Reconstruction of Punctured Convolutional Codes

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Problem:

- Starting from an intercepted noisy bitstream, one wants to recover the original message.
- The intercepter wants to decode as efficiently (regarding time and noise level) as the legitimate receiver.
- For punctured convolutional codes, this requires to recover the puncturing pattern.

Parent matrix

$$\left(1+D+D^3+D^4,\ 1+D+D^4\right) \xleftrightarrow{\text{equivalent coders}} \begin{pmatrix} 1+D^2 & 1+D^2 & 1+D & 1 \\ D+D^2 & D & 1+D^2 & 1+D^2 \end{pmatrix}$$

Grouped matrix

Puncturing is equivalent to grouping the matrix and removing some columns

$$\begin{pmatrix} 1+D^2 & 1+D^2 & 1+D & 1 \\ D+D^2 & D & 1+D^2 & 1+D^2 \end{pmatrix}$$

Case study:

source 0001111101100010010111000101010001111000111001011

convolutional coding ↓ $(1+D+D^3+D^4, 1+D+D^4)$

equivalent coding

coded 0000001100001001101001001001001100010011101101010100111101000111100000001010010100000001011000111011

puncturing ↓ repeated "1101" pattern

punctured 000001000101100010101001001001101011010111010011100000100011000001010001101

noise ↓ BSC with 0.05 crossover probability

intercepted 001001000100100010101001001001101011010111010001100000100011000011010001101

Step 1: find dualwords h verifying

$$\begin{pmatrix} 00100100010010001 0 \\ 101001001001101011 \\ 010111010001100000 \\ 100011000011010001 \end{pmatrix} \times h \approx \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

101

Using Valembois' algorithm we get:

111000011110110000
000111000011110110 →
111111011101000110

$$\left(D^5+D^2+D,\ D^5+D^3+D^2+D,\ D^5+D^3\right)$$
$$\left(D^4+D+1,\ D^4+D^2+D+1,\ D^4+D^2\right)$$
$$\left(D^5+D^4+D^2+1,\ D^5+D^4+D^3+1,\ D^5+D^4+D^3+D^2\right)$$

Step 2: find an equivalent convolutional coder...

$$\begin{pmatrix} 1+D^2+D^3+D^5 & 1+D^2+D^3+D^5 & 1+D^3 \\ 1+D^5 & 1+D^4+D^5 & 1+D^3+D^4 \end{pmatrix}$$

...and compute it's canonical form:

$$\begin{pmatrix} 1+D^2 & 1+D^2 & 1 \\ 1+D & 1+D+D^2 & D^2 \end{pmatrix}$$

Constraint length: 4, Decoding complexity: 2^6

This is where previous techniques stop. This is enough to decode, but not with optimal complexity.

Step 3: for all possible puncturing patterns, find all pairs (P,M) such that:

$$P \times \begin{pmatrix} 1+D^2 & 1+D^2 & 1 \\ 1+D & 1+D+D^2 & D^2 \end{pmatrix} = \begin{pmatrix} ZM & M \end{pmatrix} \quad \text{with} \quad Z = \begin{pmatrix} 0 & 1 \\ D & 0 \end{pmatrix}$$

Each matrix M corresponds to a possible parent code. We use a reduction algorithm to select the smallest (lowest degree).

Pattern: 0111
Parent code: $(D+D^3+D^4+D^8+D^9,\ 1+D^2+D^4+D^8)$
Constraint length: 9, Decoding complexity: 2^{10}

Pattern: 1101
Parent code: $(1+D+D^3+D^4,\ 1+D+D^4)$
Constraint length: 4, Decoding complexity: 2^5

Pattern: 1011
Parent code: $(D^4+D^8,\ 1+D+D^2+D^3+D^4+D^8+D^9)$
Constraint length: 9, Decoding complexity: 2^{10}

Pattern: 1110
Parent code: $(1+D^2+D^4+D^8,\ 1+D^2+D^3+D^7+D^8)$
Constraint length: 8, Decoding complexity: 2^9