A Physiologically-Based Model for the Active Cardiac Muscle Contraction

Dominique Chapelle, Frédérique Clément, Frank Génot,
Patrick Le Tallec, Michel Sorine, and José M. Urquiza

INRIA-Rocquencourt, BP 105, 78153 Le Chesnay, France,
Corresp. author: Dominique.Chapelle@inria.fr

Abstract. We present a 3D mechanical model for the behaviour of the cardiac muscle, based on a recently proposed model of myofibre contraction. The construction of the 3D model involves a rheological model similar to that of Hill-Maxwell. We then introduce some discretisation techniques adapted to our mechanical model, and we report on some preliminary numerical results.

1 Introduction

Like most biological materials, the cardiac muscle displays strongly nonlinear and anisotropic mechanical behaviour, see, e.g., [5]. Anisotropy is governed—in particular—by the architecture of muscle fibres and of the fibre sheets. Moreover, muscle fibres are responsible for the active stress generated by the electrical stimulations. This active stress mainly concerns the stress component along the fibre direction, which is also the primary direction of electrical propagation.

In this work we present a complete mechanical model that aims at correctly representing the cardiac muscle behaviour in the presence of the chemical/electrical stimulation. This model is based on a recently proposed myofibre model [2], which we recall before presenting the methodology allowing its incorporation into a global three-dimensional (3D) mechanical model. The last section is devoted to numerical discretisation strategies adapted to the model, and we present some preliminary numerical results obtained for a 1D simplified problem.

2 The Excitation-Contraction Myofibre Model

The myofibre model introduced and justified in [2] (see also [3]), is based on the classical Hill-Maxwell rheological model [6], depicted in Figure 1. Elastic material laws are used for the series element E_s and for the parallel element E_p. Based on empirical results, the corresponding stress-strain laws are generally assumed to be of exponential type [9, 12].

The element E_c accounts for the contractile electrically-activated part of the behaviour. In order to represent this behaviour, we use the constitutive law recently proposed in [2] (a refinement of the model presented in [3]), based on the microstructural *sliding filament* model of Huxley [8] and the *distribution-moment* approach of Zahalak [13]. This (time-dependent) constitutive law reads

$$\begin{cases} \dot{\tilde{\sigma}}_c = -(|\dot{\varepsilon}_c| + |u|), \tilde{\sigma}_c + k_c \dot{\varepsilon}_c + \sigma_0 |u|_+, \\ \dot{k}_c = -(|\dot{\varepsilon}_c| + |u|) k_c + k_0 |u|_+ \\ \sigma_c = k_c \varepsilon_0 + \tilde{\sigma}_c + \nu \dot{\varepsilon}_c, \end{cases} \tag{1}$$

where u is the excitation, a chemical quantity that mainly depends on the calcium concentration, $\dot{\varepsilon}_c$ stands for the time derivative of ε_c, and σ_0, k_0, ε_0, ν are local parameters such that $\sigma_0, k_0, \nu > 0$. This model is also in accordance with observations made on the passive behaviour of the muscle [10].

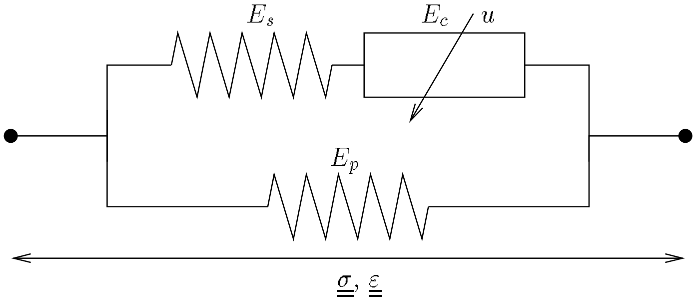

Fig. 1. Hill-Maxwell rheological model

3 Geometry, Fibre Orientations and Governing Equations of Activated Heart Contractions

In order to construct our global 3D mechanical model we need:

- Anatomical data allowing for a definition of the heart geometry, with an accurate description of the microstructural anatomy, i.e. of the orientations of muscle fibres and fibre sheets.
- The governing equations of the 3D mechanical behaviour.

We used the anatomical data provided by the Auckland biomedical engineering research group [7]. We refined the original mesh composed of hexahedral elements by using the softwares YAMS and GHS3D developped at INRIA in the Gamma team [4, 11]. The refined mesh, composed of tetrahedral elements, is shown in Figure 2.

The orientation of the fibres is available at each integration point in the mesh. The corresponding streamline representation is shown in Figure 3.

