

## tr\_extendedCRR

This is a nice improvement of the standard Cox-Ross-Rubinstein algorithm (cf [Routine tr\\_coxrossrubinstein\\_c](#)) which is described in [1]. The idea is to extend the tree 2 periods backward (ie exactly as if it started at the same underlying level  $S_0$  at time  $-2h$ ). Thus there is a node of the tree at level  $S_0$  at time 0 and also two neighbouring nodes at time 0 which allow a computation of the delta (and even the gamma) in a finite-difference manner with points at the same time step.

The price at the point  $(0, S_0)$  is exactly the same, of course, as in the CRR tree. Only the value of the delta is different.

Surprisingly in the article this way of doing is compared with the numerical-differentiation computation of the delta (ie you choose a space increment  $dx$  and shoot two different tree starting from  $S_0 - dx$  and  $S_0 + dx$ ) which obviously is a very poor numerical method since for a piecewise linear price function (like those for a Put, Call, CallSpread...) this gives a piecewise constant therefore discontinuous delta.

It should be rather compared to the usual CRR delta. It seems to give better results. The reason is not that clear. It seems that the bias introduced in the CRR method by working with samples of the price at the next (first) step is higher than the lack of accuracy in the method at hand which draws a finite-difference approximation between points  $S_0 * u^2$  and  $S_0 * d^2$ .

Let us stress that the CRR delta has an interesting superhedging property in the case of the selling of convex standard options (cf [here](#)) in case the time step is not taken to zero, which this one loses.

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/*Price, intrinsic Value arrays*/
The number of steps of the extended tree is extN=N+2.

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/*Up and Down factors*/
Here  $u = e^{\sigma\sqrt{h}}$ ,  $d = e^{-\sigma\sqrt{h}}$ . These are the usual CRR  $u$  and  $d$ .

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/\*Risk-Neutral Probability\*/

Computation of  $p_{\star} = \frac{e^{(r-\delta)h}-d}{u-d}$  (the value computed from.

/\*Intrinsic Value initialization\*/

Storage of the  $2extN + 1$  possible values of the intrinsic value since this is a flat tree. Notice that this is exactly as in a CRR tree starting two periods before.

/\*Terminal Values\*/

The price of the option at maturity. It involves only the values  $iv[2*j]$ .

/\*Backward Resolution\*/

Note that we don't re-compute the intrinsic value. Of course we stop at time step 2 (with step time 0 at the time origin  $-2h$  of the extended tree) since this is today's time.

/\*Delta\*/

This is the natural finite-difference approximation for the delta. It yields better results than the CRR delta.

/\*Price\*/

This is exactly the CRR price.

/\*Desallocation\*/

## References

- [1] A.PELSSER-T.VORST. The binomial model and the greeks. *The Journal Of Derivatives*, Spring:45–49, 1994. [1](#)