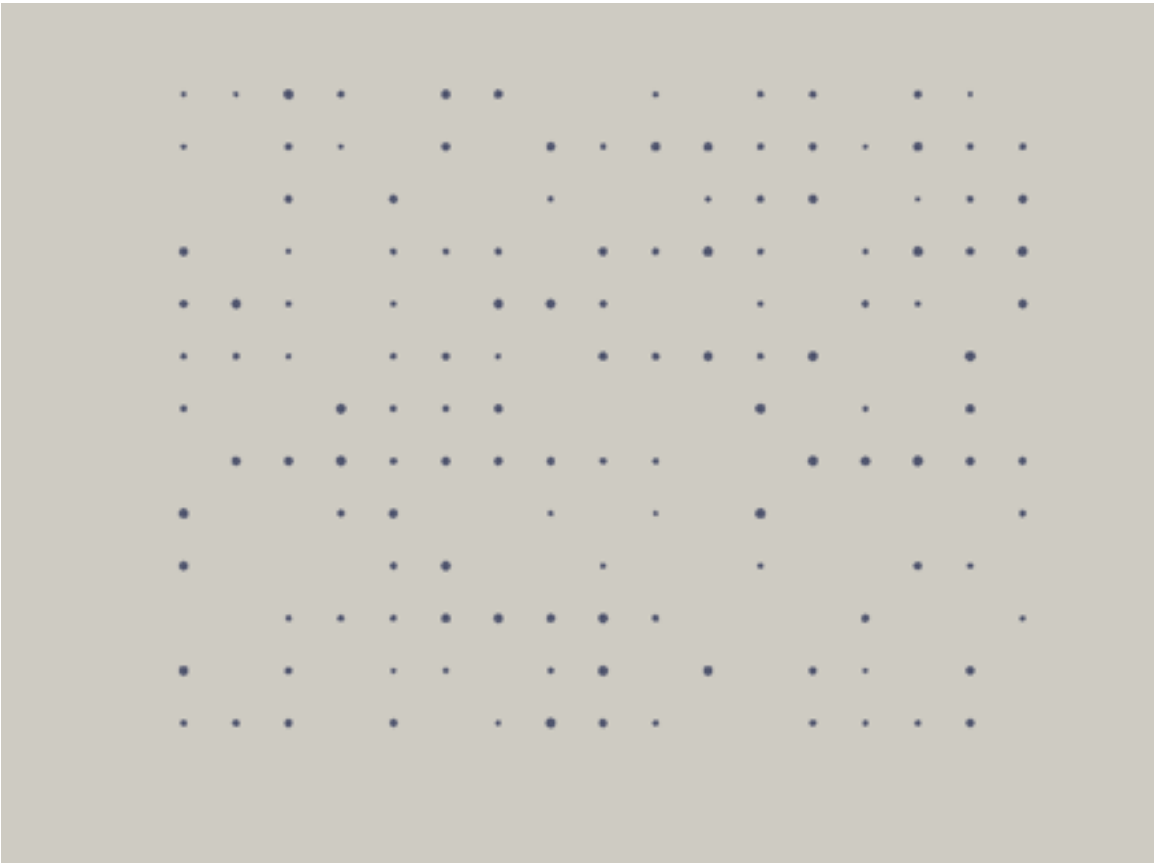
Simulation time (μs)	35
CPU time (s)	3527
Degrees of freedom	465132
$\Delta t \quad (\mu s)$	0.0013

CPU time for simulation in a medium of same size without defects is about 5 minutes!!!

