

cf_put_merton

Let

- T = maturity date ($T > t$)
- K = strike price
- x = spot price
- t = pricing date
- σ = volatility
- r = interest rate
- δ = dividend yields
- λ = jump intensity
- $\kappa = \mathbf{E}U - 1$ where $\mathbf{E}U$ is the expectation over the random jump variable U
- $\theta = T - t$

We assume [1] that the jump variable U is log-normally distributed with constant mean and variance. Specifically, let $\mathbf{E}[\ln U] = \mu$ and $\mathbf{E}[(\ln U)^2] = \mu^2 + v$, such that $\mathbf{E}U = e^{\mu + \frac{v}{2}}$.

Set:

$$d_1 = \frac{\log\left(\frac{x}{K}\right) + \left(r_n + \frac{\sigma_n^2}{2}\right)\theta}{\sigma_n\sqrt{\theta}} \quad d_2 = d_1 - \sigma_n\sqrt{\theta},$$

where

$$\sigma_n^2 = \sigma^2 + \frac{nv}{\theta} \quad r_n = r - \delta - \lambda\kappa + \frac{n}{\theta}\ln(1 + \kappa).$$

and N as the cumulative normal distribution function:

$$N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^d e^{-x^2/2} dx.$$

Put Option

PAYOFF	$P_T = (K - S_T)_+$
PRICE	$C(t, x; K) = \sum_{n=0}^{\infty} \frac{(\lambda(1+\kappa)\theta)^n}{n!} e^{-\lambda(1+\kappa)\theta - \delta\theta} [K e^{-r_n\theta} N(-d_2) - x N(-d_1)]$
DELTA	$\frac{\partial C(t, x; K)}{\partial x} = \sum_{n=0}^{\infty} \frac{(\lambda(1+\kappa)\theta)^n}{n!} e^{-\lambda(1+\kappa)\theta - \delta\theta} N(-d_1)$

References

- [1] R.C.MERTON. Option pricing when the underlying stocks returns are discontinuous. *Journ. Financ. Econ.*, 5:125–144, 1976. [1](#)