A Physiologically-Based Model for the Active Cardiac Muscle Contraction

Dominique CHAPELLE
INRIA-Rocquencourt, Projet MACS

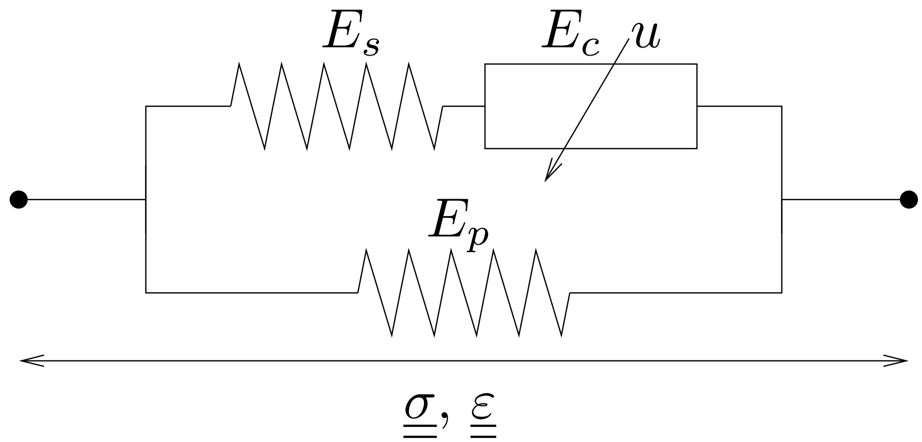
Joint work with:
F. Clément (INRIA-SOSSO), F. Génot (INRIA-MACS),
P. Le Tallec (Ecole Polytechnique), **J. Sainte-Marie** (INRIA-MACS),
M. Sorine (INRIA-SOSSO), J. Urquiza (INRIA-MACS)

(outline)

Outline

- **Excitation-contraction myofibre model**
- **3D non-linear constitutive and equilibrium equations**
- **Coupling with blood circulation**
- **Preliminary 1D simulations**

Excitation-contraction myofibre model



Hill-Maxwell rheological model

- E_s and E_p: series and parallel elements $\longrightarrow$ elastic (Wong 71, Mirsky & Parmley 73);

- E_c: contractile (electrically activated) element.

E_c as proposed in (Bestel & Sorine 2000; Bestel, Clément & Sorine 2001)

$$\left\{ \begin{array}{l} \dot{\tilde{\sigma}}_c = -(|\dot{\varepsilon}_c| + |u|)\tilde{\sigma}_c + k_c \dot{\varepsilon}_c + \sigma_0 |u|_+, \\ \dot{k}_c = -(|\dot{\varepsilon}_c| + |u|)k_c + k_0 |u||_+ \\ \sigma_c = k_c \varepsilon_0 + \tilde{\sigma}_c + \nu \dot{\varepsilon}_c, \end{array} \right. \tag{1}$$

u (input): electrical excitation related to chemical quantities (in particular calcium concentration).

Note:

- (1) based on sliding filament model of Huxley (57) and distribution-moment approach of Zahalak (81).
- Compatible with molecular nanomotor theory (Prost 94).

3D constitutive and equilibrium equations

We need to address 3D behaviour and large displacements/strains.

Use of rheological model with 3D non-linear problem:

- Parallel branches
  - ⋆ Addition of (2nd Piola-Kirchhoff) stresses

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}_p + \sigma_{1\mathrm{D}}\, \underline{n} \otimes \underline{n},$$

$\underline{n}$: unit vector tangent to muscle fibre direction.

  - ⋆ Equality of (Green-Lagrange) strains

$$\underline{\underline{\varepsilon}}_p = \underline{\underline{\varepsilon}}, \qquad \varepsilon_{1\mathrm{D}} = \sum_{i,j} \varepsilon_{ij} n_i n_j.$$

- Series branch ("1D")

$$1 + \varepsilon_{1\mathrm{D}} = (1 + \varepsilon_c)(1 + \varepsilon_s),$$

(composition of deformations)

$$\sigma_{1\mathrm{D}} = \frac{\sigma_c}{1 + \varepsilon_s} = \frac{\sigma_s}{1 + \varepsilon_c},$$

(formal thermodynamical considerations).