PhD motivations

Homogeneous medium

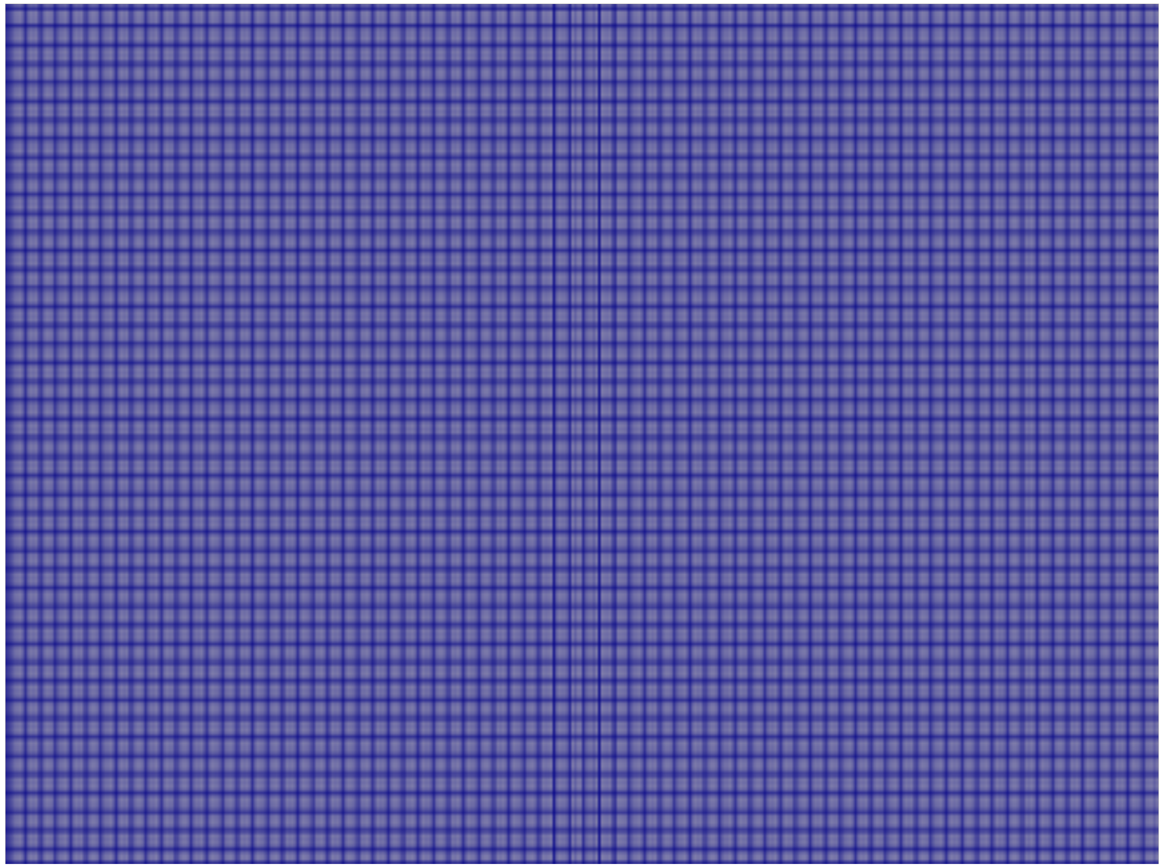
Medium with defects



PhD motivations

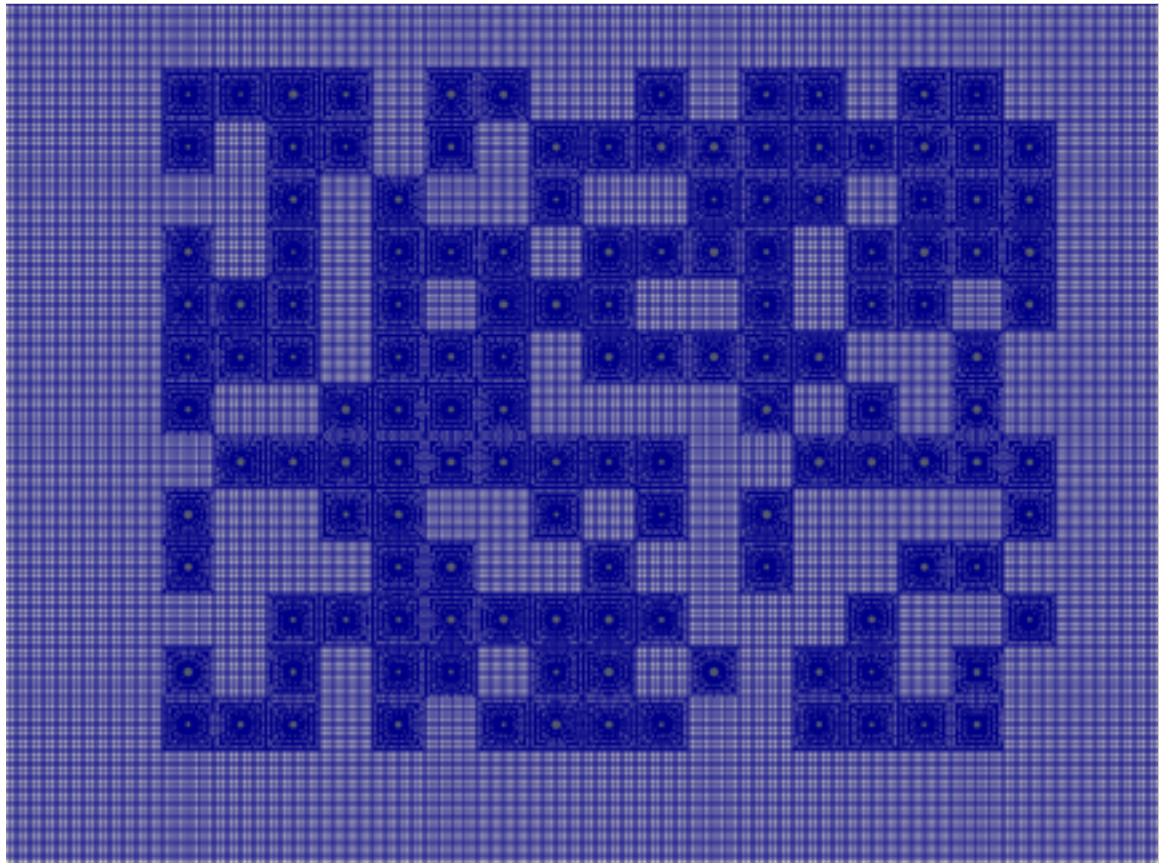
Homogeneous medium

about 90 000 degrees of freedom



Medium with defects

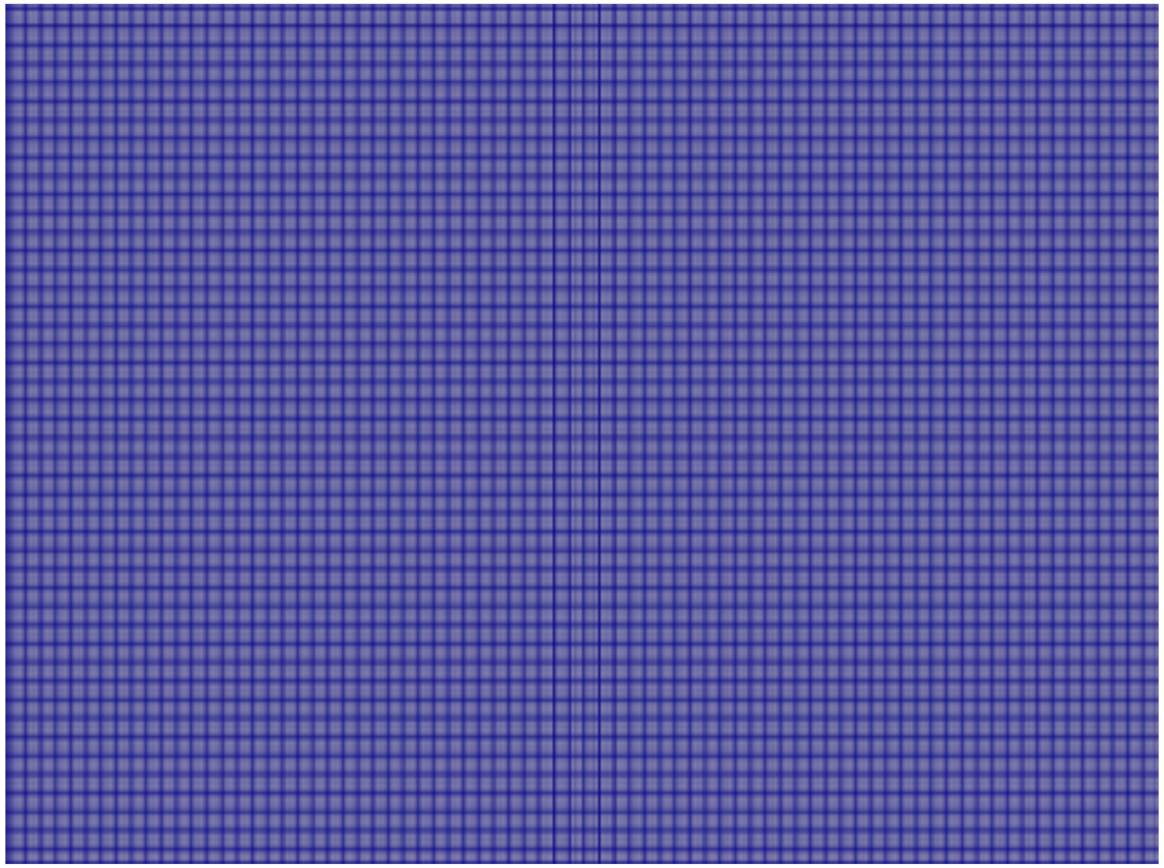
about 450 000 degrees of freedom



PhD motivations

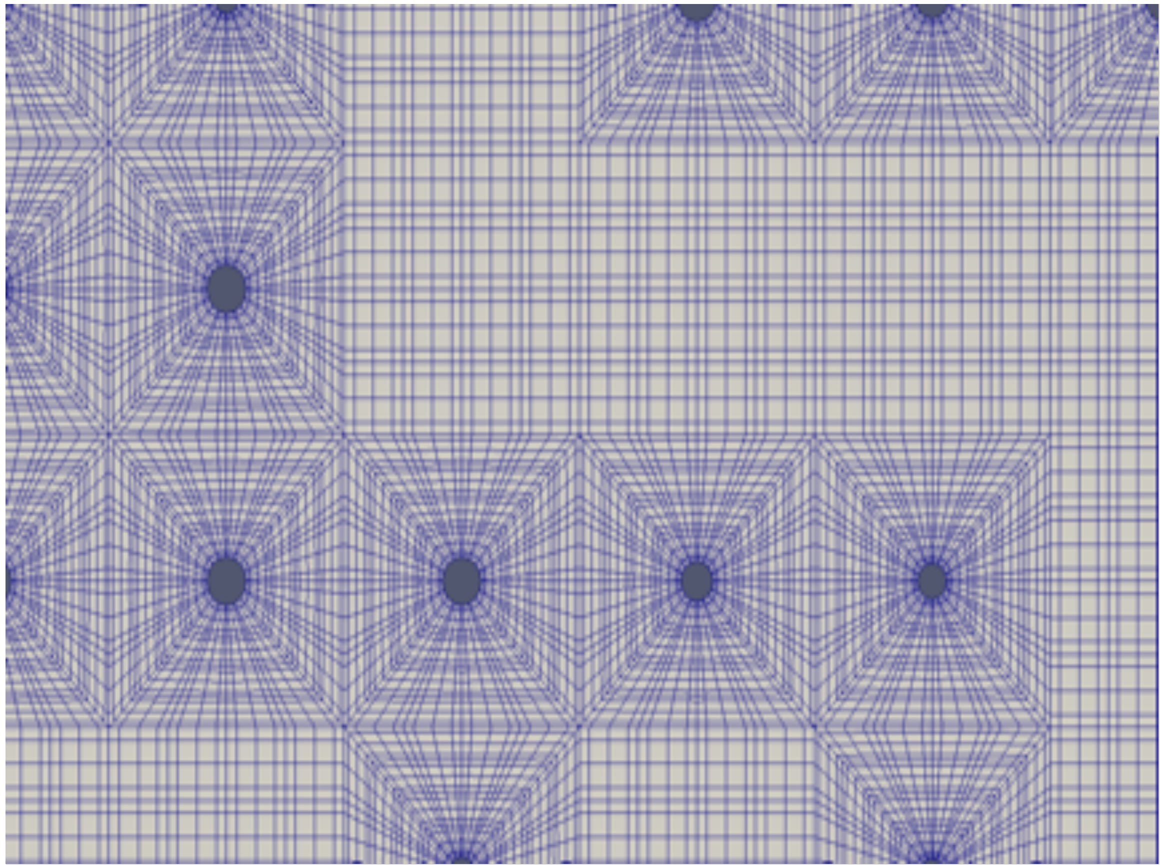
Homogeneous medium

about 90 000 degrees of freedom



Medium with defects

about 450 000 degrees of freedom



PhD motivations

We deal with time domain waves propagating in a smooth media with

- defects of characteristic size ε
- central wavelength λ

Assumption: $\varepsilon << \lambda$

PhD motivations

We deal with time domain waves propagating in a smooth media with

- defects of characteristic size ε
- central wavelength λ

Assumption: $\varepsilon << \lambda$

How to choose Δx to reach satisfying accuracy?

PhD motivations

We deal with time domain waves propagating in a smooth media with

- defects of characteristic size ε
- central wavelength λ

Assumption: $\varepsilon << \lambda$

How to choose Δx to reach satisfying accuracy?

Homogeneous medium

$$\Delta x \simeq \lambda$$

PhD motivations

We deal with time domain waves propagating in a smooth media with

- defects of characteristic size ε
- central wavelength λ

Assumption: $\varepsilon << \lambda$

How to choose Δx to reach satisfying accuracy?

Homogeneous medium

$$\Delta x \simeq \lambda$$

Medium with defects

$$\Delta x \simeq \varepsilon$$

PhD motivations

We deal with time domain waves propagating in a smooth media with

- defects of characteristic size ε
- central wavelength λ

Assumption: $\varepsilon << \lambda$

How to choose Δx to reach satisfying accuracy?

<u>*Homogeneous medium*</u>

$$\Delta x \simeq \lambda$$

<u>*Medium with defects*</u>

$$\boxed{\Delta x \simeq \varepsilon}$$

Very bad since we have to choose (*stability condition*)

$$\Delta t \leq C\ \Delta x$$

PhD motivations

We deal with time domain waves propagating in a smooth media with

- defects of characteristic size ε
- central wavelength λ

Assumption: $\varepsilon << \lambda$

How to choose Δx to reach satisfying accuracy?

Homogeneous medium

$$\Delta x \simeq \lambda$$

Medium with defects

$$\Delta x \simeq \varepsilon$$

PhD motivations

We deal with time domain waves propagating in a smooth media with

- defects of characteristic size ε
- central wavelength λ

Assumption: $\varepsilon << \lambda$

How to choose Δx to reach satisfying accuracy?

Homogeneous medium

$$\Delta x \simeq \lambda$$

Medium with defects

$$\Delta x \simeq \varepsilon$$

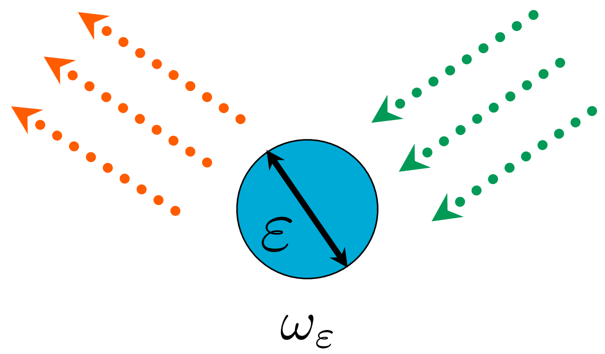
What we want:

$$\Delta x \simeq \lambda$$

take ε -scale into account via special numerical feature based on asymptotics

A model problem

Wave acoustic equation: look for $u_\varepsilon(x,t)$ such that

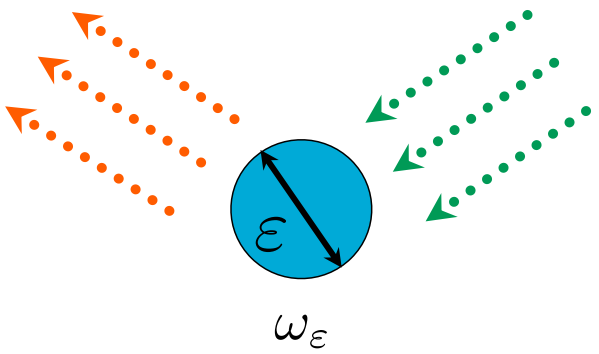


$$\rho_\varepsilon \frac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla \cdot (\mu_\varepsilon \nabla u_\varepsilon) = f, \qquad x \in \mathbb{R}^d, \quad t > 0.$$

A model problem

Wave acoustic equation: look for $u_\varepsilon(x,t)$ such that

$$\rho_\varepsilon \frac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla\cdot(\mu_\varepsilon \nabla u_\varepsilon) = f, \qquad x \in \mathbb{R}^d, \quad t > 0.$$

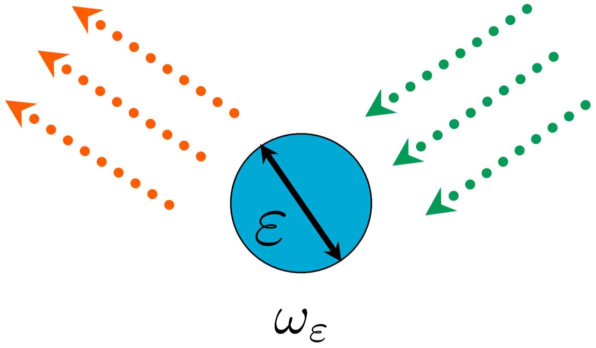


$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_d} \end{pmatrix} \qquad \nabla \cdot \vec{g} = \sum_{i=1}^{d} \frac{\partial g_i}{\partial x_i}$$

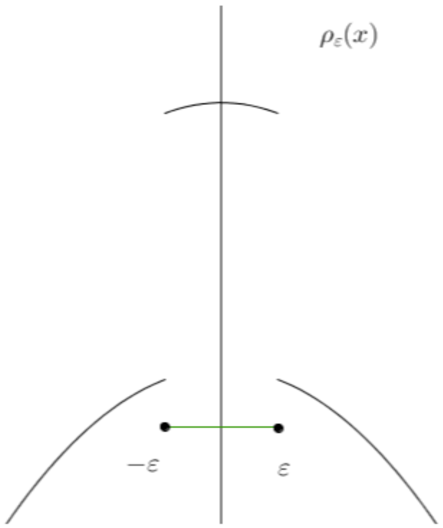
A model problem

Wave acoustic equation: look for $u_\varepsilon(x,t)$ such that

$$\rho_\varepsilon \frac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla\cdot(\mu_\varepsilon \nabla u_\varepsilon) = f, \qquad x \in \mathbb{R}^d, \quad t > 0.$$



The physical parameters $\rho_\varepsilon,\ \mu_\varepsilon$ admit a jump on the defect boundary.



