Autonomous Quantum Error Correction

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QUANTIC

Outline

I. Why quantum information processing ?

II. Classical bits vs. Quantum bits

III. Quantum Error Correction

WHY QUANTUM INFORMATION PROCESSING ?

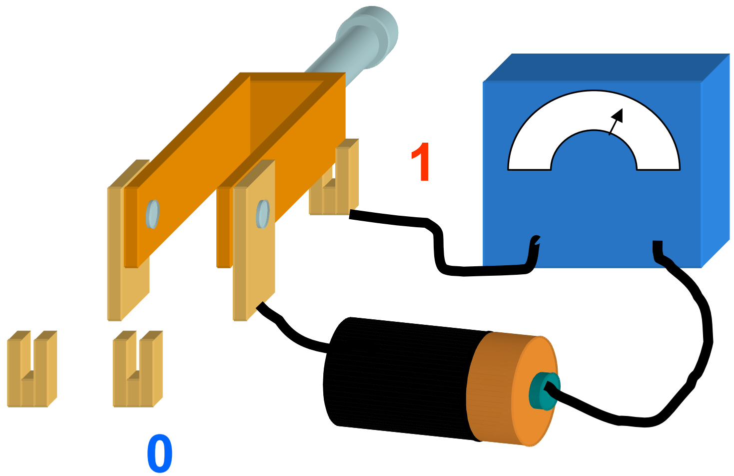
Why quantum information processing ?

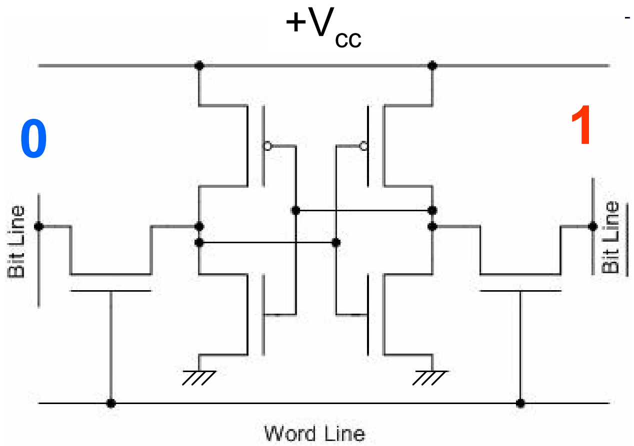
- Cryptography : quantum key distribution
- Quantum simulator : simulation of quantum systems
- Quantum algorithms : Shor's algorithm on prime number factorization in polynomial time

