

[Help](#)

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#include "
href../../common/math/mc_lmm_glassermanzhao_h_src.pdfmc_lmm_glassermanzhao.h"

/** "Arbitrage-Free Discretization Of Lognormal Forward Libor Model" by Glasserman
* We consider a tenor structure  $0=T_0 < T_1 < \dots < T_N < T_{N+1}$  equally spaced
* and Libor rates  $L(t, T_0), L(t, T_2), \dots, L(t, T_N)$  for a certain date  $t$ .  $L(\cdot, T_i)$ 
* Convention: for  $t > T_i$   $L(t, T_i) = L(T_i, T_i)$ 
* Simulation can be done with the function "Sim_Libor_Glasserman" under two measures
* @param start_index index of time from which simulation starts.  $T(\text{start\_index})$ 
* @param end_index index of last date of simulation.
* @param ptLOld Libor structure, contains initial value of libor rates
* @param ptVol Volatility structure contains libor volatility deterministic functions
* @param generator the index of the random generator to be used
* @param NbrMCsimulation the number of samples
* @param NbrStepPerTenor number of steps of discretization between  $T(i)$  and  $T(i+1)$ 
* @param save_all_paths flag to decide whether we store the simulated value of libor rates
* @param LiborPathsMatrix PnlMat contains libor simulated paths.
* @param save_brownian flag to decide to store brownian motion values.
* save_brownian==0. we don't store the value brownian motion used in the simulation.
* save_brownian==1. we store the value brownian motion used in the simulation, at each step.
* save_brownian==2. we also store intermediate steps, between  $T(i)$  and  $T(i+1)$ .
* @param BrownianMatrixPaths PnlMat contains brownian motion simulated paths.
* @param flag_numeraire measure under which simulation is done.
* flag_numeraire=0->Terminal measure, flag_numeraire=1->Spot measure
*/

void Sim_Libor_Glasserman(int start_index, int end_index, Libor *ptLOld, Volatility *ptVol,
{
    if (flag_numeraire == 0)
    {
        Sim_Libor_Glasserman_TerminalMeasure(start_index, end_index, ptLOld, ptVol, generator, NbrMCsimulation, NbrStepPerTenor, save_all_paths, LiborPathsMatrix, BrownianMatrixPaths, save_brownian);
    }
    else
    {
        Sim_Libor_Glasserman_SpotMeasure(start_index, end_index, ptLOld, ptVol, generator, NbrMCsimulation, NbrStepPerTenor, save_all_paths, LiborPathsMatrix, BrownianMatrixPaths, save_brownian);
    }
}

/** "Swaption_Payoff_Discounted" computes the swaption payoff discounted with re
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* @param ptL Libor structure, contains value of libor rates.
* @param ptSwpt Swaption structure contains swaption information.
* @param flag_numeraire flag decide the numeraire we discounte with
* flag_numeraire=0->Terminal measure numeraire, flag_numeraire=1->Spot measure n
*/
double Swaption_Payoff_Discounted(Libor *ptL, Swaption *ptSwpt, NumFunc_1 *p, in
{
    if (flag_numeraire == 0)
    {
        return Swaption_Payoff_TerminalMeasure(ptL, ptSwpt, p);
    }
    else
    {
        return Swaption_Payoff_SpotMeasure(ptL, ptSwpt, p);
    }
}

int Sim_Libor_Glasserman_TerminalMeasure(int start_index, int end_index, Libor *
{
    double tenor;
    double Di, tk, Ti, dt, sqrt_dt;
    int i, j, k, m, l, N, Nfac, Nsteps;

    PnlVect *Xvalue_0, *Xvalue_t;
    PnlVect *sigma;
    PnlVect *lambda1; // Volatility vector of the the process L(.,Ti,Ti+1) valued
    PnlVect *lambda2; // Volatility vector of the the process L(.,Ti+1,Ti+2) value
    PnlVect *Xprev;
    PnlVect *ptW, *ptW_sum;

