# Numerical Methods for Fluid-Structure Interaction

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REO team
Inria de Paris & Université Pierre et Marie Curie (Paris 6) - LJLL

PhD Advisor: ***Miguel A. Fernández***

Junior Seminar, Inria, Paris, February 16, 2016

# A widespread problem...

- Framework: **interaction** of
  - **Structure**: elastic (non-linear,…)

  with an internal or surrounding
  - **Fluid**: incompressible (viscous,...)

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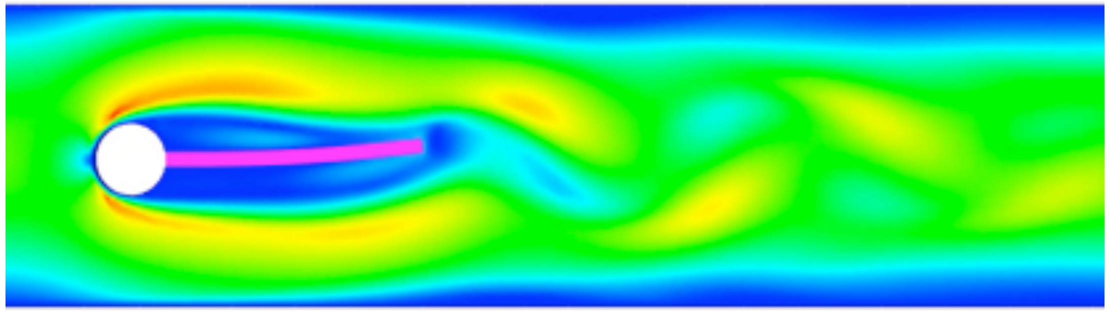

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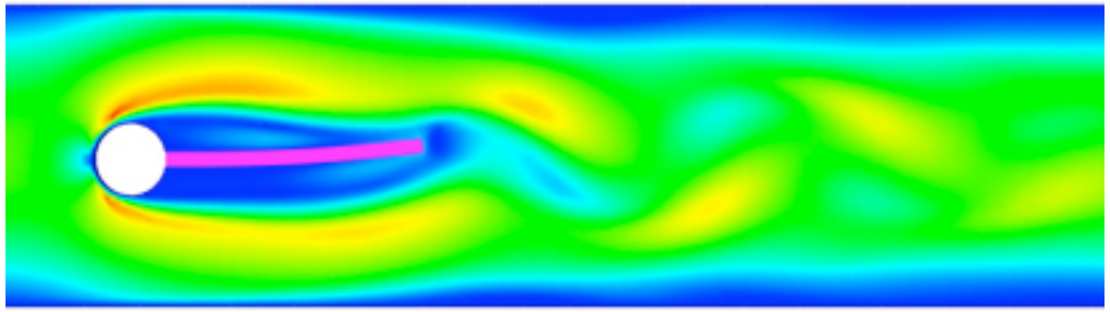

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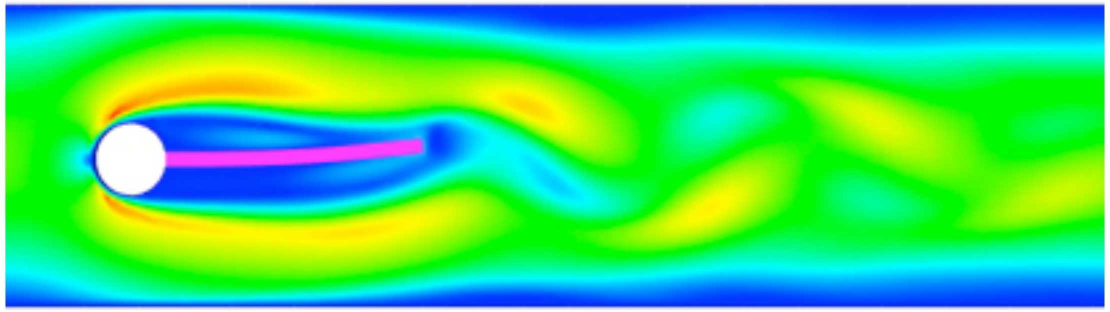

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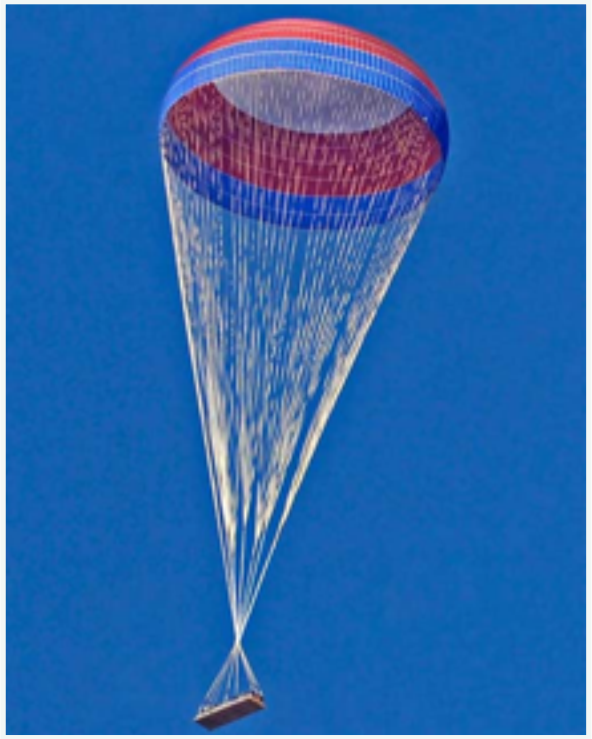

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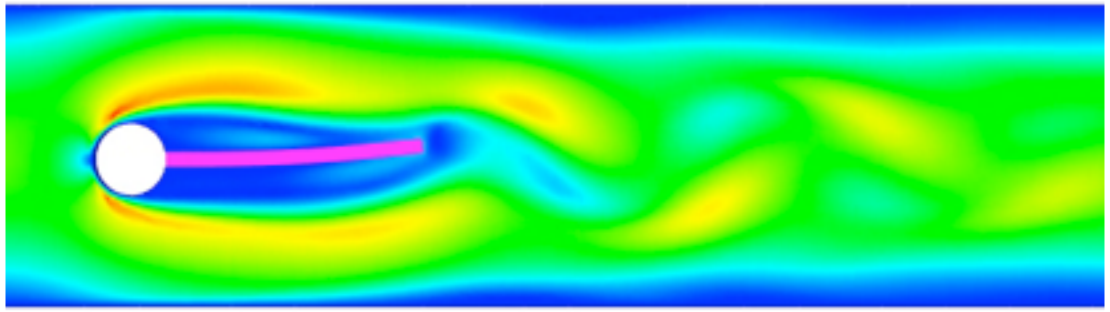

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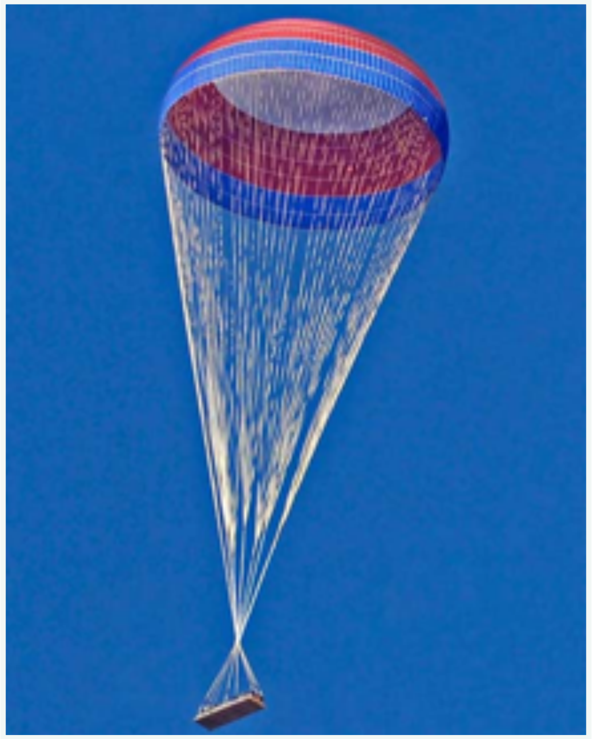

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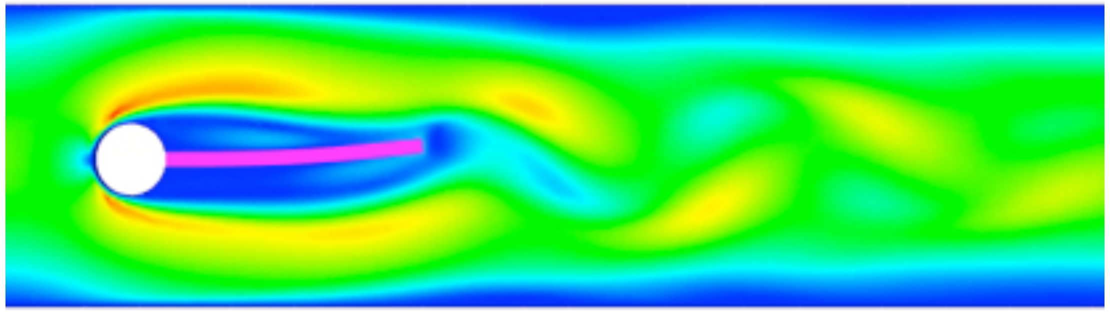

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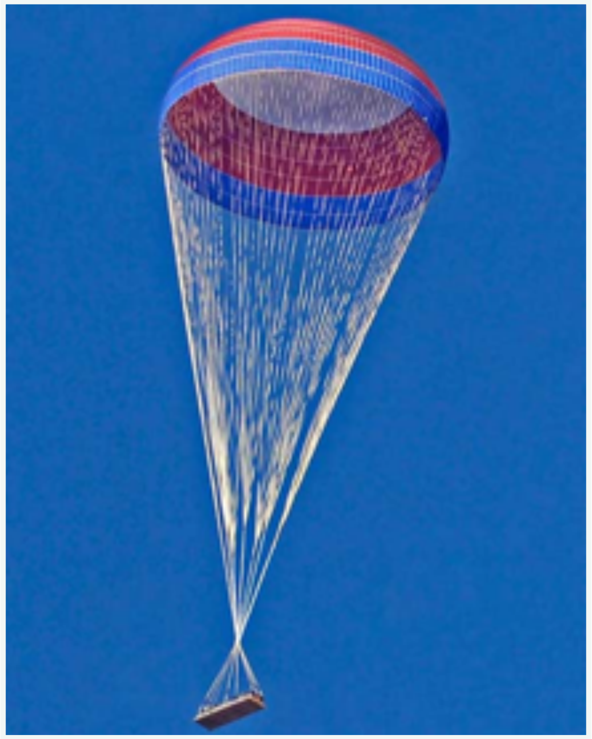

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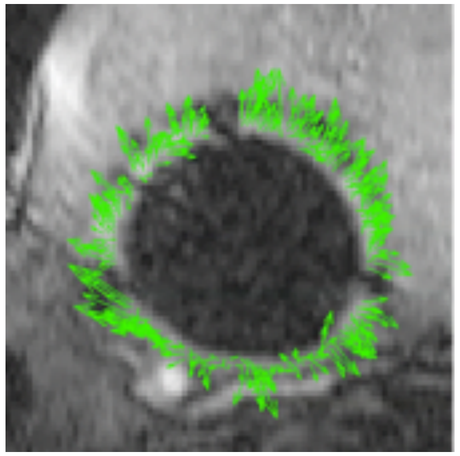

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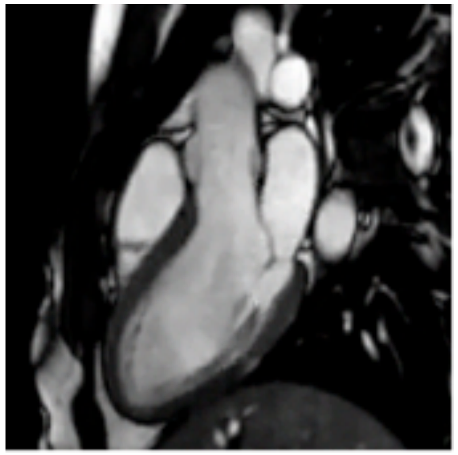

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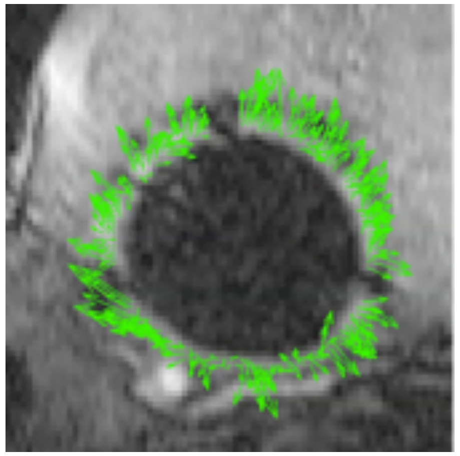

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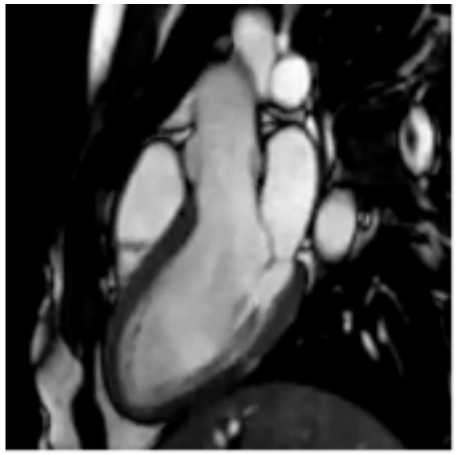

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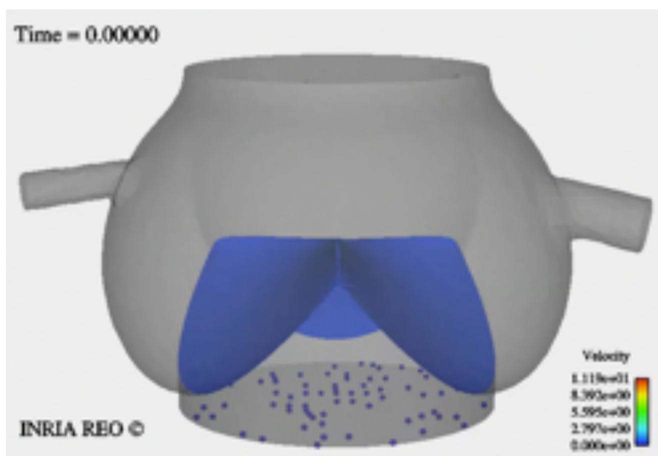


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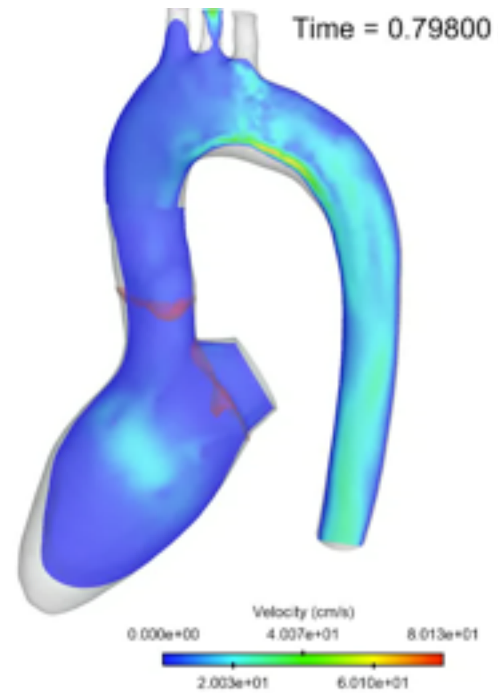


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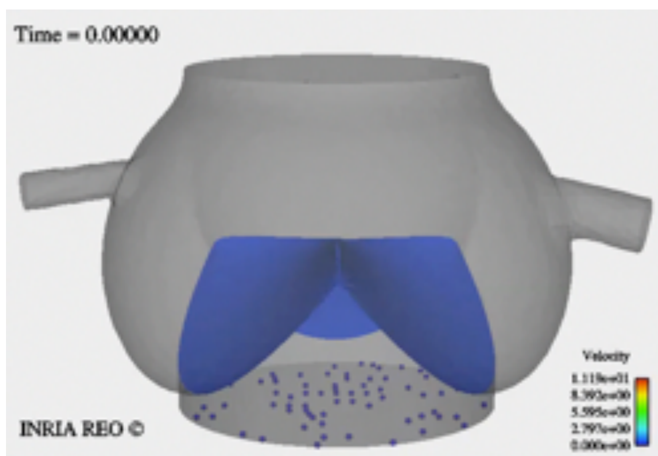


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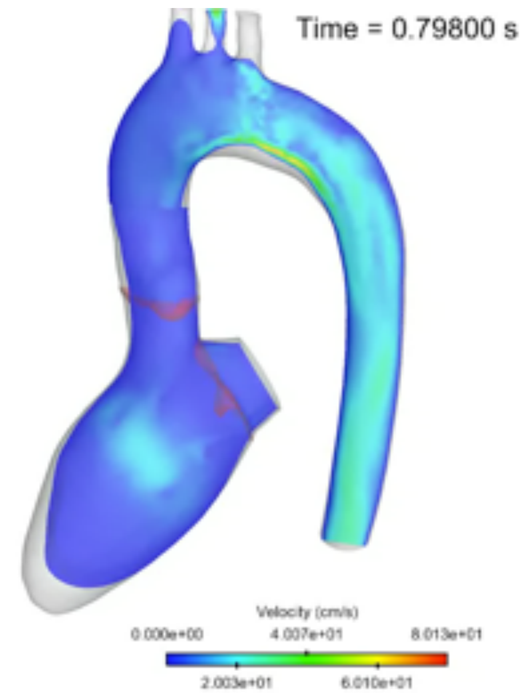


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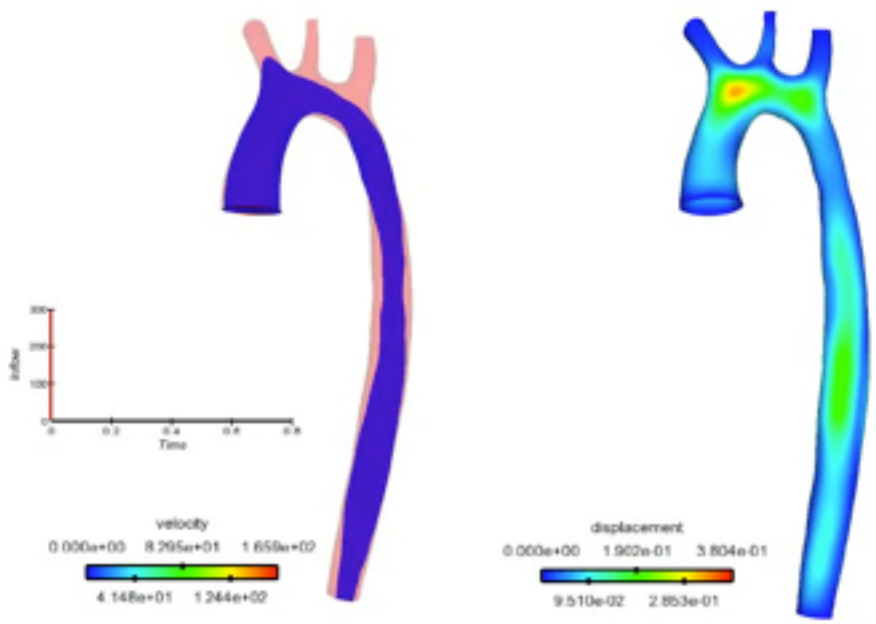


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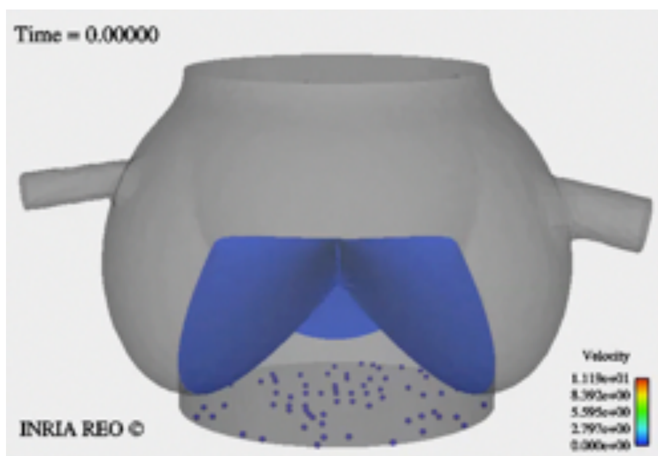


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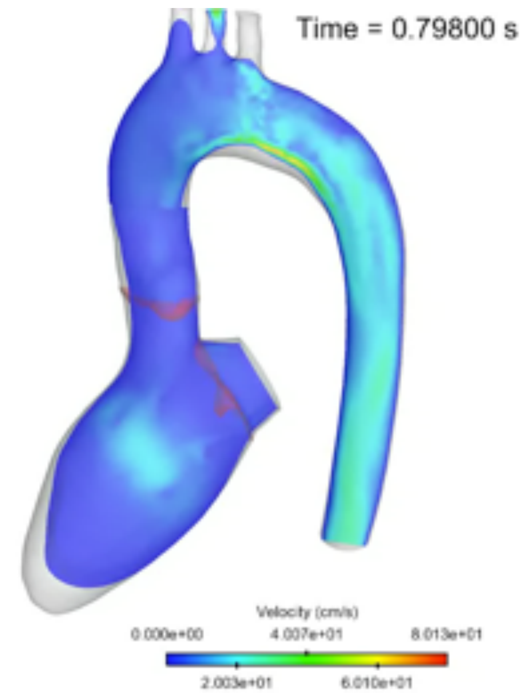