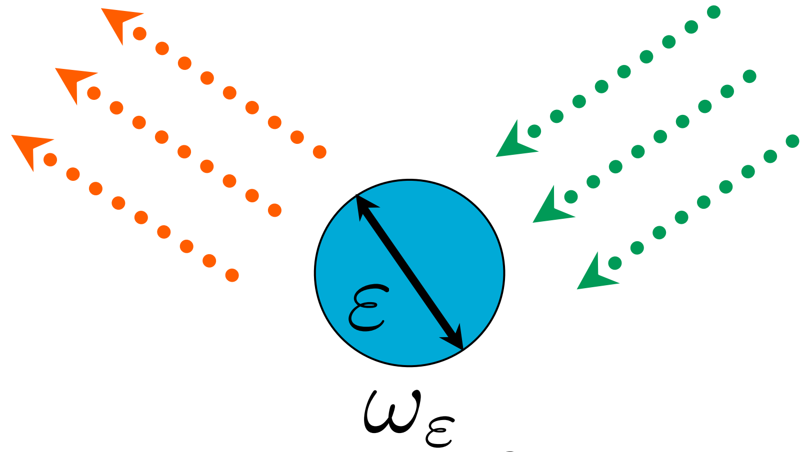
Construction of an approximate model: defect taken into account via auxiliary sources.

MATCHED ASYMPTOTIC METHOD

Matched asymptotics

Look for the solution u_ε of the following model problem:

$$(P_\varepsilon)\left\{\begin{array}{ll} \rho_\varepsilon \dfrac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla\cdot(\mu_\varepsilon \nabla u_\varepsilon) = f, & x \in \mathbb{R}^d, \quad t > 0, \\ u_\varepsilon(x,0) = \dfrac{\partial u_\varepsilon}{\partial t}(x,0) = 0, & x \in \mathbb{R}^d. \end{array}\right.$$

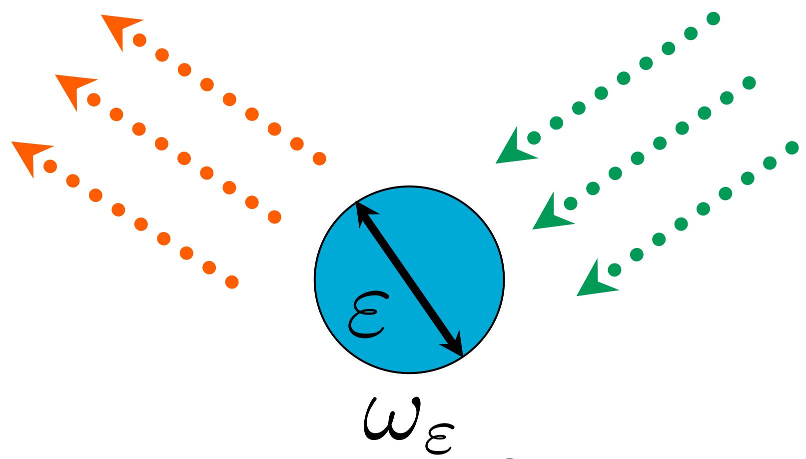


Asymptotic behavior of u_ε as $\varepsilon \to 0$?

Matched asymptotics

Look for the solution u_ε of the following model problem:

$$(P_\varepsilon)\ \begin{cases} \rho_\varepsilon \dfrac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla\cdot(\mu_\varepsilon \nabla u_\varepsilon) = f, & x \in \mathbb{R}^d, \quad t > 0, \\ u_\varepsilon(x,0) = \dfrac{\partial u_\varepsilon}{\partial t}(x,0) = 0, & x \in \mathbb{R}^d. \end{cases}$$



Asymptotic behavior of u_ε as $\varepsilon \to 0$?

We would like something like

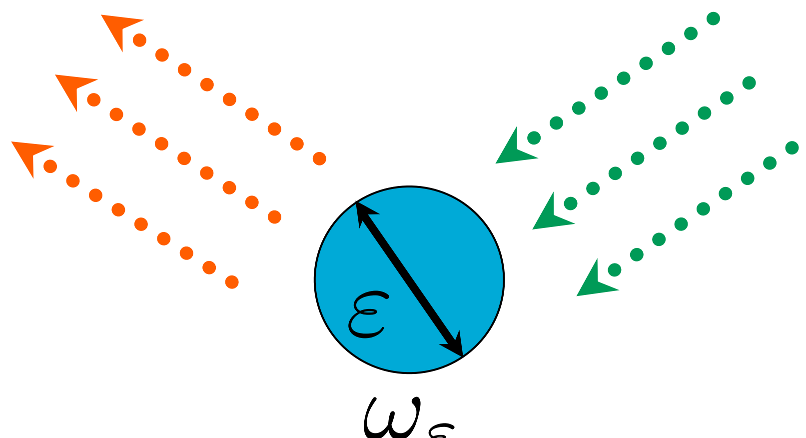
$$u_\varepsilon(x,t) = \sum_{n=0}^{\infty} \varepsilon^n u_n(x,t)$$

and compute the u_n (independent of ε).

Matched asymptotics

Look for the solution u_ε of the following model problem:

$$(P_\varepsilon)\ \begin{cases} \rho_\varepsilon \dfrac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla\cdot(\mu_\varepsilon \nabla u_\varepsilon) = f, & x \in \mathbb{R}^d, \quad t > 0, \\ u_\varepsilon(x,0) = \dfrac{\partial u_\varepsilon}{\partial t}(x,0) = 0, & x \in \mathbb{R}^d. \end{cases}$$



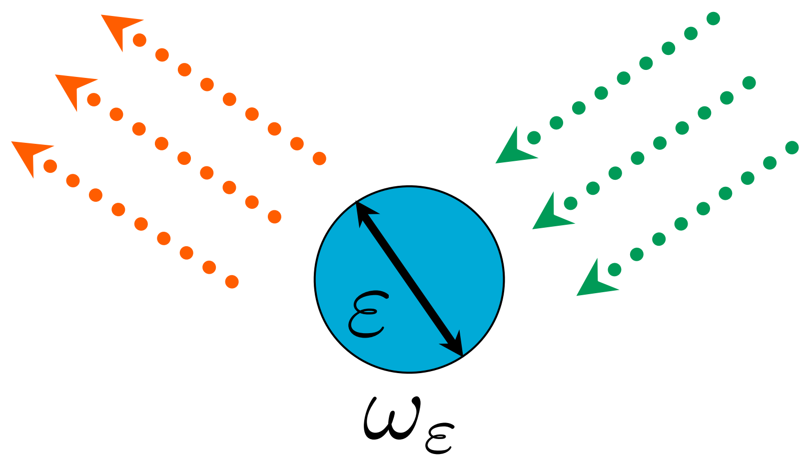
Asymptotic behavior of u_ε as $\varepsilon \to 0$?

- No uniform development of u_ε in function of ε !
- Need to distinguish a far-field zone and a near-field zone inside which development holds.

Matched asymptotics

Look for the solution u_ε of the following model problem:

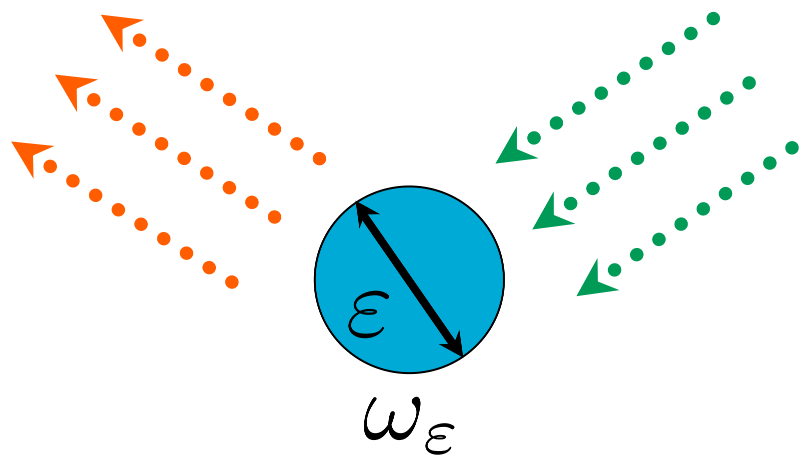
$$(P_\varepsilon)\left\{\begin{aligned} &\rho_\varepsilon \frac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla\cdot(\mu_\varepsilon \nabla u_\varepsilon) = f, && x \in \mathbb{R}^d, \quad t > 0,\\ &u_\varepsilon(x,0) = \frac{\partial u_\varepsilon}{\partial t}(x,0) = 0, && x \in \mathbb{R}^d.\end{aligned}\right.$$



Matched asymptotics

Look for the solution u_ε of the following model problem:

$$(P_\varepsilon)\left\{\begin{array}{ll} \rho_\varepsilon \dfrac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla\cdot(\mu_\varepsilon \nabla u_\varepsilon) = f, & x \in \mathbb{R}^d, \quad t>0, \\ u_\varepsilon(x,0) = \dfrac{\partial u_\varepsilon}{\partial t}(x,0) = 0, & x\in\mathbb{R}^d. \end{array}\right.$$



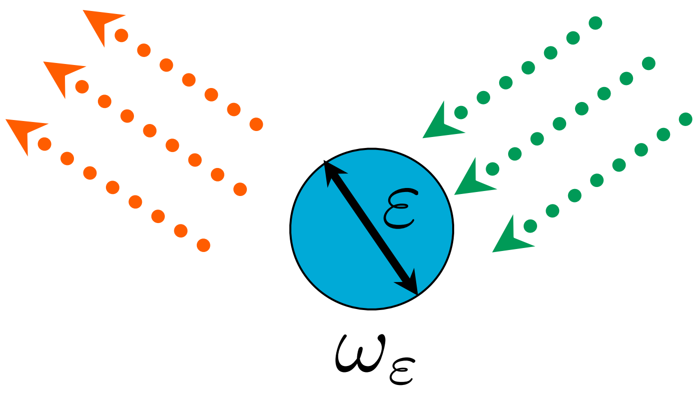
We denote

$$\rho_\varepsilon(x) = \left\{\begin{array}{ll} \rho^-(x) & \text{if } x\in\omega_\varepsilon \\ \rho^+(x) & \text{if } x\in\mathbb{R}^d\setminus\omega_\varepsilon \end{array}\right. \qquad \mu_\varepsilon(x) = \left\{\begin{array}{ll} \mu^-(x) & \text{if } x\in\omega_\varepsilon \\ \mu^+(x) & \text{if } x\in\mathbb{R}^d\setminus\omega_\varepsilon \end{array}\right.$$

