

Smart expansion and fast calibration for jump diffusion

Ahmed KEBAIER

March 3, 2020

Premia 22

1 Model specification

The log asset price is solution to

$$X_t = x_0 + \int_0^t \mu(u, X_{u-}) du + \int_0^t \sigma(u, X_{u-}) dW_u + J_t \quad (1)$$

where W is a standard Brownian motion and J is a compund Poisson process independent of W , $\mu(t, x) = \lambda \left(1 - e^{\eta_j + \frac{\gamma_j^2}{2}} \right) - \frac{\sigma^2(t, x)}{2}$ and $\sigma(t, x) = \nu(t) e^{(\beta(t)-1)x}$. Here ν et β are deterministic picewise functions (For more details on the practical choice of ν and β see section 4.2 in [1]). The proxy model is nothing but the Merton model given by

$$X_t^M = x_0 + \int_0^t \mu(u, x_0) du + \int_0^t \sigma(u, x_0) dW_u + J_t$$

2 Second order approximation formula for pricing European call

We use the smart expansion introduced by Benhamou, Gobet and Miri [1] to give an analytical accurate approximation of a call European price, written as the expected value under the risk neutral probability measure of the payoff function $h(x) = e^{rT}(e^{(r-q)T}e^x - K)_+$ evaluated at the maturity time T :

$$Call Price = \mathbb{E}[h(X_T)]$$

where r and q denote respectively the interest rate and the dividend. More precisely, for r , q and ν constants, we use the second order approximation price formula given by Theorem 2.1 of [1], to write

$$Call Price \simeq \mathbb{E}h(X_T^M) + \sum_{i=1}^3 \alpha_{i,T} Greek_i^h(X_T^M) + \sum_{i=1}^3 \beta_{i,T} \overline{Greek}_i^h(X_T^M) \quad (2)$$

where $\mathbb{E}h(X_T^M)$ is nothing but the price of an European Call asociated to the Merton model (See the formula in Remark 2.2 in [1]). Explicit forms of the greeks in formula (2) are given in Proposition 4.1 of [1].

References

- [1] E.Benhamou, E. Gobet and M. Miri Smart expansion and fast calibration for jump diffusion. Finance and Stochastics 13, 4 (2009) 563-589 [1](#)