Private Stream Search at Almost the Same Communication Cost as a Regular Search

Matthieu Finiasz and Kannan Ramchandran

- A stream search consists in filtering data according to a set of keywords:
  - the data is a stream (it could also be a database)
    - → every piece of data is treated independently
  - the filtering is done externally
    - → you retrieve only the matching data.
- A typical scenario is Google Alerts:
  - get an alert for each new page matching your interests.
- Private Stream Search does this without revealing the keywords (your interests) to the filtering server.

- Protect your privacy:
  - Google Alerts,
  - web search in general...
- Protect your financial interests:
  - when searching for patents,
    → reveals what your company is focusing on.
- Global surveillance systems:
  - search for keywords in emails.

- But PSS is only worth it if it is efficient:
  - no one is ready to lose efficiency for privacy...

- To preserve privacy, the user sends a masked query:
  - a public list of possible keywords is needed,
  - the query is an encrypted selection of keywords.
- The server filters according to the encrypted query:
  - all documents/all keywords are treated symmetrically,
  - it accumulates matches in an encrypted data buffer,
  - only the user can extract the matches.
- PSS requires computations on encrypted data:
  - possible using (simple) homomorphic encryption,

    → here we use Paillier's cryptosystem.

The First PSS Scheme
[Ostrovsky-Skeith 2005]

- Requirements for this scheme:
  - a public dictionary of keywords $\Omega = \{k_1, ..., k_{|\Omega|}\}$,
  - the users asks OR queries on words of Ω,
  - a database/stream of t documents $(f_1, ..., f_t)$,
  - the users has an estimate of the number m of matches.
- We consider an example with:

$\Omega =$ dog brown cat black bird white

$f_1 =$ "the dog is black" $f_3 =$ "the bird is white"

$f_2 =$ "the cat is white" $f_4 =$ "the bird is black"

The First PSS Scheme
Query Construction

Ω = dog brown cat black bird white

↓ ↓ ↓ ↓ ↓ ↓

$Q = \mathcal{E}(0)\ \mathcal{E}(0)\ \mathcal{E}(1)\ \mathcal{E}(0)\ \mathcal{E}(0)\ \mathcal{E}(1)$

USER

- The user wants to query "cat OR white",
  - he computes a tuple Q of $\mathcal{E}(0)$ and $\mathcal{E}(1)$ accordingly.

$$\Omega = \text{dog} \quad \text{brown} \quad \text{cat} \quad \text{black} \quad \text{bird} \quad \text{white}$$

$$Q = \mathcal{E}(0) \quad \mathcal{E}(0) \quad \mathcal{E}(1) \quad \mathcal{E}(0) \quad \mathcal{E}(0) \quad \mathcal{E}(1)$$

SERVER

buffer B

- The server prepares a response buffer B,
  - the matches will be accumulated in B.

The First PSS Scheme

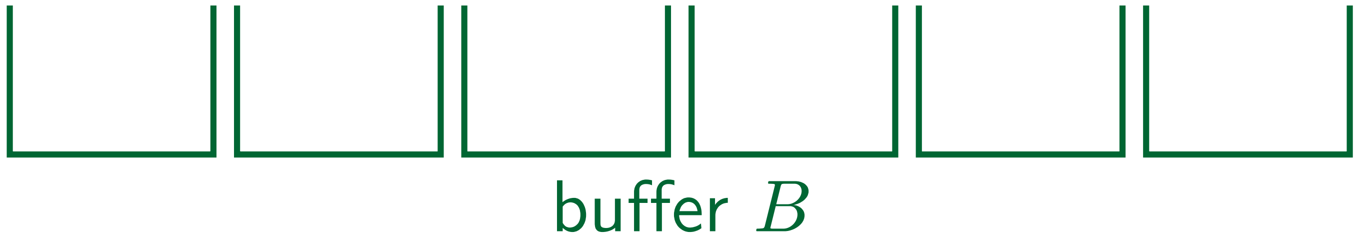
Query Execution

Ω = dog brown cat black bird white

$Q = \mathcal{E}(0)\ \mathcal{E}(0)\ \mathcal{E}(1)\ \mathcal{E}(0)\ \mathcal{E}(0)\ \mathcal{E}(1)$