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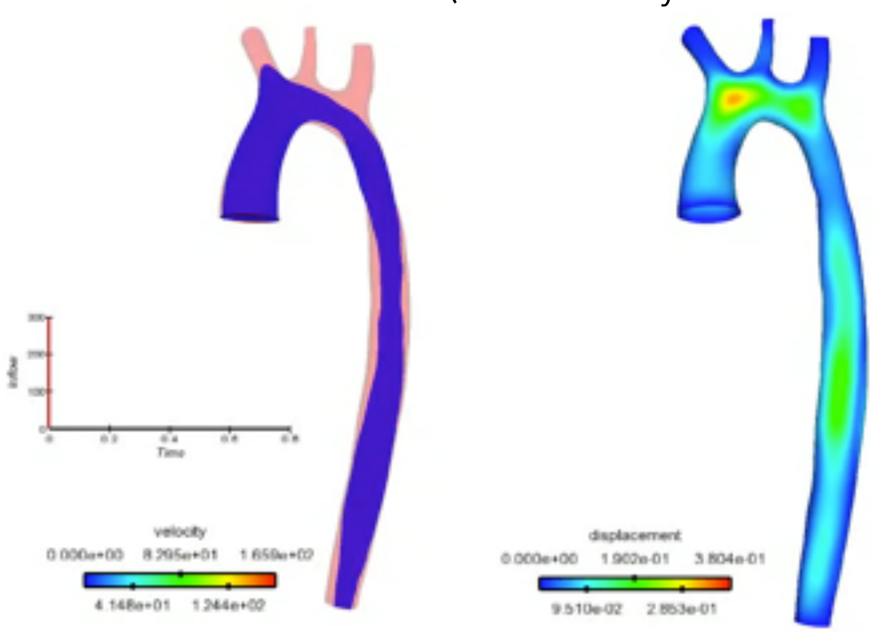


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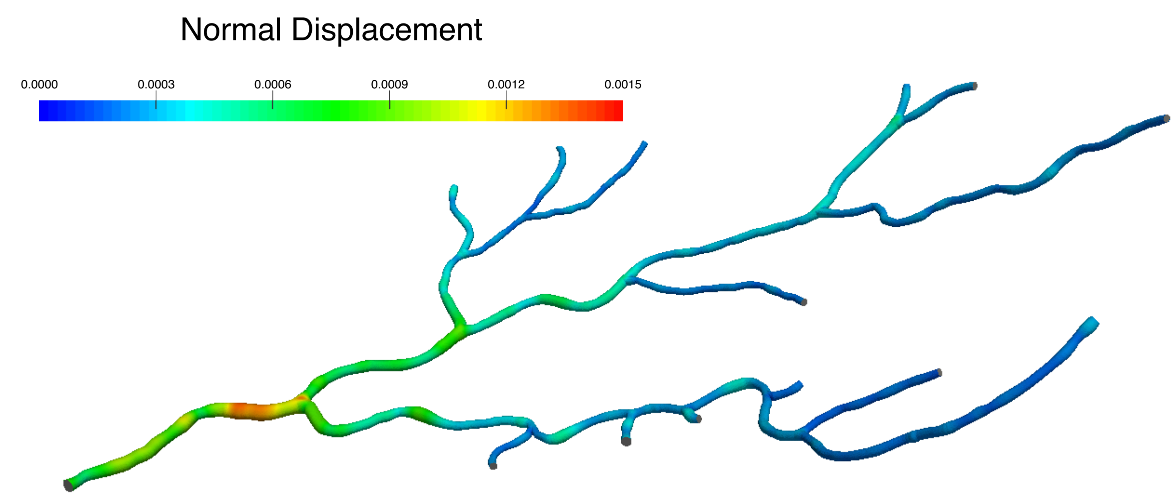


**Retinal Hemodynamics** (simulation by M. Aletti)

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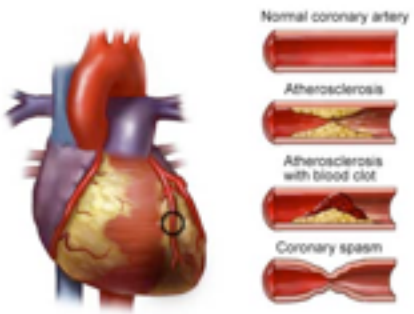

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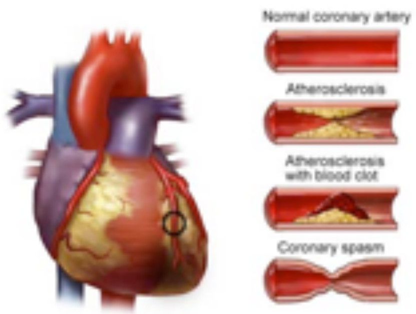
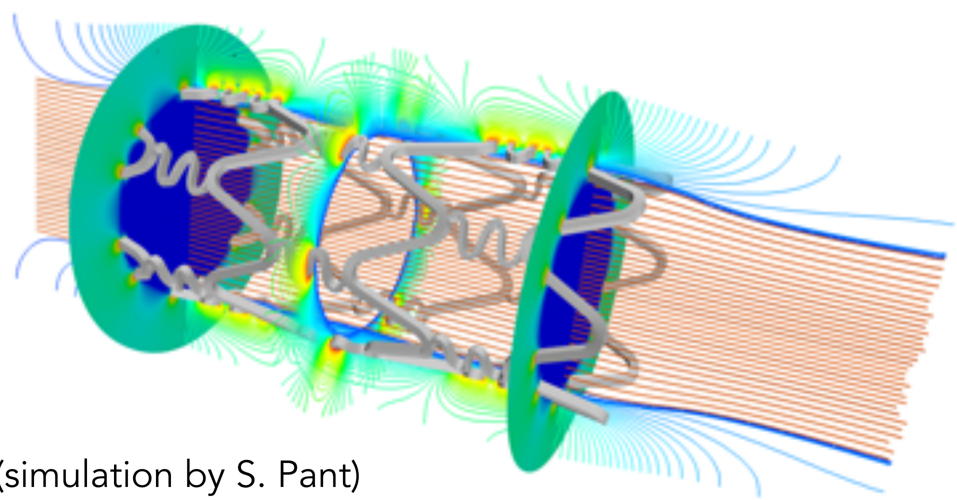
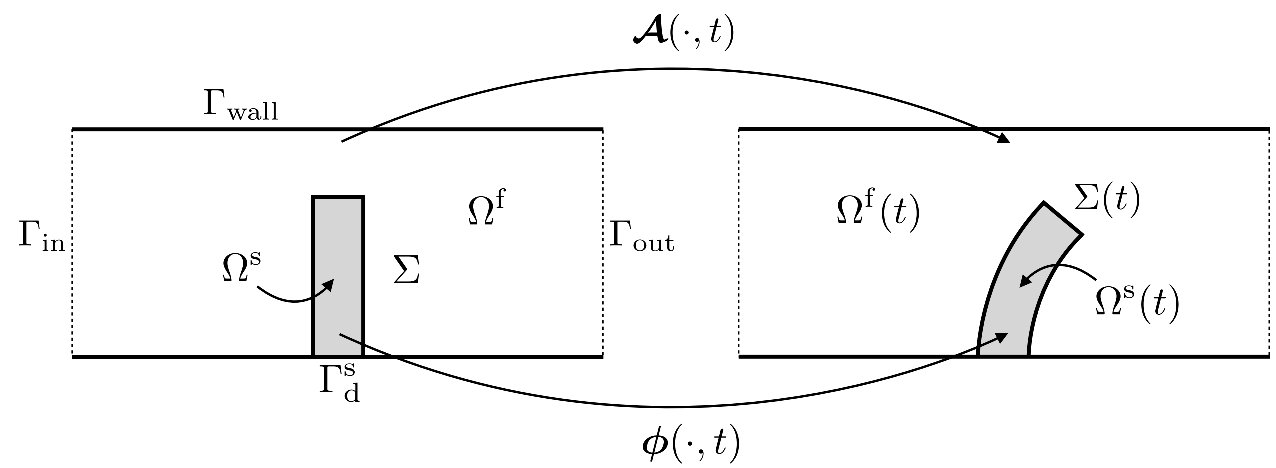



- **Control & optimization** of medical devices and therapies
  - *What would be the best?*



**Coronary stent design** (simulation by S. Pant)

# The Non-Linear Coupled Problem



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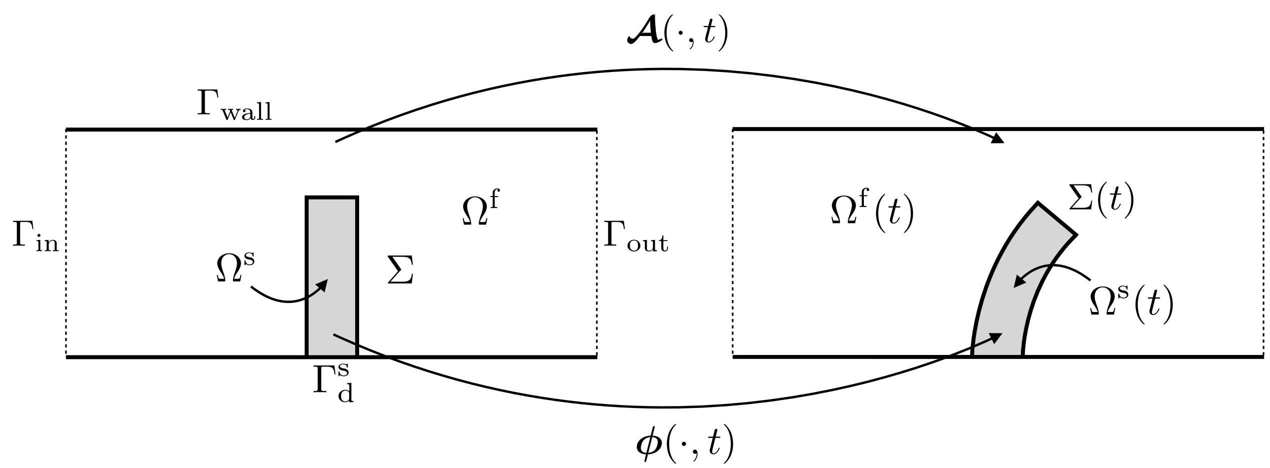


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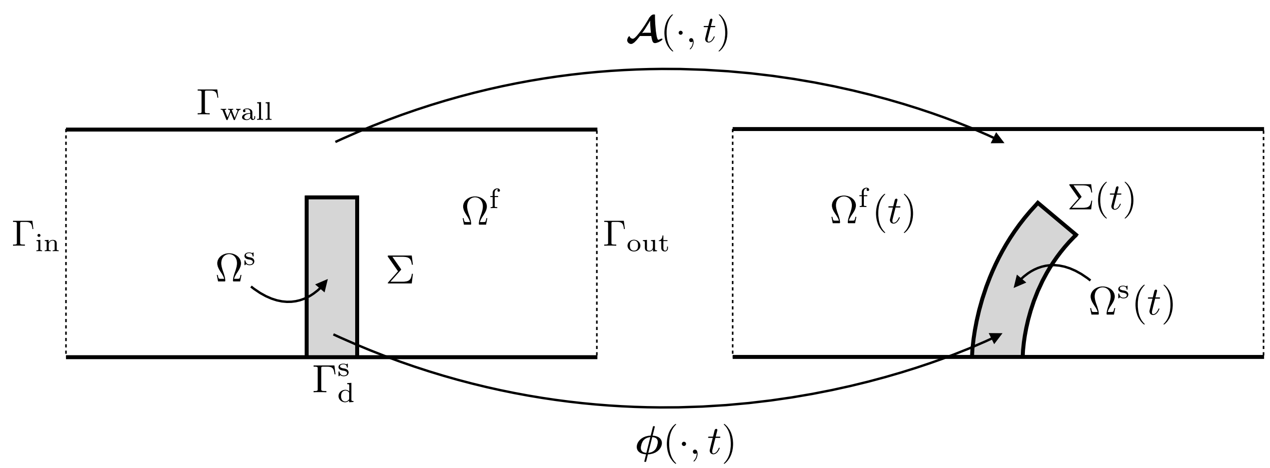


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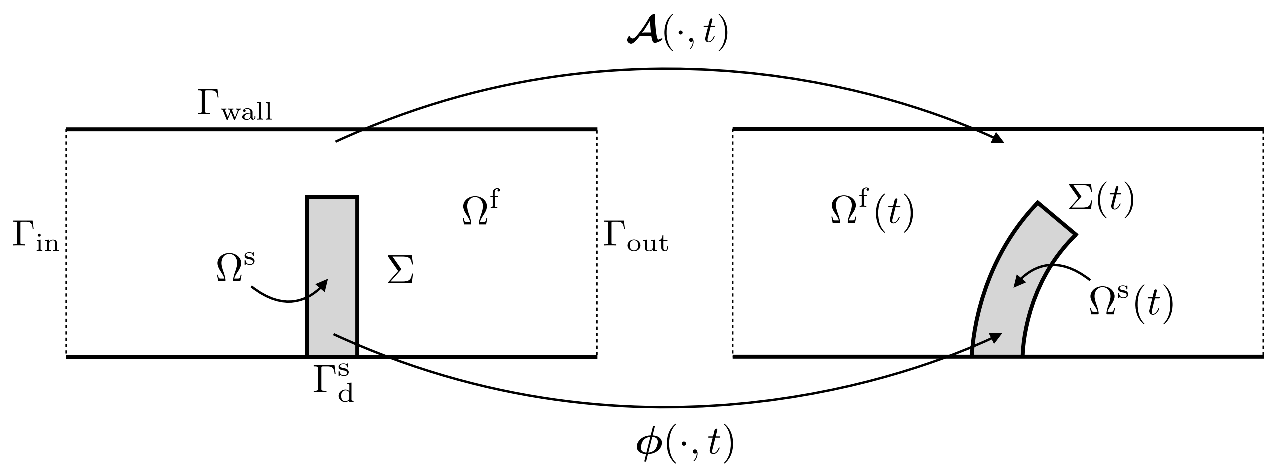


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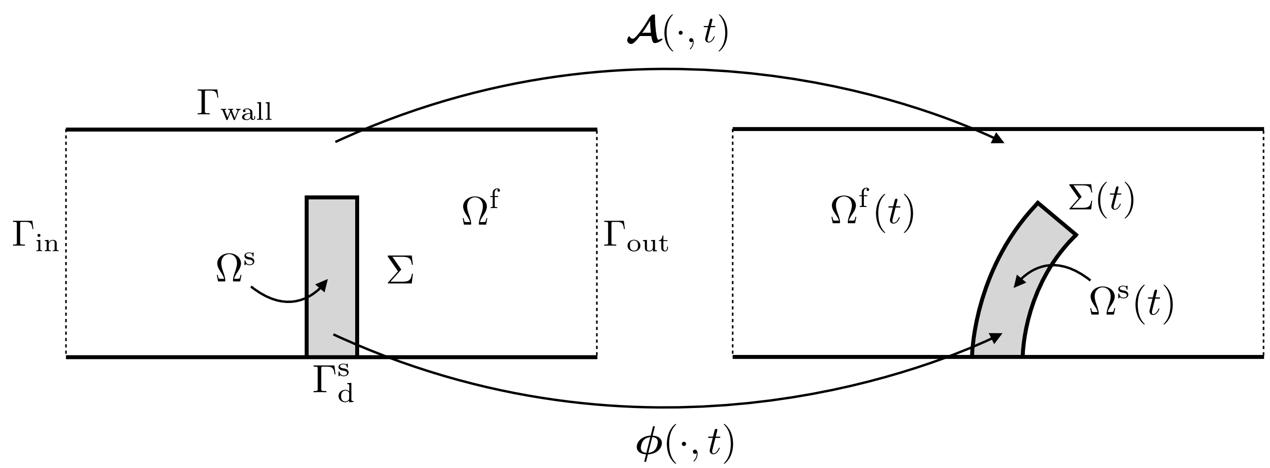


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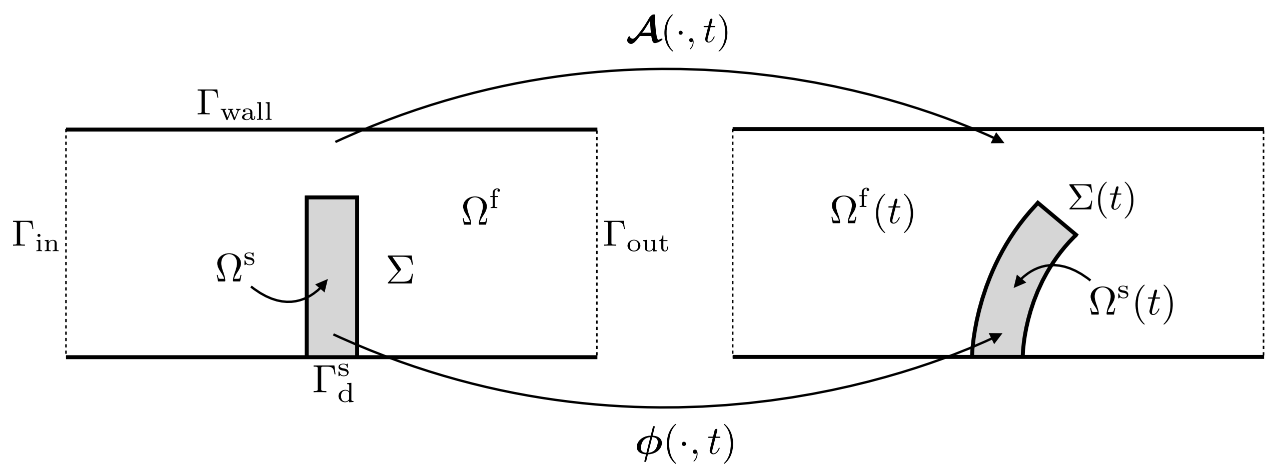


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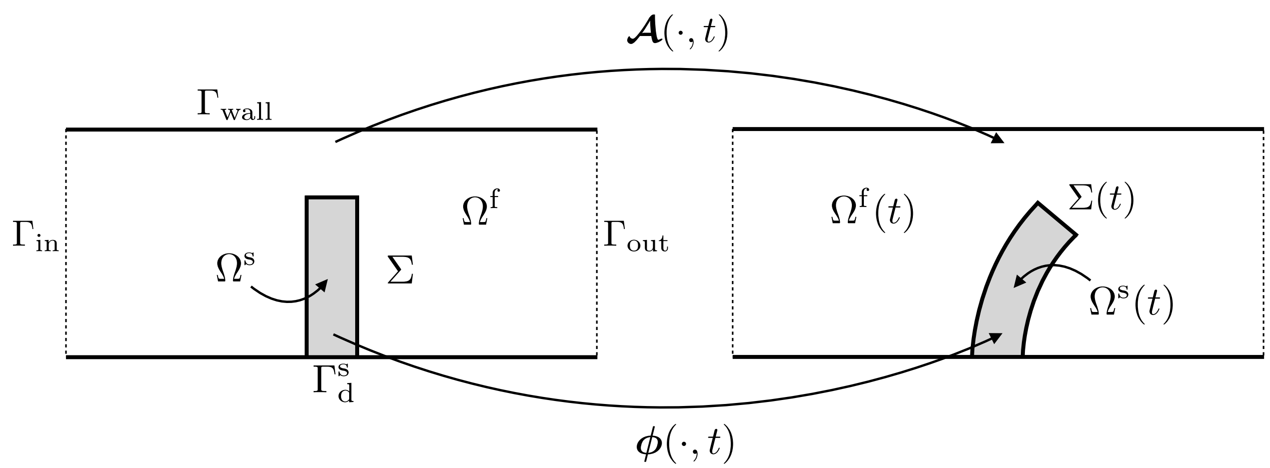


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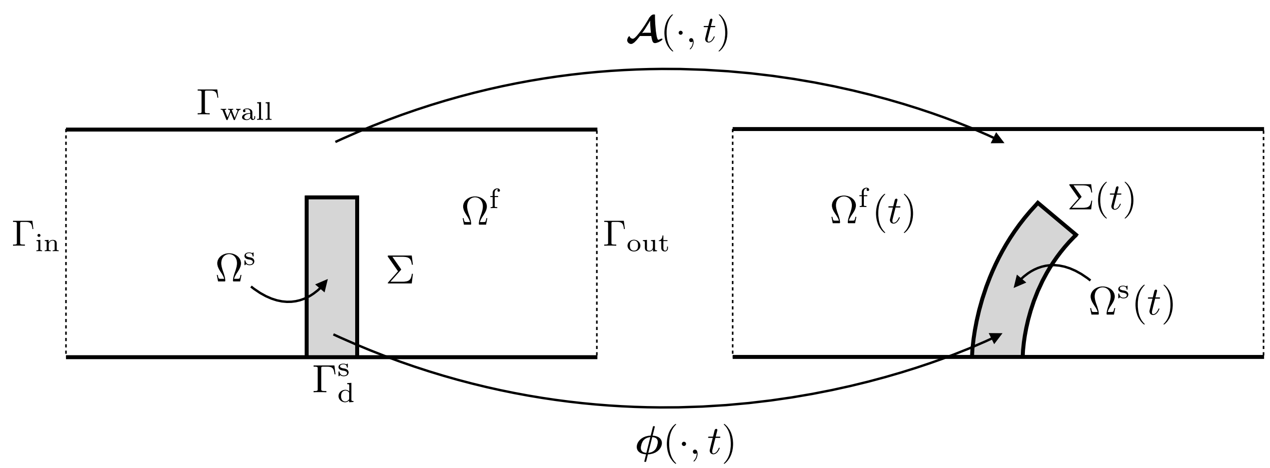


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A lot of effort has been devoted, over the last decades, to the numerical approximation of these kind of coupled problems:

(Mok et al. '01, Gerbeau, Vidrascu. '03, Heil '04, Fernández, Moubachir '05, Deparis et al. '06, Dettmer, Peric '06, Badia et al.'08, MF, Gerbeau, Grandmont '07, Quarteroni, Quaini '08, Guidoboni, Glowinski, Cavallini, Canic '09, Gee et al. '11, ...)