CLASSICAL BITS VS. QUANTUM BITS

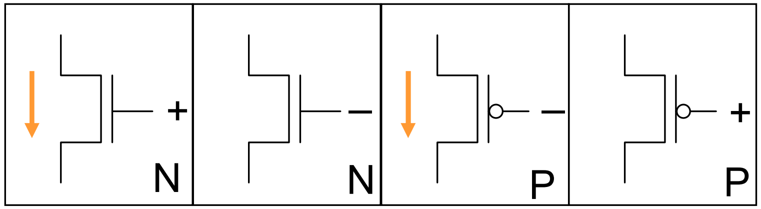
Classical Bit : Bistable system

Bit : State 0 ou 1



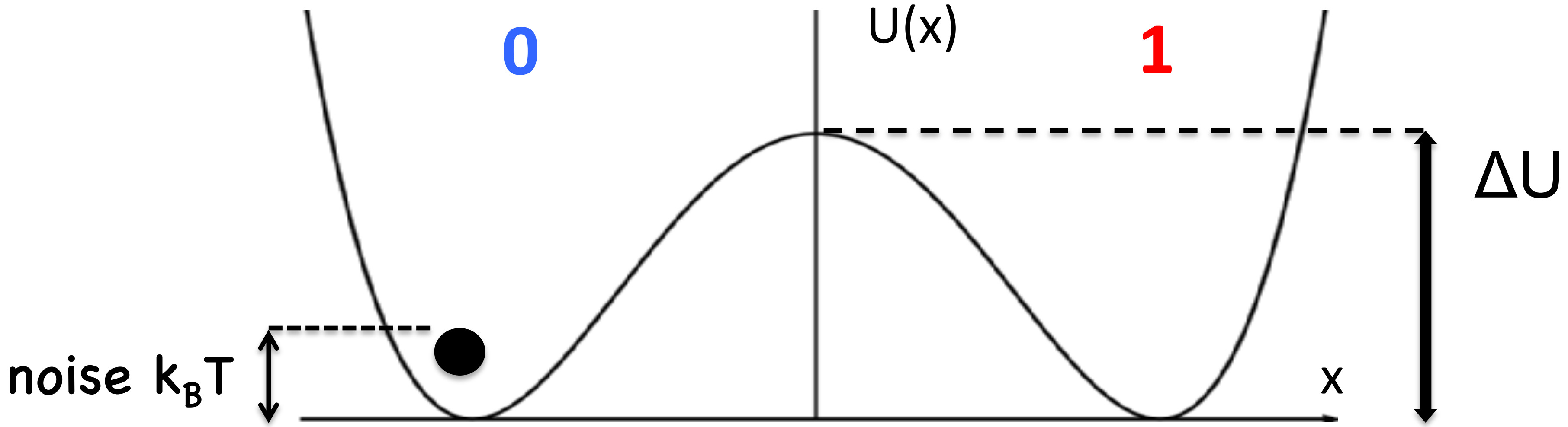


CMOS Transistors:



Courtesy of Michel Devoret, Collège de France, 2010

Classical Bit : Bistable system



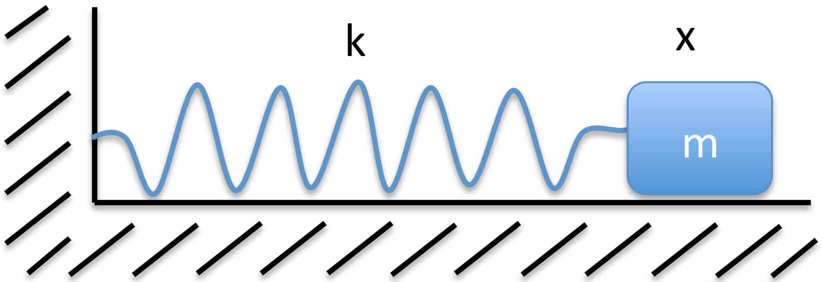
$$m\frac{d^2x}{dt^2} + \eta\frac{dx}{dt} + \frac{\partial U}{\partial x} = 0$$

η : friction coefficient

- friction
- thermal noise $k_BT \ll \Delta U$

Quantum physics ?
Quantum bit (Qubit) ?

