

ap_carr_mer

We assume that the underlying S_t evolves according to Merton's model:

$$S_t = S_0(e^{\sigma W_t + (\mu - \delta - \frac{\sigma^2}{2})t} \prod_{j=1}^{N_t} e^{m + \sqrt{v}g_j})$$

where

- W_t is a brownian motion.
- N_t is the counting function of an independent Poisson process with parameter λ .
- The $(g_j)_j$ are independent normal variables with mean 0 and variance 1, independent of both W and N .
- σ : volatility.
- μ : trend.
- δ : dividend rate.
- (m, v) : parameters of the jump law.

In other words, the spot evolves according to the Black-Scholes model between the jump times (τ_j) of the Poisson process, and jumps at times τ_j :

$$S_{\tau_j} = S_{\tau_j-} (e^{m + \sqrt{v}g_j})$$

We choose Merton's risk neutral probability measure in order to price the call option with maturity T and strike K . Therefore, we define the price of this option at time t by:

$$C_t = \mathbf{E}^*(e^{-r(T-t)}(S_T - K)_+ | \mathcal{F}_t)$$

where $\frac{d\mathbf{P}^*}{d\mathbf{P}} = e^{\int_0^T \frac{r - \lambda e^{\frac{m+\frac{v}{2}}{\sigma}} - \mu}{\sigma} dW_t - \frac{1}{2} \int_0^T (\frac{r - \lambda e^{\frac{m+\frac{v}{2}}{\sigma}} - \mu}{\sigma})^2 dt}$.

[1].

References

- [1] P. CARR D.B.MADAN. Option valuation using the fast fourier transform. *Journal of Computational Finance*, 2(2):61–73, 1998. [1](#)