    Nfac = ptVol->numberOfFactors;
    N = ptLOld->numberOfMaturities;
    tenor = ptLOld->tenor;
    dt = tenor / NbrStepPerTenor;
    sqrt_dt = sqrt(dt);

    Xvalue_0 = pnl_vect_create(N - 1);
    Xvalue_t = pnl_vect_create(N - 1);
    Xprev = pnl_vect_create(N - 1);
    ptW = pnl_vect_create(Nfac);

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ptW_sum = pnl_vect_create(Nfac);
lambda1 = pnl_vect_create(Nfac);
lambda2 = pnl_vect_create(Nfac);
sigma    = pnl_vect_create(Nfac);

Nsteps = end_index - start_index;

// vector Xvalue is a transformation of libor rates. cf Glasserman Zhao (2000)
// Initial value of the vector Xvalue
LET(Xvalue_0, N - 2) = GET(ptLOld->libor, N - 1); // X(0,T(N-2)) = Libor(0, T(
for (i = N - 3; i >= 0; i--)
{
    LET(Xvalue_0, i) = GET(Xvalue_0, i + 1) * GET(ptLOld->libor, i + 1) * (1 /
}

if (save_all_paths == 1)
{
    pnl_mat_resize(LiborPathsMatrix, 1 + Nsteps * NbrMCsimulation, N);
    pnl_mat_set_row(LiborPathsMatrix, ptLOld->libor, 0);

    if (save_brownian == 1)
    {
        pnl_mat_resize(BrownianMatrixPaths, Nsteps * NbrMCsimulation, Nfac);
    }

    else if (save_brownian == 2)
    {
        pnl_mat_resize(BrownianMatrixPaths, NbrStepPerTenor * Nsteps * NbrMCsi
    }
}
else
{
    pnl_mat_resize(LiborPathsMatrix, NbrMCsimulation, N);
    if (save_brownian == 1)
    {
        pnl_mat_resize(BrownianMatrixPaths, NbrMCsimulation, Nfac);
    }
}

for (m = 0; m < NbrMCsimulation; m++)
{

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pnl_vect_set_zero(ptW_sum);
pnl_vect_clone(Xprev, Xvalue_0);
tk = start_index * tenor;

for (l = 1; l <= Nsteps; l++) // Time Loop from T(start_index) to T(end_in
{
    for (k = 1 ; k <= NbrStepPerTenor; k++)
    {
        pnl_vect_rand_normal(ptW, Nfac, generator);
        pnl_vect_axpby(sqrt_dt, ptW, 1., ptW_sum); /* y := a x + b y */

        // save_brownian==2. We also save intermediate steps.
        if (save_all_paths == 1 && save_brownian == 2)
        {
            pnl_mat_set_row(BrownianMatrixPaths, ptW_sum, k - 1 + (l - 1)*

        }

        // Last component of the vector X
        Ti = (N - 1) * tenor;
        for (j = 0; j < Nfac; j++)
        {
            LET(lambda2, j) = evalVolatility(ptVol, j, tk, Ti); // sigma(t
            LET(sigma, j) = GET(lambda2, j);
        }
        // Discretization equation for the last component of the vector X:
        LET(Xvalue_t, N - 2) = GET(Xprev, N - 2) * exp(-0.5 * pnl_vect_sca

        // For the rest of the components of vector X.
        Di = 1;
        for (i = N - 3; i >= 0; i--)
        {
            Di += tenor * GET(Xprev, i + 1);
            Ti -= tenor ;

            for (j = 0; j < Nfac; j++)
            {
                LET(lambda1, j) = evalVolatility(ptVol, j, tk, Ti);
                LET(sigma, j) = GET(sigma, j) + GET(lambda1, j) - GET(lamb
                LET(lambda2, j) = GET(lambda1, j);
            }