Spring : Classical case



$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

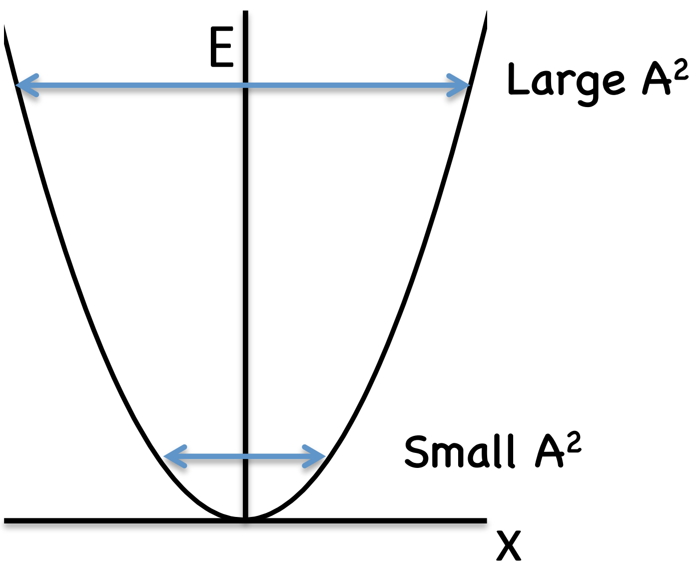
$$x(t) = A\cos(\omega t + \phi)$$

Harmonic potential

$$V(x) = \frac{1}{2}kx^2$$

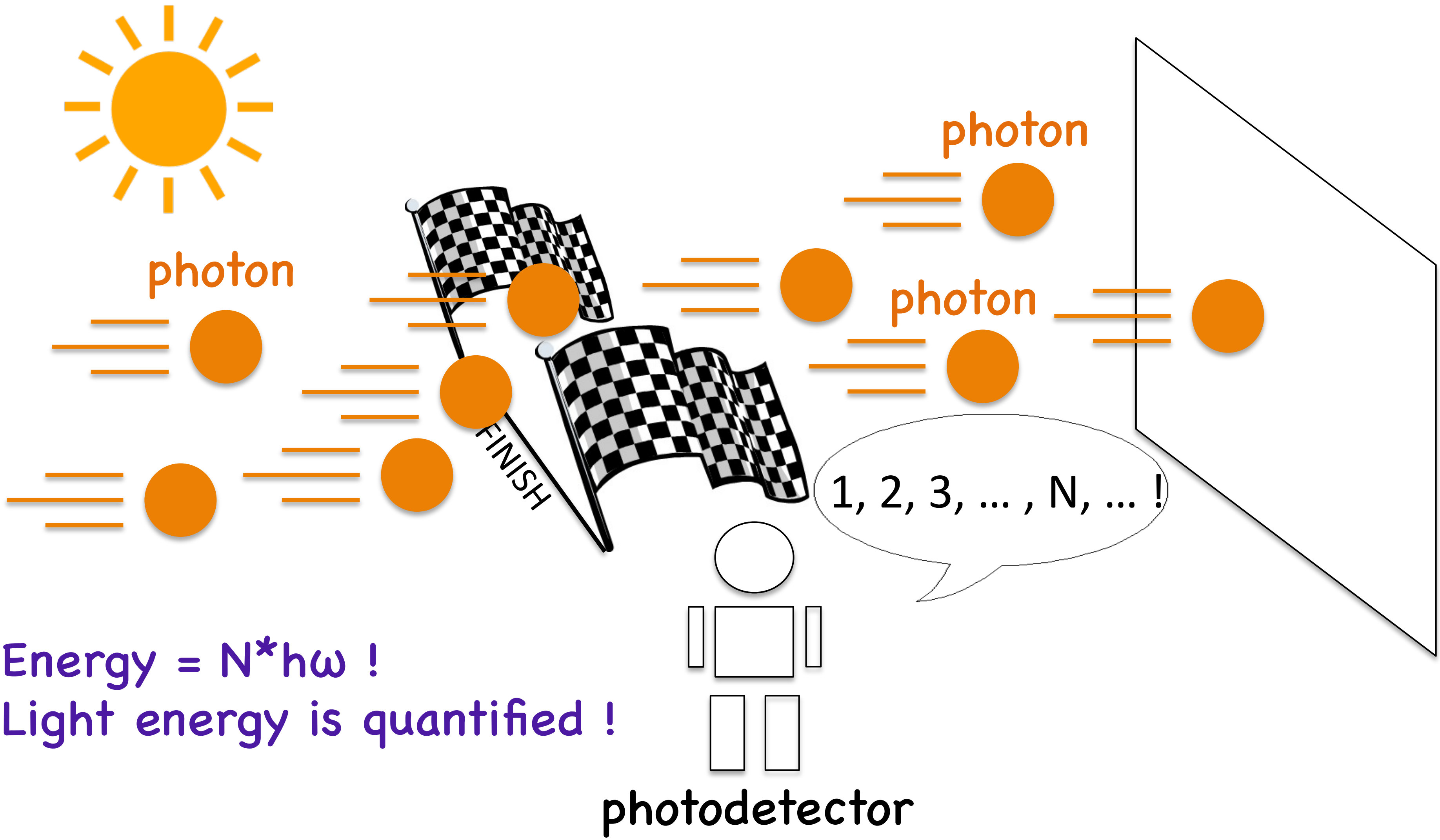
No dissipation :

$\Rightarrow$ constant energy



Quantum Mechanics : System energy is quantified !

Example : Light



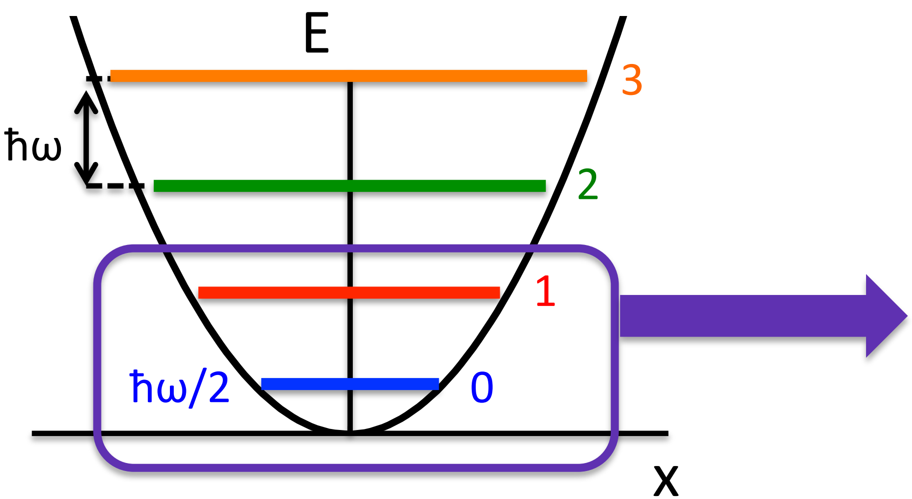
Energy = N*hω !
Light energy is quantified !