Matched asymptotics

Model problem

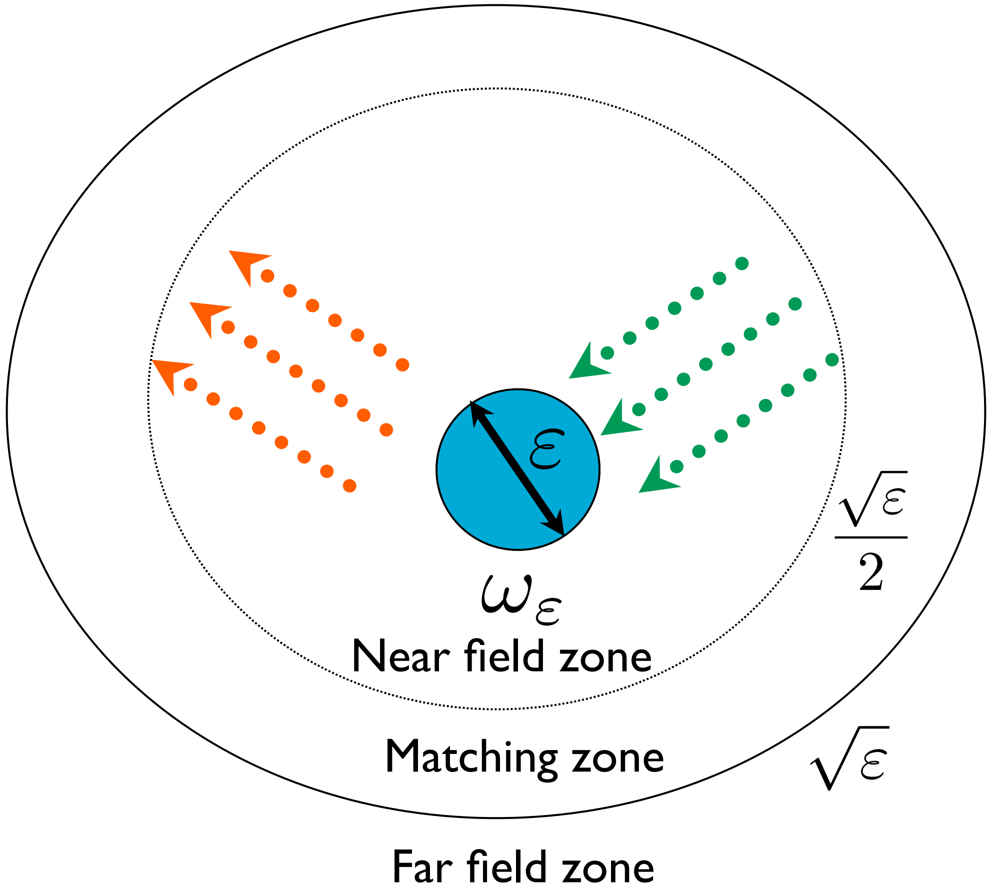
$$\rho_\varepsilon \frac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla\cdot(\mu_\varepsilon \nabla u_\varepsilon) = f,$$



Matched asymptotics

<u>Model problem</u>

$$\rho_\varepsilon \frac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla \cdot (\mu_\varepsilon \nabla u_\varepsilon) = f,$$

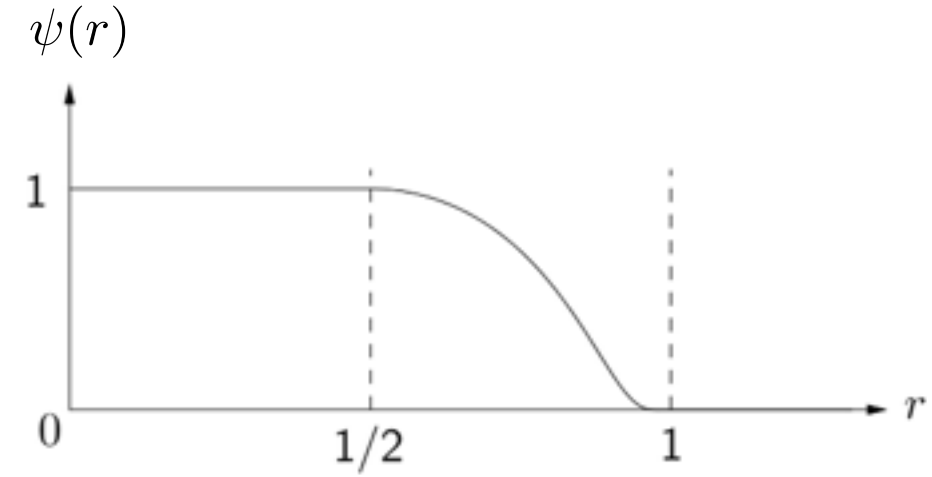


Ansatz and total field

Select a cutoff function ψ ,

scale it $\psi_\varepsilon(r) = \psi\left(\frac{r}{\sqrt{\varepsilon}}\right)$,

and represent the total field as the sum



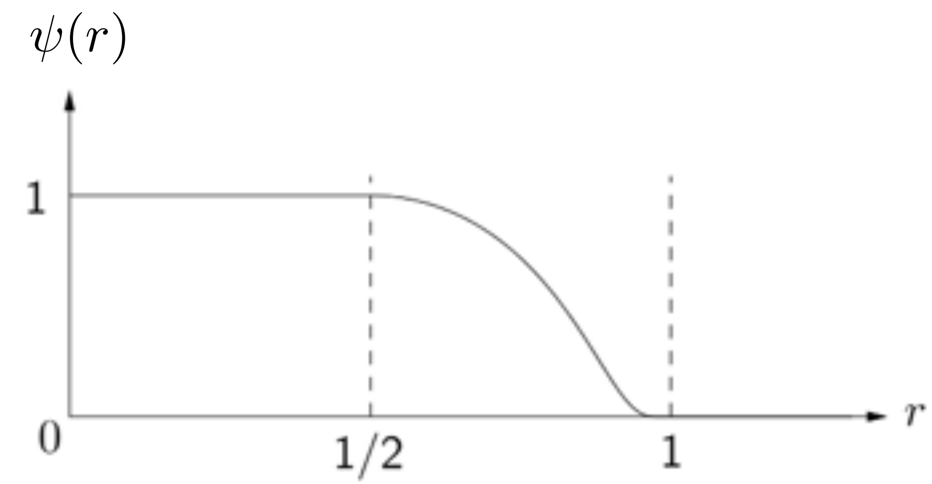
$$u_\varepsilon(x,t) = \psi_\varepsilon(|x|)\, U_\varepsilon^N(x,t) + (1 - \psi_\varepsilon(|x|))\, u_\varepsilon^F(x,t)$$

Ansatz and total field

Select a cutoff function ψ ,

scale it $\psi_\varepsilon(r) = \psi\left(\frac{r}{\sqrt{\varepsilon}}\right)$,

and represent the total field as the sum



$$u_\varepsilon(x,t) = \psi_\varepsilon(|x|)\, U_\varepsilon^N(x,t) + (1-\psi_\varepsilon(|x|))\; u_\varepsilon^F(x,t)$$

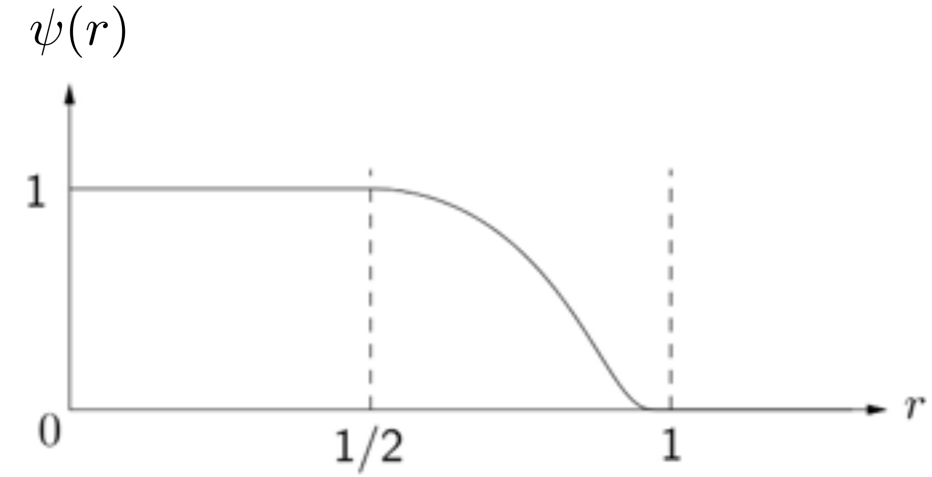
= when $|x| \leq \frac{\sqrt{\varepsilon}}{2}$

Ansatz and total field

Select a cutoff function ψ ,

scale it $\psi_\varepsilon(r) = \psi\left(\frac{r}{\sqrt{\varepsilon}}\right)$,

and represent the total field as the sum



$$u_\varepsilon(x,t) = \psi_\varepsilon(|x|)\, U_\varepsilon^N(x,t) + (1-\psi_\varepsilon(|x|))\, u_\varepsilon^F(x,t)$$

= when $|x| \geq \sqrt{\varepsilon}$

Matching principle

Far field equations

Limit field

$$\begin{cases} \rho^+ \dfrac{\partial^2 u_0}{\partial t^2} - \nabla \cdot \left(\mu^+ \nabla u_0\right) = f, \\ \rho^+ \dfrac{\partial^2 u_n}{\partial t^2} - \nabla \cdot \left(\mu^+ \nabla u_n\right) = 0, \quad n \geq 1 \end{cases}$$

Near field equations

$$\begin{cases} \nabla \cdot \left(\mu \nabla U_n\right) = 0, \quad n = 0, 1 \\ \nabla \cdot \left(\mu \nabla U_n\right) = \rho \dfrac{\partial^2 U_{n-2}}{\partial t^2}, \quad n \geq 2 \end{cases}$$

Matching principle

Far field equations

Limit field

$$\begin{cases} \rho^+ \dfrac{\partial^2 u_0}{\partial t^2} - \nabla \cdot \left(\mu^+ \nabla u_0\right) = f, \\ \rho^+ \dfrac{\partial^2 u_n}{\partial t^2} - \nabla \cdot \left(\mu^+ \nabla u_n\right) = 0, \quad n \geq 1 \end{cases}$$

Near field equations

$$\begin{cases} \nabla \cdot \left(\mu \nabla U_n\right) = 0, \quad n = 0, 1 \\ \nabla \cdot \left(\mu \nabla U_n\right) = \rho \dfrac{\partial^2 U_{n-2}}{\partial t^2}, \quad n \geq 2 \end{cases}$$

Matching principle:

The partial sums $\sum_{n=0}^{N} \varepsilon^n u^n$ and $\sum_{n=0}^{N} \varepsilon^n U^n$ must coïncide in the matching zone, $\forall N \geq 0.$

Matching principle

Far field equations

Limit field

$$\begin{cases} \rho^+ \dfrac{\partial^2 u_0}{\partial t^2} - \nabla \cdot \left(\mu^+ \nabla u_0\right) = f, \\ \rho^+ \dfrac{\partial^2 u_n}{\partial t^2} - \nabla \cdot \left(\mu^+ \nabla u_n\right) = 0, \quad n \geq 1 \end{cases}$$

Near field equations

$$\begin{cases} \nabla \cdot \left(\mu \nabla U_n\right) = 0, \quad n = 0, 1 \\ \nabla \cdot \left(\mu \nabla U_n\right) = \rho \dfrac{\partial^2 U_{n-2}}{\partial t^2}, \quad n \geq 2 \end{cases}$$

Matching principle:

The partial sums $\sum_{n=0}^{N} \varepsilon^n u^n$ and $\sum_{n=0}^{N} \varepsilon^n U^n$ must coïncide in the matching zone, $\forall N \geq 0.$

Equations + matching principle → the U_n 's can be computed.