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        // Discretization equation :
        LET(Xvalue_t, i) = GET(Xprev, i) * exp(-0.5 * pnl_vect_scalar_
    }

    pnl_vect_clone(Xprev, Xvalue_t);
    tk += dt;
}

// If save_all_paths=1, we store the simulated value of libors at each
// Transform the vector X into Libor values and store then in the matr

if (save_all_paths == 1)
{
    MLET(LiborPathsMatrix, l + m * Nsteps, N - 1) = GET(Xvalue_t, N -
    Di = 1;

    for (i = N - 2; i >= 1; i--)
    {
        Di += tenor * GET(Xvalue_t, i);
        MLET(LiborPathsMatrix, l + m * Nsteps, i) = GET(Xvalue_t, i -
    }
    for (i = 1 - 1; i >= 0; i--)
    {
        MLET(LiborPathsMatrix, l + m * Nsteps, i) = MGET(LiborPathsMat

    }

    // Store the value brownian motion used in the simulation.
    if (save_brownian == 1)
    {
        pnl_mat_set_row(BrownianMatrixPaths, ptW_sum, l - 1 + m * Nste
    }

}

}

// If save_all_paths=0, we store only the simulated value of libors at the
// Transform the vector X into Libor values and store then in the matrix L
if (save_all_paths == 0)
{
    MLET(LiborPathsMatrix, m, N - 1) = GET(Xvalue_t, N - 2);

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        Di = 1;
        for (i = N - 2; i > 0; i--)
        {
            Di += tenor * GET(Xvalue_t, i);
            MLET(LiborPathsMatrix, m, i) = GET(Xvalue_t, i - 1) / Di;
        }
        MLET(LiborPathsMatrix, m, 0) = GET(ptL0ld->libor, 0); // L(t, T(0), T(0))

        if (save_brownian == 1)
        {
            pnl_mat_set_row(BrownianMatrixPaths, ptW_sum, m);
        }
    }

    pnl_vect_free(&Xvalue_0);
    pnl_vect_free(&Xvalue_t);
    pnl_vect_free(&sigma);
    pnl_vect_free(&lambda1);
    pnl_vect_free(&lambda2);
    pnl_vect_free(&Xprev);
    pnl_vect_free(&ptW);
    pnl_vect_free(&ptW_sum);

    return (1);
}

double Swaption_Payoff_TerminalMeasure(Libor *ptL, Swaption *ptSwpt, NumFunc_1 *
{
    int j, alpha, beta, N;
    double SumDi, tenor, Di;
    double swaption_maturity, swap_maturity, swaption_strike, payoff;

    N = ptL->numberOfMaturities;
    tenor = ptL->tenor;

    swaption_maturity = ptSwpt->swaptionMaturity;
    swap_maturity = ptSwpt->swapMaturity;
    swaption_strike = ptSwpt->strike;

    alpha = pnl_iround(swaption_maturity / tenor); // T_alpha is the swaption matu

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    beta = pnl_iround(swap_maturity / tenor); // T_beta is the swap maturity

    p->Par[0].Val.V_DOUBLE = 0.0;

    // Di discounted bond.
    // D_beta
    Di = 1.;
    for (j = beta; j < N; j++)
    {
        Di *= (1 + tenor * GET(ptL->libor, j));
    }
    payoff = Di;
    SumDi = Di;

    // sum D_j for j from alpha+1 to beta
    for (j = beta - 1; j > alpha; j--)
    {
        Di *= (1 + tenor * GET(ptL->libor, j));
        SumDi += Di;
    }

    payoff += swaption_strike * tenor * SumDi;

    // D_alpha
    Di *= (1 + tenor * GET(ptL->libor, alpha));

    payoff -= Di;

    payoff = ((p->Compute)(p->Par, payoff)); // Payoff

    return payoff;
}

int Sim_Libor_Glasserman_SpotMeasure(int start_index, int end_index, Libor *ptL0
{
    double tenor, tk, Ti, dt, SumVi, Prod_i, sqrt_dt;
    int i, j, k, m, l, N, Nfac, Nsteps, eta;

    PnlVect *Vvalue_0, *Vvalue_t;
    PnlVect *sigma;

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PnlVect *lambda;
PnlVect *ci;
PnlVect *Vprev;
PnlVect *ptW, *ptW_sum;

Nfac = ptVol->numberOfFactors;
N = ptLOld->numberOfMaturities;
tenor = ptLOld->tenor;
dt = tenor / NbrStepPerTenor;
sqrt_dt = sqrt(dt);