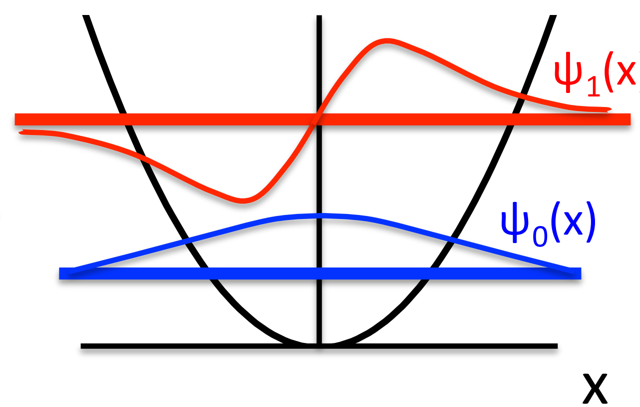
Spring : Quantum case

- Discrete set of stationnary energy states :
  $\psi_0(x)$, $\psi_1(x)$, $\psi_2(x)$, $\psi_3(x)$ …
  L^2 functions associated to energies E_0, E_1, E_2, E_3 …

  ψ_k satisfying $[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + U(x)]\psi_k = E_k\psi_k$

  $|\psi(x)|^2$ = density of probability to find the system in x
- Restrict to the first two energy levels : $|0\rangle := \psi_0$
  and $|1\rangle := \psi_1$





Postulate 1 : Quantum superposition

- $\{|0\rangle, |1\rangle\}$ form an orthonormal basis of a 2D-Hilbert space with $\langle 0|1\rangle = \int_D dx \psi_0^*(x)\psi_1(x)$
- Quantum superposition : general state is given by

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad \alpha, \beta \in \mathbb{C} \qquad |\alpha|^2 + |\beta|^2 = 1$$

- $|\alpha|^2$ probability to find the system in state $|0\rangle$
- $|\beta|^2$ probability to find the system in state $|1\rangle$

Qubit!

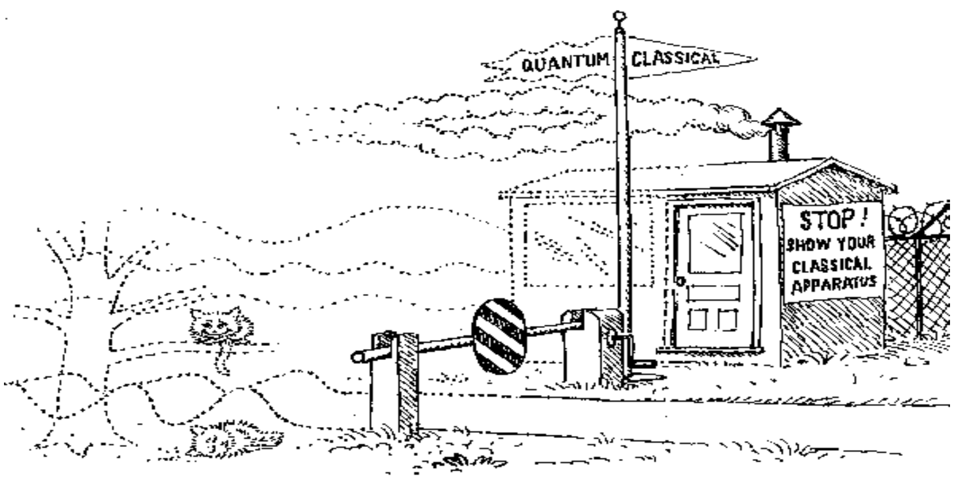
$|0\rangle$

$|1\rangle$

$\frac{|0\rangle - |1\rangle}{\sqrt{2}}$

$\frac{|0\rangle + 2i|1\rangle}{\sqrt{5}}$

$\vdots$



$|0\rangle$

$|1\rangle$

Postulate 2 : Quantum Measurement

- Quantum measurement :
  - Consider the qubit in state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
  - Ask the system : are you in $|0\rangle$ or $|1\rangle$?
  - With probability $|\alpha|^2$ the answer is $|0\rangle$
  - The system is **projected** in state $|\psi\rangle = |0\rangle$!

Measurement modifies the qubit state !

Quantum measurement can be DESTRUCTIVE !

