

# Multilevel Monte Carlo for Pricing European Call Option in Scott's Stochastic Volatility Model

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## Premia 22

### 1 Model specification

We consider the following stochastic volatility model with a general form given by

$$\begin{cases} S_t &= s_0 + \int_0^t r S_u du + \int_0^t f(Y_u) S_u (\rho dW_u + \sqrt{1 - \rho^2} dB_t) \\ Y_t &= y_0 + \int_0^t b(Y_u) du + \int_0^t \sigma(Y_t) dW_t, \end{cases}$$

where  $(S_t)_{t \in [0, T]}$  is the asset price,  $r$  the instantaneous interest rate,  $(B_t)_{t \in [0, T]}$  and  $(W_t)_{t \in [0, T]}$  are independent standard one-dimensional Brownian motions,  $\rho \in [-1, 1]$  is the correlation between the Brownian motions respectively driving the asset price and the process  $(Y_t)_{t \in [0, T]}$ . The volatility process is  $(f(Y_t))_{t \in [0, T]}$  where the transformation function  $f$  is usually taken positive and strictly monotonic in order to ensure that the effective correlation between the stock price and the volatility keeps the same sign. In the particular case where  $f(y) = \sqrt{y}$ ,  $b(y) = \mu y$  and  $\sigma(y) = \xi Y$ , we recover the Hull & White model see [2]. In this work, we focus on the Scott's stochastic volatility model given by

$$\begin{cases} S_t &= s_0 + \int_0^t r S_u du + \int_0^t \sigma_0 e^{Y_u} S_u (\rho dW_u + \sqrt{1 - \rho^2} dB_t) \\ Y_t &= y_0 + \int_0^t \kappa(\theta - Y_u) du + \int_0^t \nu dW_t, \end{cases} \quad (1)$$

which corresponds to choose  $f(y) = \sigma_0 e^y$ ,  $b(y) = \kappa(\theta - y)$  and  $\sigma(y) = \nu$ .

### 2 European Call Price

Our aim, consists in implementing the paper by Jourdain & Sbair [?] in order to compute the European call price  $\mathbb{E}(e^{-rT}(S_T - K)_+)$ , where the stock price process  $(S_t)_{t \in [0, T]}$  follows the Scott's stochastic volatility model (1). A common way to compute this quantity is to use a Monte Carlo method given by

$$\frac{1}{M} \sum_{i=1}^M \text{BS}_T \left( s_0 e^{\rho(F(\bar{Y}_T^{N,i}) - F(y_0)) + \bar{m}_T^{N,i} + \left( \frac{(1-\rho^2)\bar{v}_T^{N,i}}{2T} - r \right) T}, \frac{(1-\rho^2)\bar{v}_T^{N,i}}{T} \right),$$

where  $\text{BS}_T(s, v)$  stands for the price of a European Call option with maturity  $T$  in the Black & Scholes model with initial stock price  $s$ , volatility  $v$  and constant interest rate  $r$ . The number  $M$  is the total number of Monte Carlo samples and the index  $i$  refers to independent draws. The quantities  $\bar{Y}_T^{N,i}$ ,  $\bar{m}_T^{N,i}$  and  $\bar{v}_T^{N,i}$  are respectively discretization schemes of the quantities

$$Y_T, \quad m_T = \int_0^T r - \frac{\sigma_0^2 e^{2Y_s}}{2} - \rho \sigma_0 e^{Y_s} \left( \frac{\kappa}{\nu} (\theta - Y_s) + \frac{\nu}{2} \right) dt \quad \text{and} \quad v_T = \int_0^T \sigma_0^2 e^{2Y_s} ds.$$

The function  $F$  is given by  $F(y) = \frac{\sigma_0 e^y}{\nu} - 1$ . In the Premia code we use the Euler scheme to approximate  $Y_T$  and Riemann schemes to approximate both  $m_T$  and  $v_T$ . In their paper, Jourdain & Sbair [?] use a Multilevel Monte Carlo method instead of the classical Monte Carlo one (described above) to compute the European Call price. The multi-level Monte Carlo method is introduced by Giles [1] as an extended method of the

statistical Romberg one of Kebaier [4]. When approximating the expected value of a function of a stochastic differential equation solution, these methods improve efficiently the computational complexity of a standard Monte Carlo algorithm.

## References

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