Vvalue_0 = pnl_vect_create(N);
Vvalue_t = pnl_vect_create(N);
Vprev     = pnl_vect_create(N);
ptW       = pnl_vect_create(Nfac);
ptW_sum   = pnl_vect_create(Nfac);
lambda    = pnl_vect_create(Nfac);
ci        = pnl_vect_create(Nfac);
sigma     = pnl_vect_create(Nfac);

Nsteps = end_index - start_index;

// Initial value of the vector V
Prod_i = 1. / (1 + tenor * GET(ptLOld->libor, 1));
LET(Vvalue_0, 0) = tenor * GET(ptLOld->libor, 1) * Prod_i;

for (i = 1; i < N - 1; i++)
{
    Prod_i /= (1 + tenor * GET(ptLOld->libor, i + 1));
    LET(Vvalue_0, i) = tenor * GET(ptLOld->libor, i + 1) * Prod_i;
}
LET(Vvalue_0, N - 1) = Prod_i;

if (save_all_paths == 1)
{
    pnl_mat_resize(LiborPathsMatrix, 1 + Nsteps * NbrMCsimulation, N);
    pnl_mat_set_row(LiborPathsMatrix, ptLOld->libor, 0);

    if (save_brownian == 1)
    {
        pnl_mat_resize(BrownianMatrixPaths, Nsteps * NbrMCsimulation, Nfac);
    }
}

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    }

    else if (save_brownian == 2)
    {
        pnl_mat_resize(BrownianMatrixPaths, NbrStepPerTenor * Nsteps * NbrMCsi
    }
else
{
    pnl_mat_resize(LiborPathsMatrix, NbrMCsimulation, N);
    if (save_brownian == 1)
    {
        pnl_mat_resize(BrownianMatrixPaths, NbrMCsimulation, Nfac);
    }
}

for (m = 0; m < NbrMCsimulation; m++)
{
    pnl_vect_set_zero(ptW_sum);
    tk = start_index * tenor;
    pnl_vect_clone(Vprev, Vvalue_0);

    for (l = 1; l <= Nsteps; l++) // Time Loop from T(1) to T(N-1)
    {
        for (k = 1 ; k <= NbrStepPerTenor; k++)
        {
            SumVi = pnl_vect_sum(Vprev);
            eta = (int) ceil(tk / tenor);
            pnl_vect_rand_normal(ptW, Nfac, generator);
            pnl_vect_axpby(sqrt_dt, ptW, 1., ptW_sum); /* y := a x + b y */

            // save_brownian==2. We also save intermediate steps.
            if (save_all_paths == 1 && save_brownian == 2)
            {
                pnl_mat_set_row(BrownianMatrixPaths, ptW_sum, k - 1 + (l - 1)*

            // Fisrt component of the vector V
            Ti = tenor;
            for (j = 0; j < Nfac; j++)
            {

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        LET(lambda, j) = evalVolatility(ptVol, j, tk, Ti); // vol(tk,
        LET(sigma, j) = GET(lambda, j) * (SumVi - GET(Vprev, 0)) / Sum
    }
    // Discretization equation for the first component of the vector X
    LET(Vvalue_t, 0) = GET(Vprev, 0) * exp(-0.5 * pnl_vect_scalar_prod

pnl_vect_set_double(ci, 0.0);

/// For the rest of the components of vector V
for (i = 1; i < N; i++)
{
    Ti += tenor;

    if (i >= eta)
    {
        for (j = 0; j < Nfac; j++)
        {
            LET(ci, j) += GET(Vprev, i - 1) / SumVi * GET(lambda,
        }
    }

    SumVi = SumVi - GET(Vprev, i - 1);

    for (j = 0; j < Nfac; j++)
    {
        if (i == N - 1)
        {
            LET(lambda, j) = 0;
        }
        else
        {
            LET(lambda, j) = evalVolatility(ptVol, j, tk, Ti);
        }