f_1 = "the dog is black" $\mathcal{E}(0+0)$

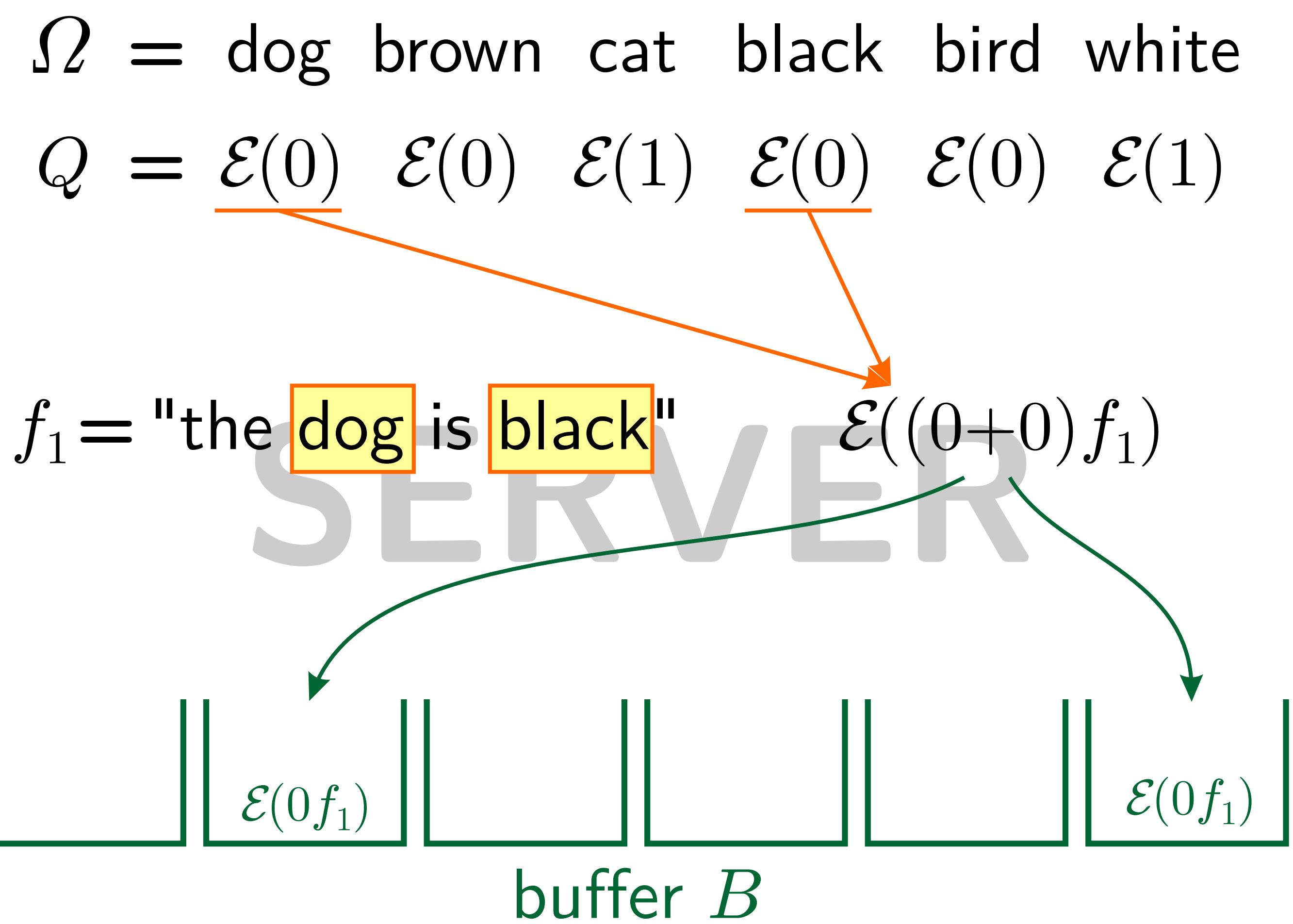
SERVER



- For every document f_i, the server computes:

$$\mathcal{E}(c_i) = \prod_{k_j \in f_i} q_j .$$

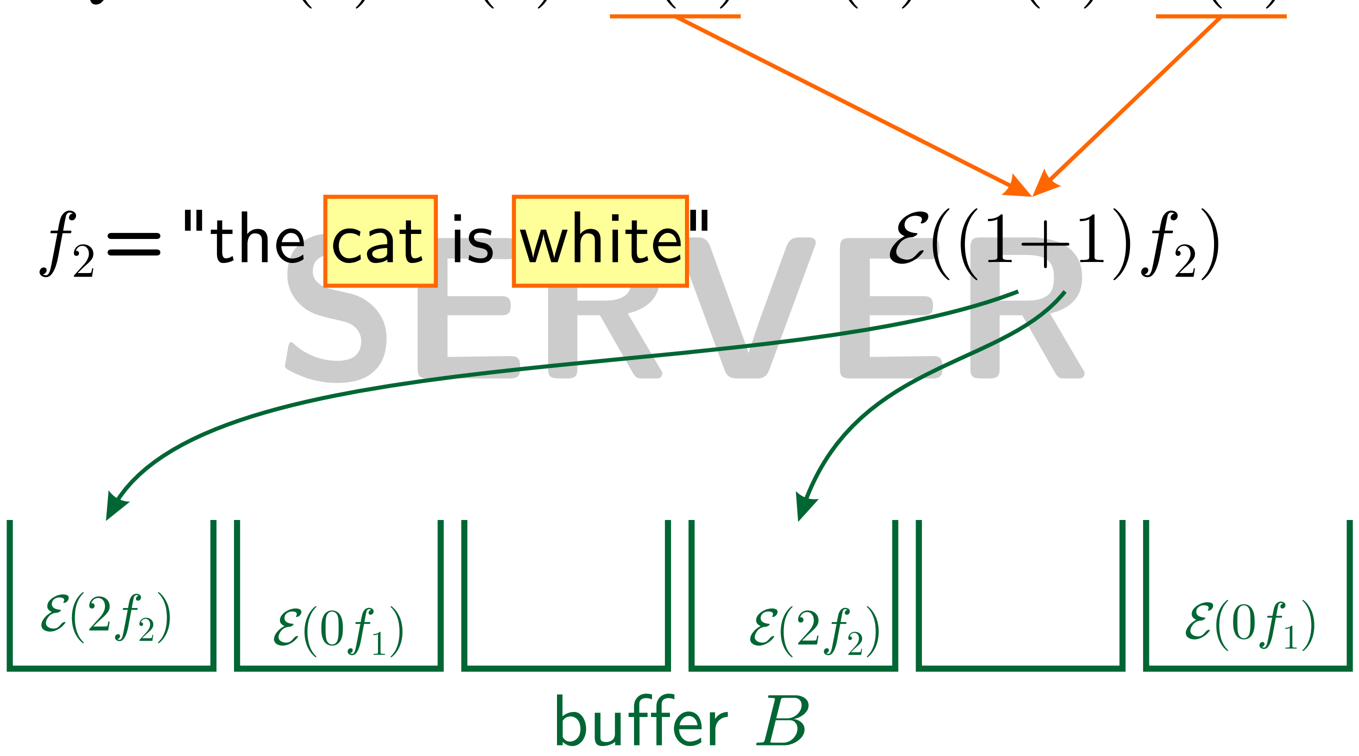
→ c_i is the number of matching keywords in f_i.