With $\sigma_s = \sigma_s(\varepsilon_s)$, $\underline{\underline{\sigma}}_p = \underline{\underline{\sigma}}_p(\underline{\underline{\varepsilon}}_p)$ and (1) the behaviour is defined: "$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}(\underline{y})$".

$\longrightarrow$ Equation of dynamics: $$\boxed{\mathrm{div}(\underline{\underline{F}} \cdot \underline{\underline{\sigma}}) - \rho \ddot{\underline{y}} = \underline{0}}$$

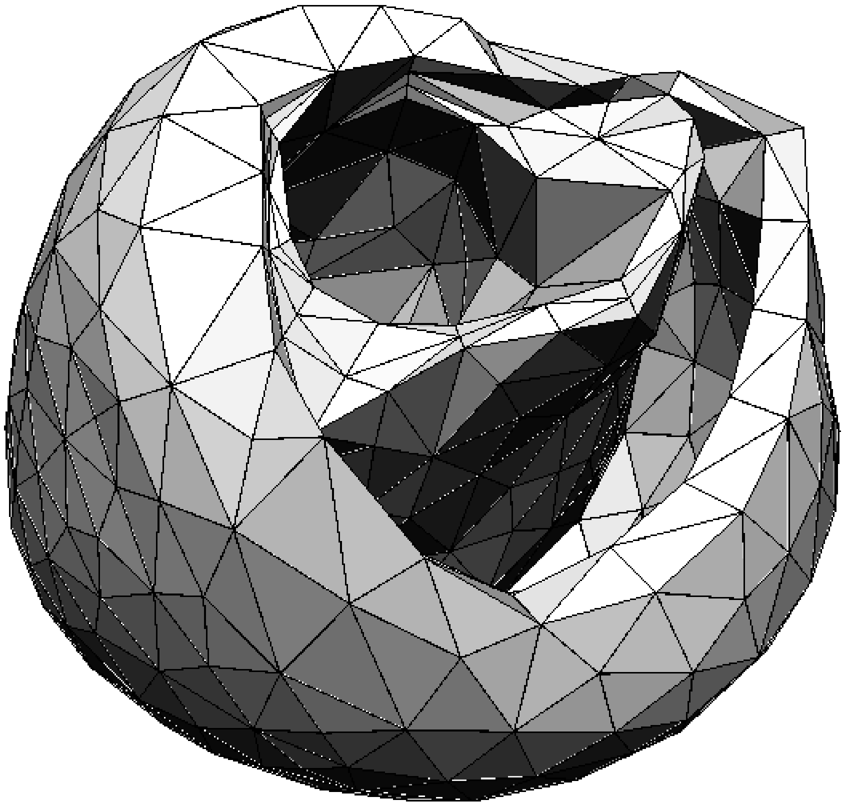
($\underline{\underline{F}}$ deformation gradient).