TOWARDS AN ASYMPTOTIC METHOD

An asymptotic model

From the initial model problem

$$(P_\varepsilon) \qquad \rho_\varepsilon \frac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla\cdot(\mu_\varepsilon \nabla u_\varepsilon) = f, \qquad x \in \mathbb{R}^d, \quad t > 0,$$

An asymptotic model

From the initial model problem

$$(P_\varepsilon) \qquad \rho_\varepsilon \frac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla\cdot(\mu_\varepsilon \nabla u_\varepsilon) = f, \qquad x \in \mathbb{R}^d, \quad t > 0,$$

seeing $\rho_\varepsilon, \ \mu_\varepsilon$ as perturbations of $\rho^+, \ \mu^+$ we derive the following approximate model:

An asymptotic model

From the initial model problem

$$(P_\varepsilon) \qquad \rho_\varepsilon \frac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla \cdot (\mu_\varepsilon \nabla u_\varepsilon) = f, \qquad x \in \mathbb{R}^d, \quad t > 0,$$

seeing $\rho_\varepsilon, \ \mu_\varepsilon$ as perturbations of $\rho^+, \ \mu^+$ we derive the following approximate model:

auxiliary source

$$(\widetilde{P}_\varepsilon) \begin{cases} \rho^+ \dfrac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla \cdot \left(\mu^+ \nabla u_\varepsilon\right) = f + S_\varepsilon^N v_\varepsilon, \quad x \in \mathbb{R}^d, \quad t > 0, \\ v_\varepsilon = \left(S_\varepsilon^N\right)^\star u_\varepsilon, \end{cases}$$

coupling the auxiliary source with the unknown

An asymptotic model

ε - wave operator, not suited for discretization

From the initial model problem

$$(P_\varepsilon) \quad \boxed{\rho_\varepsilon \frac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla\cdot(\mu_\varepsilon \nabla u_\varepsilon)} = f, \qquad x \in \mathbb{R}^d, \quad t > 0,$$

seeing $\rho_\varepsilon,\ \mu_\varepsilon$ as perturbations of $\rho^+,\ \mu^+$ we derive the following approximate model:

$$(\widetilde{P}_\varepsilon) \quad \begin{cases} \boxed{\rho^+ \frac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla\cdot\left(\mu^+ \nabla u_\varepsilon\right)} = f + S_\varepsilon^N v_\varepsilon, \quad x \in \mathbb{R}^d, \quad t > 0, \\ v_\varepsilon = \left(S_\varepsilon^N\right)^\star u_\varepsilon, \end{cases}$$

ε - independent wave operator, well suited for discretization

- S_ε^N is an operator involving the first N near-field functions
- N is the order of approximation of the method:

$$\|\mathrm{sol}_{(P)_\varepsilon} - \mathrm{sol}_{(\widetilde{P})_\varepsilon}\| \leq C\varepsilon^N$$

An asymptotic model

ε - wave operator, not suited for discretization

From the initial model problem

$$(P_\varepsilon) \quad \boxed{\rho_\varepsilon \frac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla\cdot(\mu_\varepsilon \nabla u_\varepsilon)} = f, \qquad x \in \mathbb{R}^d, \quad t > 0,$$

seeing $\rho_\varepsilon,\ \mu_\varepsilon$ as perturbations of $\rho^+,\ \mu^+$ we derive the following approximate model:

$$(\widetilde{P}_\varepsilon) \begin{cases} \boxed{\rho^+ \dfrac{\partial^2 u_\varepsilon}{\partial t^2} - \nabla\cdot\left(\mu^+ \nabla u_\varepsilon\right)} = f + S_\varepsilon^N v_\varepsilon, & x \in \mathbb{R}^d, \quad t > 0, \\ v_\varepsilon = \left(S_\varepsilon^N\right)^\star u_\varepsilon, \end{cases}$$

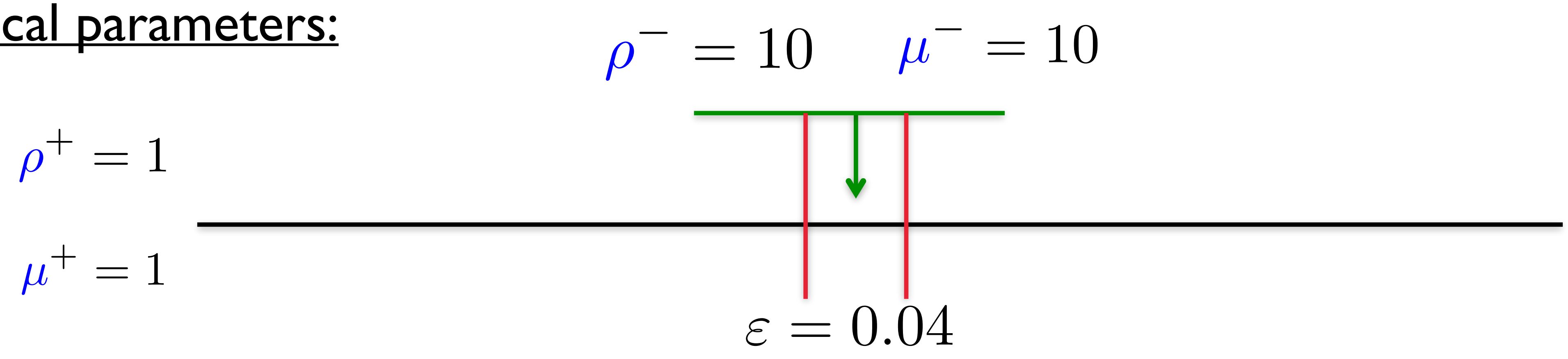
ε - independent wave operator, well suited for discretization

- S_ε^N is an operator involving the first N near-field functions
- N is the order of approximation of the method:

$$\|\mathrm{sol}_{(P)_\varepsilon} - \mathrm{sol}_{(\widetilde{P})_\varepsilon}\| \le C\varepsilon^N$$

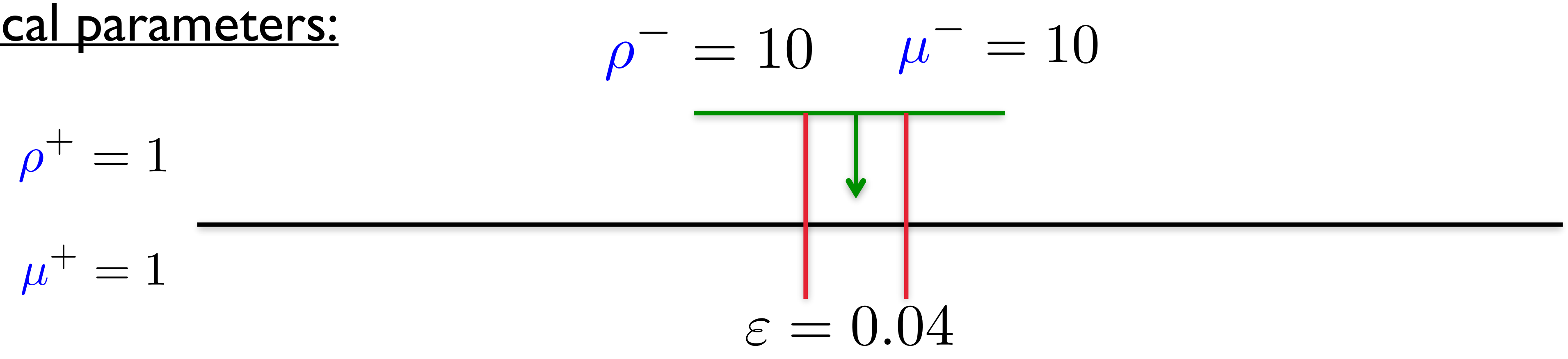
1D Numerical example

<u>Physical parameters:</u>

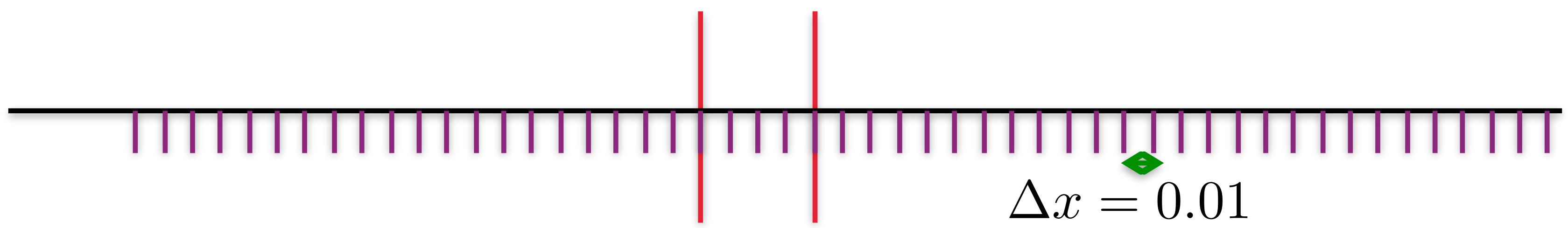


1D Numerical example

Physical parameters:



Computation of a reference solution via classical numerical method



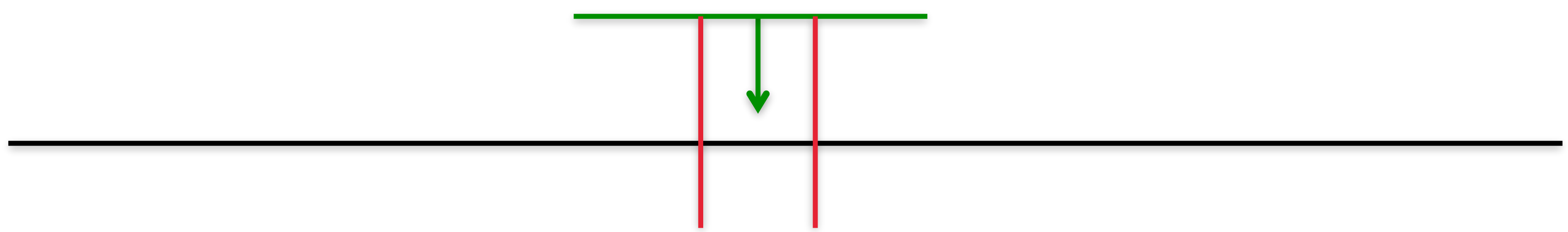
1D Numerical example

Physical parameters:

$\rho^- = 10 \qquad \mu^- = 10$

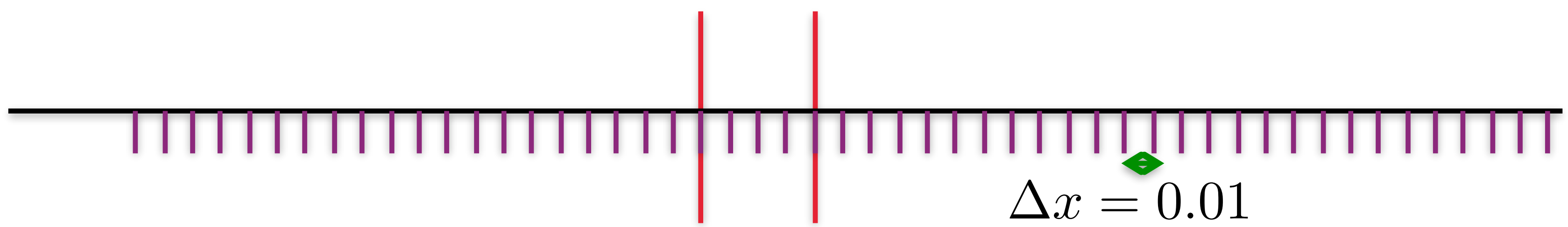
$\rho^+ = 1$

$\mu^+ = 1$



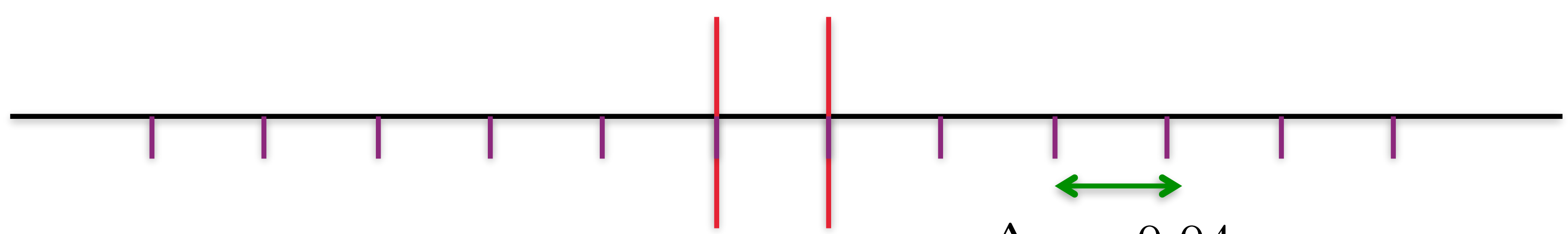
$\varepsilon = 0.04$

Computation of a reference solution via classical numerical method



$\Delta x = 0.01$

Computation of an approximate solution via asymptotic numerical method



$\Delta x = 0.04$

1D Numerical example

Total field

Diffracted field

CPU time (s)

Reference solution

$h_{\mathrm{ref}} = 0.001$

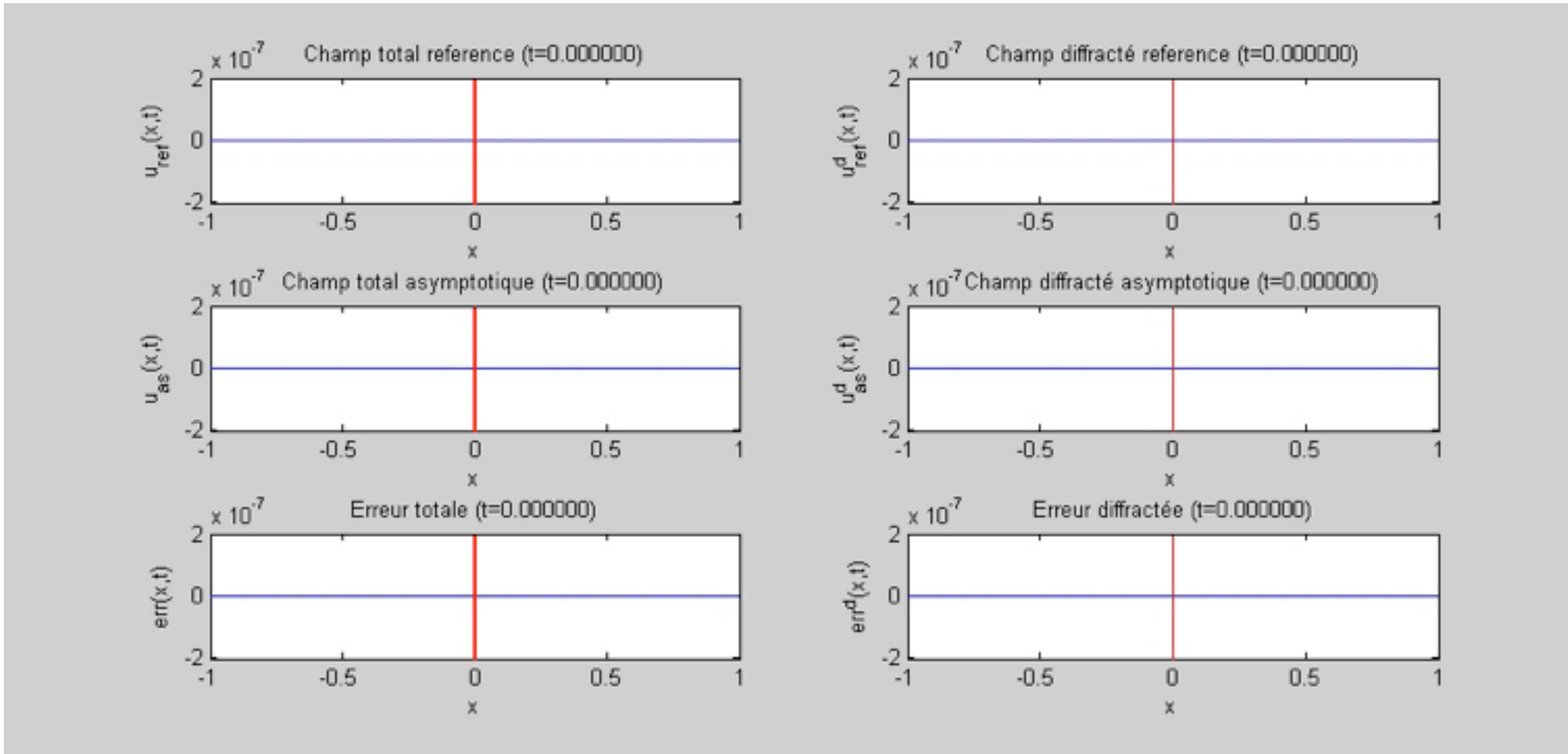
15

Asymptotic solution

$h_{\mathrm{as}} = 0.01$

1

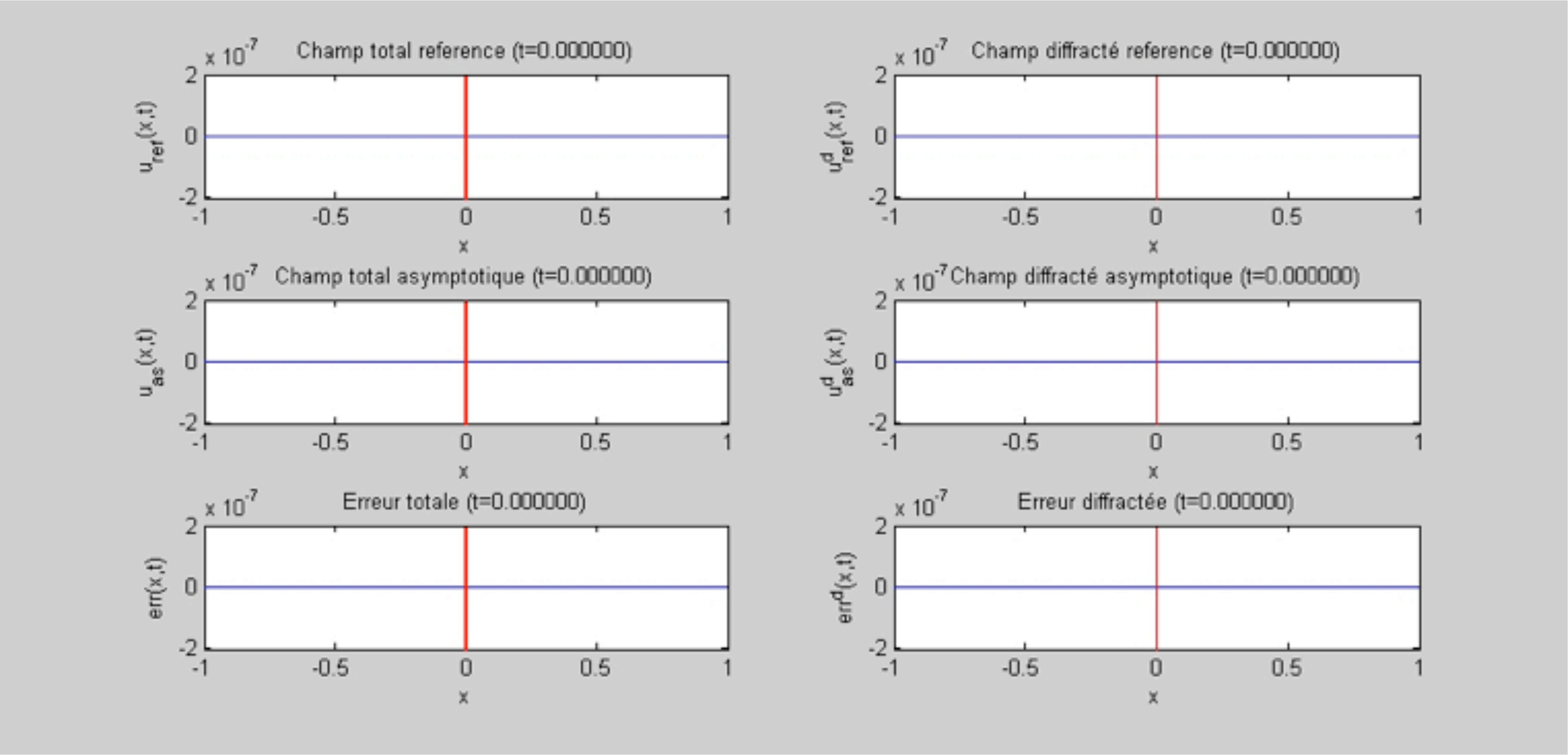
Difference



Relative L^2 error < 1%

1D Numerical example

	Total field	Diffracted field	CPU time (s)
Reference solution $h_{\text{ref}} = 0.001$			15
Asymptotic solution $h_{\text{as}} = 0.01$			1
Difference			