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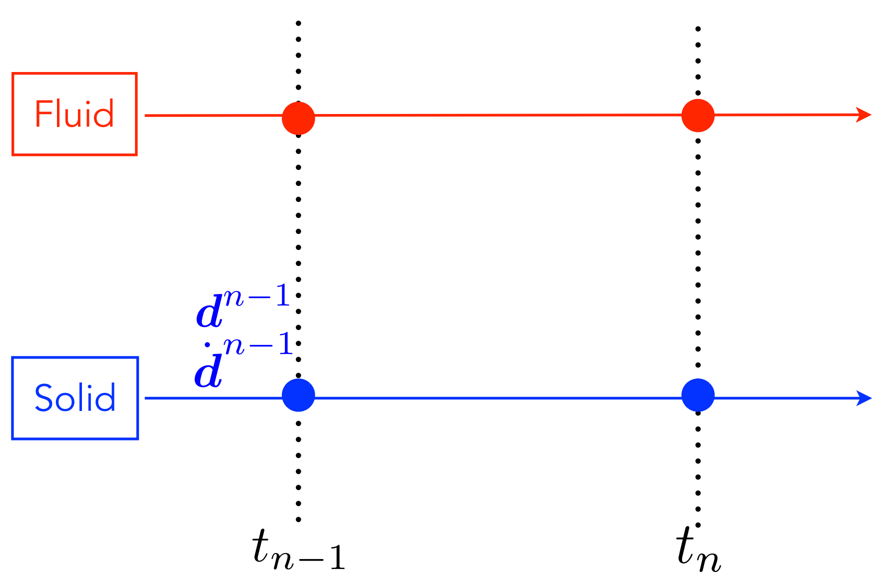


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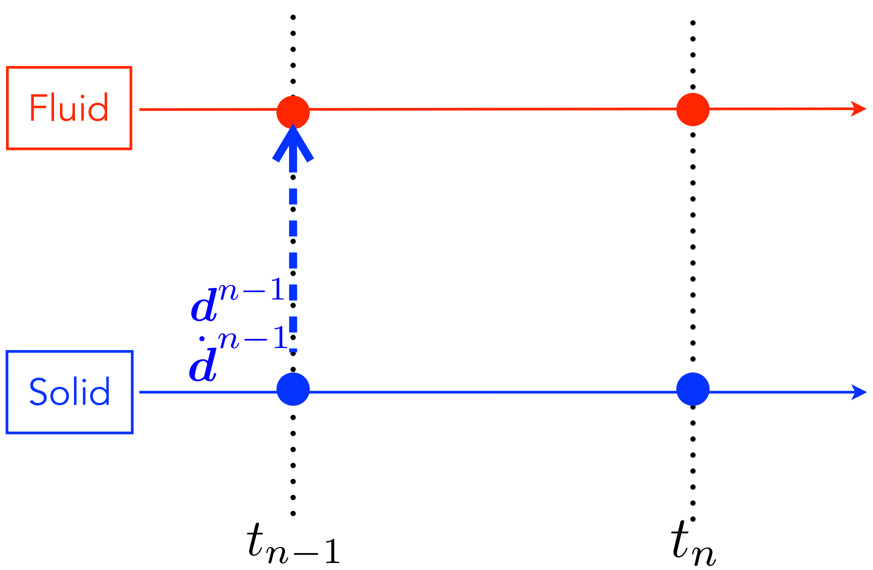


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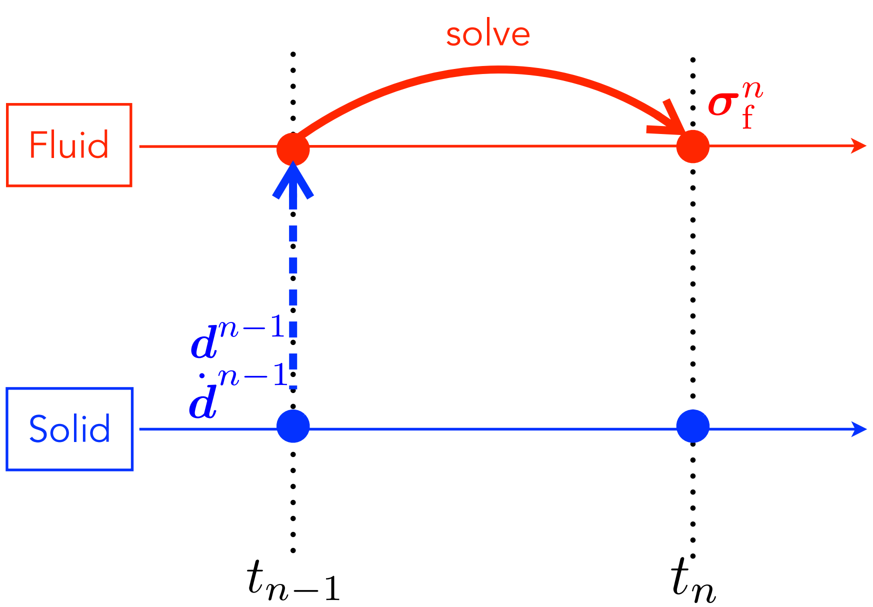


**Standard explicit time-marching strategy**

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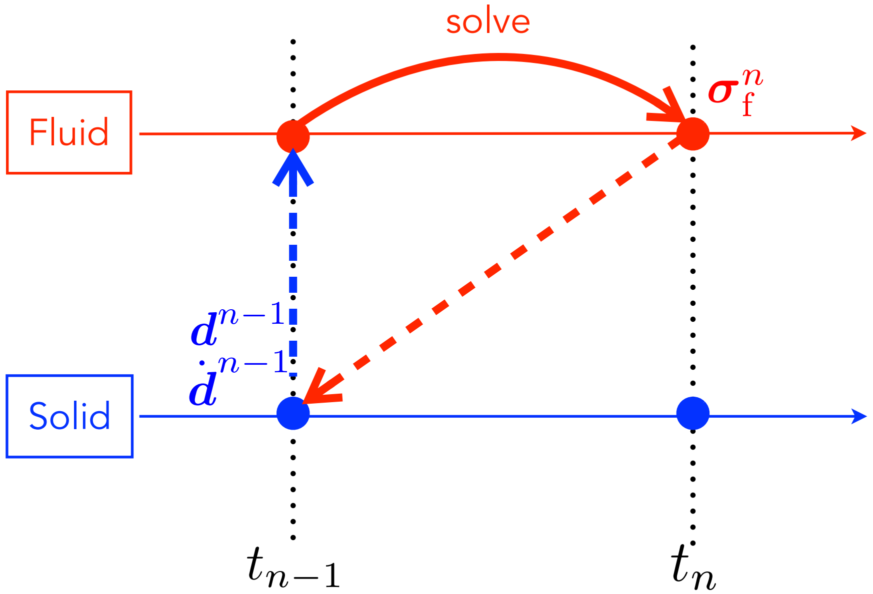


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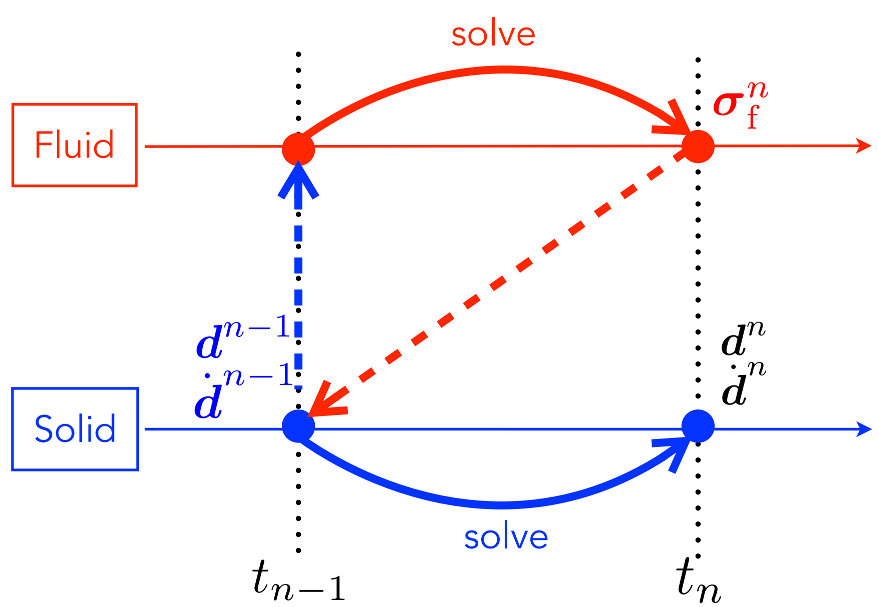


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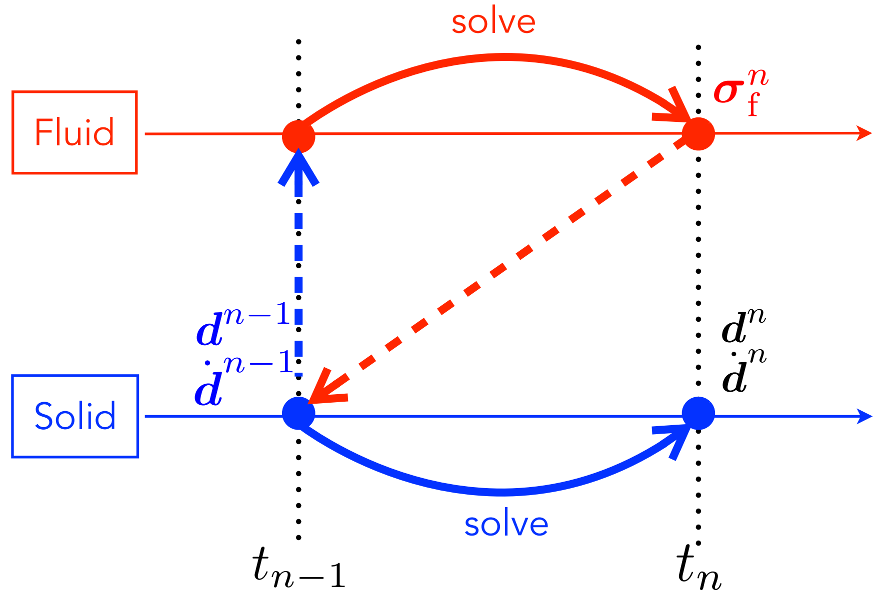


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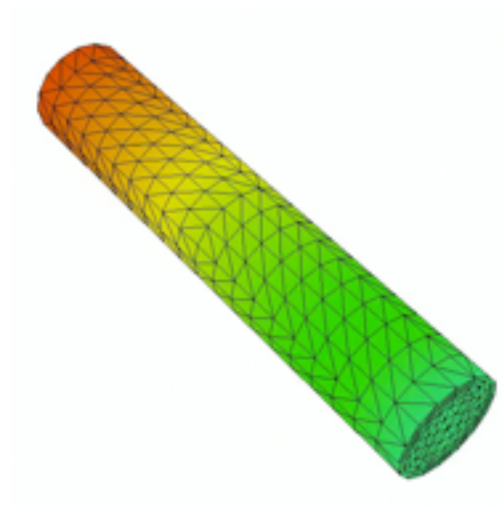

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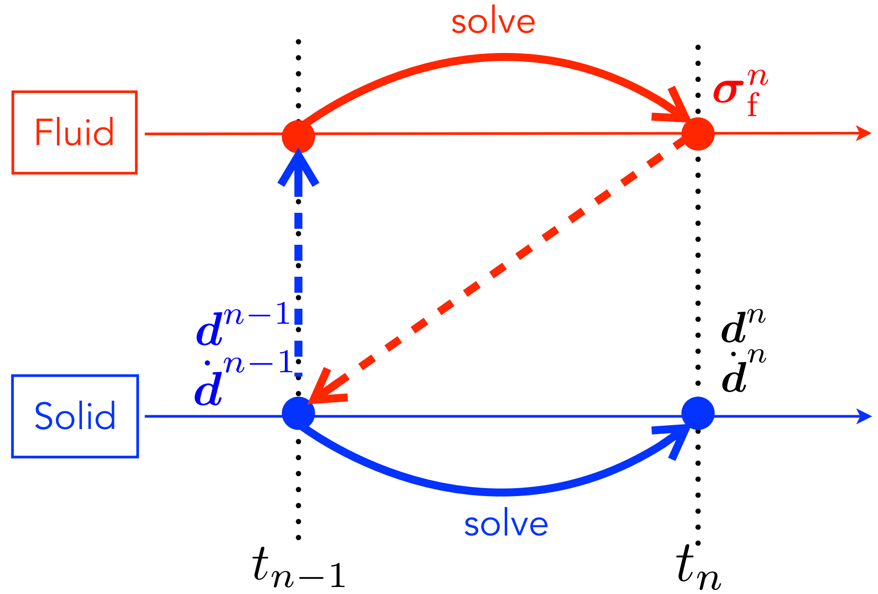


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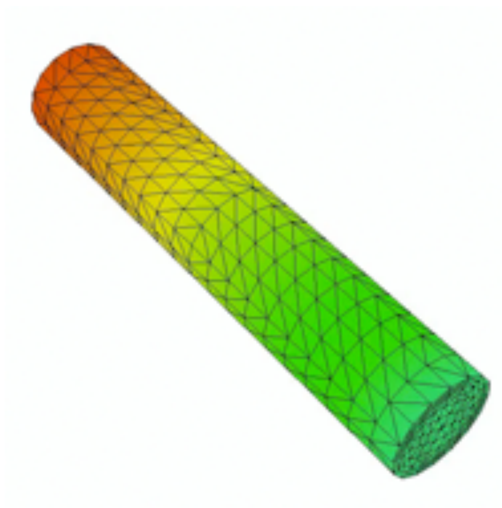

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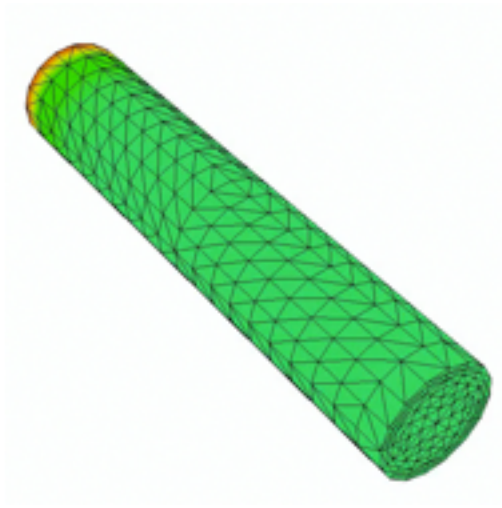

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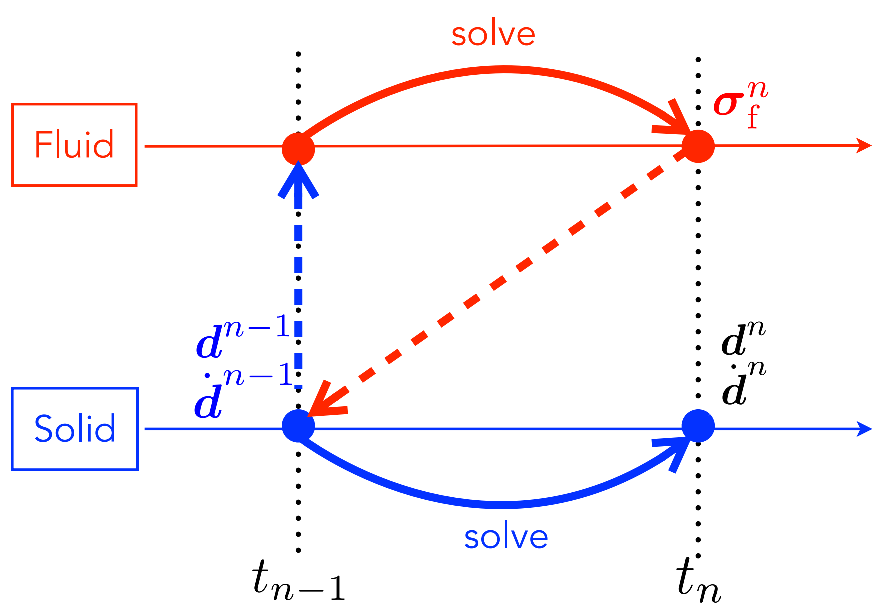


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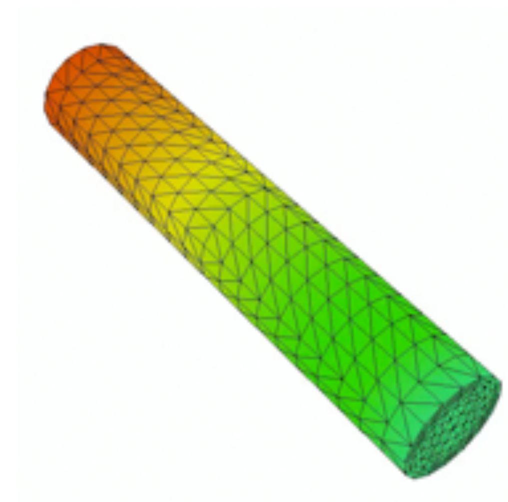

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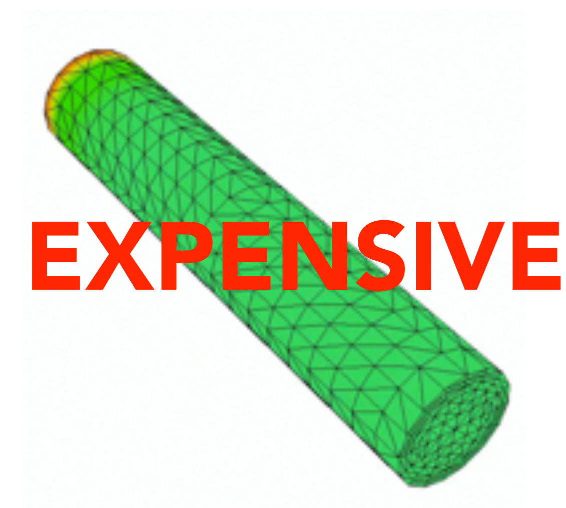


**implicit coupling**

  - Large number of sub-iterations required in partitioned approaches for implicit coupling

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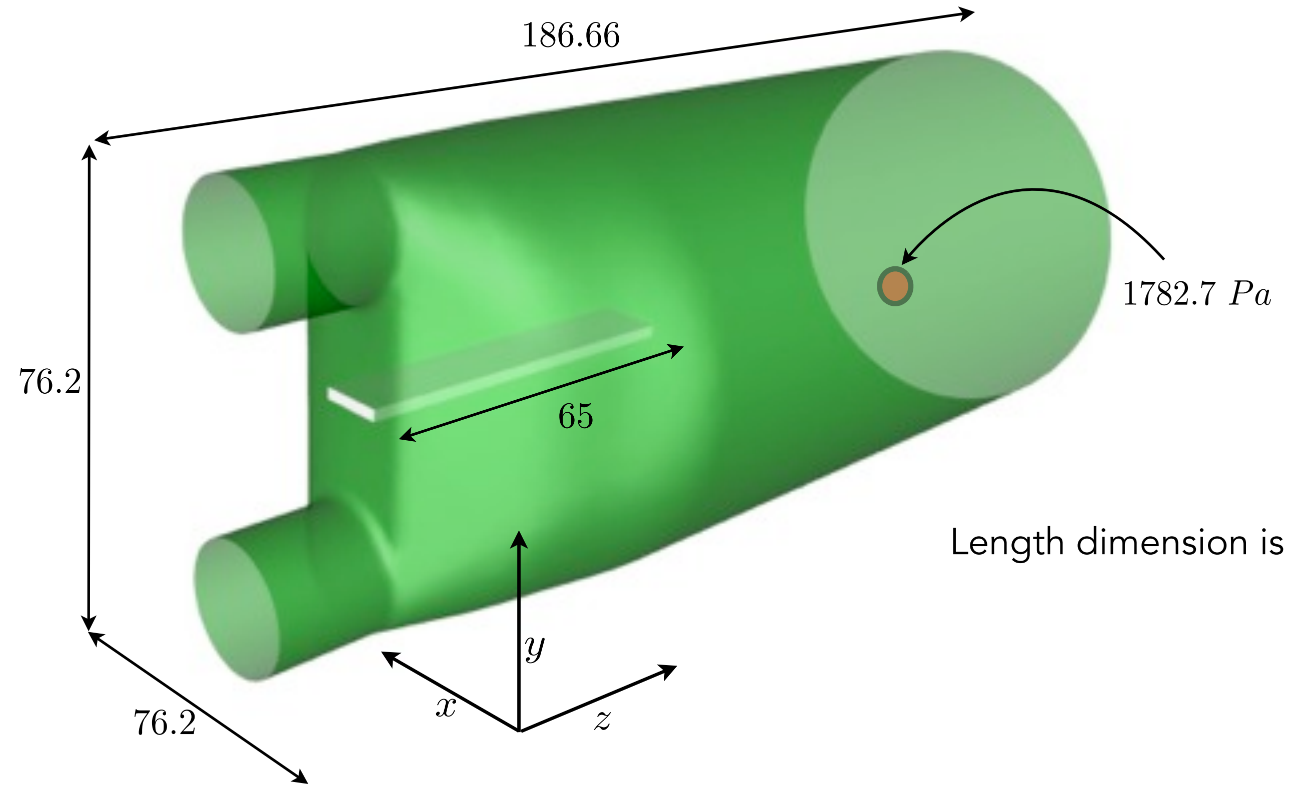
*Joint work with* ***M. Vidrascu***, ***D. Chapelle***, ***M. A. Fernández***