Composite system and Quantum Entanglement

- Composite system : Consider two qubits A and B
  - Qubit A lives in $\mathcal{H}_A$
  - Qubit B lives in $\mathcal{H}_B$
  - Joint system qubits A+B lives in $\mathcal{H}_A \otimes \mathcal{H}_B$

$$\mathcal{H}_A \otimes \mathcal{H}_B = \mathrm{vec}_C\{|00\rangle, |10\rangle, |01\rangle, |11\rangle\}$$

$$|00\rangle := |0\rangle_A \otimes |0\rangle_B \qquad |01\rangle := |0\rangle_A \otimes |1\rangle_B \qquad \ldots$$

- Entangled state :
  - Consider $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$
  - First, we measure qubit A : we find qubit A in $|0\rangle$ (with 50% probability)
  - The joint state collapses to $|\psi\rangle = |00\rangle$
  - qubit B is in $|0\rangle$ with probability 1 !

Quantum « rules » : Summary

1) Quantum superposition : general qubit state

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

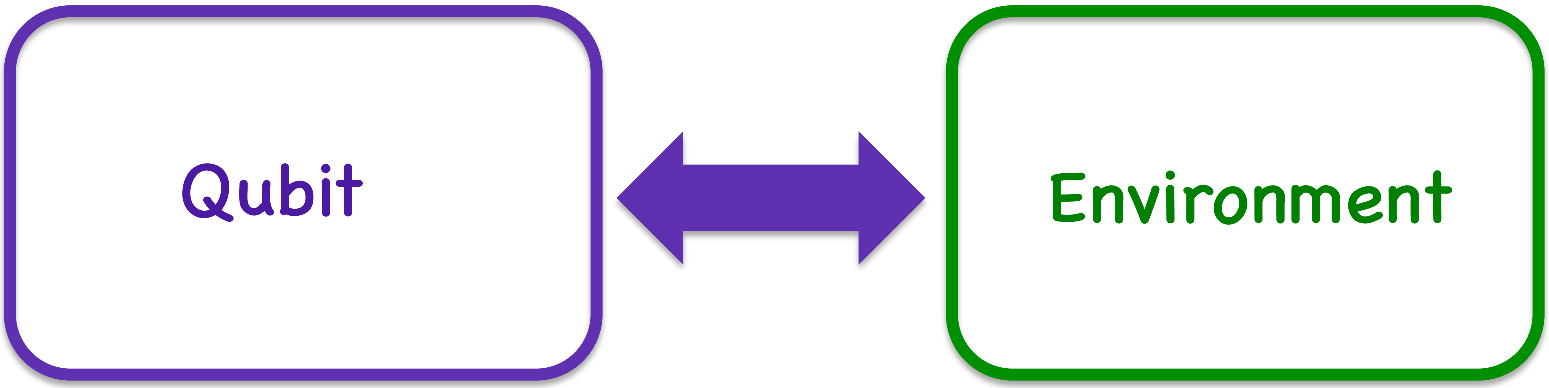
$$\alpha, \beta \in \mathbb{C} \qquad |\alpha|^2 + |\beta|^2 = 1$$

2) Measurement : revealing information about the state can destroy the superposition

3) Quantum Entanglement : possibility of having strongly correlated states between two qubits

Consequence : Decoherence

Unwanted coupling with the environment :



-The environment measures the qubit
and this measurement destroys the quantum superposition !

-> lifetime of typically 100us (for superconducting circuits)

- Lifetime decreases with the number of qubits

How can we fight decoherence ?

QUANTUM ERROR CORRECTION (QEC)

Bit vs. Qubit errors

- Errors on classical bits : bit-flip errors 0 → 1
  1 → 0
- Errors on qubits :

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \longrightarrow |\psi'\rangle = \alpha'|0\rangle + \beta'|1\rangle$$

Errors can be cast in two error channels :

Bit-flip errors

$$|0\rangle \rightarrow |1\rangle$$
$$|1\rangle \rightarrow |0\rangle$$

Rest of the talk

Phase-flip errors

$$|0\rangle \rightarrow |0\rangle$$
$$|1\rangle \rightarrow -|1\rangle$$
$$\frac{1}{\sqrt{2}}[|0\rangle + |1\rangle] \longrightarrow \frac{1}{\sqrt{2}}[|0\rangle - |1\rangle]$$

Quantum error correction

- Classical error correction:
  information redundantly encoded

  Ex : 0 $\longrightarrow$ 000
  1 $\longrightarrow$ 111

  such that error on bit 1 : 100 $\longrightarrow$ 000
  error on bit 2 : 101 $\longrightarrow$ 111

Quantum error correction

- Quantum error correction (bit-flip errors only)
  three-qubit bit-flip code :

$$|0\rangle \rightarrow |000\rangle$$
$$|1\rangle \rightarrow |111\rangle \qquad \alpha|0\rangle + \beta|1\rangle \xrightarrow{\text{encoding}} \underbrace{\alpha|000\rangle}_{=|0_L\rangle} + \underbrace{\beta|111\rangle}_{=|1_L\rangle}$$

Error on qubit 1 :

$$\alpha\,|100\rangle + \beta\,|011\rangle \xrightarrow{\text{correction}} \alpha|000\rangle + \beta|111\rangle$$

But information about α and β
must not be revealed ...