- For every document f_i:
  - the server "adds" $\mathcal{E}(c_i)^{f_i} = \mathcal{E}(c_i f_i)$ randomly in B.

$$\Omega = \text{dog} \quad \text{brown} \quad \text{cat} \quad \text{black} \quad \text{bird} \quad \text{white}$$

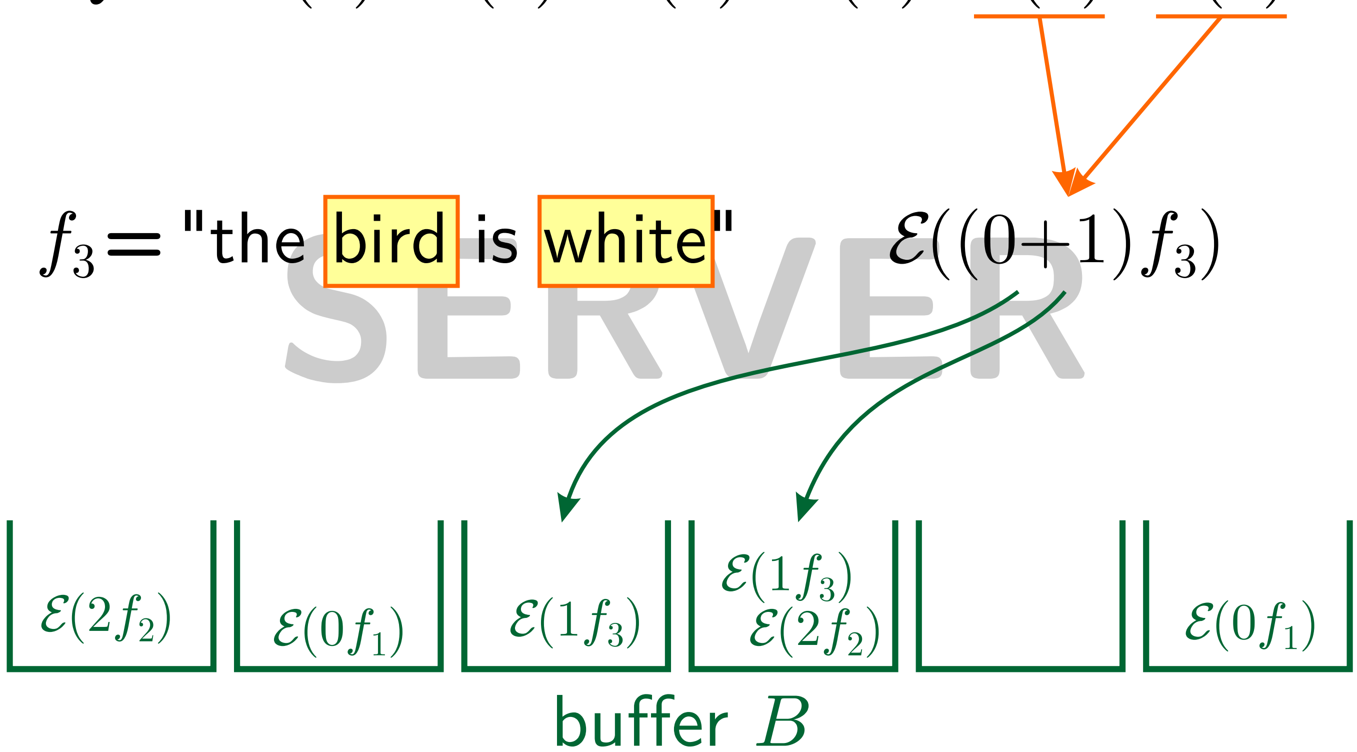
$$Q = \mathcal{E}(0) \quad \mathcal{E}(0) \quad \mathcal{E}(1) \quad \mathcal{E}(0) \quad \mathcal{E}(0) \quad \mathcal{E}(1)$$



- The server repeats this for all documents.

$$\Omega = \text{dog} \quad \text{brown} \quad \text{cat} \quad \text{black} \quad \text{bird} \quad \text{white}$$

$$Q = \mathcal{E}(0) \quad \mathcal{E}(0) \quad \mathcal{E}(1) \quad \mathcal{E}(0) \quad \mathcal{E}(0) \quad \mathcal{E}(1)$$



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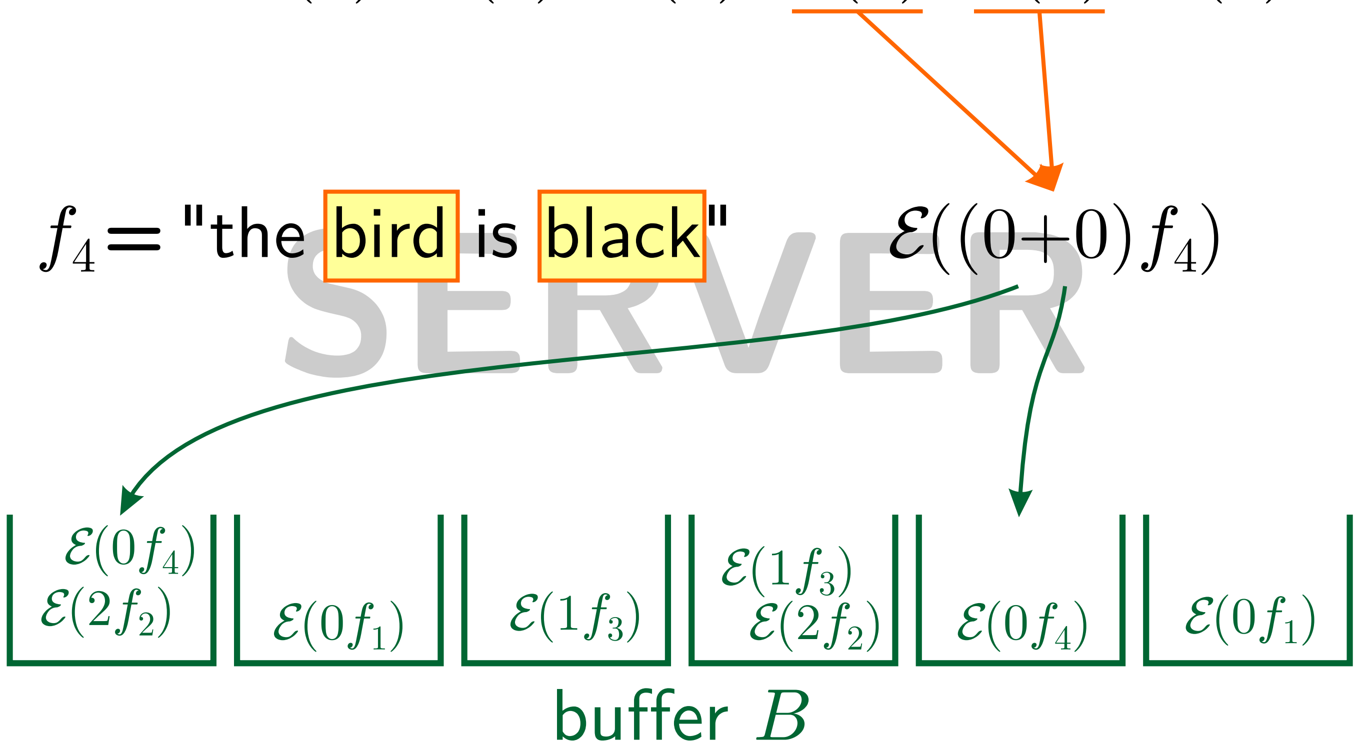