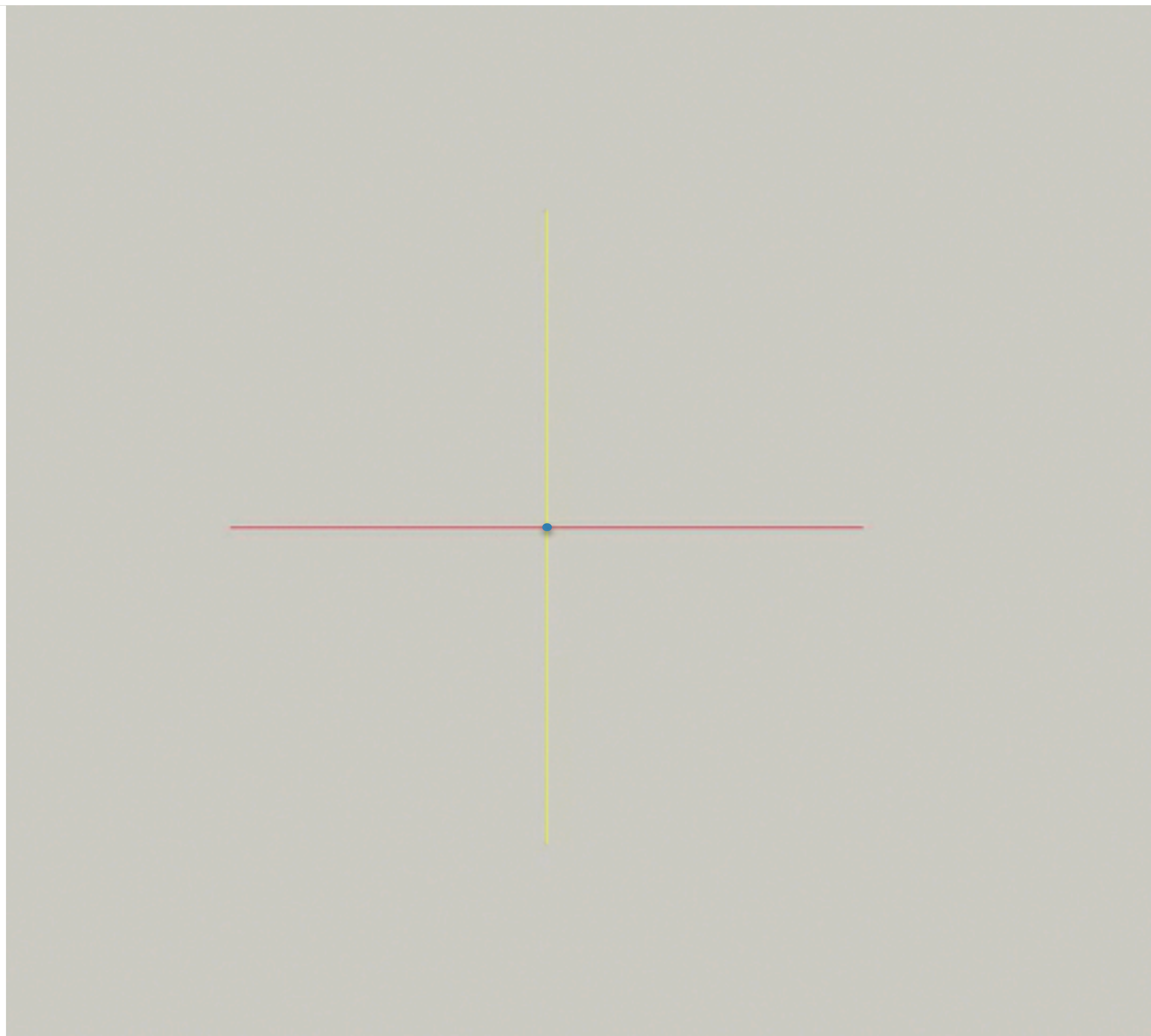
Relative L^2 error < 1%

Numerical example

Case of a unique defect

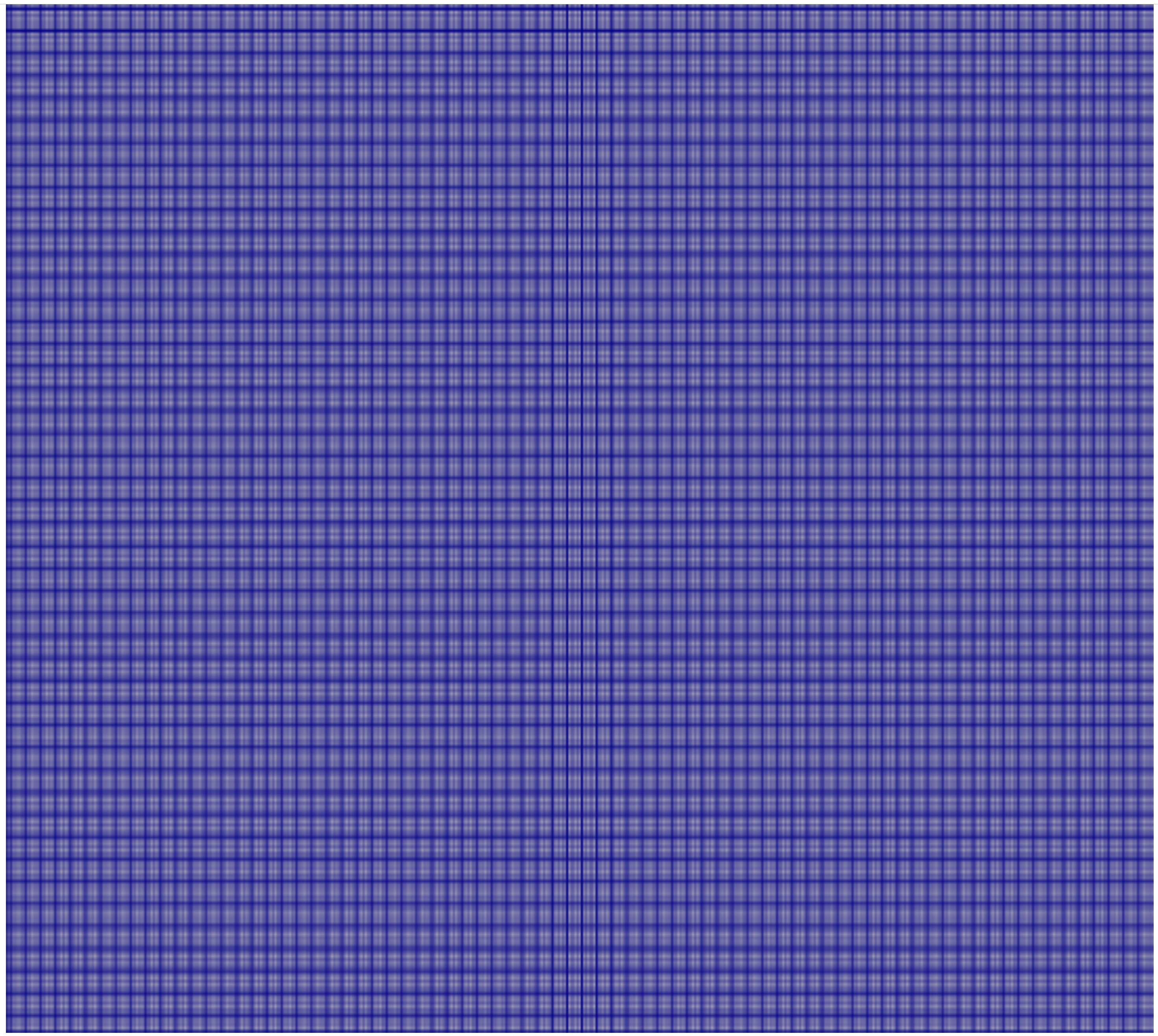
Number of degrees of freedom
90 000

CPU time
5 minutes



Numerical example

Case of a unique defect



<u>Number of degrees of freedom</u>
90 000

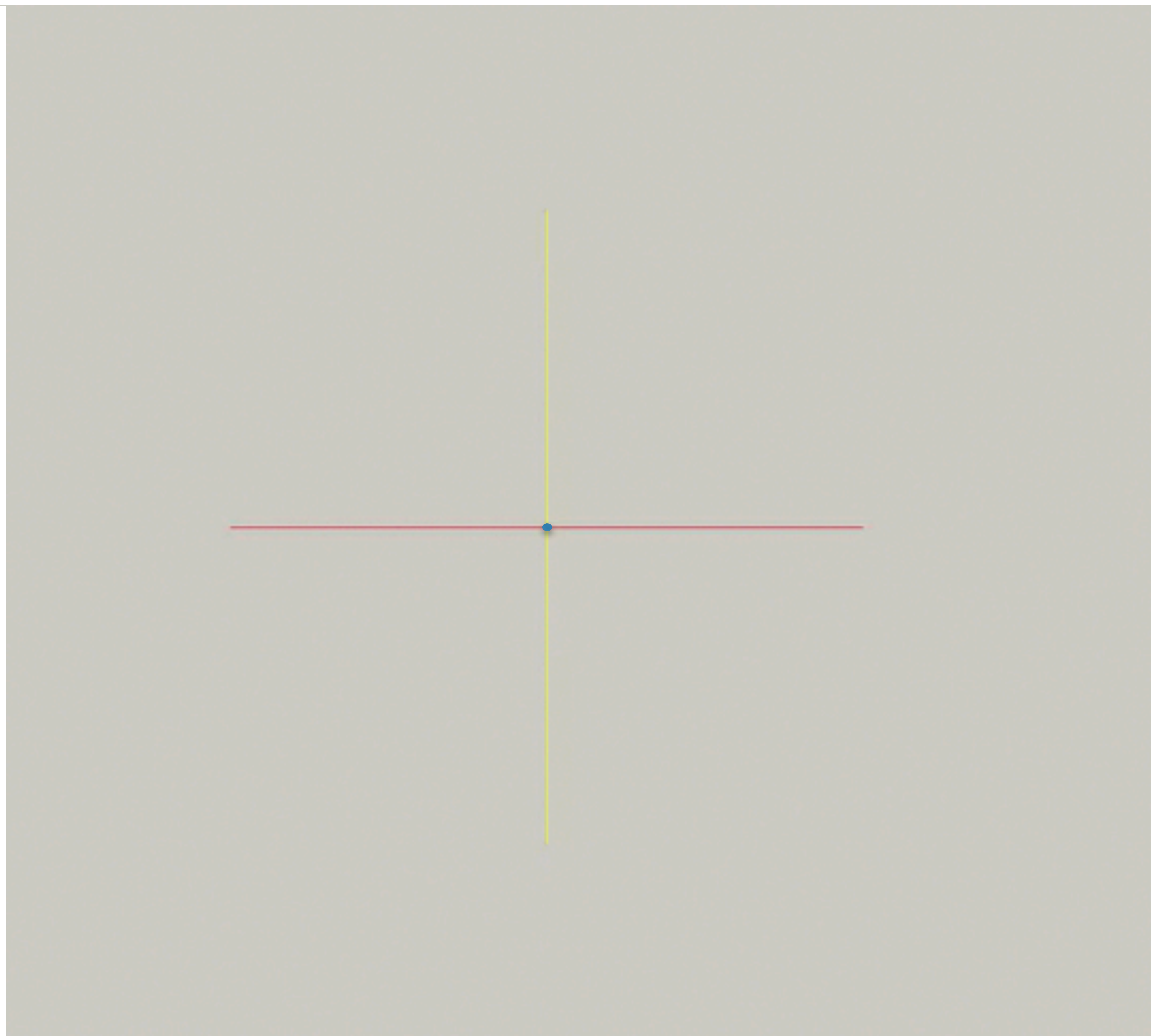
<u>CPU time</u>
5 minutes

Numerical example

Case of a unique defect

Number of degrees of freedom
90 000

CPU time
5 minutes

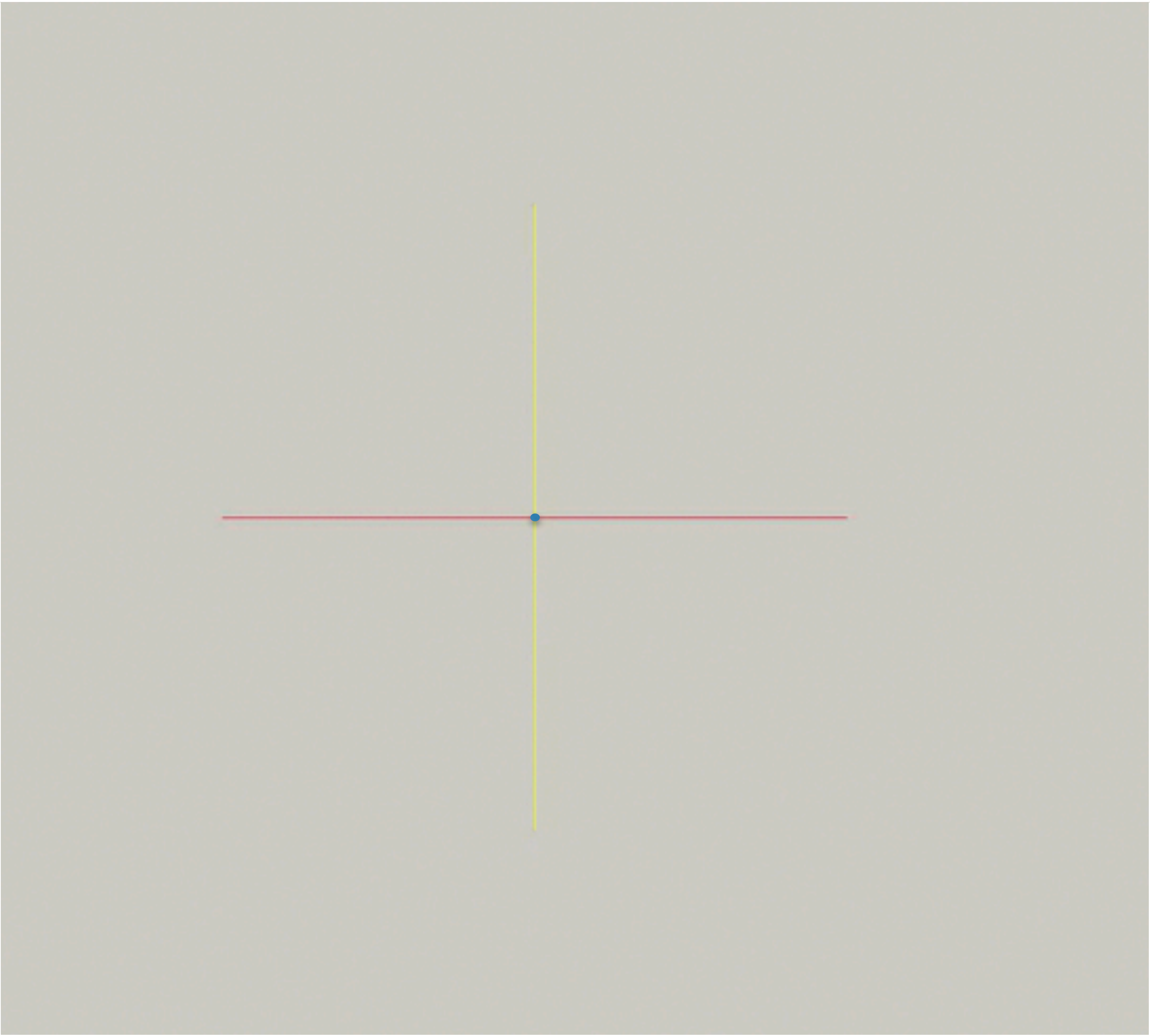


Numerical example

Case of a unique defect

Number of degrees of freedom
90 000

CPU time
5 minutes

