Monitoring surface currents from uncertain image observations

Isabelle Herlin[a,b], Etienne Huot[a,b,c]

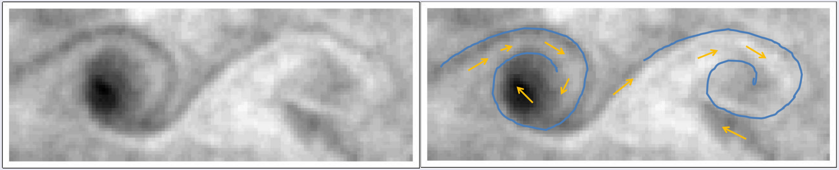
[a] INRIA, Institut National de Recherche en Informatique et Automatique, France
[b] CEREA, joint laboratory ENPC – EDF R&D, Université Paris-Est, France
[c] Université de Versailles – Saint-Quentin-en-Yvelines, France

Objective

Estimation of motion $\mathbf{w}(\mathbf{x}, t)$ on an image sequence $I(\mathbf{x}, t)$ by data assimilation.

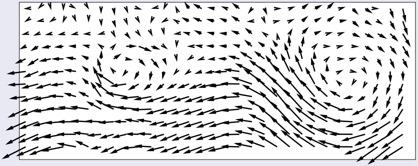
Input Data

Sequence of satellite images I



Result of estimation

Estimation of apparent velocity $\mathbf{w}$:



Data Assimilation

- $\mathbf{X}(\mathbf{x}, t)$, a state vector, $\mathbf{Y}(\mathbf{x}, t)$ an observation vector, defined on $A = \Omega \times [0, t_N]$
- System to be solved w.r.t. $\mathbf{X}$:

Evolution equation	$\dfrac{\partial \mathbf{X}}{\partial t} + \mathbb{M}(\mathbf{X})$	$=$	0
Background equation	$\mathbf{X}(0) - \mathbf{X}_b$	$=$	$\epsilon_B(\mathbf{x})$
Observation equation	$\mathbb{H}(\mathbf{X}, \mathbf{Y})$	$=$	$\epsilon_R(\mathbf{x}, t)$

- The optimization problem is defined as the minimization of:

$$J_1(\mathbf{X}(0)) = \frac{1}{2} \int_A \frac{(\epsilon_R(\mathbf{x}, t))^2}{R(\mathbf{x}, t)} d\mathbf{x} dt + \frac{1}{2} \int_\Omega \frac{(\epsilon_B(\mathbf{x}))^2}{B(\mathbf{x})} d\mathbf{x}$$

Direct approach

State vector $\mathbf{X} = \mathbf{w}$

Evolution equations $\frac{\partial \mathbf{w}}{\partial t} + (\mathbf{w} \cdot \nabla)\mathbf{w} = 0$

Observation equation $\mathbb{H}_1 = I_t + \mathbf{w} \cdot \nabla I$

I_t stands for the discrete temporal derivative: $I_t = \frac{I_{k+1} - I_k}{t_{k+1} - t_k}$.

Pseudo-observations approach

State vector $\mathbf{X} = (\mathbf{w} \quad I_s)^T$

Evolution equations

$$\frac{\partial \mathbf{w}}{\partial t} + (\mathbf{w} \cdot \nabla)\mathbf{w} = 0$$

$$\frac{\partial I_s}{\partial t} + \mathbf{w} \cdot \nabla I_s = 0$$

Observation equation $\mathbb{H}_2 = I_s - I$

Combined approach

Observation equations

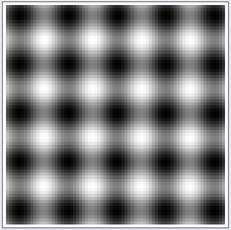
$$\mathbb{H}_1 = I_t + \mathbf{w} \cdot \nabla I$$

$$\mathbb{H}_2 = I_s - I$$

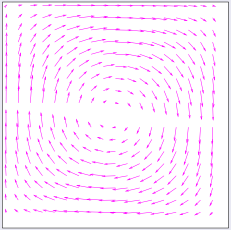
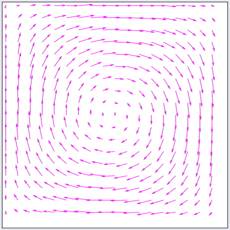
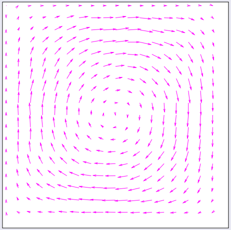
Comparison: direct *vs* pseudo-image approaches