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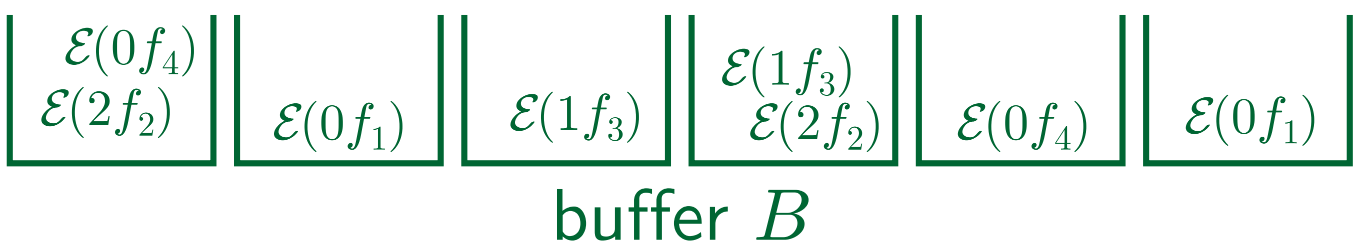
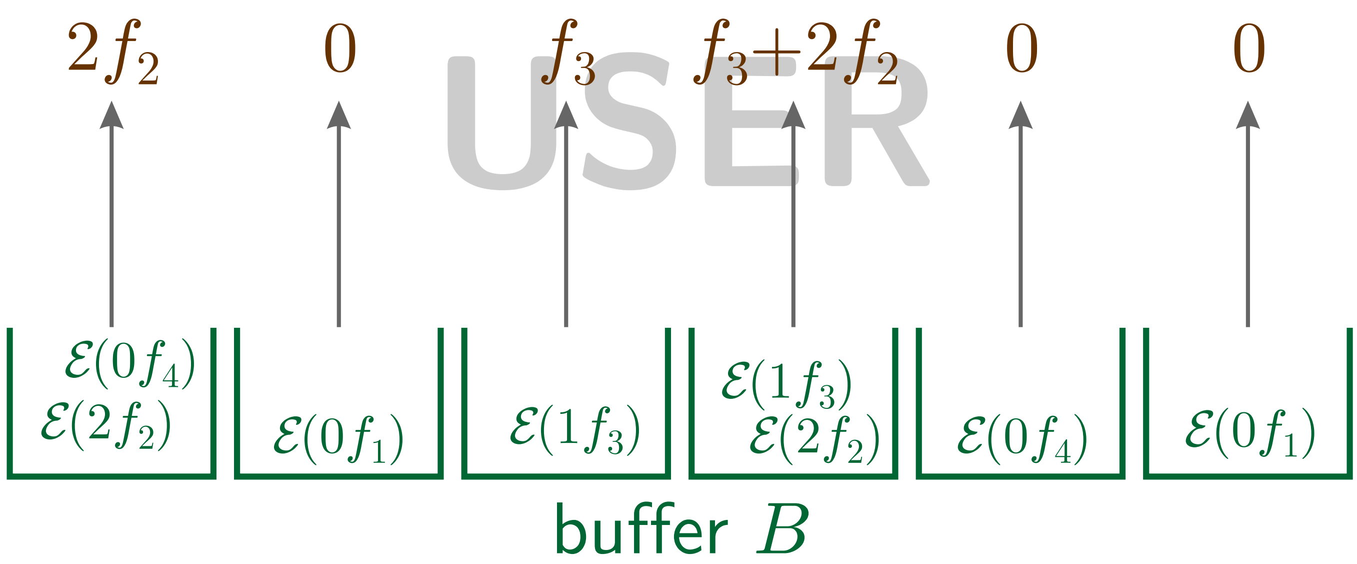
USER



- The user receives the encrypted buffer B.

Ω = dog brown cat black bird white

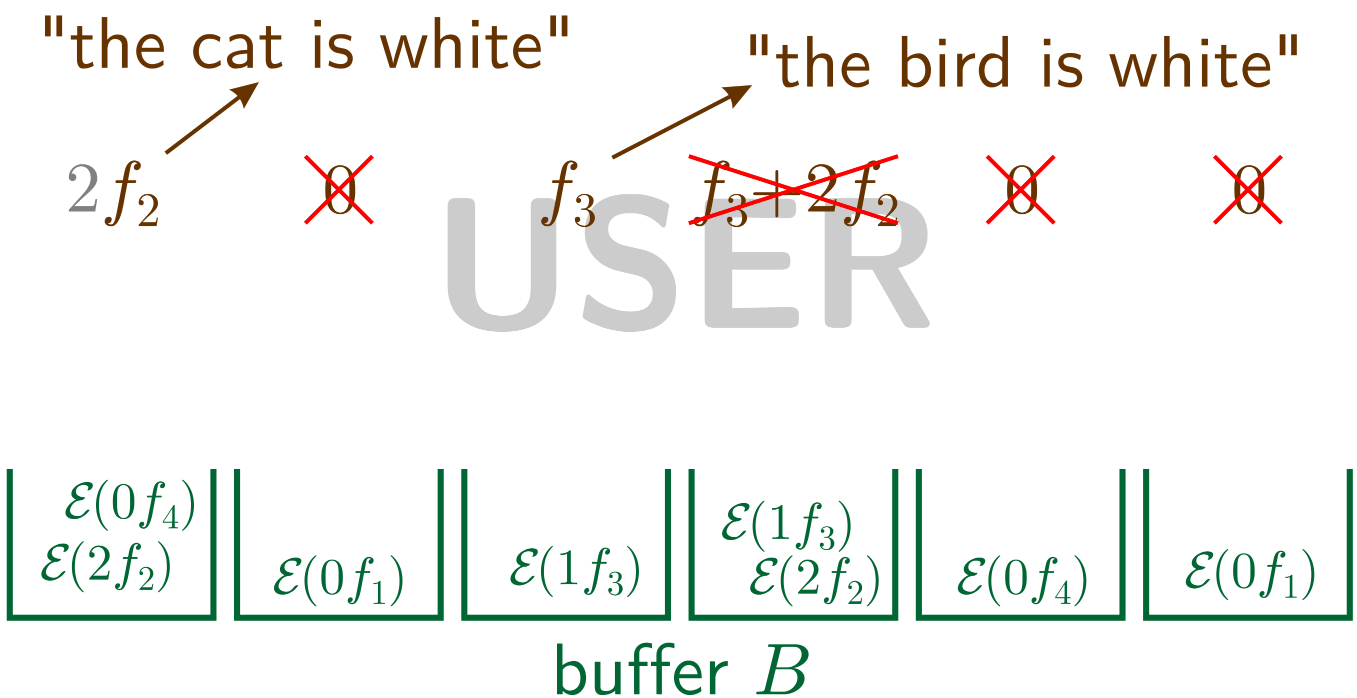
$Q = \mathcal{E}(0)\ \ \mathcal{E}(0)\ \ \mathcal{E}(1)\ \ \mathcal{E}(0)\ \ \mathcal{E}(0)\ \ \mathcal{E}(1)$