- **Geometry**

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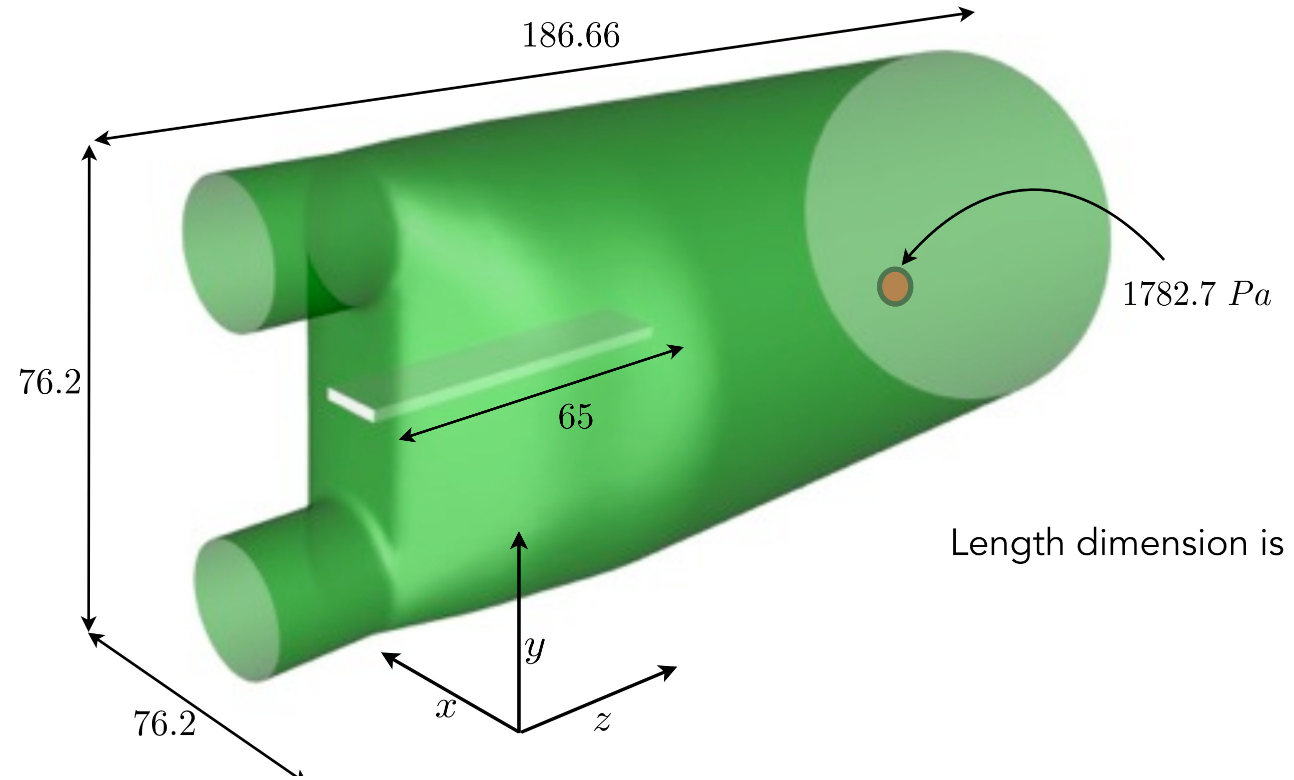


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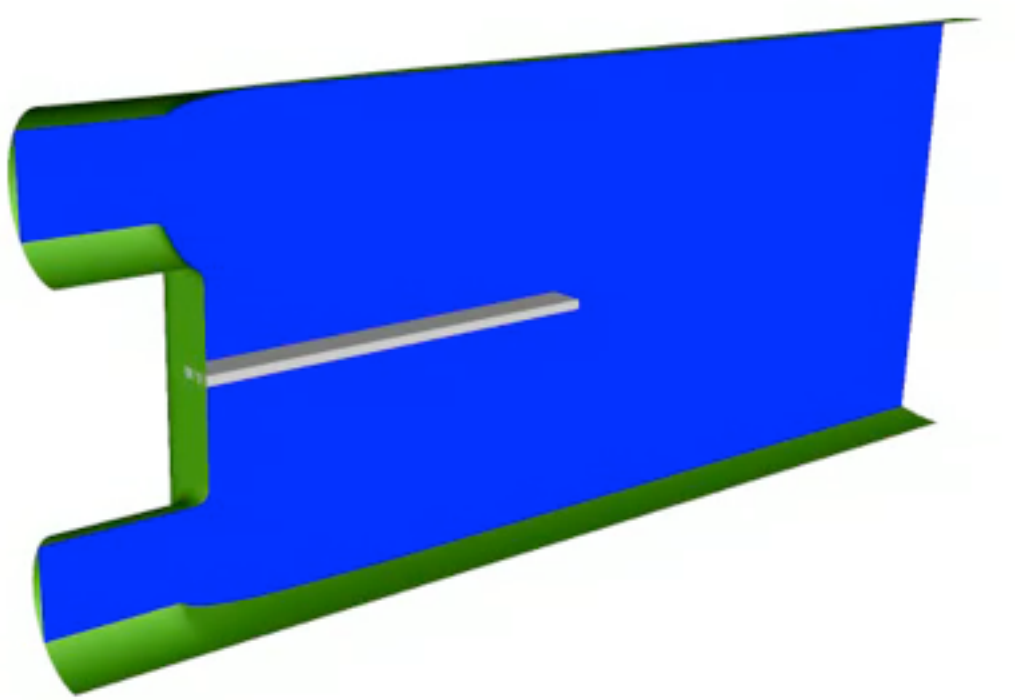

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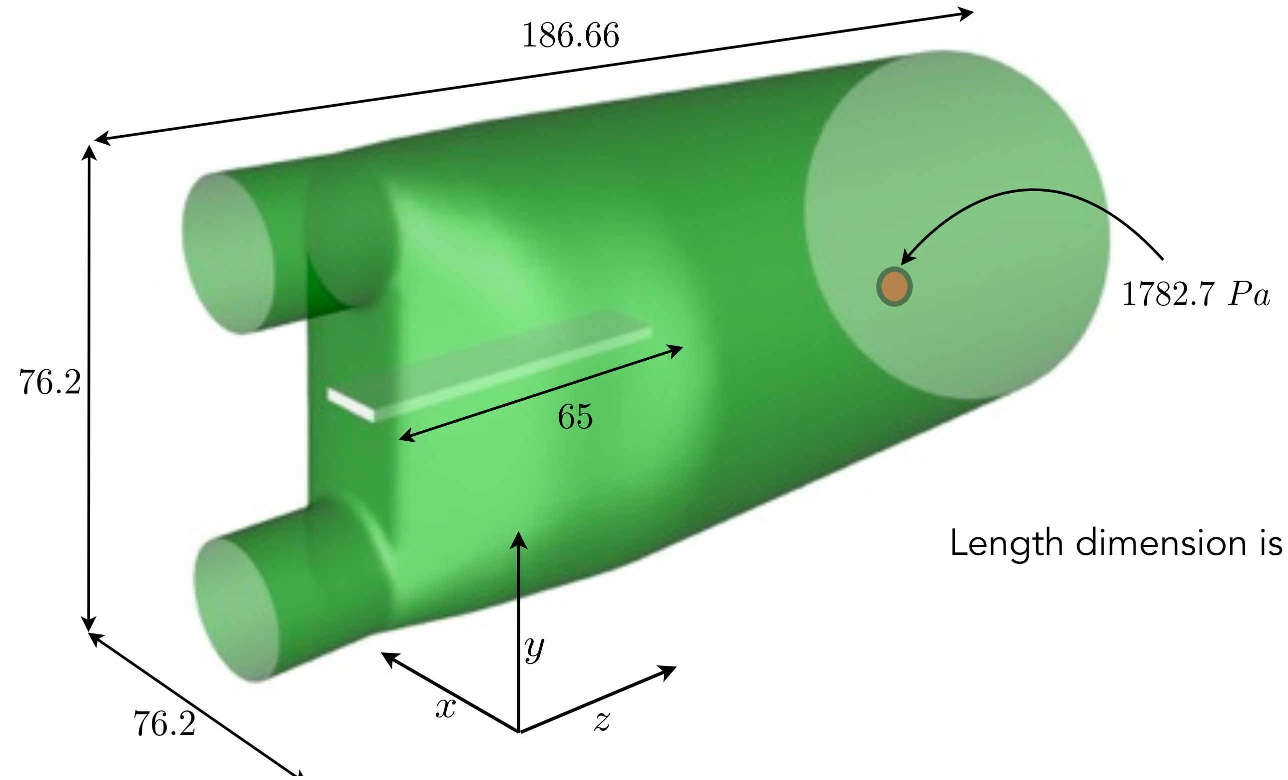


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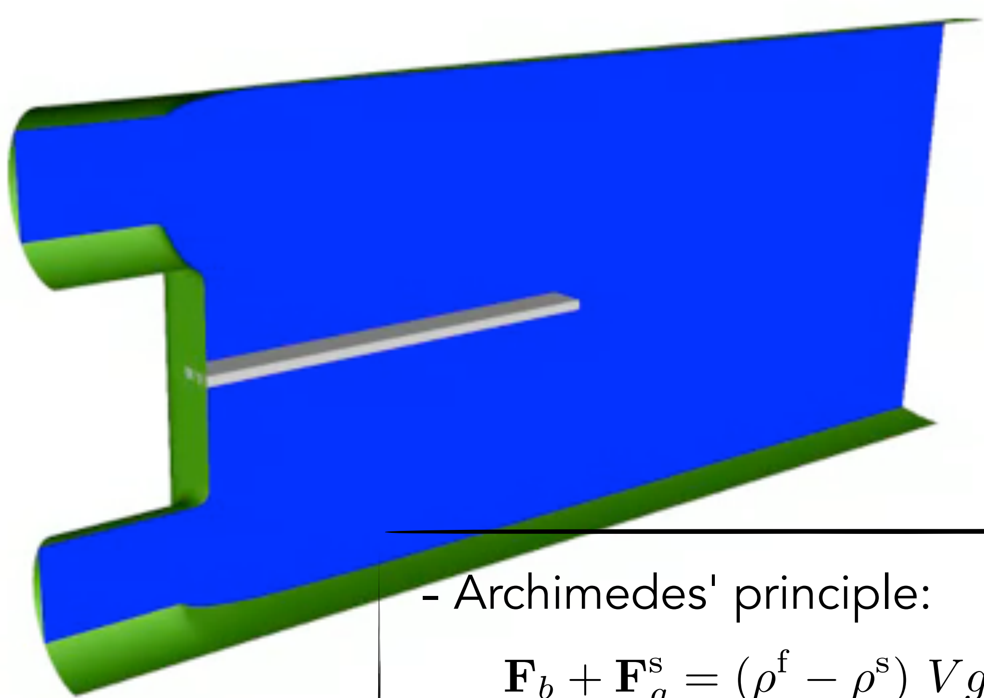

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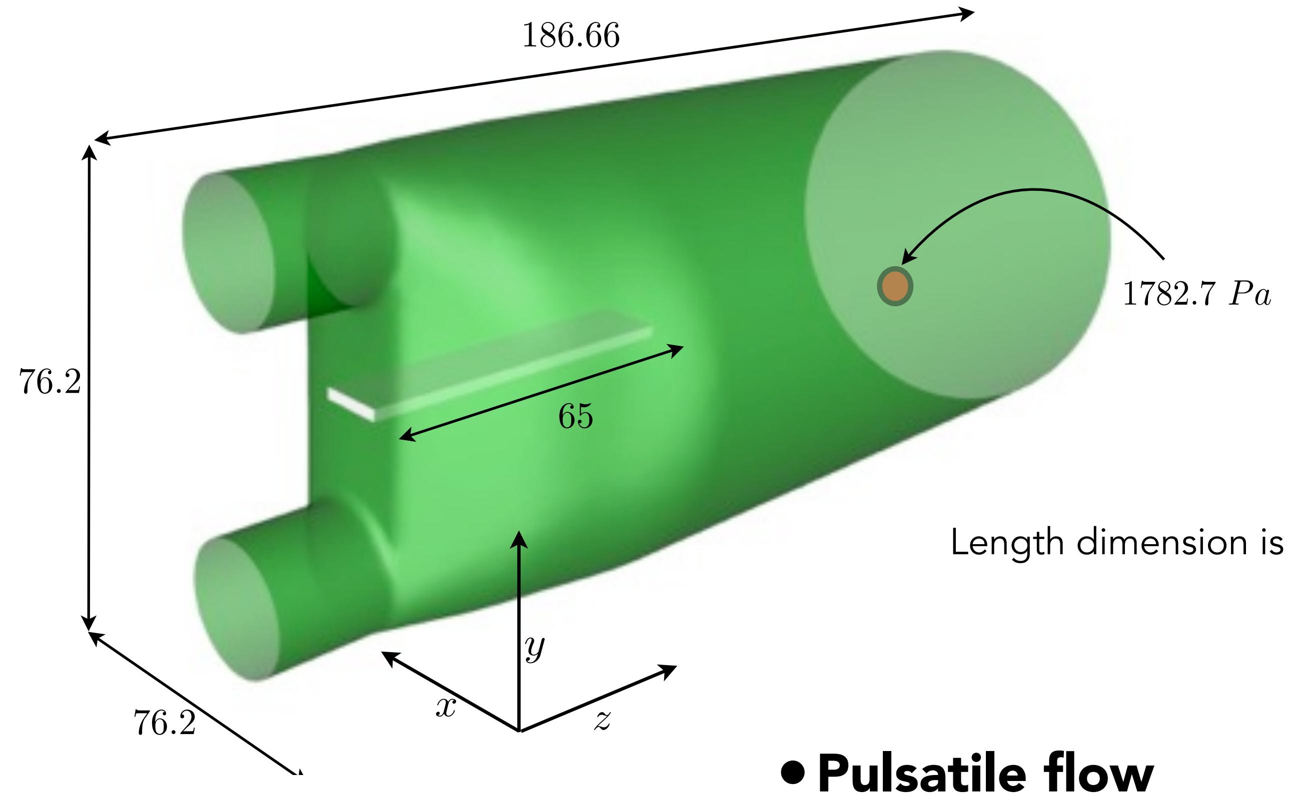


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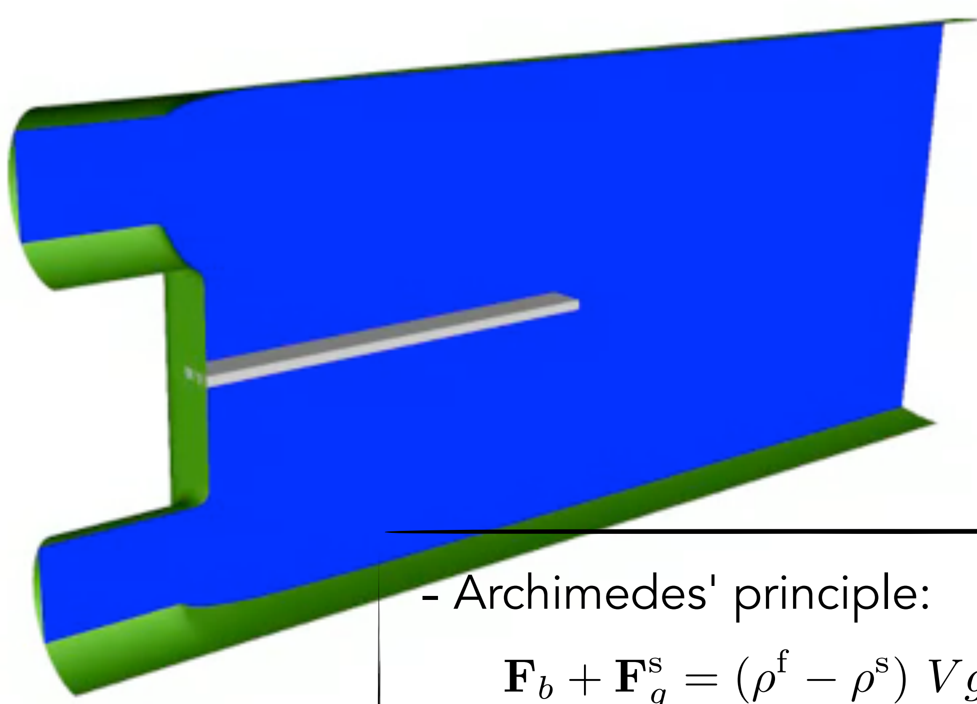

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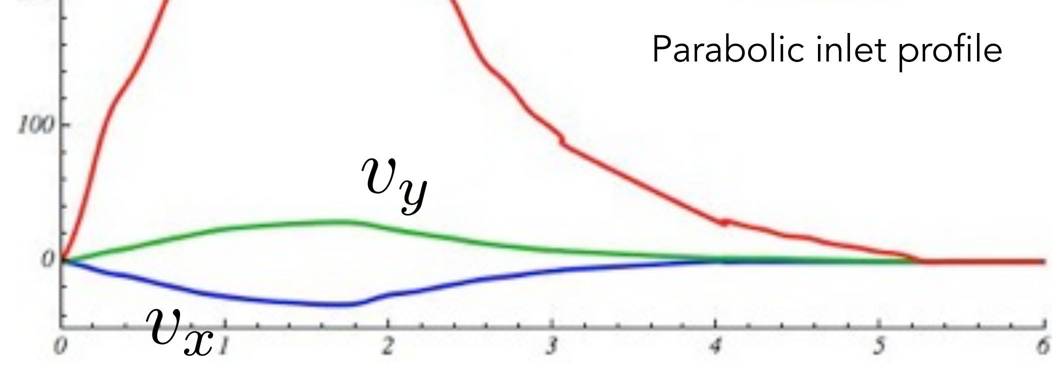
- **Pulsatile flow**




Parabolic inlet profile

# The Simulation

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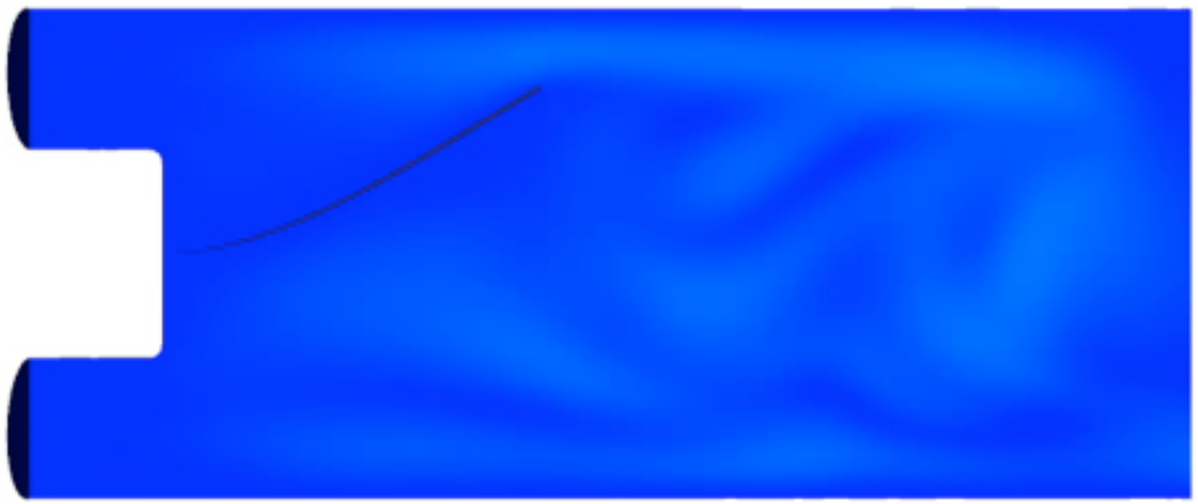