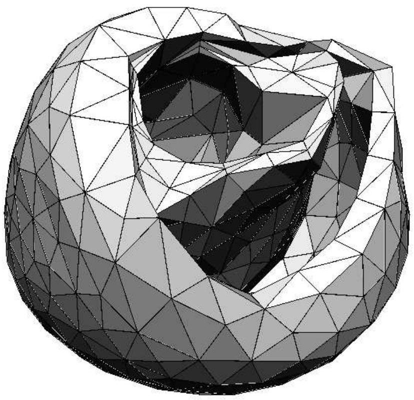

Fig. 2. Refined tetrahedral mesh

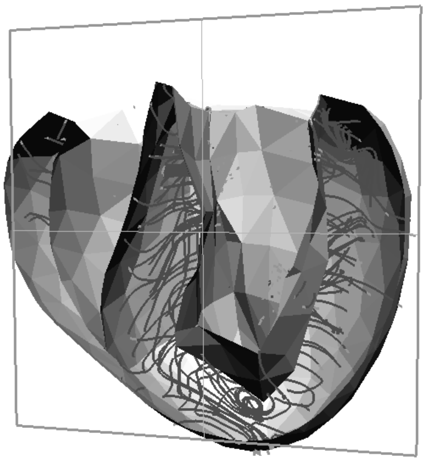

Fig. 3. Heart fibre directions

We now introduce the governing 3D mechanical equations. We denote by $\underline{\underline{F}}$ and $\underline{\underline{\varepsilon}}$ the deformation gradient and the Green-Lagrange strain tensor, respectively. We then use the second Piola-Kirchhoff stress tensor, denoted by $\underline{\underline{\sigma}}$. Recalling the rheological model of Figure 1, the total stress is the sum of the stresses in the two branches. The branch containing the contractile element, however, only contributes to the stress tensor in the direction of the fibre, given by the unit tangent vector $\underline{n}$. Namely, we have

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}_p + \sigma_{1\mathrm{D}}\, \underline{n} \otimes \underline{n}, \tag{2}$$

where $\sigma_{1\mathrm{D}}$ is a scalar quantity that represents the (second Piola-Kirchhoff) stress in the series branch. The series kinematics and the one-dimensional character of the deformation in this branch imply that the corresponding (scalar) Green-Lagrange strain

$$\varepsilon_{1\mathrm{D}} = \sum_{i,j} \varepsilon_{ij} n_i n_j \tag{3}$$

satisfies the multiplicative law

$$1 + \varepsilon_{1\mathrm{D}} = (1 + \varepsilon_c)(1 + \varepsilon_s), \tag{4}$$

where ε_c and ε_s denote the strains in the contractile and series element, respectively (note that this relation reduces to the usual additive law in the case of small strains). Furthermore, the associated second Piola-Kirchhoff stresses satisfy, by formal thermodynamical considerations,

$$\sigma_{1\mathrm{D}} = \frac{\sigma_c}{1 + \varepsilon_s} = \frac{\sigma_s}{1 + \varepsilon_c}. \tag{5}$$

On the other hand, the parallel kinematics directly implies

$$\underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}_p. \tag{6}$$

Combining the above equations with the constitutive laws of the elastic elements

$$\sigma_s = \sigma_s(\varepsilon_s), \qquad \underline{\underline{\sigma}}_p = \underline{\underline{\sigma}}_p(\underline{\underline{\varepsilon}}_p), \tag{7}$$

and with the equations (1), the 3D material behaviour is completely defined.

The global 3D mechanical model is obtained by using these constitutive relations (involving the input u) together with the equation of dynamics

$$\mathrm{div}(\underline{\underline{F}} \cdot \underline{\underline{\sigma}}) - \rho \ddot{\underline{y}} = \underline{0}, \tag{8}$$

where $\underline{y}$ denotes the displacement vector and ρ the mass density (in the original configuration).

4 A One-Dimensional Activated Fibre Contraction

As a model problem we consider a one-dimensional homogeneous medium governed by a 1D reduction of equations (1)-(8).

In this first step to the numerical exploration of the excitation-contraction model we use the infinitesimal displacement assumption: with $y(x,t)$ denoting the longitudinal displacement of a material point located at position x, $0 < x < 1$, in a reference stress-free state, the deformation is given by $\varepsilon(x,t) = \frac{\partial y(x,t)}{\partial x}$. Moreover, we assume that series and parallel elements are linear and that $\nu = 0$, $\varepsilon_0 = 0$. The considered equations read

$$\begin{cases} \rho \ddot{y} - \frac{\partial}{\partial x}(k_p \varepsilon + \sigma_c) = 0, \\ \dot{\sigma_c} = -(|\dot{\varepsilon_c}| + |u|)\sigma_c + k_c \dot{\varepsilon_c} + \sigma_0 |u|_+, \\ \dot{k_c} = -(|\dot{\varepsilon_c}| + |u|)k_c + k_0 |u|_+, \\ \sigma_c = k_s(\varepsilon - \varepsilon_c), \end{cases} \tag{9}$$

with positive constants k_p and k_s. We also consider that the body is fixed at both ends, that is $y(0,t) = y(1,t) = 0$, corresponding to an isometric contraction or relaxation.

Performing a variational finite element space discretization with piecewise-linear continuous functions for the displacement and piecewise-constant functions for stress and deformation variables, we use a midpoint scheme for solving the resulting system of differential-algebraic equations (see [1]).