- The user receives the encrypted buffer B,
  - he decrypts it.

$$\Omega = \text{dog} \quad \text{brown} \quad \text{cat} \quad \text{black} \quad \text{bird} \quad \text{white}$$

$$Q = \mathcal{E}(0) \quad \mathcal{E}(0) \quad \mathcal{E}(1) \quad \mathcal{E}(0) \quad \mathcal{E}(0) \quad \mathcal{E}(1)$$



- The user receives the encrypted buffer B,
  - he gets one document for each singleton.

What can be improved in this scheme?

- Computations:
  - PSS requires one operation for each message,
  - difficult to improve,
    - → requires more efficient homomorphic encryption.
- Communications:
  - the query is linear in the dictionary size,
    - → fully homomorphic encryption could help,
  - the reply is linear in the buffer size,
    - → the buffer size should be the number of matches.
- In the Ostrovsky-Skeith scheme, the size is $O(m \log m)$,
  - Bethencourt *et al.* and Danezis-Díaz improve this.

Our Contribution

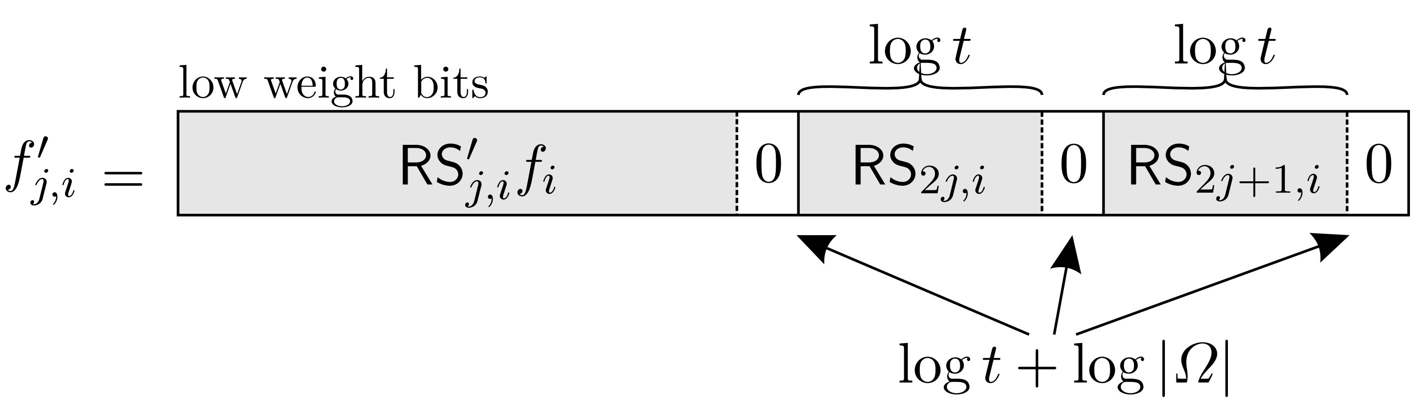
- Take an information theory look at the problem:
  - the server computes $\mathcal{E}(c_i f_i)$ an encrypted sparse vector
    → the problem is to compress it,
  - possible to compute it's syndrome for any linear code
    → compatible with homomorphic encryption.

- We propose two different approaches:
  - Using Reed-Solomon codes,
    → allows a "zero-error" guarantee (if m is known).
  - Using irregular LDPC codes,
    → gives optimal asymptotic performances.

Using Reed-Solomon Codes

- The straightforward solution uses:
  - a buffer B of size $2m$ for m matching documents,
  - each $\mathcal{E}(c_i f_i)$ is multiplied by a Reed-Solomon parity check matrix column and added to B.
- The code length (database size) is much smaller than the error space (symbol size),

  → possible to combine erasure and error correction.



Using Reed-Solomon Codes

- This solution gives:
  - a buffer of size m,
    - ⟶ with some loss in each symbol,
  - a zero-error guarantee,
    - ⟶ if the number of matches is known in advance.

- It has two main drawbacks:
  - it is computationally (very) heavy on the server side,
    - ⟶ each document requires m exponentiations,
  - the reply size still depends on the database size,
    - ⟶ we get the documents and their position.