Twin experiments

Observations / direct approach / pseudo-image approach

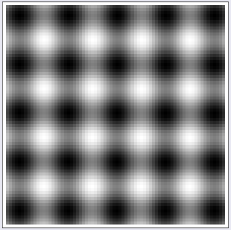


Ground truth / direct approach / pseudo-image approach

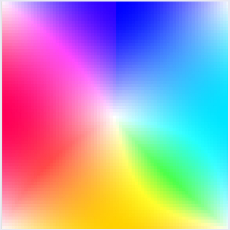




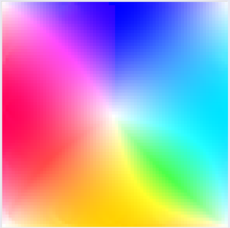
Comparison: direct *vs* pseudo-image approaches

Observations / direct approach / pseudo-image approach



Ground truth / direct approach / pseudo-image approach





Comparison: direct *vs* pseudo-image approaches

Twin experiments statistics

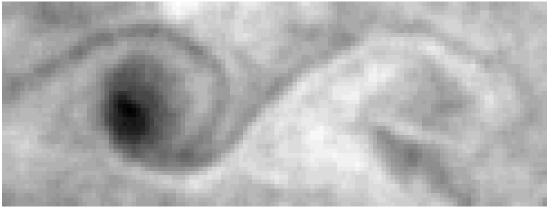
Error statistics

	magnitude (%)	orientation (in ^{o})
direct assimilation	0.279	2.128
pseudo-image approach	0.098	0.792

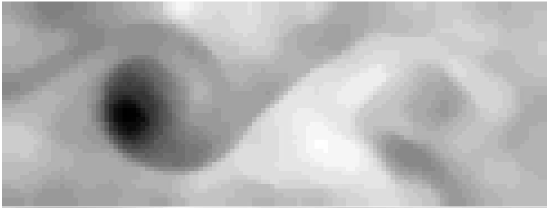
Comparison: direct *vs* pseudo-image approaches

Satellite sequence

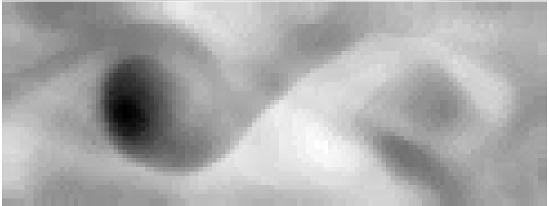
Observations



Direct assimilation



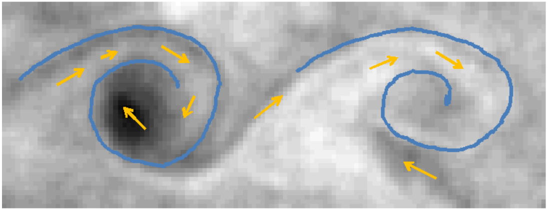
Pseudo-image approach



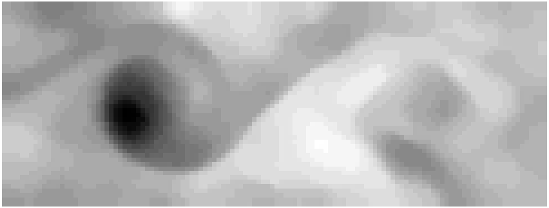
Comparison: direct *vs* pseudo-image approaches

Satellite sequence

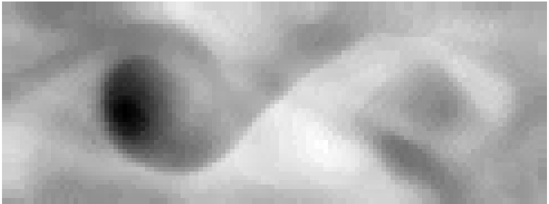
Observations



Direct assimilation



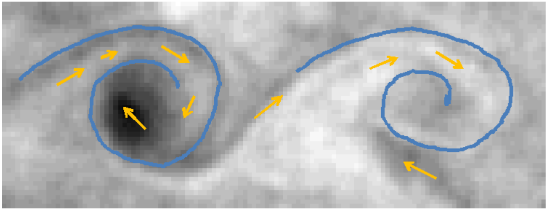
Pseudo-image approach



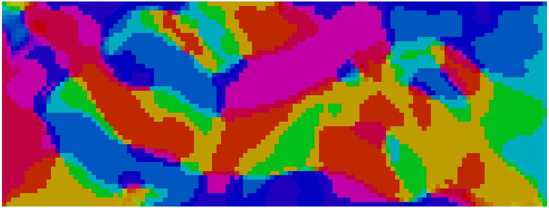
Comparison: direct *vs* pseudo-image approaches