Geometrical data

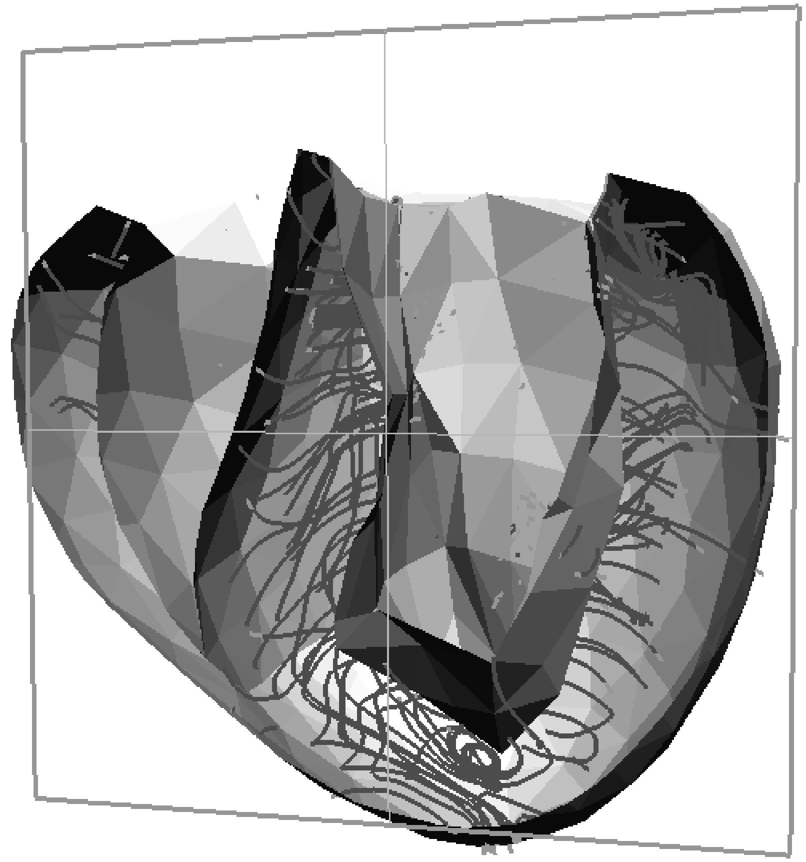
Based on data from **Auckland Bioengineering Institute**.

Mesh refinement performed by using:

- Surface mesh refinement: YAMS (**INRIA-GAMMA**);
- 3D automatic mesh generation: GHS3D (**INRIA-GAMMA**);
- Interpolation of fiber data.



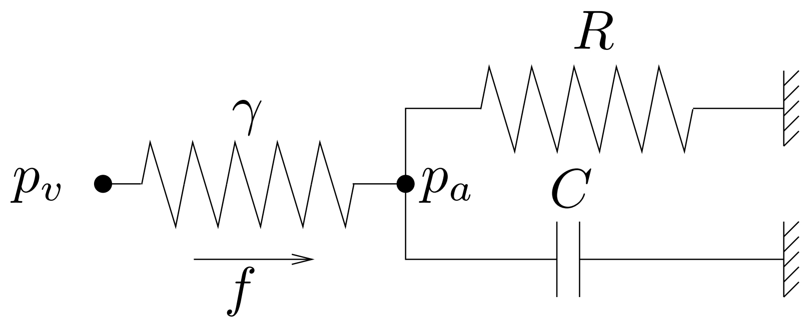
Refined mesh



Streamline representation of fibers

Coupling with blood circulation

Focus on (typically) left ventricle: internal volume V, pressure p_v.**Note:** p_v assumed uniform inside cavity.



Electrical analogy for circulation

External circulation: $f = C\dot{p}_a + \dfrac{p_a}{R}$, (simplified "windkessel");

Behaviour of valve: $f = \gamma\, |p_v - p_a|_+$;

Fluid conservation: $f = -\dot{V}$, (systolic and isovolumetric phases).

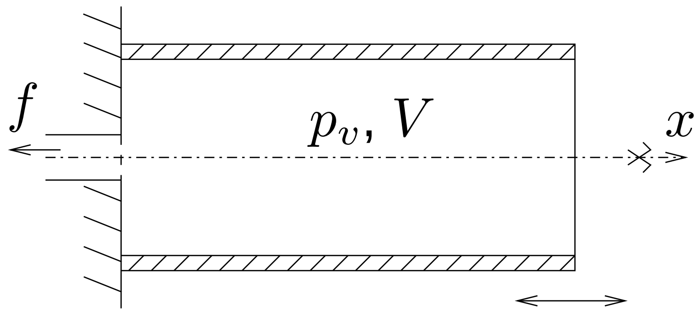
Note:

- In practice: γ “big” ($p_v \sim p_a$).
- During isovolumetric stages ($f = 0$, $p_v < p_a$): constrained deformation$\longrightarrow$ p_v Lagrange multiplier.
- In fact R and C vary: controlled by nervous system.