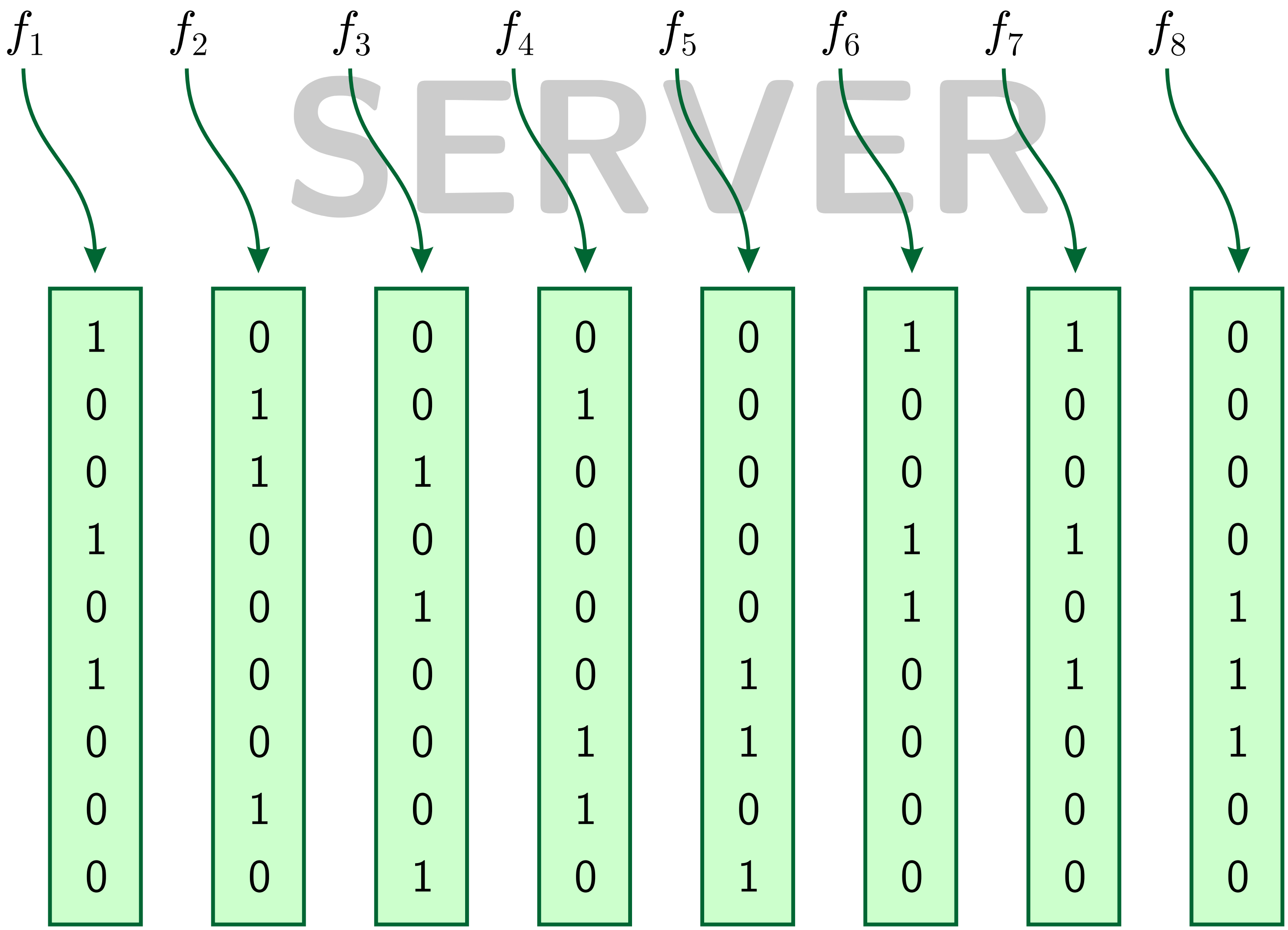
Using LDPC codes

- To obtain an optimal reply size:
  - the user should only get the documents,
  - changing their order should not change the syndrome.

- Each document defines its own parity check column:
  - use the document as a seed to a PRNG,
  - use the PRNG to generate a “random” LDPC column.

⚠ This can’t be done in a standard communication,
→ we define the code from the values of the error.

PSS with LDPC codes



- Use a PRNG to generate LDPC columns.

$$Q = \mathcal{E}(0) \quad \mathcal{E}(0) \quad \mathcal{E}(1) \quad \mathcal{E}(0) \quad \mathcal{E}(0) \quad \mathcal{E}(1)$$

SERVER

$\mathcal{E}(0f_1)$	$\mathcal{E}(1f_2)$	$\mathcal{E}(1f_3)$	$\mathcal{E}(0f_4)$	$\mathcal{E}(1f_5)$	$\mathcal{E}(0f_6)$	$\mathcal{E}(0f_7)$	$\mathcal{E}(1f_8)$
1	0	0	0	0	1	1	0
0	1	0	1	0	0	0	0
0	1	1	0	0	0	0	0
1	0	0	0	0	1	1	0
0	0	1	0	0	1	0	1
1	0	0	0	1	0	1	1
0	0	0	1	1	0	0	1
0	1	0	1	0	0	0	0
0	0	1	0	1	0	0	0

- Compute the encrypted sparse vector $\mathcal{E}(c_i f_i)$ as before.

$$Q = \mathcal{E}(0) \quad \mathcal{E}(0) \quad \mathcal{E}(1) \quad \mathcal{E}(0) \quad \mathcal{E}(0) \quad \mathcal{E}(1)$$

SERVER

$\mathcal{E}(0f_1)$ $\mathcal{E}(1f_2)$ $\mathcal{E}(1f_3)$ $\mathcal{E}(0f_4)$ $\mathcal{E}(1f_5)$ $\mathcal{E}(0f_6)$ $\mathcal{E}(0f_7)$ $\mathcal{E}(1f_8)$

