

## cf\_putin\_kunitomoiked

Let

- $T$  = maturity date ( $T > t$ )
- $K$  = strike price
- $L$  = lower barrier
- $U$  = upper barrier
- $x$  = spot price
- $t$  = pricing date
- $\sigma$  = volatility
- $r$  = interest rate
- $\delta$  = dividend yields
- $\theta = T - t$
- $b = r - \delta$

The exact value for double barrier call/put options is given by the Ikeda-Kunitomo formula [1], which allows to compute exactly the price when the boundaries suitably depend on the time variable  $t$ . More precisely, set

$$U(s) = Ue^{\delta_1 s} \quad L(s) = Le^{\delta_2 s}$$

where the constants  $U, L, \delta_1, \delta_2$  are such that  $L(s) < U(s)$ , for every  $s \in [t, T]$ . The functions  $U(s)$  and  $L(s)$  play the role of *upper* and *lower* barrier respectively.  $\delta_1$  and  $\delta_2$  determine the curvature and the case of  $\delta_1 = 0$  and  $\delta_2 = 0$  corresponds to two flat boundaries.

*In the software, we consider only flat boundaries.*

The numerical studies suggest that in most cases it suffices to calculate the leading five terms of the series giving the price of the knock-out and knock-in double barrier call options.

Let  $\tau$  stand for the first time at which the underlying asset price  $S$  reaches at least one barrier, i.e.

$$\tau = \inf\{s > t; S_s \leq L(s) \text{ or } S_s \geq U(s)\}.$$

We define the following coefficients:

- $\mu_1 = 2 \frac{b - \delta_2 - n(\delta_1 - \delta_2)}{\sigma^2} + 1$
- $\mu_2 = 2 \frac{n(\delta_1 - \delta_2)}{\sigma^2}$
- $\mu_3 = 2 \frac{b - \delta_2 + n(\delta_1 - \delta_2)}{\sigma^2} + 1$

## Knock-In Put Option

### References

- [1] N.KUNIMOTO N.IKEDA. Pricing options with curved boundaries. *Mathematical finance*, 2:275–298, 1992. [1](#)