Preliminary 1D simulations

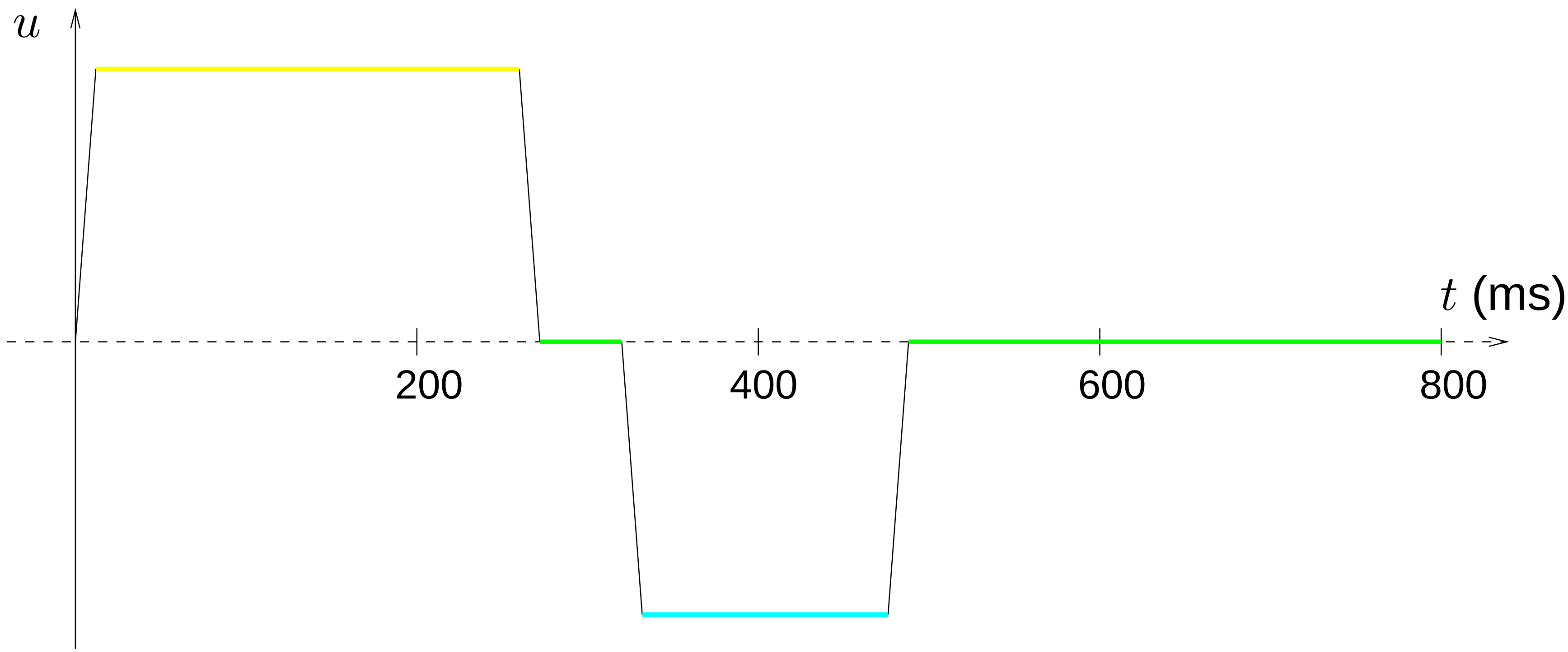
Objective: experiment with (1D) contractile constitutive equations$\longrightarrow$ “pre-identify” parameters.

“1D left ventricle”:



Use linearized elasticity (large displacements non-linearities “standard”).

$$\begin{cases} \rho\ddot{y} - \frac{d}{dx}(k_p\varepsilon + \sigma_c) = 0, \\ \dot{\sigma_c} = -(|\dot{\varepsilon}_c| + |u|)\sigma_c + k_c\dot{\varepsilon}_c + \sigma_0|u|_+, \\ \dot{k_c} = -(|\dot{\varepsilon}_c| + |u|)k_c + k_0|u|_+, \\ \sigma_c = k_s(\varepsilon - \varepsilon_c), \end{cases}$$



Electrical excitation (propagating from right to left)

Global stress σ Contractile stress σ_c