$\times$

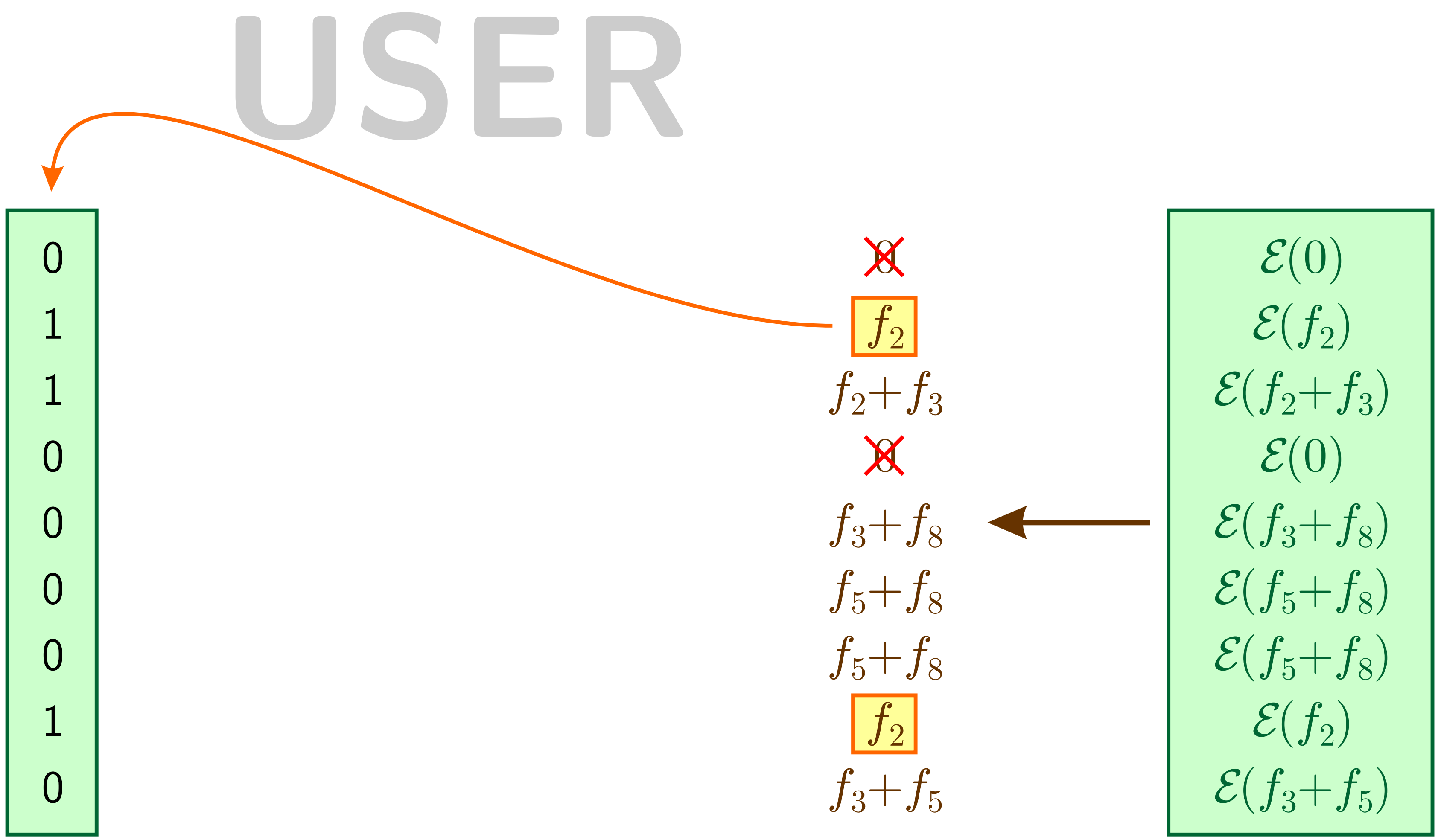
1	0	0	0	0	1	1	0		$\mathcal{E}(0)$
0	1	0	1	0	0	0	0		$\mathcal{E}(f_2)$
0	1	1	0	0	0	0	0		$\mathcal{E}(f_2+f_3)$
1	0	0	0	0	1	1	0		$\mathcal{E}(0)$
0	0	1	0	0	1	0	1	=	$\mathcal{E}(f_3+f_8)$
1	0	0	0	1	0	1	1		$\mathcal{E}(f_5+f_8)$
0	0	0	1	1	0	0	1		$\mathcal{E}(f_5+f_8)$
0	1	0	1	0	0	0	0		$\mathcal{E}(f_2)$
0	0	1	0	1	0	0	0		$\mathcal{E}(f_3+f_5)$

- Compute it's syndrome and send it to the user.

USER

0	$\mathcal{E}(0)$
f_2	$\mathcal{E}(f_2)$
f_2+f_3	$\mathcal{E}(f_2+f_3)$
0	$\mathcal{E}(0)$
f_3+f_8 ←	$\mathcal{E}(f_3+f_8)$
f_5+f_8	$\mathcal{E}(f_5+f_8)$
f_5+f_8	$\mathcal{E}(f_5+f_8)$
f_2	$\mathcal{E}(f_2)$
f_3+f_5	$\mathcal{E}(f_3+f_5)$

- The user first decrypts the buffer.



- For each singleton, he can generate its column.