How do we detect errors
without destroying the state ?

Quantum error correction

- Error detection : Parity measurement

$\alpha|000\rangle + \beta|111\rangle$

$P_{12} = 0 \quad P_{23} = 0$

qubit 1 flips → $\alpha\,|100\rangle + \beta\,|011\rangle$

$P_{12} = 1 \quad P_{23} = 0$

qubit 2 flips → $\alpha\,|010\rangle + \beta\,|101\rangle$

$P_{12} = 1 \quad P_{23} = 1$

qubit 3 flips → $\alpha\,|001\rangle + \beta\,|110\rangle$

$P_{12} = 0 \quad P_{23} = 1$

What we can measure :

Joint parities

P_{12} := [Q1+Q2] mod 2

P_{23} := [Q2+Q3] mod 2

NON-destructive measurements !

Quantum error correction

- Error Correction : simply apply inverse operation

$$\alpha|000\rangle + \beta|111\rangle$$

$P_{12} = 0 \quad P_{23} = 0$

$$\alpha|100\rangle + \beta|011\rangle \xrightarrow{\text{flip qubit 1}} \alpha|000\rangle + \beta|111\rangle$$

$P_{12} = 1 \quad P_{23} = 0$

$$\alpha|010\rangle + \beta|101\rangle \xrightarrow{\text{flip qubit 2}} \alpha|000\rangle + \beta|111\rangle$$

$P_{12} = 1 \quad P_{23} = 1$

$$\alpha|001\rangle + \beta|110\rangle \xrightarrow{\text{flip qubit 3}} \alpha|000\rangle + \beta|111\rangle$$

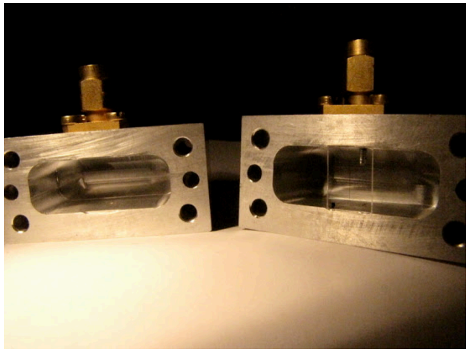
$P_{12} = 0 \quad P_{23} = 1$

Quantum error correction : implementation

1st option

- Build a feedback loop
  - real-time data analysis takes time
  - quantum systems are short-lived