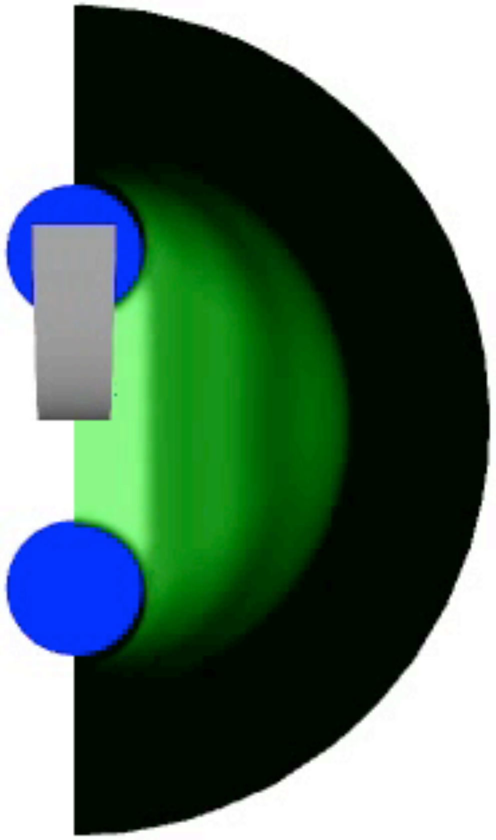

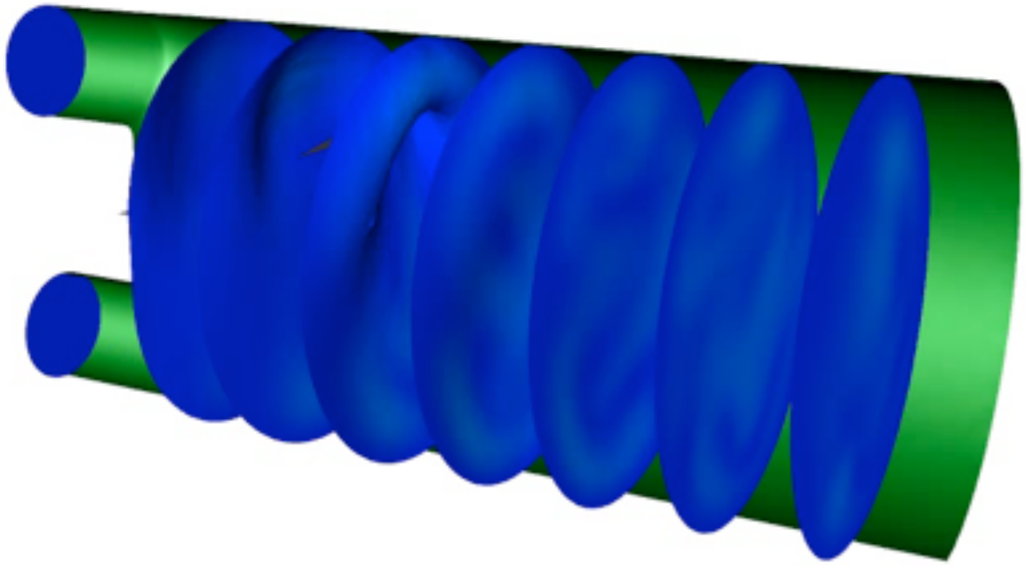

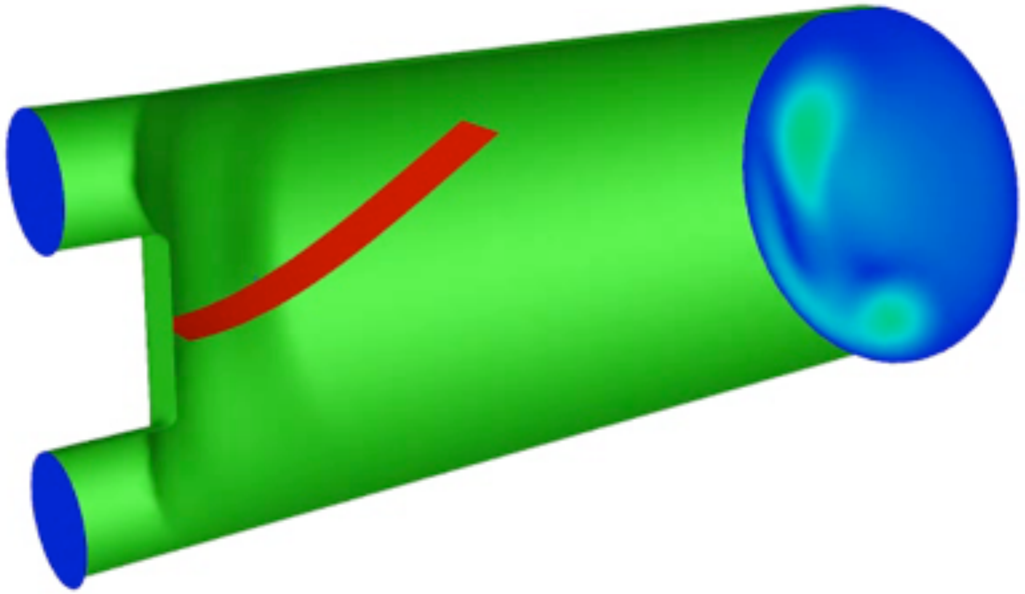

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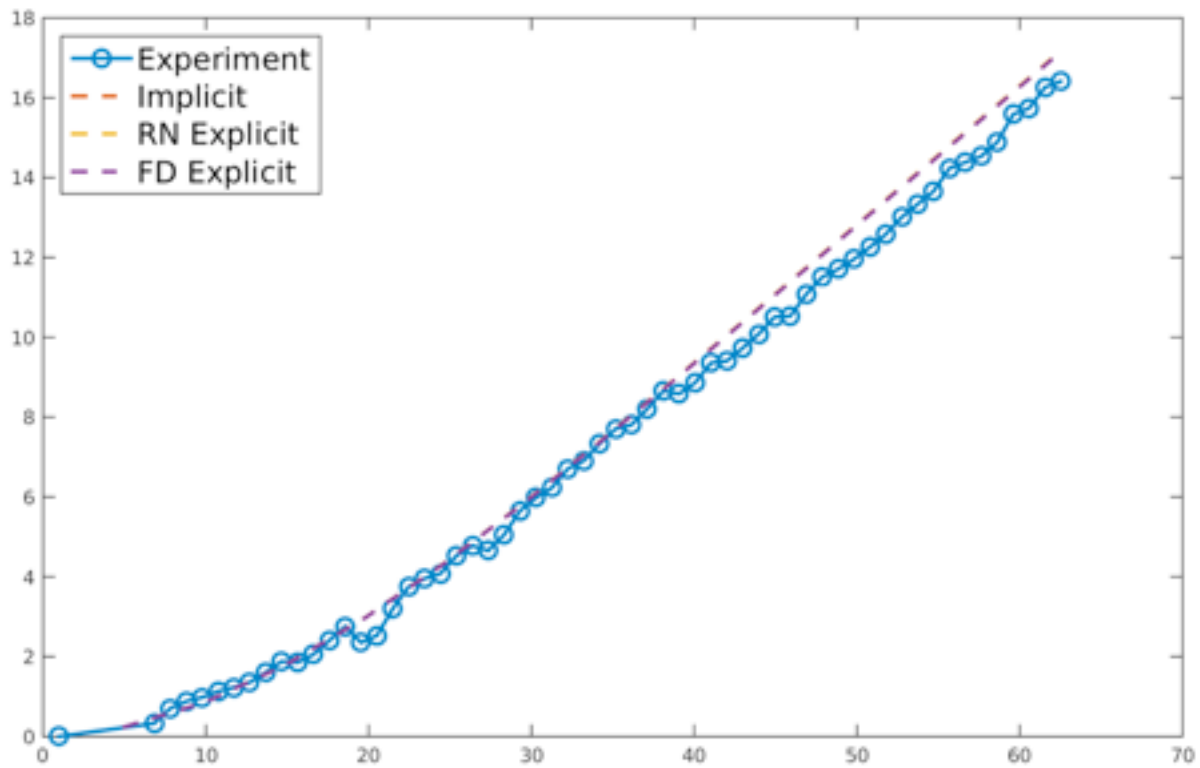

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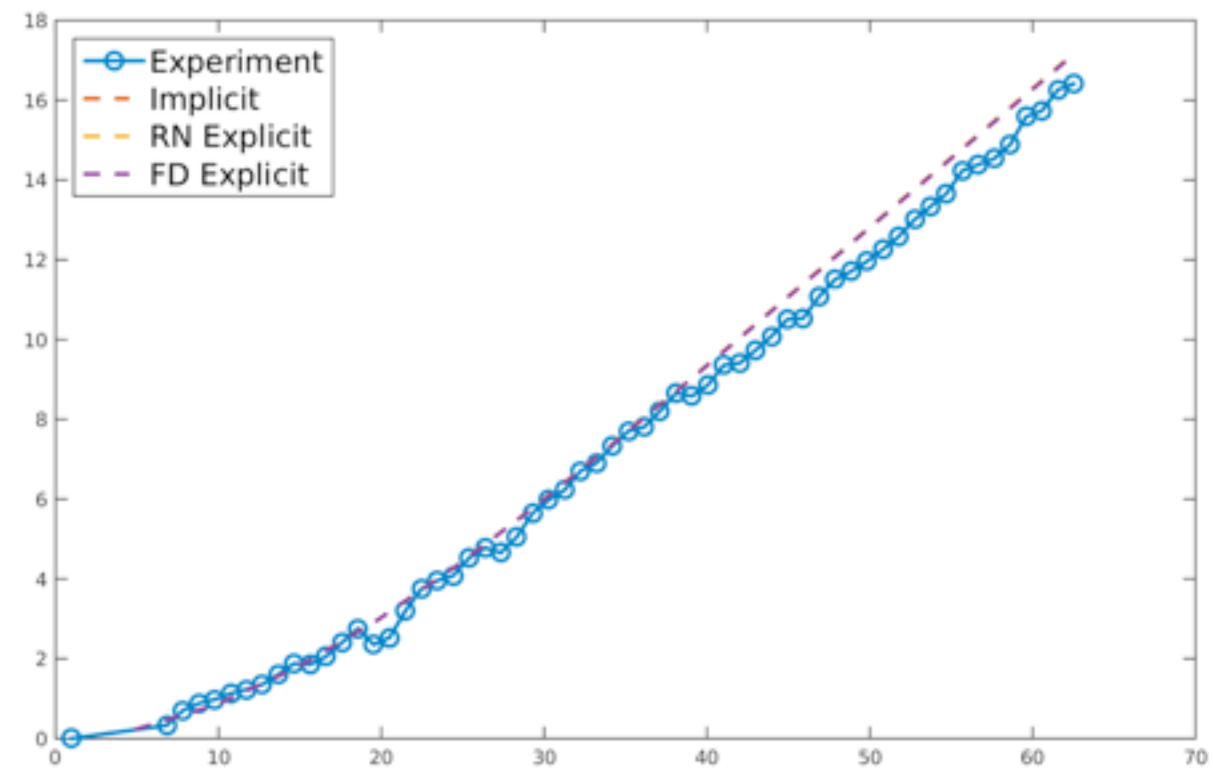


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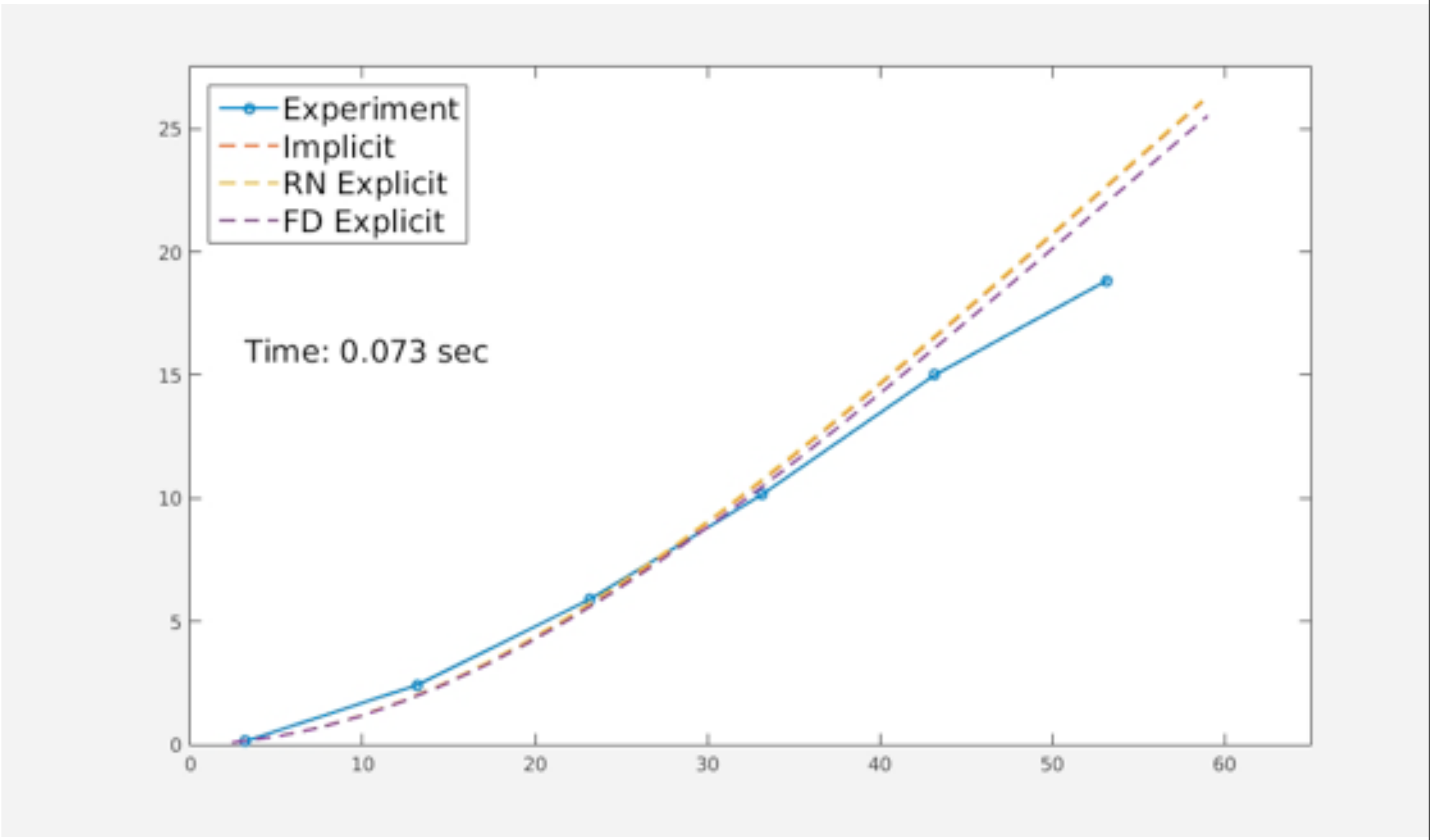


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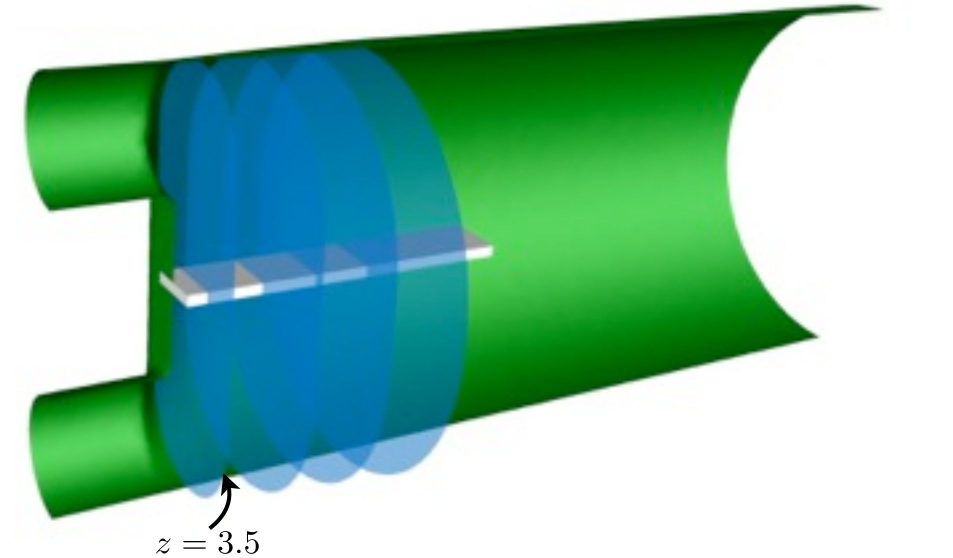


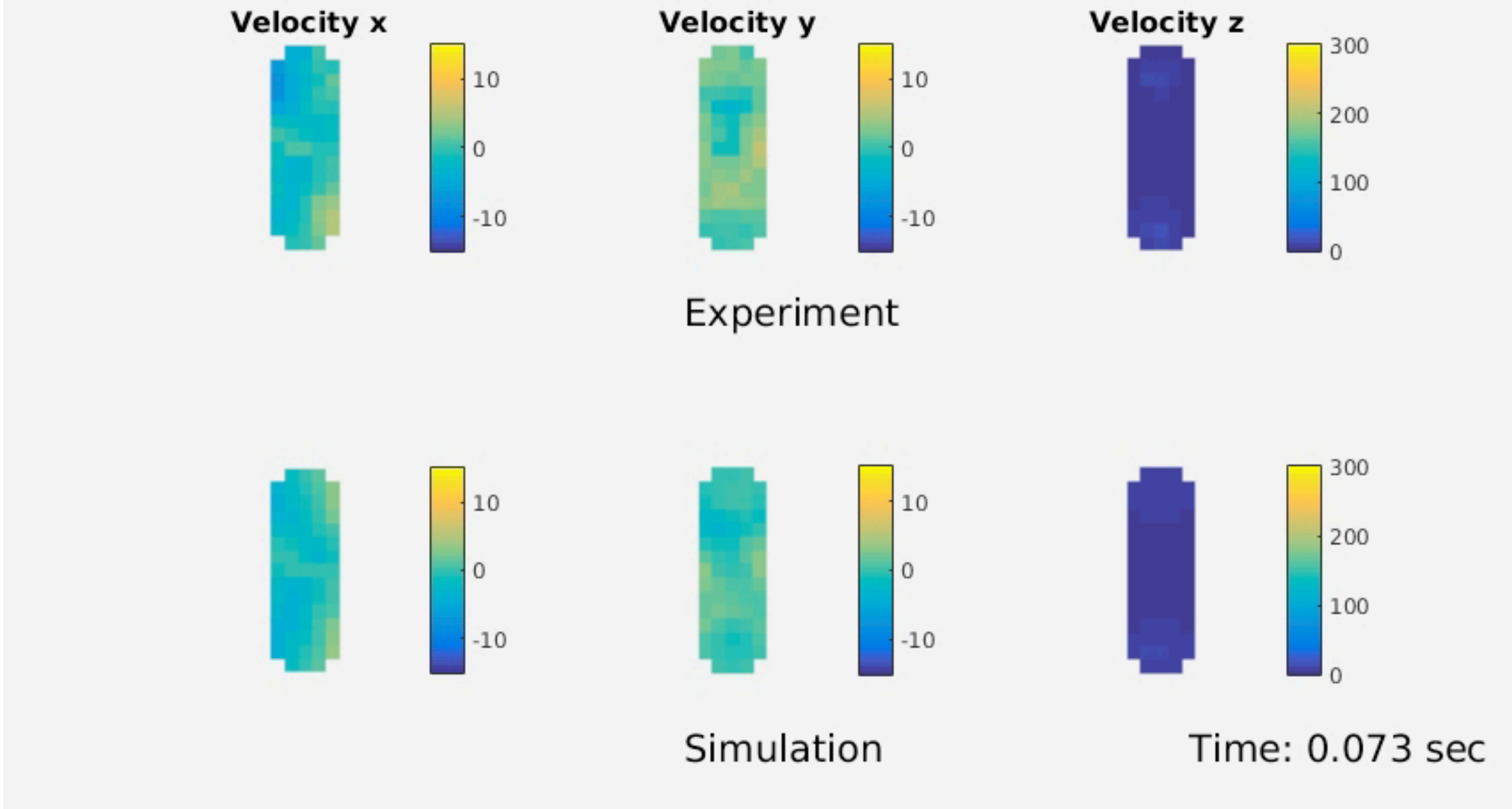

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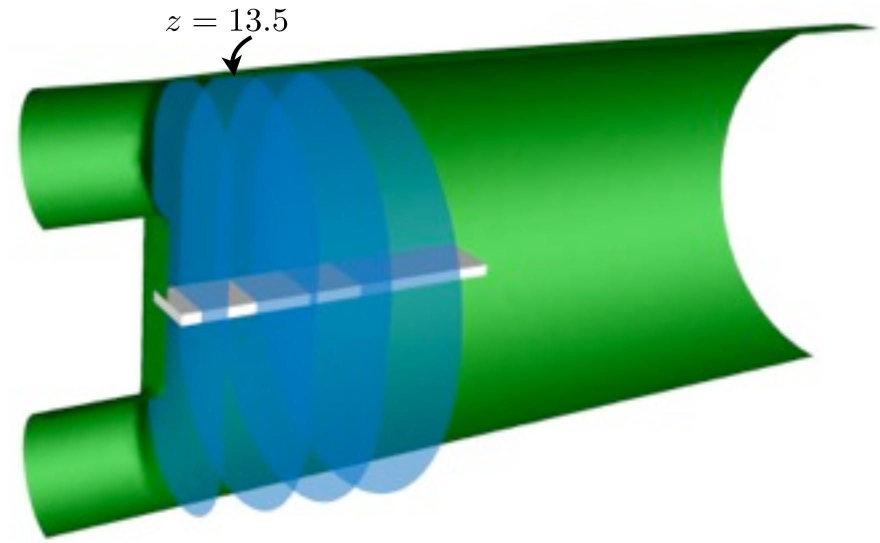