Satellite sequence

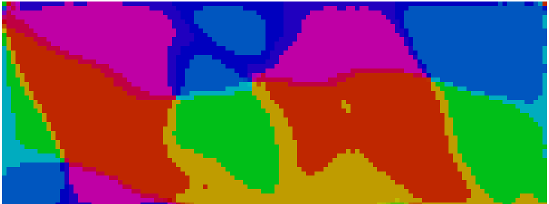
Observations



Direct assimilation



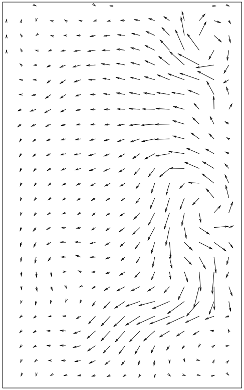
Pseudo-image approach



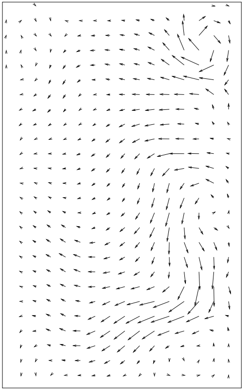
Comparison: pseudo-image *vs* combined approaches

Motion estimation

Observations



pseudo-observations approach



combined approach

Comparison: pseudo-image *vs* combined approaches

Motion estimation

Observations

pseudo-observations approach

combined approach

Comparison: pseudo-image *vs* combined approaches

Tracer

Observations

pseudo-observations approach

combined approach

Uncertainty on evolution laws

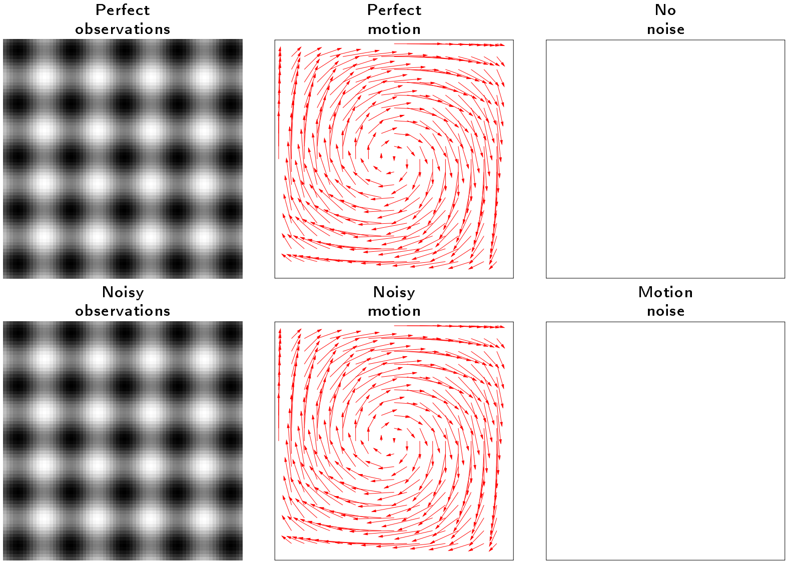
Imperfect model

Evolution equation $$\frac{\partial \mathbf{X}}{\partial t} + \mathbb{M}(\mathbf{X}) = \epsilon$$

New cost function

$$J_2(\mathbf{X}(0), \epsilon(t)) = \frac{1}{2}\int_A \frac{(\epsilon_R(\mathbf{x},t))^2}{R(\mathbf{x},t)} d\mathbf{x}dt + \frac{1}{2}\int_\Omega \frac{(\epsilon_B(\mathbf{x}))^2}{B(\mathbf{x})} d\mathbf{x}$$
$$\frac{1}{2}\int_A (\epsilon(\mathbf{x},t))^T \, Q^{-1}(\mathbf{x},t)\epsilon(\mathbf{x},t) d\mathbf{x}dt$$

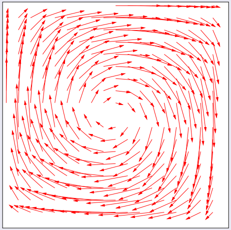
Synthetic experiment



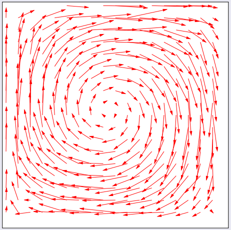
Motion estimation

Qualitative comparison

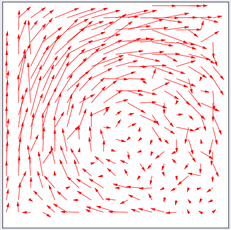
Ground truth



Result with Imperfect Model



Result with Perfect Model



Statistics

		$\lvert\theta_r - \theta_e\rvert$		$\lvert\lVert w_r\rVert - \lVert w_e\rVert\rvert / \lVert w_r\rVert$			
method	date	mean	stdev	min	mean	max	stdev
PM	$t = 0$	31.4	34.4	0.0002	0.49	7.7	0.44
IM	$t = 0$	6.0	12.1	0.00005	0.13	3.0	0.14