        LET(sigma, j) = GET(lambda, j) * (SumVi - GET(Vprev, i)) /
    }

    // Discretization equation :
    LET(Vvalue_t, i) = GET(Vprev, i) * exp(-0.5 * pnl_vect_scalar_
}

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        pnl_vect_clone(Vprev, Vvalue_t);
        tk += dt;
    }

// If save_all_paths=1, we store the simulated value of libors at each
// Transform the vector V into Libor values and store then in the matrix L
if (save_all_paths == 1)
{
    SumVi = GET(Vvalue_t, N - 1);
    for (i = N - 1; i > 0; i--)
    {
        MLET(LiborPathsMatrix, l + m * Nsteps, i) = GET(Vvalue_t, i - 1);
        SumVi += GET(Vvalue_t, i - 1);
    }
    MLET(LiborPathsMatrix, l + m * Nsteps, 0) = GET(ptLOld->libor, 0);

    // Store the value brownian motion used in the simulation.
    if (save_brownian == 1)
    {
        pnl_mat_set_row(BrownianMatrixPaths, ptW_sum, l - 1 + m * Nsteps, 0);
    }
}

// If save_all_paths=0, we store only the simulated value of libors at the end
// Transform the vector V into Libor values and store then in the matrix L
if (save_all_paths == 0)
{
    SumVi = GET(Vvalue_t, N - 1);
    for (i = N - 1; i > 0; i--)
    {
        MLET(LiborPathsMatrix, m, i) = GET(Vvalue_t, i - 1) / (tenor * SumVi);
        SumVi += GET(Vvalue_t, i - 1);
    }
    MLET(LiborPathsMatrix, m, 0) = GET(ptLOld->libor, 0); // L(t, T(0), T(0))

    // Store the value brownian motion used in the simulation.
    if (save_brownian == 1)
    {

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        pnl_mat_set_row(BrownianMatrixPaths, ptW_sum, m);
    }
}

pnl_vect_free(&Vvalue_0);
pnl_vect_free(&Vvalue_t);
pnl_vect_free(&sigma);
pnl_vect_free(&lambda);
pnl_vect_free(&ci);
pnl_vect_free(&Vprev);
pnl_vect_free(&ptW);
pnl_vect_free(&ptW_sum);

return (1);
}

double Swaption_Payoff_SpotMeasure(Libor *ptL, Swaption *ptSwpt, NumFunc_1 *p)
{
    int j, alpha, beta;
    double SumDi, tenor, Di;
    double swaption_maturity, swap_maturity, swaption_strike, payoff;

    tenor = ptL->tenor;

    swaption_maturity = ptSwpt->swaptionMaturity;
    swap_maturity = ptSwpt->swapMaturity;
    swaption_strike = ptSwpt->strike;

    alpha = pnl_iround(swaption_maturity / tenor); // T_alpha is the swaption maturity
    beta = pnl_iround(swap_maturity / tenor); // T_beta is the swap maturity

    p->Par[0].Val.V_DOUBLE = 0.0;

    // D_alpha
    Di = 1;
    for (j = 0; j < alpha; j++)
    {
        Di /= (1 + tenor * GET(ptL->libor, j));
    }
    payoff = Di;
}

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// sum D_j for j from alpha+1 to beta
SumDi = 0;
for (j = alpha; j < beta; j++)
{
    Di /= (1 + tenor * GET(ptL->libor, j));
    SumDi += Di;
}

payoff = payoff - Di - swaption_strike * tenor * SumDi;

payoff = ((p->Compute)(p->Par, -payoff)); // Payoff

return payoff;
}

double european_swaption_ap_rebonato(double valuation_date, NumFunc_1 *p, Libor
{
    int i, j, k, l, e, alpha, beta, n, Nfac, Nstep_integration, payer_or_receiver;
    double t, swaption_maturity, swap_maturity, swaption_strike, tenor, swap_rate,
    double sum_discount_factor, black_volatility, d1, d2, price, zc;