Superconducting circuits

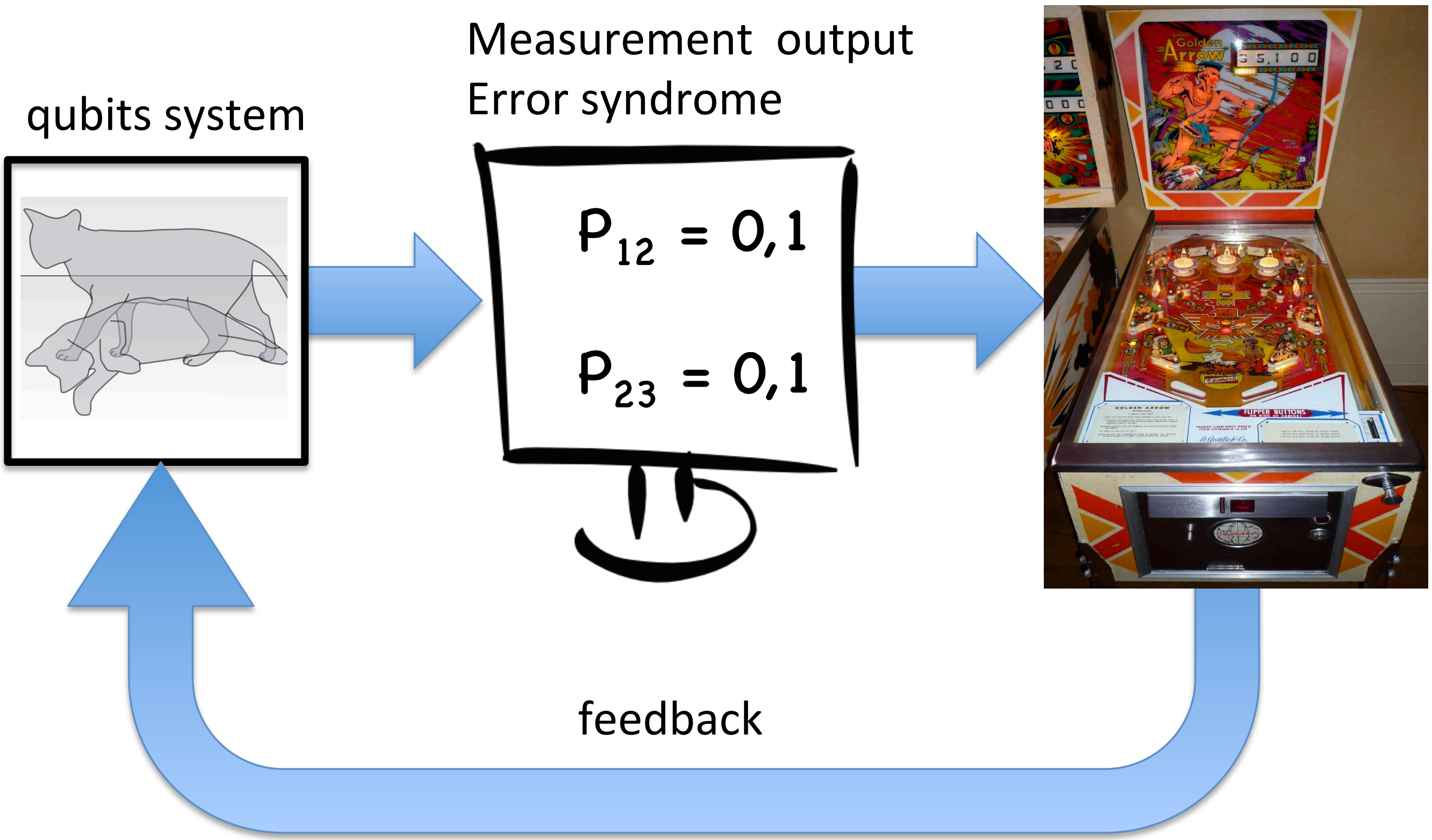


100 us

Courtesy of Quantum Electronics group, LPA, ENS (Paris)

Quantum error correction : feedback loop

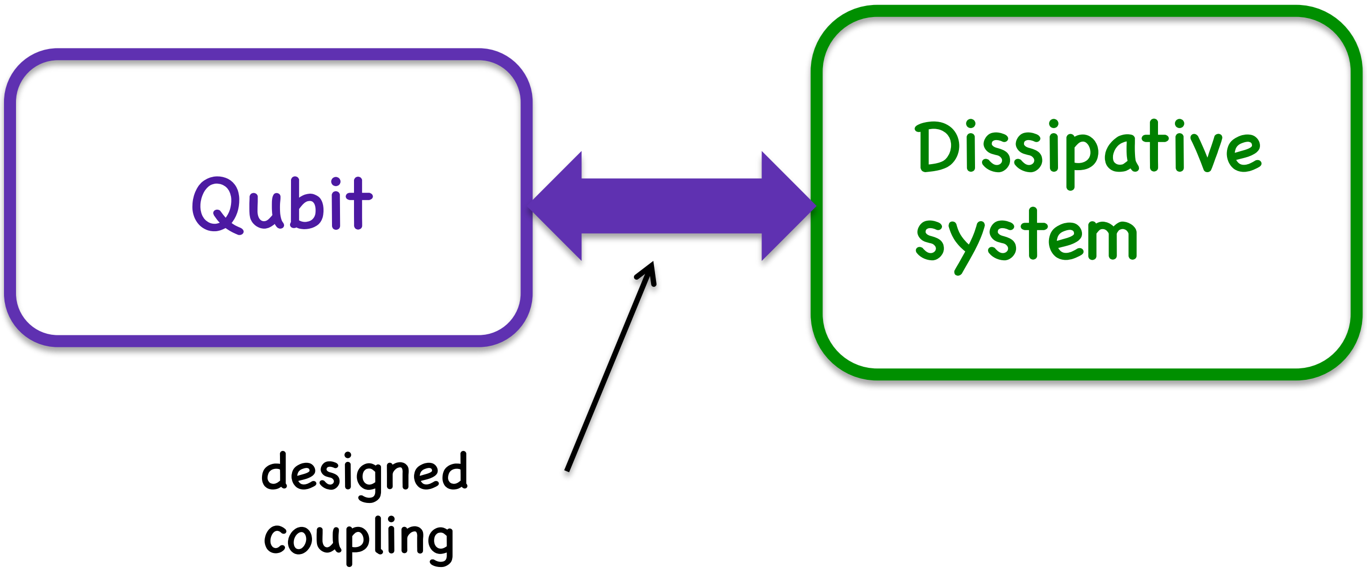
Error Correction ? Use a flipper !



