

fd_gauss_vasicek1d_swaption

Input parameters:

- Time StepNumber M

Output parameters:

- Price

The stochastic differential equation representing the short rate is given by

$$dr_t = k(\theta - r_t)dt + \sigma dW(t)$$

The price of the zero-coupon bond with maturity $S > T$ is solution of the following PDE

$$u_t + \frac{1}{2}\sigma^2 u_{rr} + [k(\theta - r)]u_r - ru = 0, u(r, S, S) = 1$$

that we solve using explicit scheme of Hull-White[1]. The price of the option is obtained solving the same PDE with boundary condition at the maturity of the option T , the price of the Zero Coupon Bond. The pricing procedure is in two steps: in the first step(INITPROBA) we compute probabilities associated to the explicit scheme; this is done simply matching the first and the second moment of the change in r over time step Δt . The branching in the lattice is modified at boundary points $r = r_{min}$ and $r = r_{max}$ to ensure that the probabilities associated with all three branches remain positive. For this purpose Hull-White[1] propose alternative branching procedures in the explicit finite difference method.

The second step is standard dynamic programming backward pricing algorithm. The price of the coupon bearing is obtained as linear combination of zero-coupon prices, taking in account properly of the coupon adjustment. A swaption can also be seen as an option of strike 1 over a certain coupon bearing.

References

- [1] J.Hull and A.WHITE. Valuing derivative securities using the explicit finite difference method. *Journal of Financial and Quantitative Analysis*, 25:87–100, 1990. [1](#)