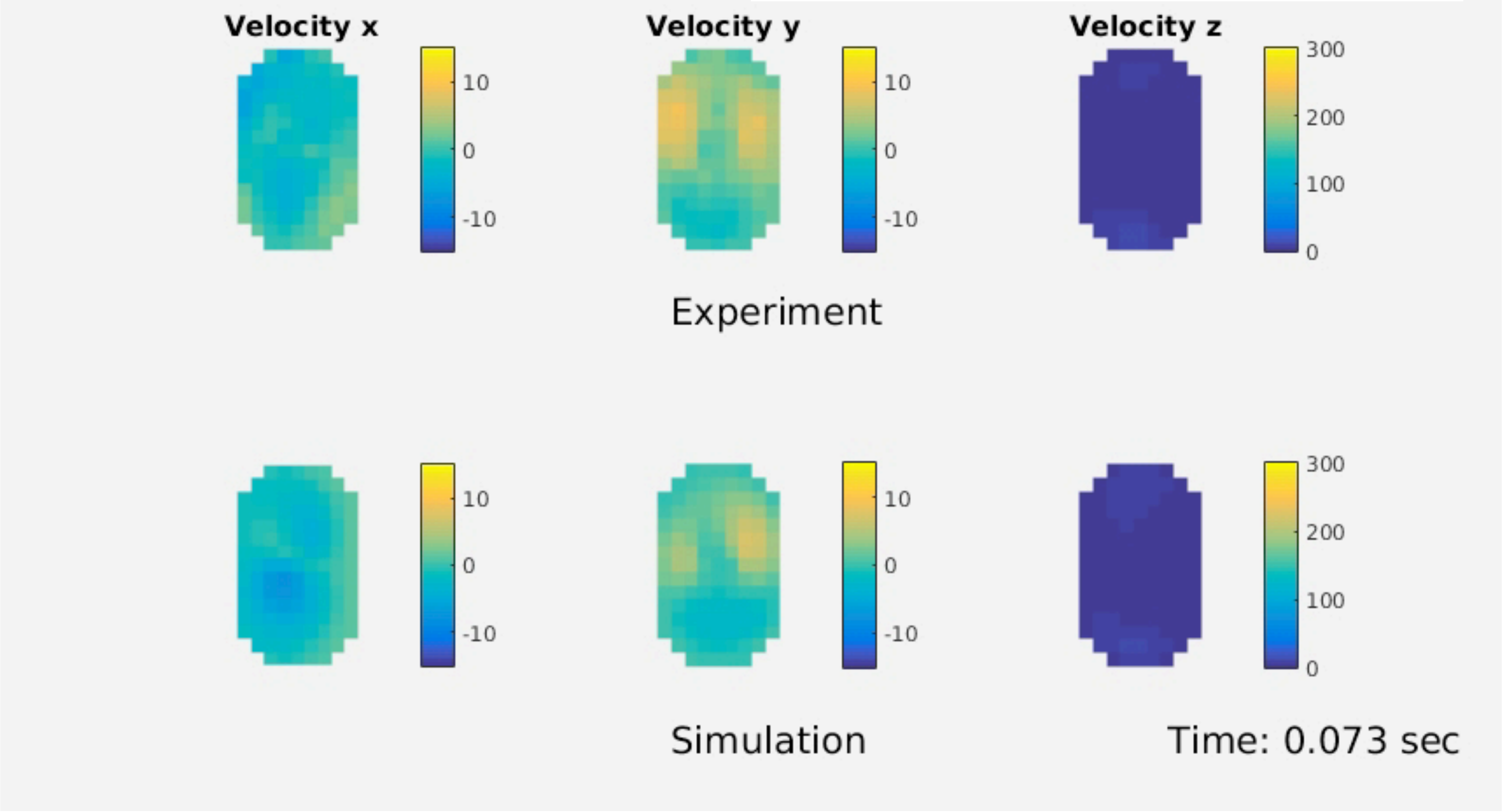

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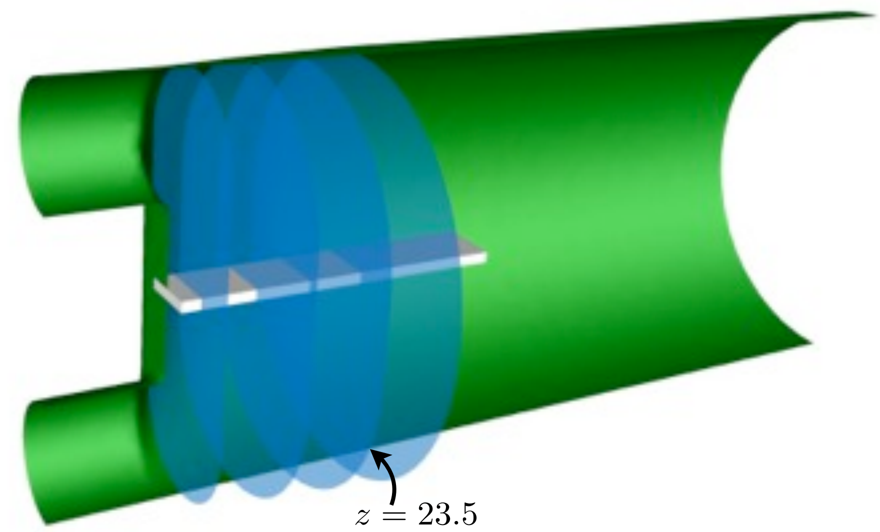


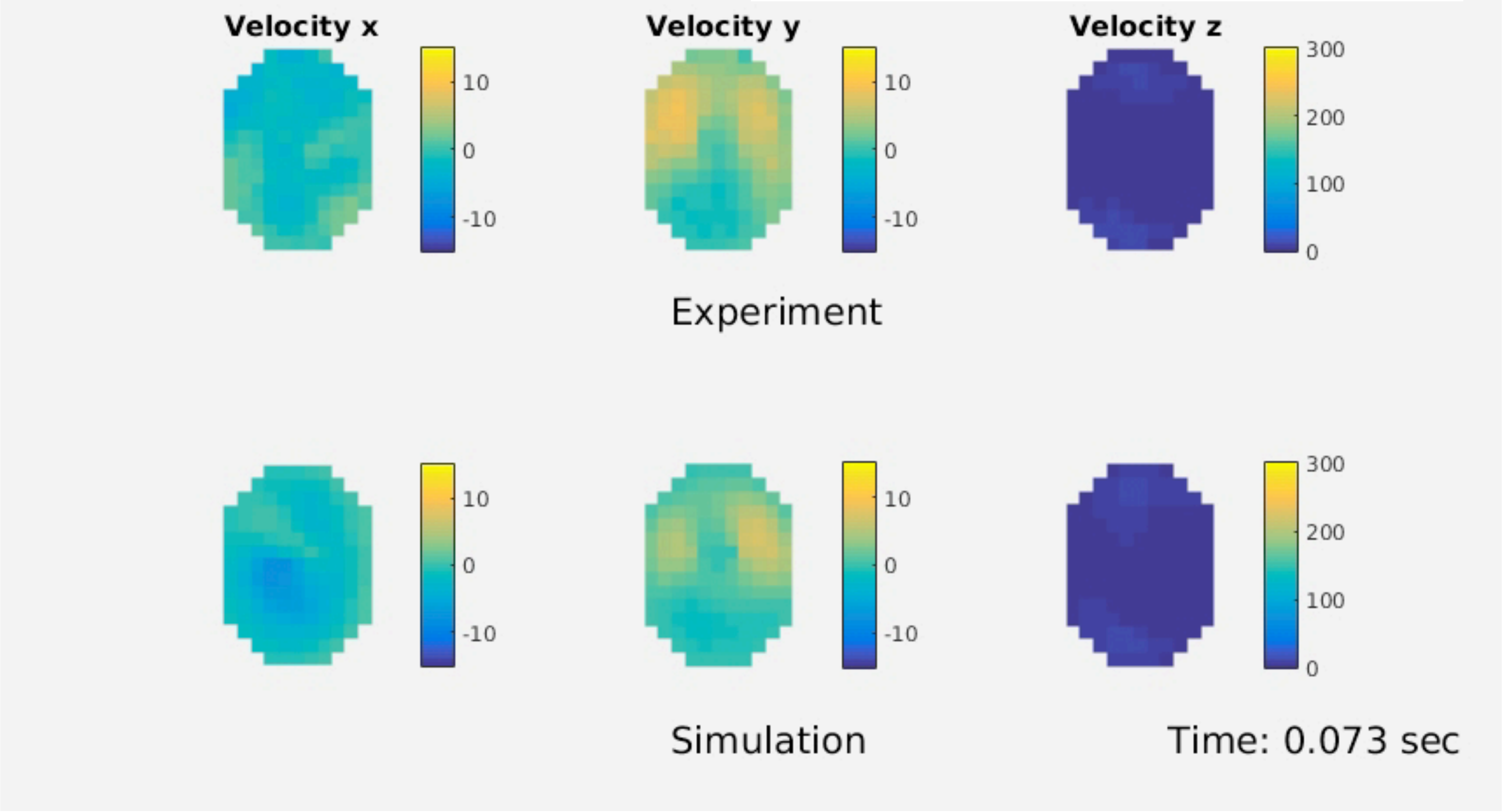

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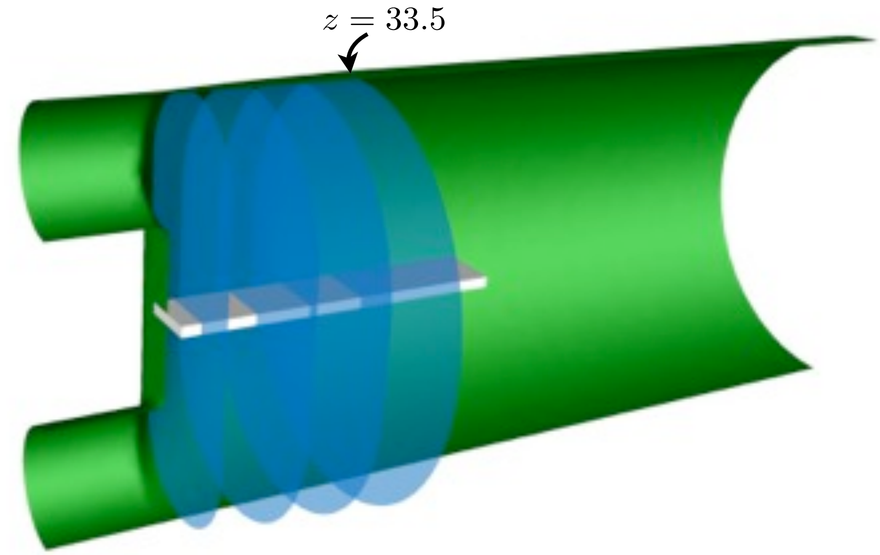


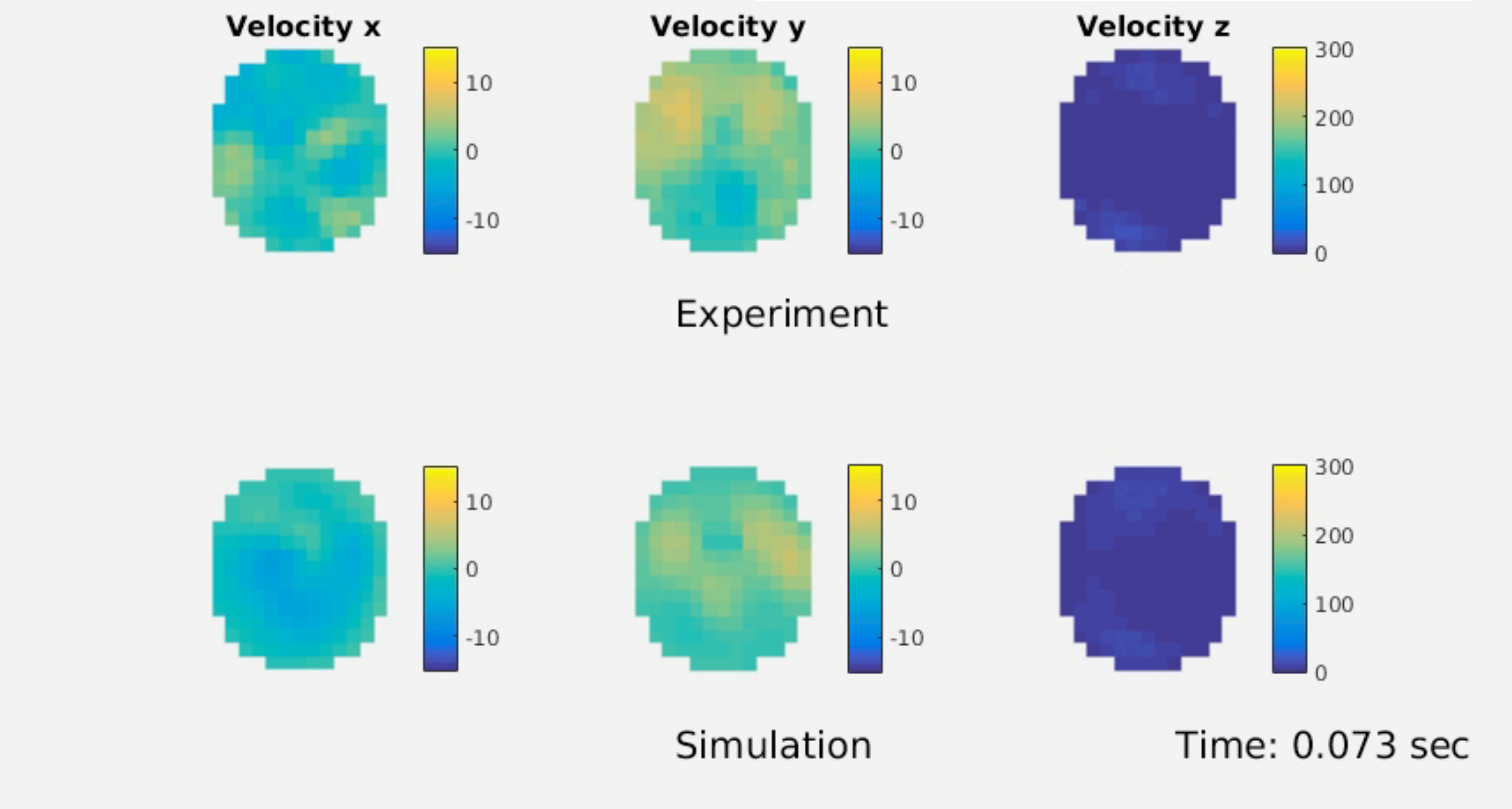

# Elapsed CPU–times comparisons

| Alg. 6 | Alg. 5 | Alg. 4 | Alg. 3 | Alg. 2 | Alg. 1 |
|---|---|---|---|---|---|
| 1 | 2 | 2.5 | 2.5 | 18 | 17.5 |

Table V. Elapsed CPU–time (dimensionless) for Algorithms 1–6 in Phase II.

# Large structural deflections, contact, …

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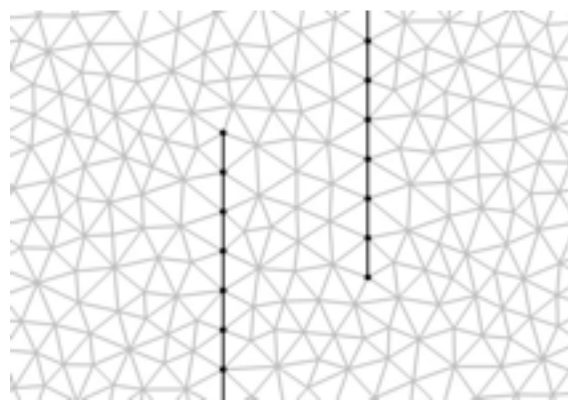
- **Fitted mesh methods**:
  - Pros: natural, accurate, adequate fluid time-stepping
  - Cons: moderate displacements, solid contact not possible



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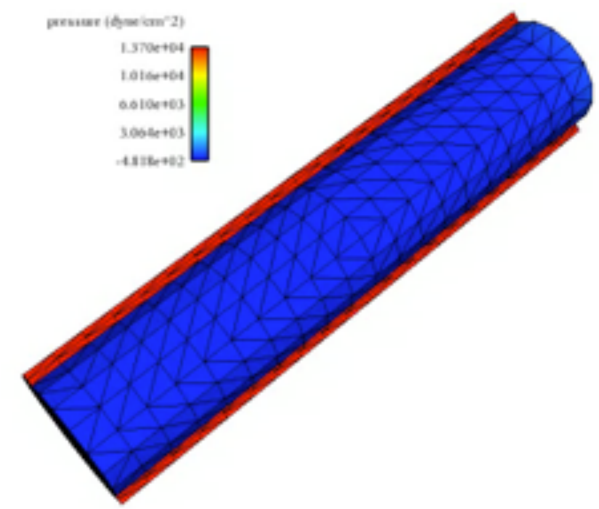

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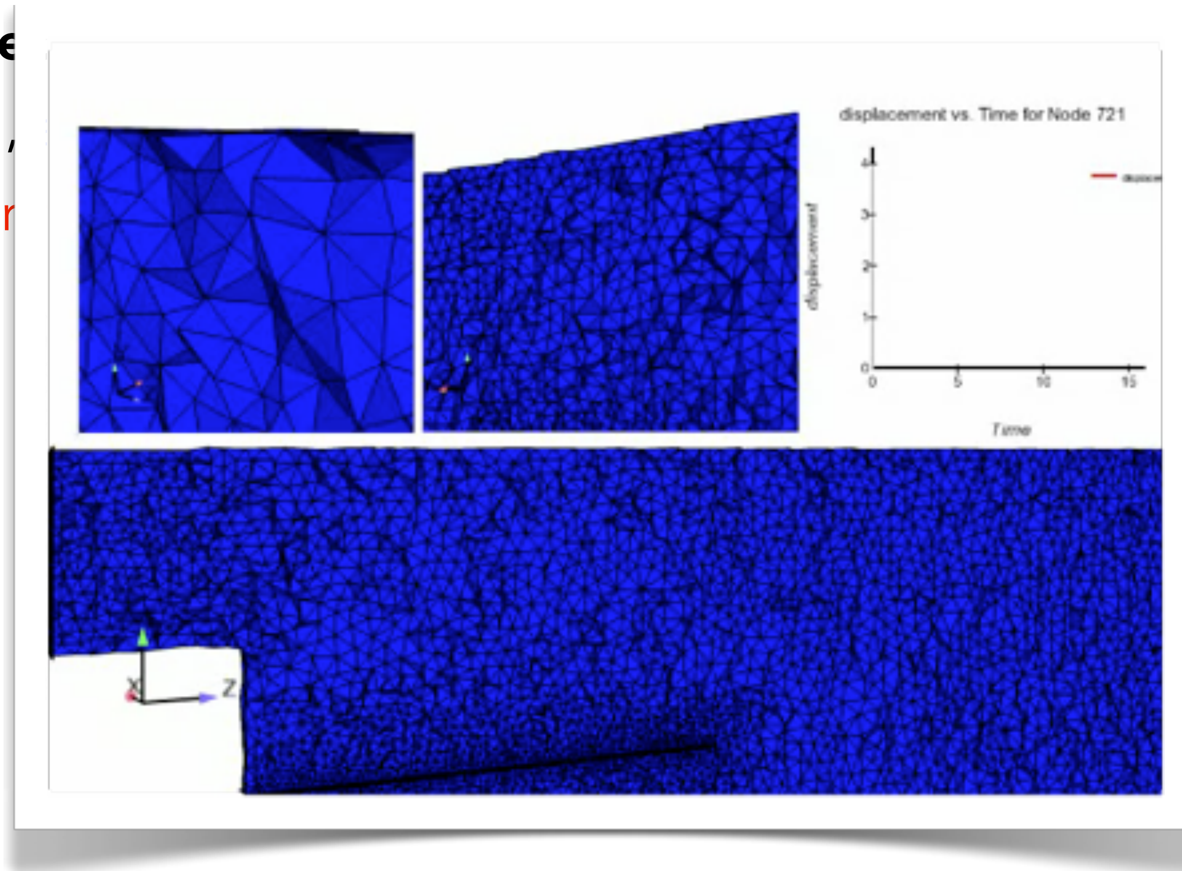


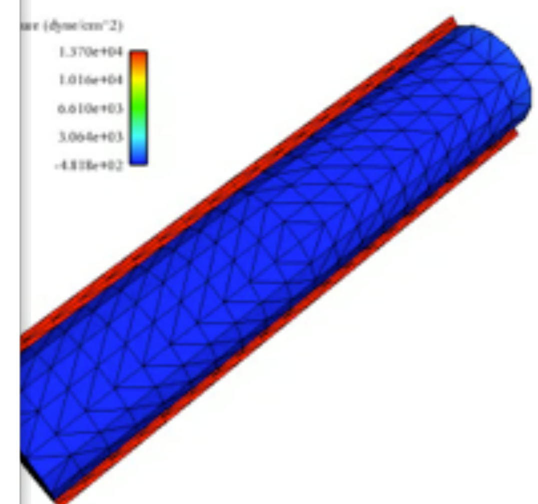

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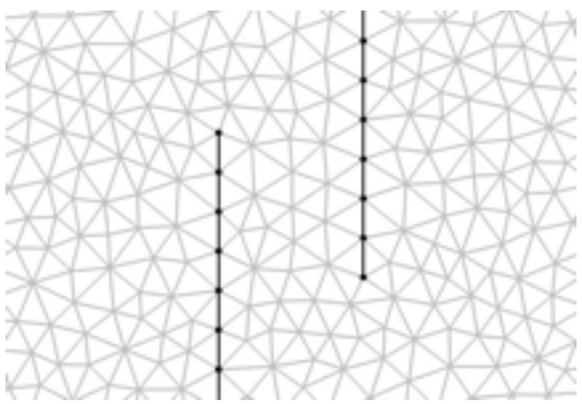

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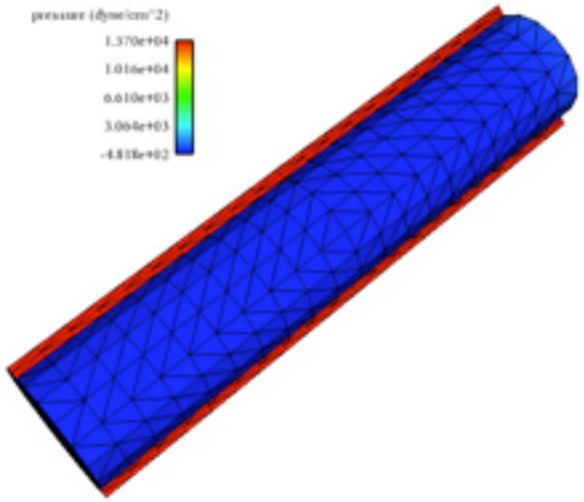

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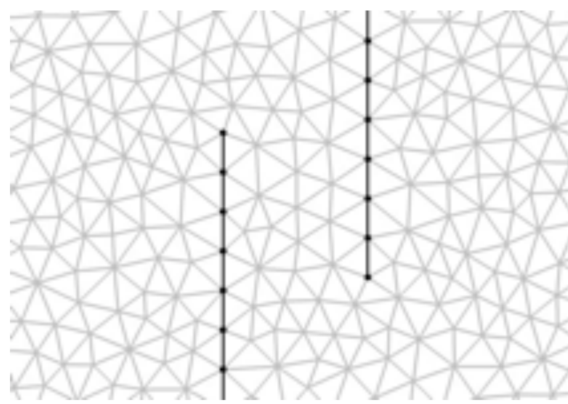

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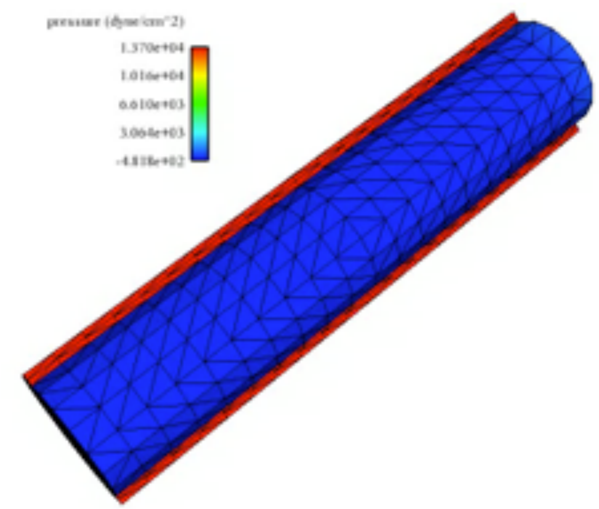
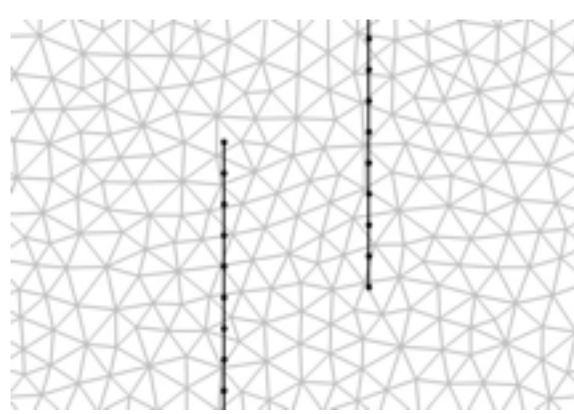


- **Unfitted mesh methods:**
  - Pros: arbitrary interface displacements, unfitted meshes, solid contact
  - Cons: robustness, local discontinuities, accurate interface computations,...