    PnlVect *weight;

    swaption_maturity = ptSwpt->swaptionMaturity;
    swap_maturity = ptSwpt->swapMaturity;
    swaption_strike = ptSwpt->strike;
    tenor = ptSwpt->tenor;
    payer_or_receiver = ((p->Compute) == &Put);

    e = pnl_iround(valuation_date / tenor);
    alpha = pnl_iround(swaption_maturity / tenor); // index of swaption_maturity
    beta = pnl_iround(swap_maturity / tenor); // index of swap_maturity
    n = beta - alpha; // Nbr of payments dates
    Nfac = ptVol->numberOfFactors; // Nbr of factors in diffusion process

    weight = pnl_vect_create(n);

    LET(weight, 0) = 1.;

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for (i = alpha; i < beta ; i++)
{
    LET(weight, i - alpha) /= (1 + tenor * GET(ptLib->libor, i));
}

sum_discount_factor = pnl_vect_sum(weight);

pnl_vect_div_double(weight, sum_discount_factor); // Normilization of the weight

// swap_rate(0) = sum over i of weight(i)*LiborRate(0,Ti,Ti+1) , see Brigo&Mercurio
swap_rate = 0;
for (i = alpha; i < beta ; i++)
{
    swap_rate += GET(weight, i - alpha) * GET(ptLib->libor, i);
}

Nstep_integration = 40; // number of step used to compute the integral of volatility
dt = (swaption_maturity - valuation_date) / Nstep_integration; // step for the integration
vol_swaption = 0; // Black's volatility of the swaption

for (i = 0; i < n ; i++)
{
    Ti = swaption_maturity + i * tenor;
    for (j = 0; j < n ; j++)
    {
        Tj = swaption_maturity + j * tenor;
        somme_integrale = 0;
        for (k = 0; k < Nfac; k++) // computation of the integral of volatility
        {
            // We use the simple trapezoidal rule
            integrale = evalVolatility(ptVol, k, valuation_date, Ti) * evalVolatility(ptVol, k, Tj, Ti);
            integrale += evalVolatility(ptVol, k, swaption_maturity, Ti) * evalVolatility(ptVol, k, Tj, swaption_maturity);
            integrale *= 0.5;

            for (l = 1 ; l < Nstep_integration; l++)
            {
                t = valuation_date + l * dt;
                integrale += evalVolatility(ptVol, k, t, Ti) * evalVolatility(ptVol, k, t, Tj);
            }
            integrale *= dt;
        }
    }
}

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        somme_integrale += integrale;
    }

    vol_swaption += GET(weight, i) * GET(weight, j) * GET(ptLib->libor, al
    }
}

vol_swaption = vol_swaption / SQR(swap_rate) ;

black_volatility = sqrt(vol_swaption);

d1 = (log(swap_rate / swaption_strike)) / black_volatility + 0.5 * black_volat
d2 = d1 - black_volatility;

zc = 1;
for (i = e; i < alpha ; i++)
{
    zc /= (1 + tenor * GET(ptLib->libor, i));
}

sum_discount_factor = 0;
for (i = alpha; i < beta ; i++)
{
    zc /= (1 + tenor * GET(ptLib->libor, i));
    sum_discount_factor += tenor * zc;
}

if (payer_or_receiver == 1) // Case of Payer Swaption
{
    price = sum_discount_factor * (swap_rate * cdf_nor(d1) - swaption_strike *

else // if (payer_or_receiver==0) Case of Receiver Swaption
{
    price = sum_discount_factor * (swap_rate * (cdf_nor(d1) - 1) - swaption_st

}

pnl_vect_free(&weight);

return price;
}

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double Numeraire(int i, Libor *ptLib_current, int flag_numeraire)
{
    int j, N;
    double B_j, tenor;

    N = ptLib_current->numberOfMaturities;
    tenor = ptLib_current->tenor;

    B_j = 1.;

    if (flag_numeraire == 0)
    {
        for (j = i; j < N; j++)
        {
            B_j /= (1 + tenor * GET(ptLib_current->libor, j));
        }

        return B_j;
    }

    else
    {
        for (j = 0; j < i; j++)
        {
            B_j *= (1 + tenor * GET(ptLib_current->libor, j));
        }

        return B_j;
    }
}

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