Quantum error correction : implementation

2nd option : (what we have proposed)

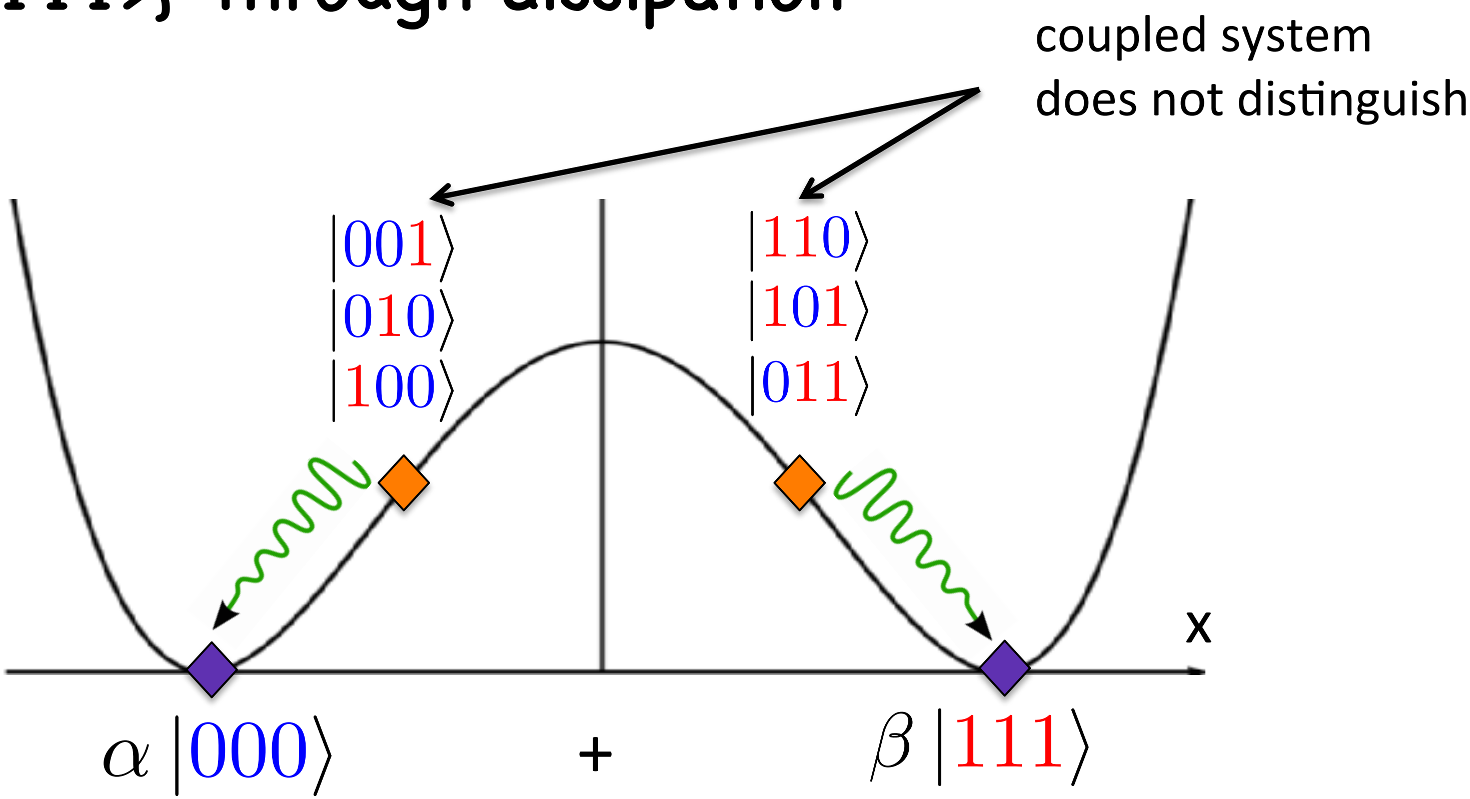
- Autonomous QEC by coupling the qubits with another strongly dissipative quantum system :



Autonomous quantum error correction

Main idea :

Coherent stabilization of the manifold $\{|000\rangle, |111\rangle\}$ through dissipation



In practice

- Complete codes (correct for all types of errors) exist but have never been physically implemented
- Qubits of many kinds : trapped ions, superconducting qubits, NV centers …
- Quantum computer : 10 qubits max so far. Limited by decoherence !

-> QEC remains a challenge to overcome !

Thanks !

Questions ?