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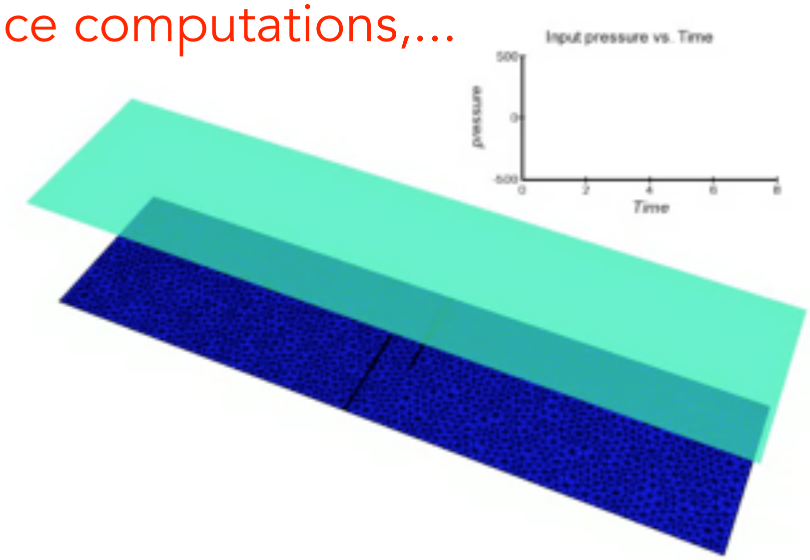
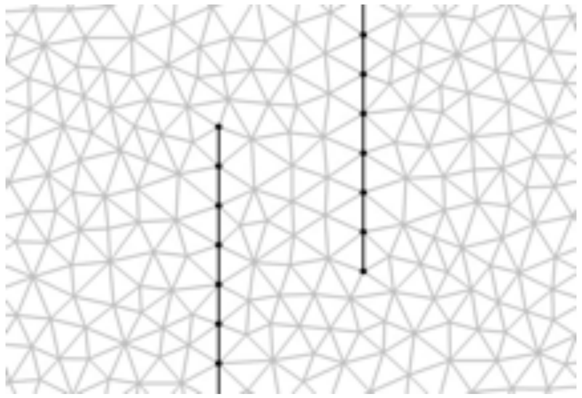


# Large structural deflections, contact, ...

- **Fitted mesh methods**:
    - Pros: natural, accurate, adequate fluid time-stepping
    - Cons: moderate displacements, solid contact not possible



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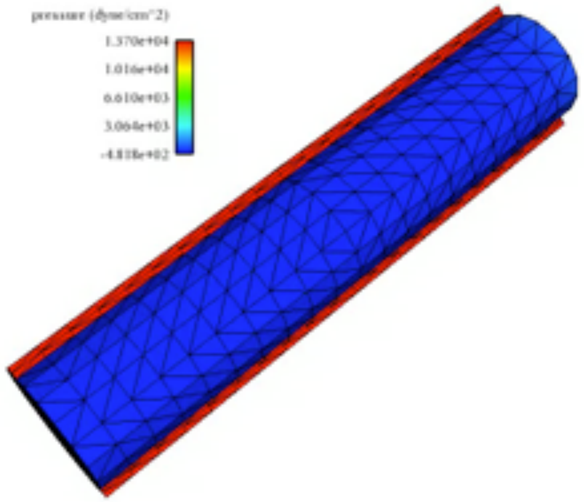

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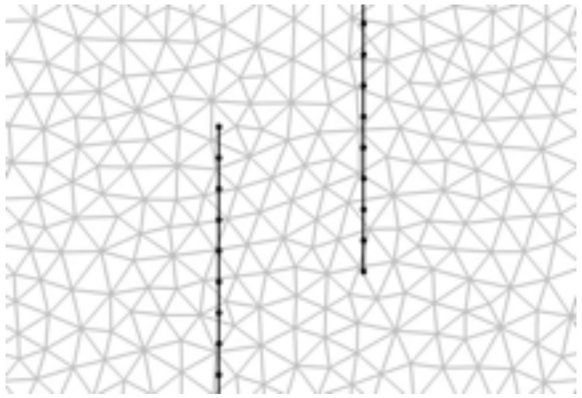

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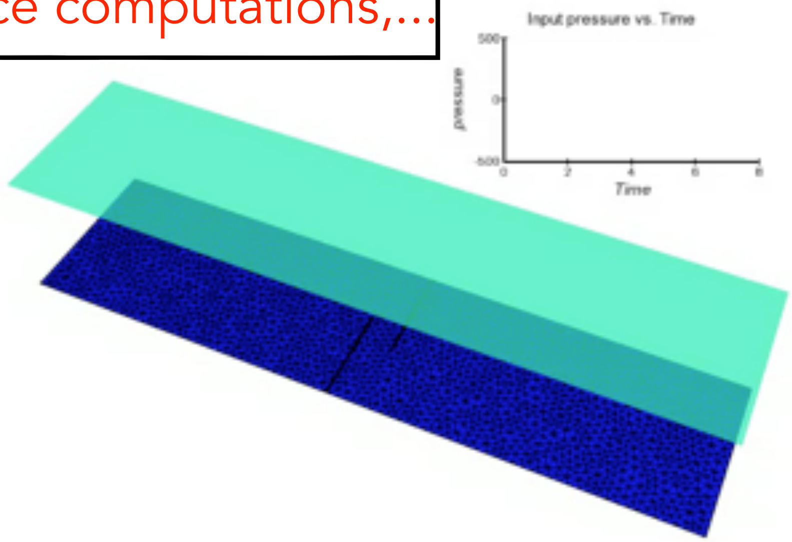
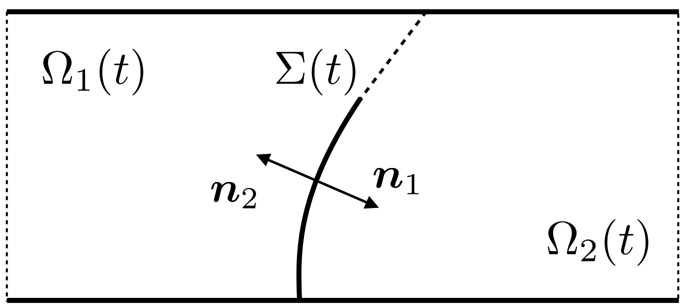
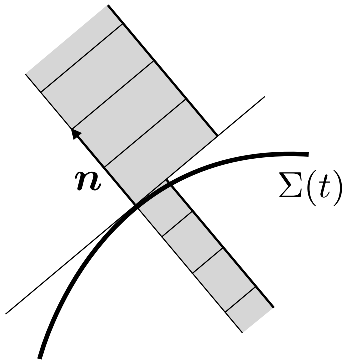


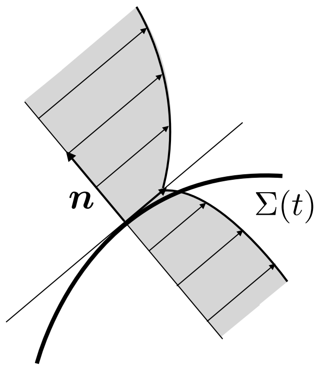
# Standard Unfitted Methods: Accuracy Issues

- Immersed thin-walled solids (e.g., cardiac valves) introduce jumps on the fluid stresses which, respectively, results in weak and strong discontinuities of the velocity and pressure fields




**Pressure field**




**Velocity field**

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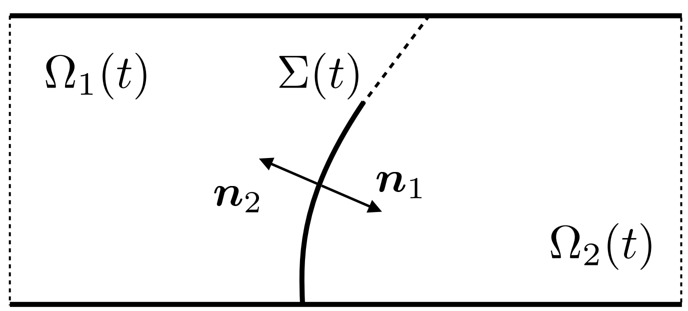


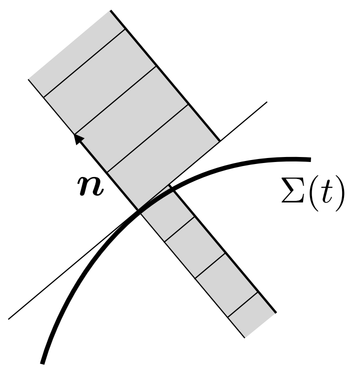


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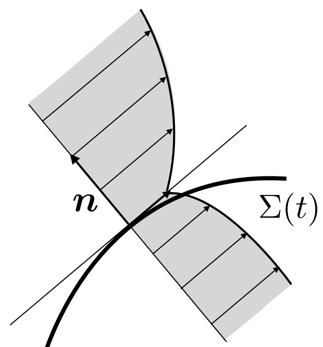
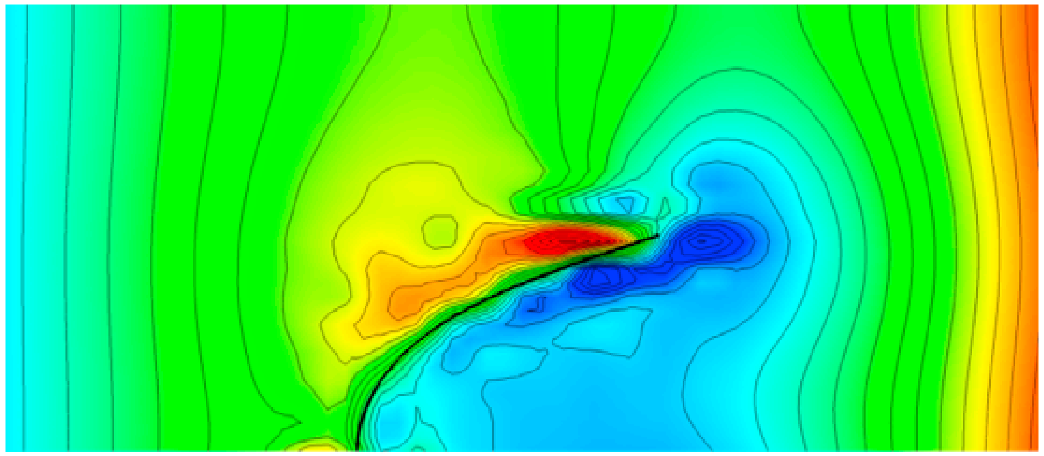



**Velocity field**

- Standard unfitted mesh approaches are inaccurate in space due to the continuous nature of the fluid approximations across the interface or to the discrete treatment of the interface conditions.



(Kamensky *et al.* '15)

# A consistent, accurate and robust method

***In my thesis work, I have introduced a new unfitted mesh method by combining:***

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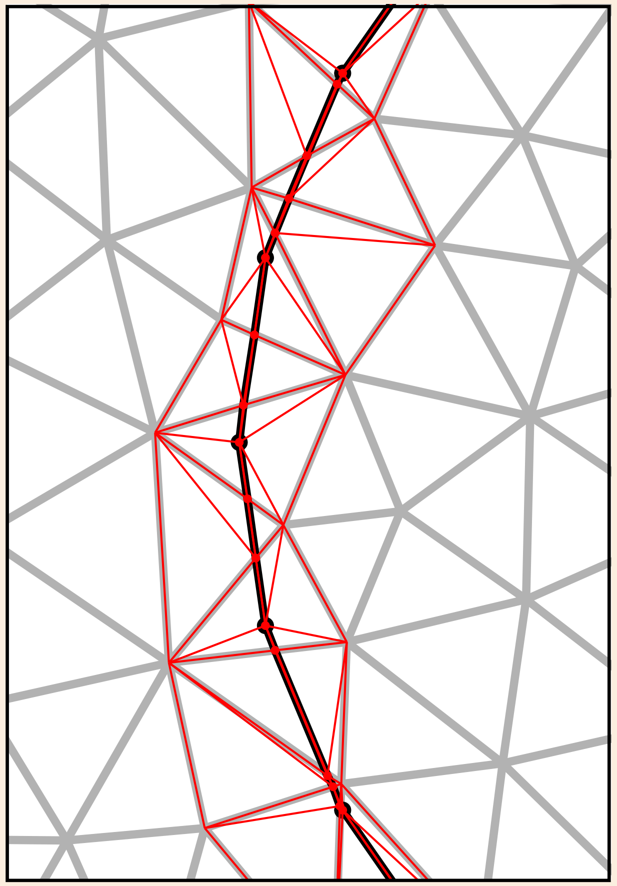

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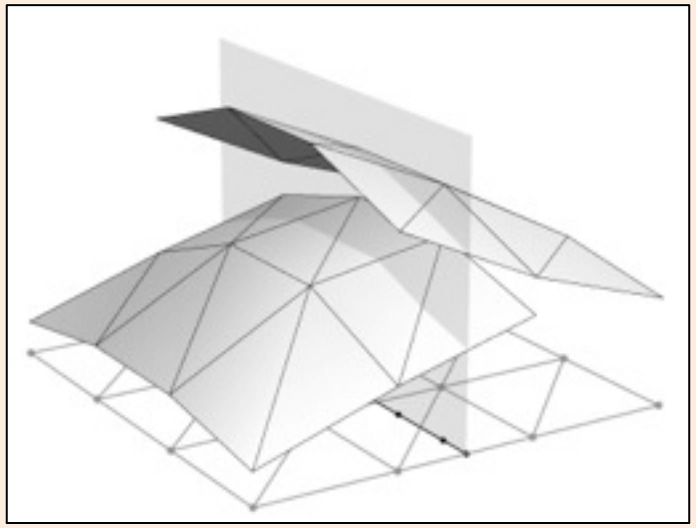

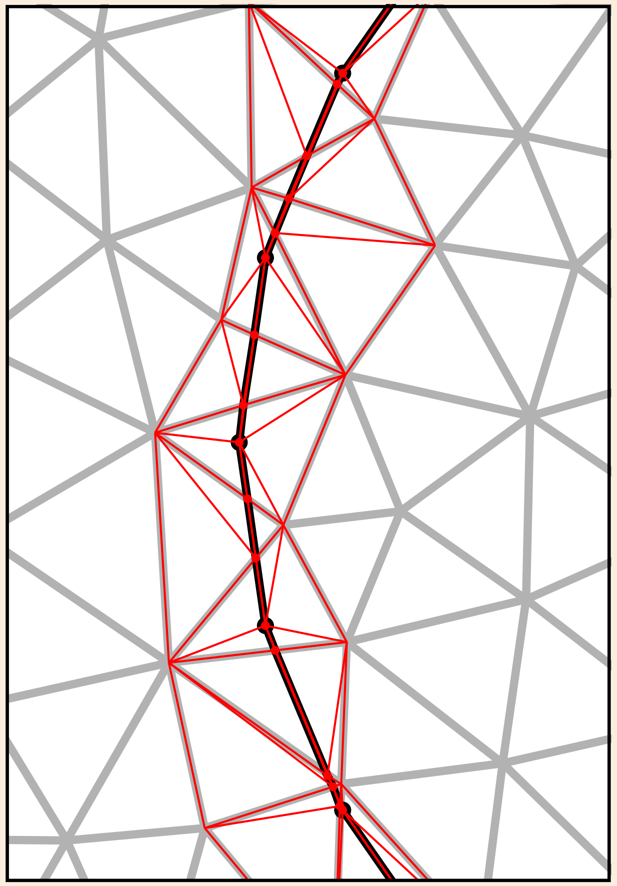

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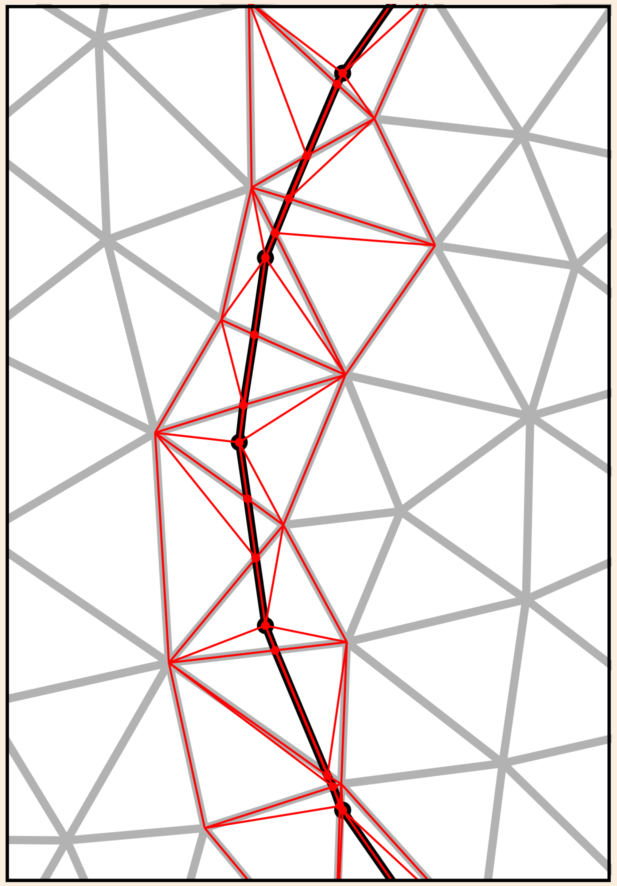
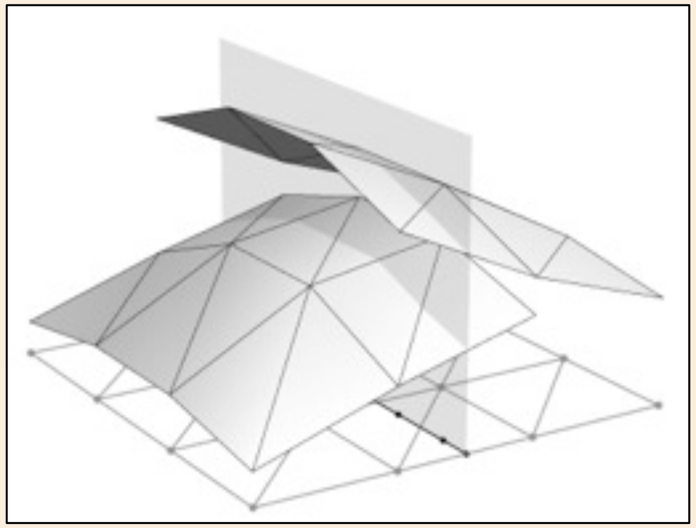
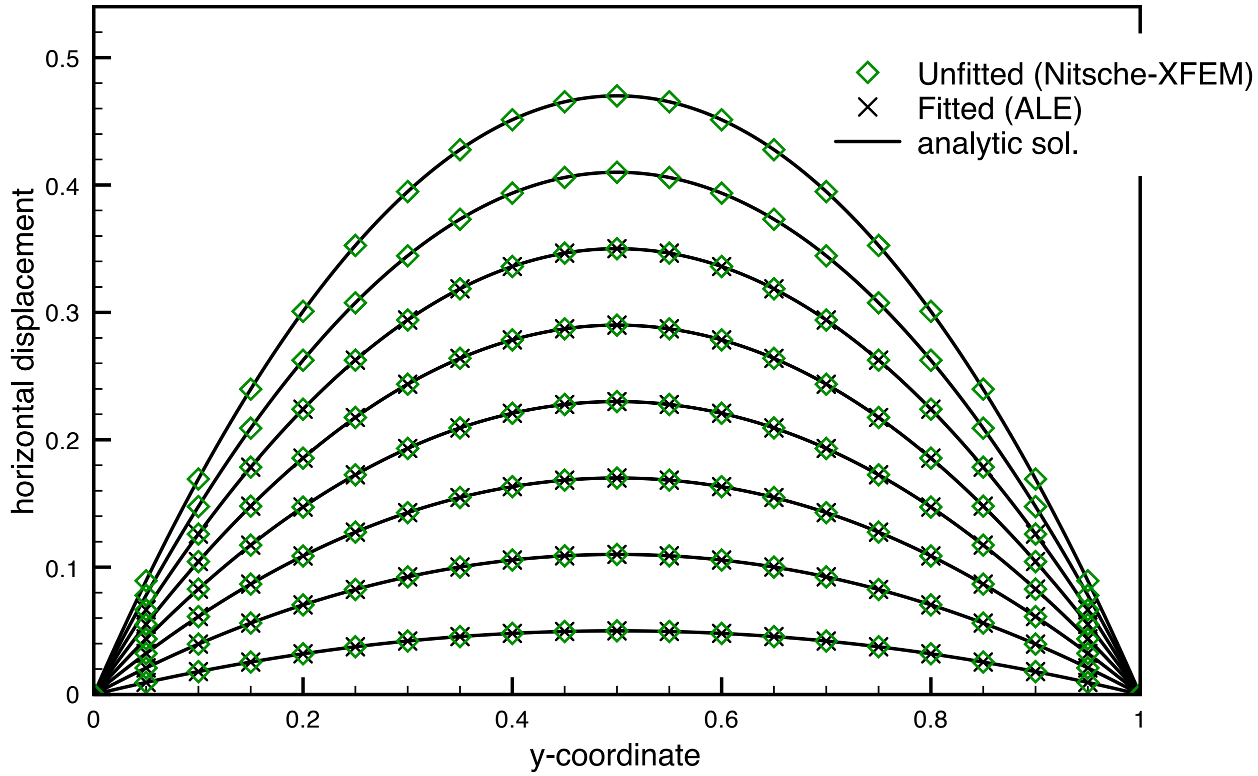