In the numerical tests presented in Figures 4 to 7 we chose all initial unknowns to be zero. These numerical experiments illustrate the active contraction (positive u) and active relaxation (negative u) abilities of the model.

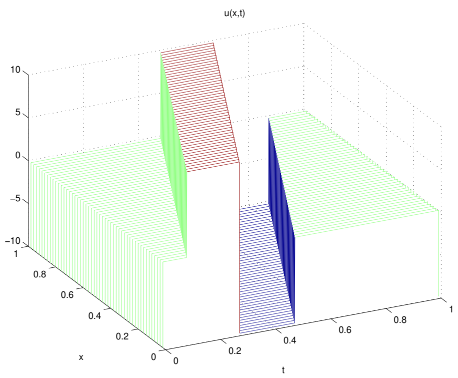

Fig. 4. Electric excitation

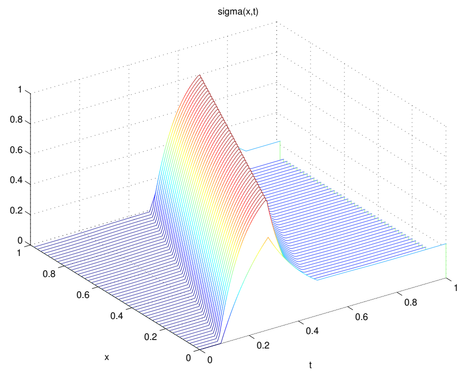

Fig. 5. Resulting stress

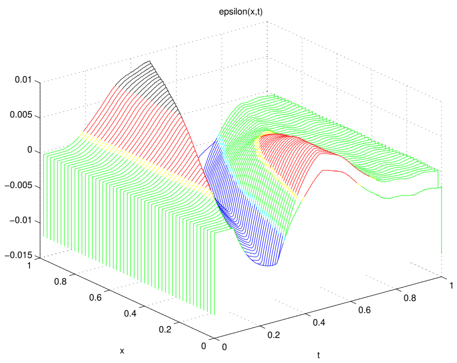

Fig. 6. Deformation

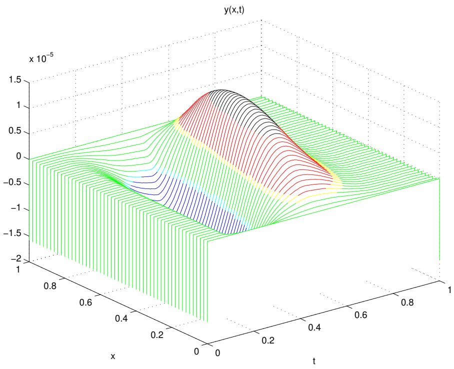

Fig. 7. Displacement

References

1. Ascher, U. M., Petzold, L. M.: Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations. SIAM Publ., Philadelphia (1998)
2. Bestel, J., Clément F., Sorine, M.: A biomechanical model of muscle contraction. Fourth Int. Conf. on Medical Image Computing and Computer-Assisted Intervention, Utrecht, The Netherlands (2001)
3. Bestel, J., Sorine, M.: A differential model of muscle contraction and applications. Schlœssman Seminar on Mathematical Models in Biology, Chemistry and Physics Max Planck Society, Bad Lausick, Germany (2000)
4. Frey, P. J., Borouchaki, H.: Geometric surface mesh optimization. Comput. Visual. in Science **1** (1998) 113–121
5. Fung, Y. C.: Biomechanics: Mechanical Properties of Living Tissues. Springer-Verlag, 2nd Ed. (1993)
6. Hill, A. V.: The heat of shortening and the dynamic constants in muscle. Proc. Roy. Soc. London (B) **126** (1938) 136–195
7. Hunter, P. J., Nash, M. P., Sands, G. B.: Computational electromechanics of the heart. In Panfilov, A.V. and Holden, A. V., editors, Computational Biology of the Heart, chap. 12 , John Wiley and Sons, (1997) 345–407
8. Huxley, A. F.: Muscle Structure and Theory of Contraction. Progr. Biomech. Biophys. Chem. **7** (1957) 255–318
9. Mirsky, I., Parmley, W. W.: Assessment of passive elastic stiffness for isolated heart muscle and the intact heart. Circul. Research **33** (1973) 233–243
10. Mirsky, I., Parmley, W. W.: Cardiac Mechanics: Physiological, Clinical and Mathematical Considerations. Pergamon Press (1974) chap. 4
11. TetMesh-GHS3D: http://www.simulog.fr/itetmeshf.htm
12. Wong, A. Y. K.: Mechanics of cardiac muscle based on Huxley's model: mathematical simulation of isometric contraction. J. Biomech. **4** (1971) 529–540
13. Zahalak, G. I.: A distribution-moment approximation for kinetic theories of muscular contraction. Math. Biosci. **55** (1981) 89–114