Conclusion

Uncertainties

- Uncertainties on data acquisition
- Uncertainties on evolution equations

Perspectives

- Which image information to assimilate? Structures.
- Where or when the dynamic model is wrong? Need of the weak 4D-Var formulation.

Aknowledgments

- E. Plotnikov and G. Korotaev (Marine Hydrophysical Institute of Sevastopol, Ukraine)
- Dominique Béréziat (Université Pierre et Marie Curie)
- Yann Lepoittevin (INRIA)

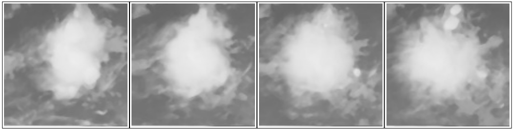
Pseudo-observation approach + object tracking

New observation operator

State vector	$\mathbf{X}$	$=$	$\big(\mathbf{w}(\mathbf{x},t)\ I_s(\mathbf{x},t)\ \phi(\mathbf{x},t)\big)$
Observation vector	$\mathbf{Y}$	$=$	$\big(I(\mathbf{x},t)\ D_e(\mathbf{x},t)\big)$
Evolution equation	$\frac{\partial \mathbf{w}}{\partial t} + (\mathbf{w}\cdot\nabla)\mathbf{w}$	$=$	0
	$\frac{\partial I_s}{\partial t} + \mathbf{w}\cdot\nabla I_s$	$=$	0
	$\frac{\partial \phi_s}{\partial t} + \mathbf{w}\cdot\nabla \phi$	$=$	0
Observation equation	$I_s - I$	$=$	ϵ_{o_I}
	$S_a(\phi)\big(D_e - \lvert\phi\rvert\big)$	$=$	ϵ_{o_ϕ}
Background equation	$\begin{pmatrix} I_s \\ \phi \end{pmatrix}$	$=$	$\begin{pmatrix} I_{t_1} \\ \phi_{t_1} \end{pmatrix} + \begin{pmatrix} \epsilon_{b_I} \\ \epsilon_{b_\phi} \end{pmatrix}$

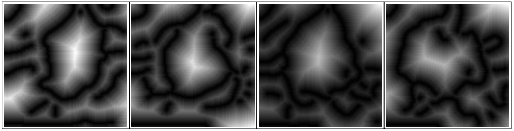
Observations

Satellite sequence



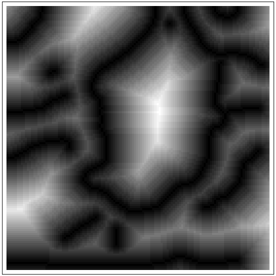
Edges

Distance map to the closer edge

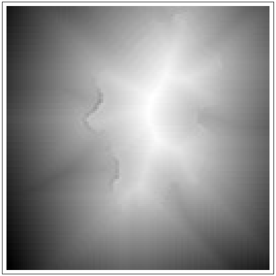


Result

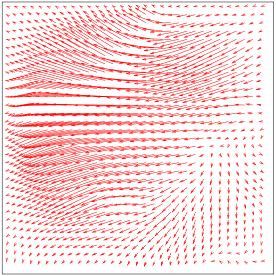
Satellite sequence



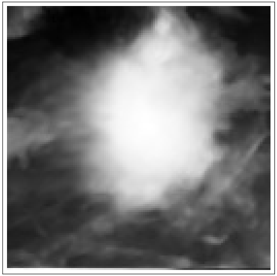
Distance map



ϕ

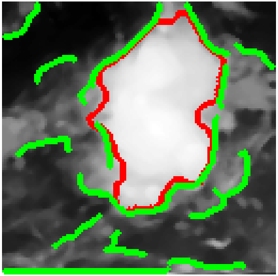


motion

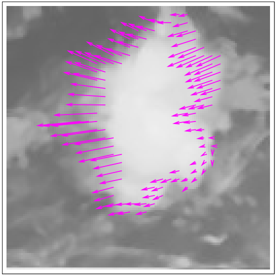


Pseudo-image

Result



Green: edges
Red: $\phi = 0$



Pseudo-image
Motion estimated on the border of the object