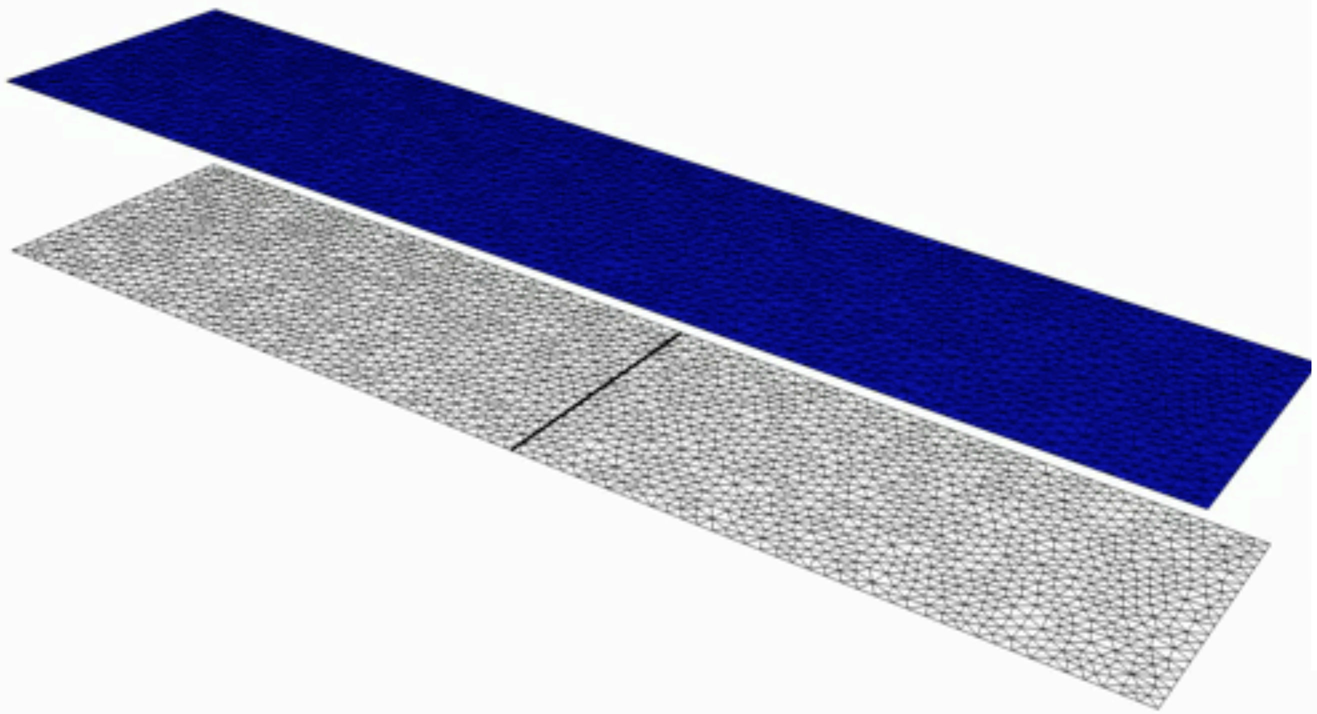
***This results in a robust and accurate method for fluid-structure interaction problems involving a thin-walled elastic structure immersed in an incompressible viscous fluid. Besides, it provides solid mathematical foundations, i.e., stability and convergence results are available (at least for linear problems).***

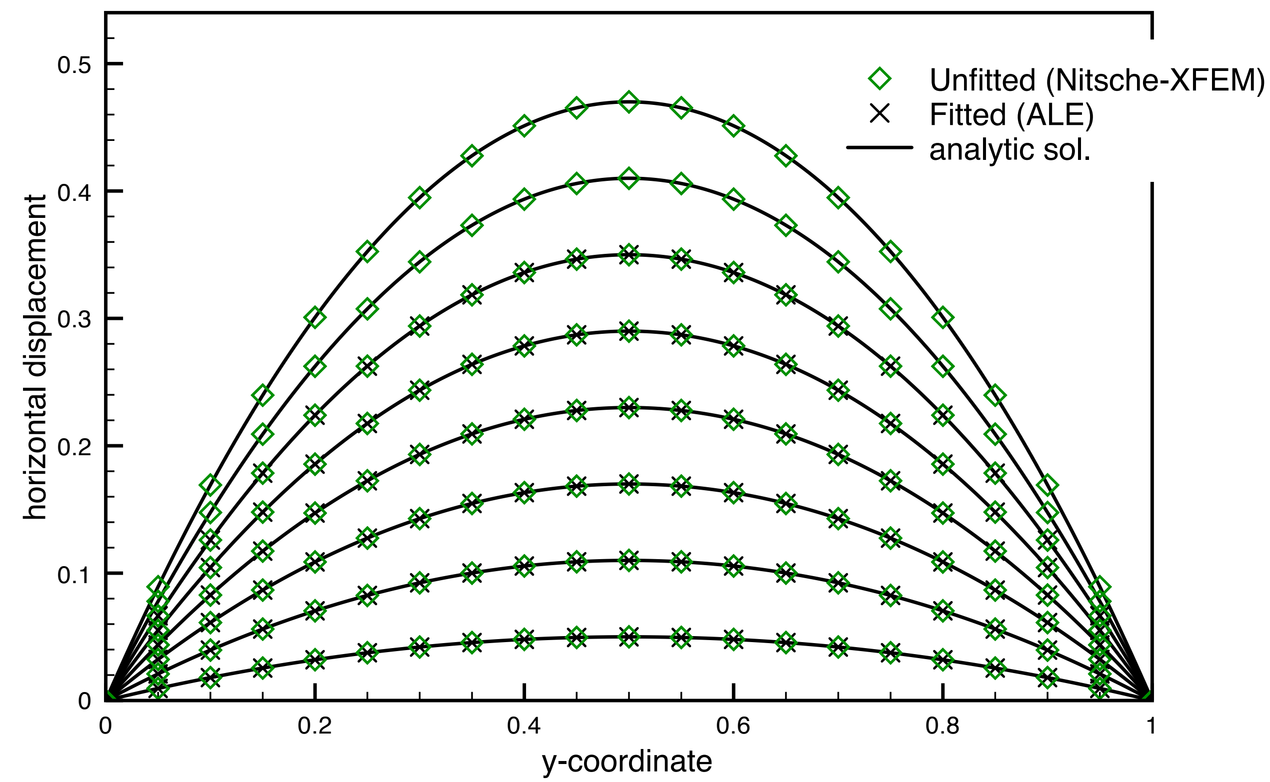
# Idealized Closed Valve




Unfitted meshes (Nitsche-XFEM)

# Idealized Closed Valve






Unfitted meshes (Nitsche-XFEM)

# Ongoing work ...

**Simulations by Benoit Fabrèges**

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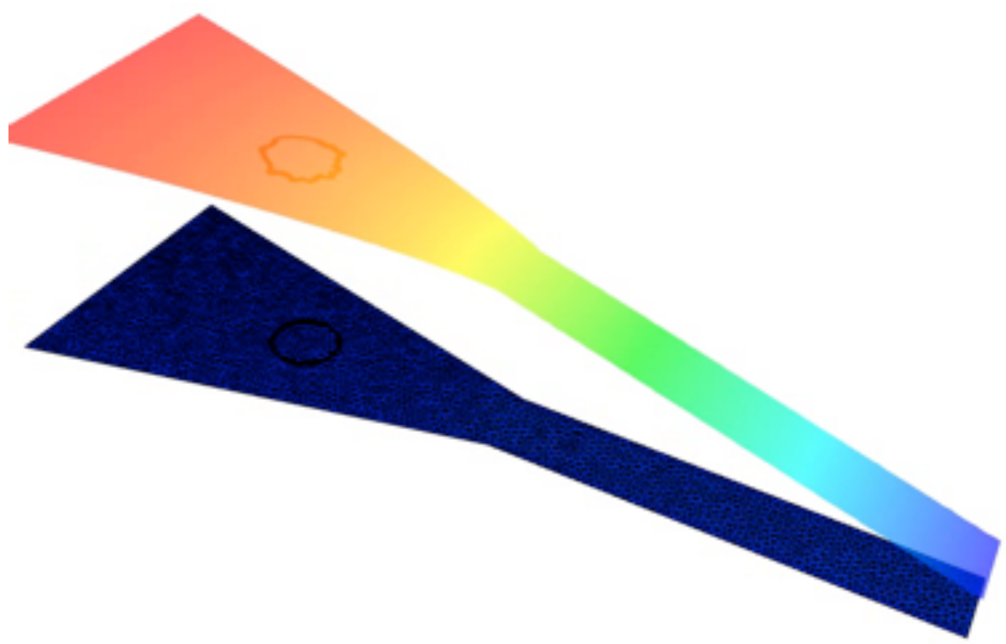

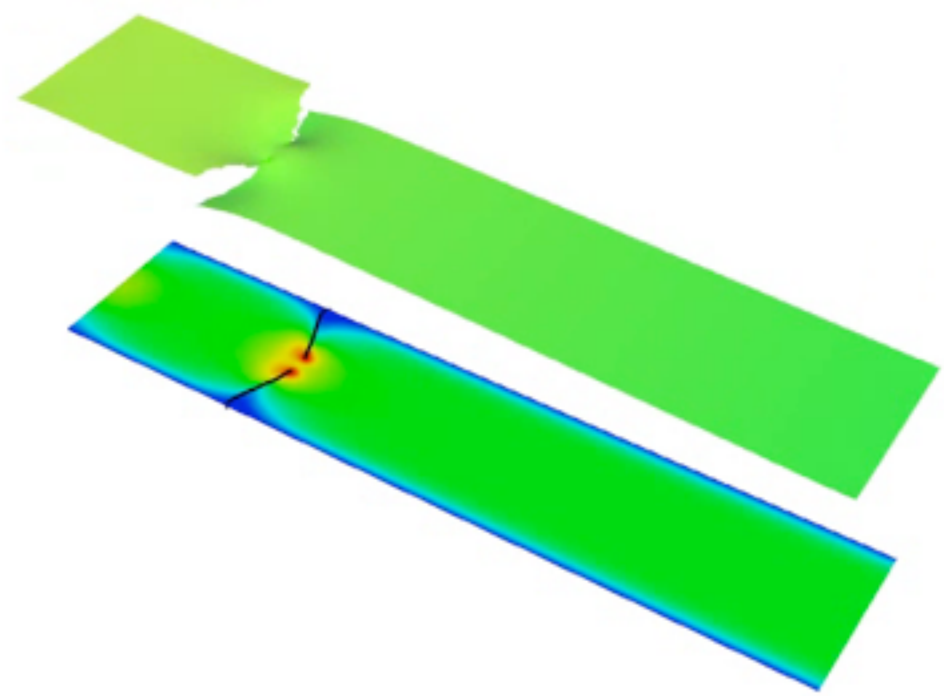

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**Simulations by Benoit Fabrèges**

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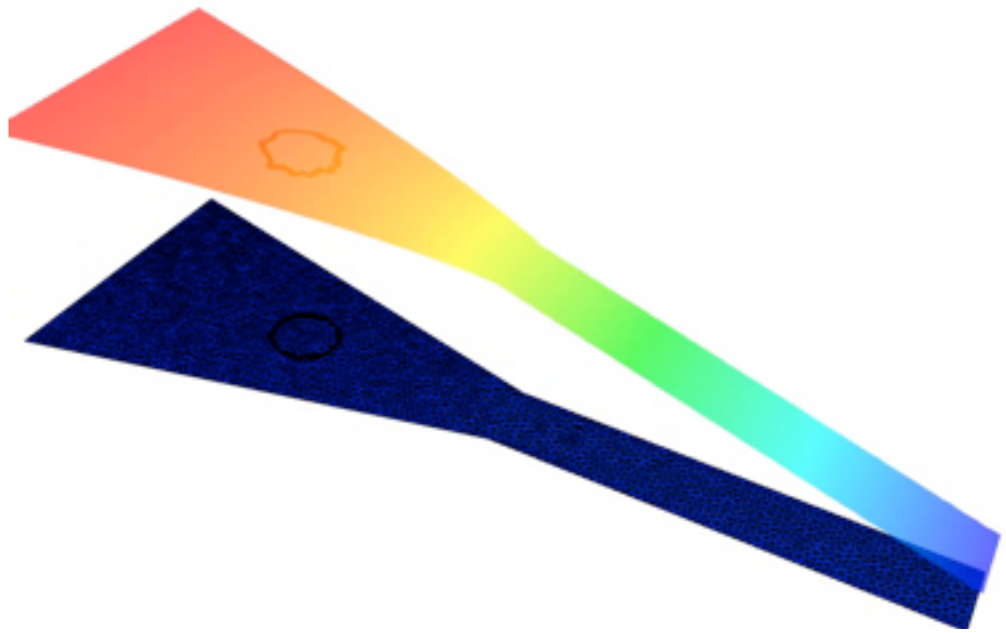
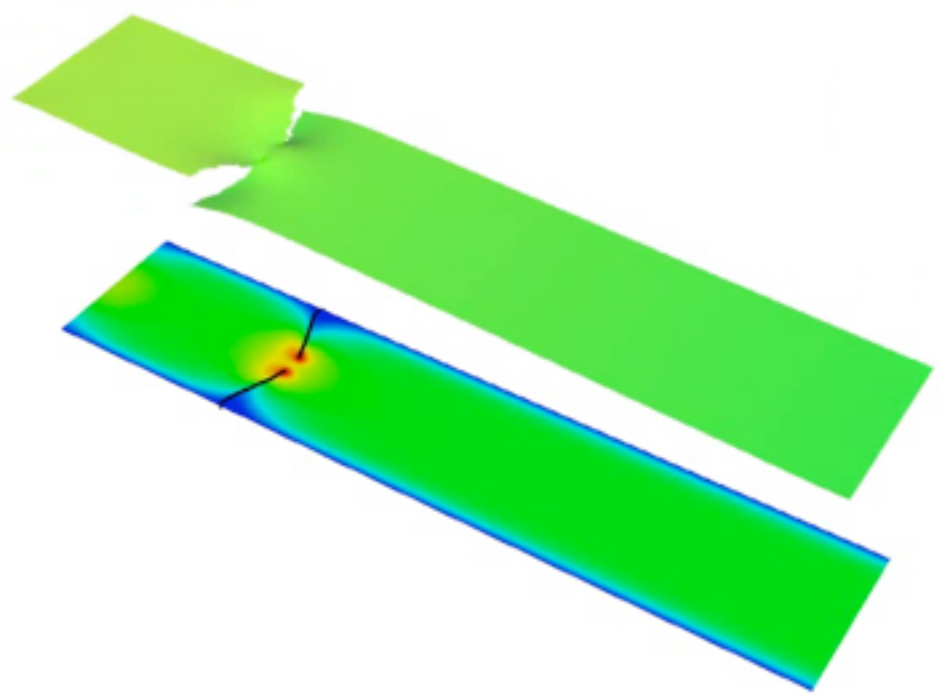
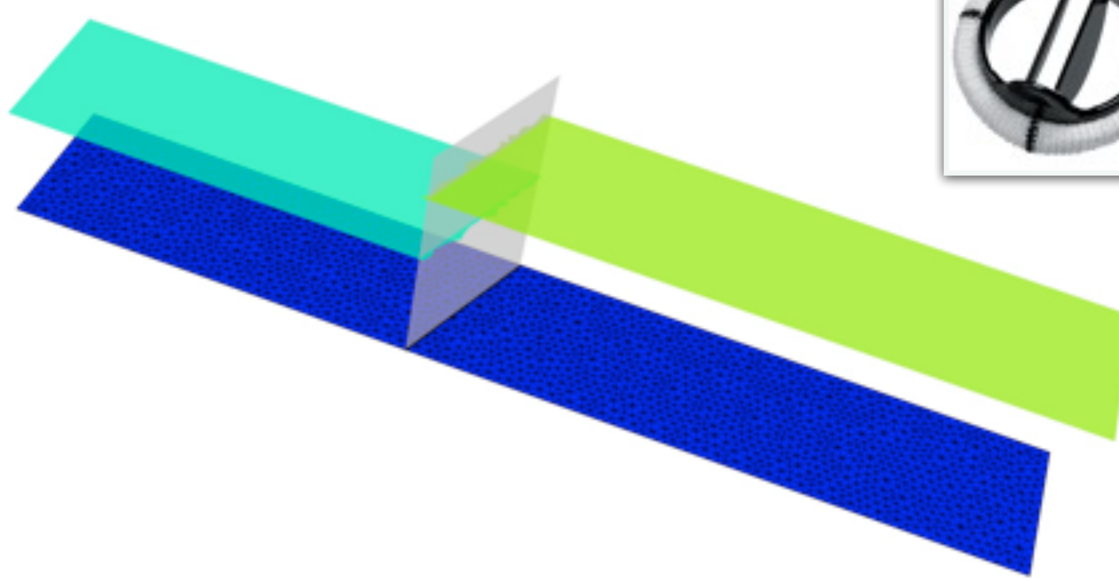




- first basic contact problems ...



# Concluding remarks

- **Main points:**
  - Fluid-structure interaction is a **widespread phenomena**.
  - Its computer-based **simulation** raises many **numerical challenges**.
  - In my work, I have focused on:
    - **Efficient splitting schemes**
    - **Accurate unfitted mesh methods**

- **Perspectives**:
  - Contacting solids, solid break-up (e.g., micro-encapsulation, drug delivery)
  - 3D simulations, non-symmetric penalty free Nitsche's method
  - Mesh intersection in 3D & HPC
  - Contact with multiple structures

# Bibliography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• M. A. Fernández, M. Landajuela, J. Mullaert, M. Vidrascu, *Robin- Neumann schemes for incompressible fluid-structure interaction*. In Domain Decomposition Methods in Science and Engineering XXII, Lecture Notes in Computer Science (LNCS). Dickopf, Th., Gander, M.J., Halpern, L., Krause, R., Pavarino, L.F. (Eds.), Lugano, Switzer- land, 2015, DOI: 10.1007/978-3-319-18827-0, https://hal.inria.fr/ hal-01113088.

• M. A. Fernández, M. Landajuela, M. Vidrascu, *Fully decoupled time- marching schemes for incompressible fluid/thin-walled struc- ture interaction*. Journal of Computational Physics, 297:156-181, 2015.

## • FSI Challenge

• M. Landajuela, M. Vidrascu, D. Chapelle, M. A. Fernández, Coupling schemes for the FSI forward prediction challenge: comparative study and validation. Submitted to International Journal for Nu- merical Methods in Biomedical Engineering. https://hal.inria.fr/ hal-01239931.

## • Unfitted mesh methods

• F. Alauzet, B. Fabrèges, M. A. Fernández, M. Landajuela, *Nitsche- XFEM for the coupling of an incompressible fluid with im- mersed thin-walled structures*. Computer Methods in Applied Me- chanics and Engineering, To appear, 2015, https://hal.inria.fr/ hal-01149225 .

• M. A. Fernández, M. Landajuela, *Unfitted formulations and split- ting schemes for incompressible fluid/thin-walled structure in- teraction.* Submitted to ESAIM: Mathematical Modelling and Numeri- cal Analysis.