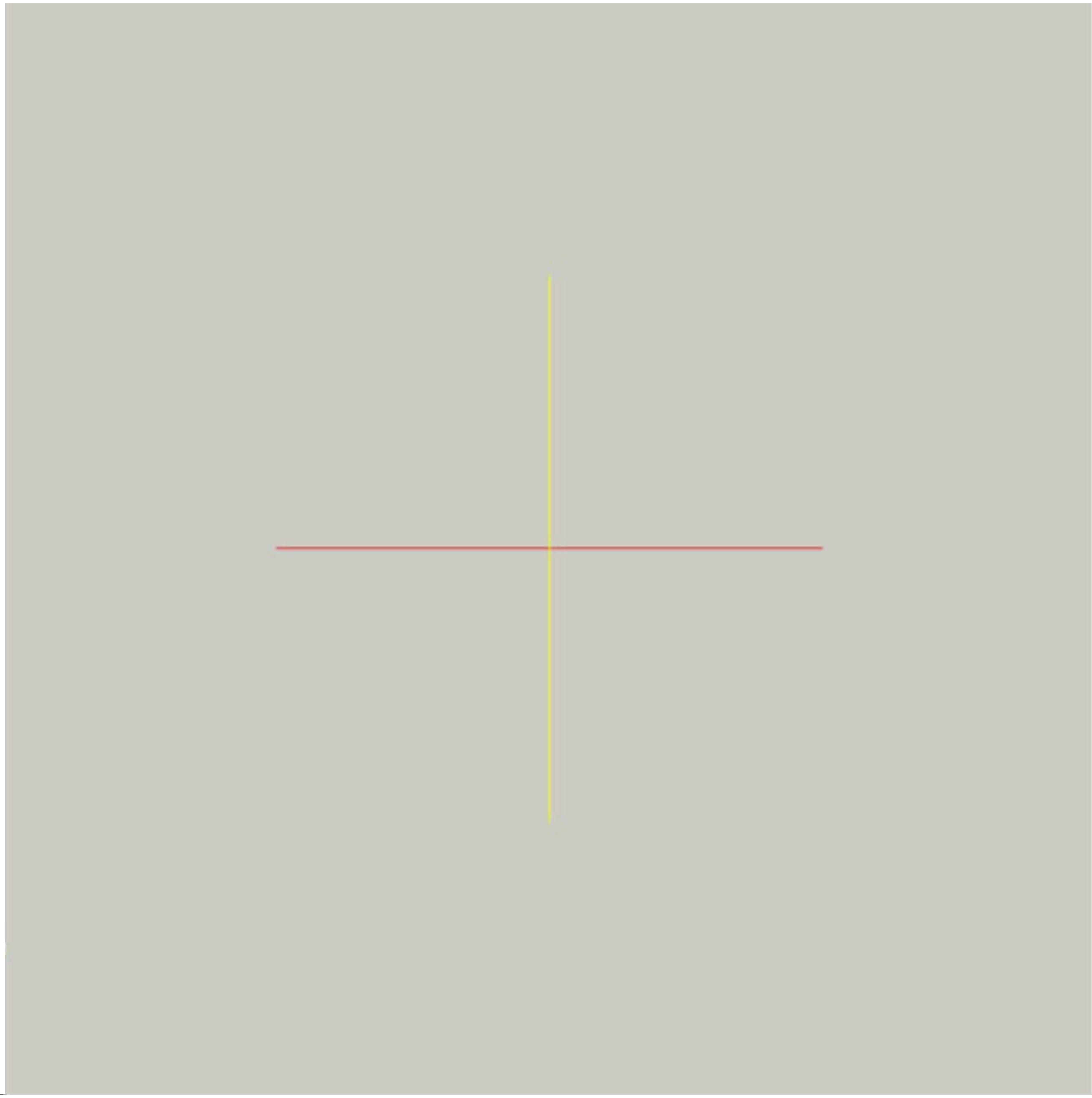


Numerical example

Case of a network of defects

Number of degrees of freedom
90 000

CPU time
5 minutes

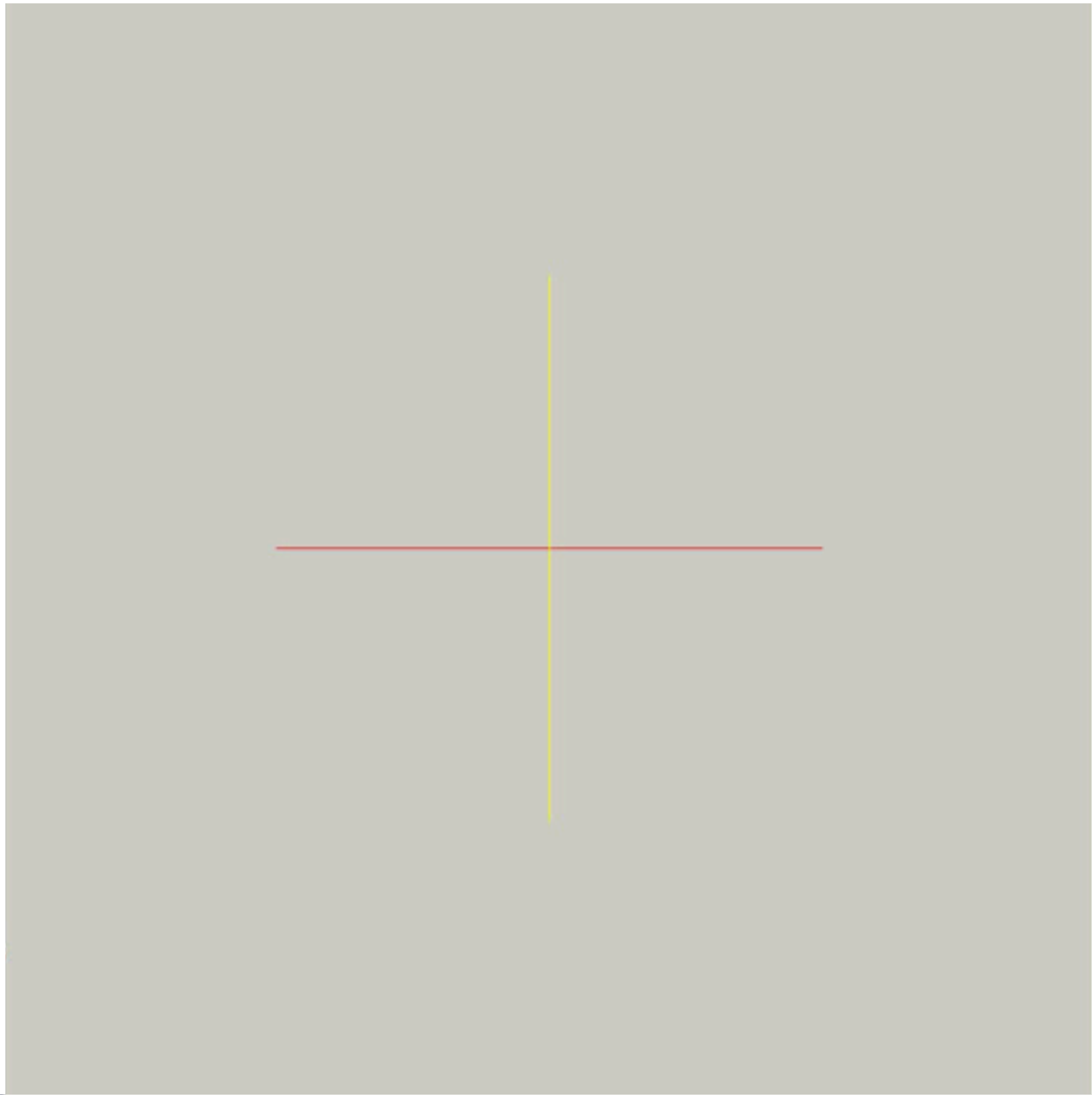


Numerical example

Case of a network of defects

Number of degrees of freedom
90 000

CPU time
5 minutes



Conclusion and prospects

- Efficient and accurate numerical method for computing the scattering of waves by small defects.
- Rigorous error estimates.

To be done:

- Adapt the method to the 3D case.
- Extension to defects of arbitrary shape.
- Extension to the elastodynamic wave equation.

Thank you for your attention !