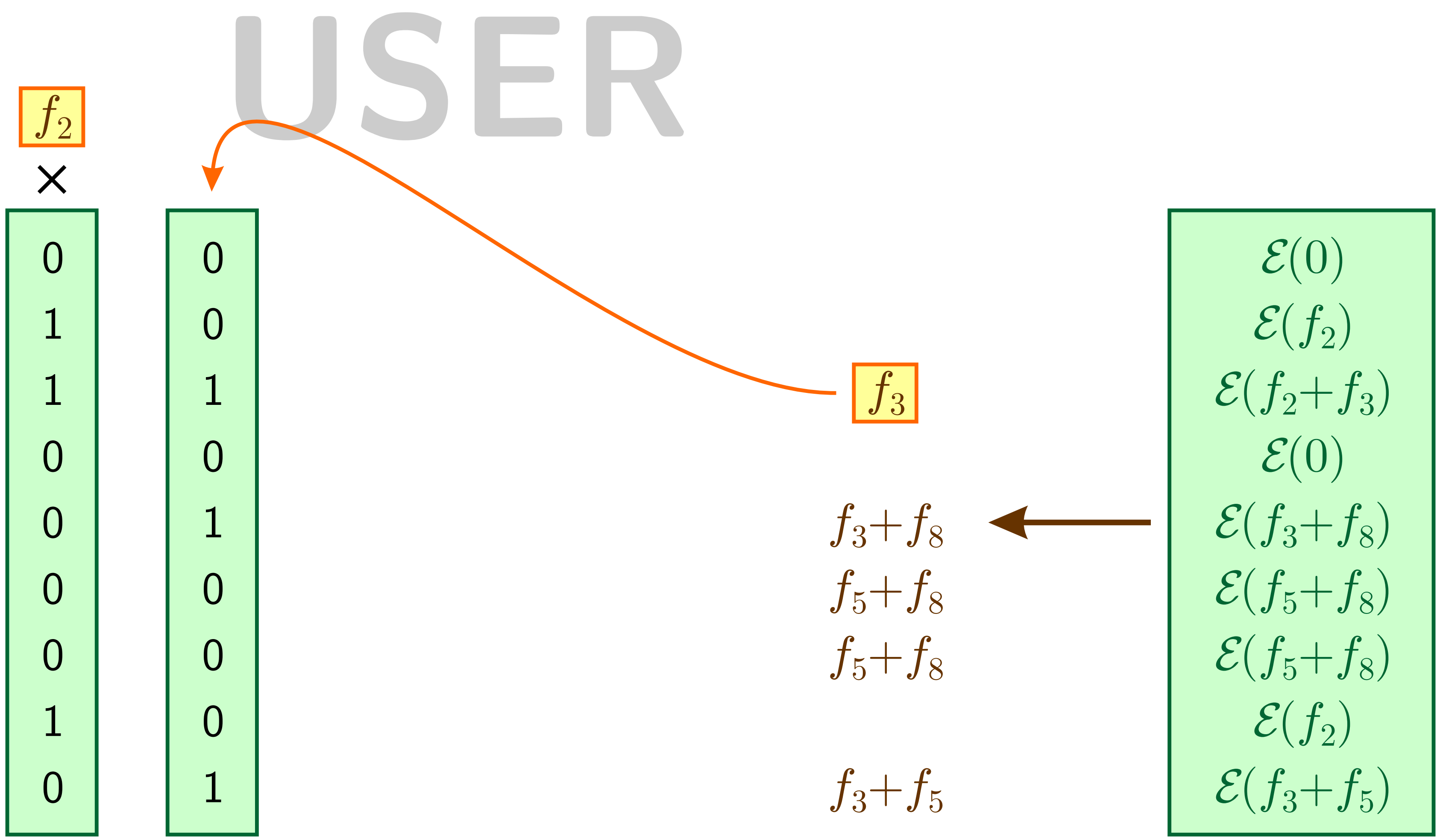
USER

f_2

$\times$

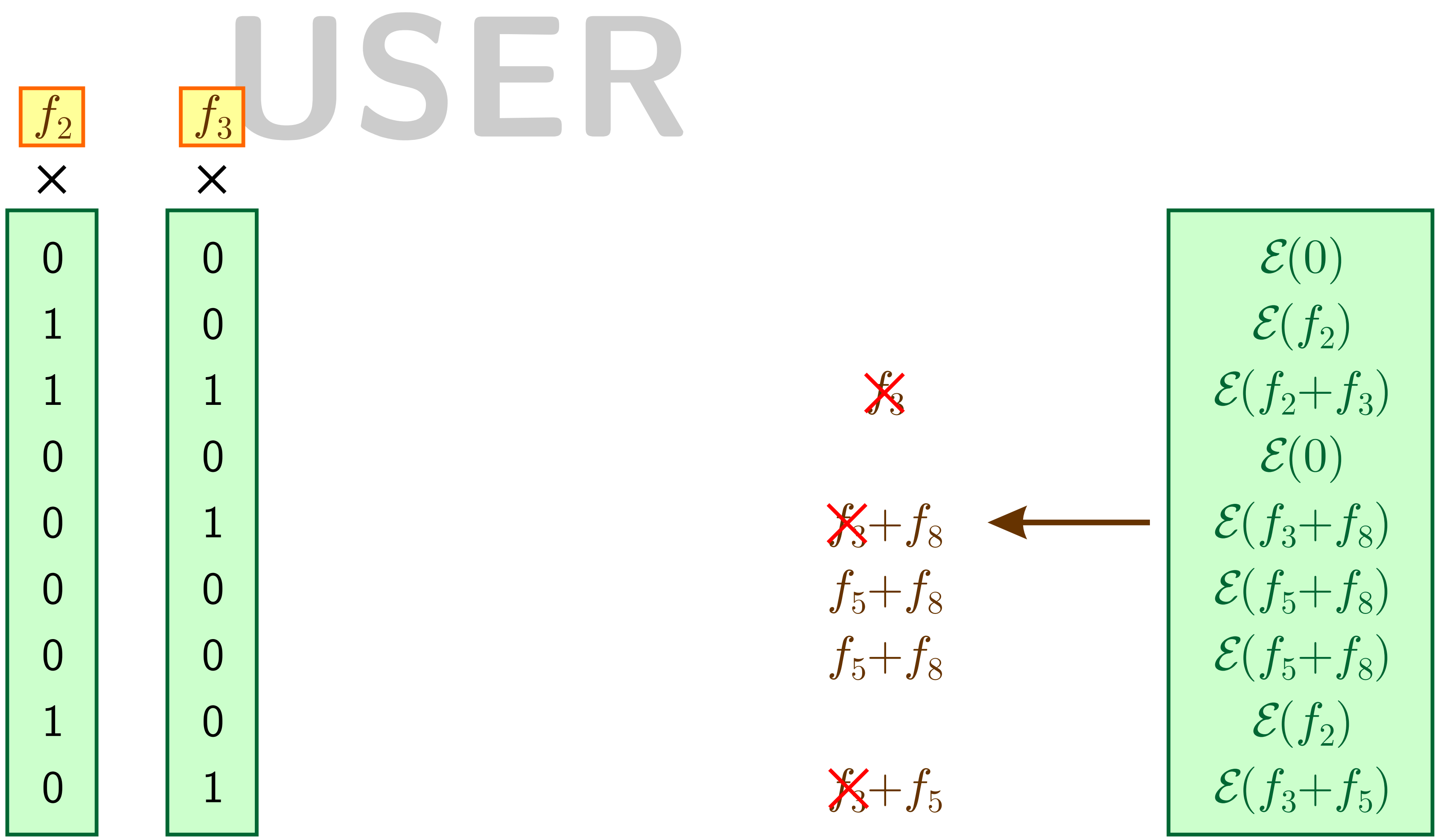
0			$\mathcal{E}(0)$
1	~~f_2~~		$\mathcal{E}(f_2)$
1	~~f_2~~$+f_3$		$\mathcal{E}(f_2+f_3)$
0			$\mathcal{E}(0)$
0	f_3+f_8	$\leftarrow$	$\mathcal{E}(f_3+f_8)$
0	f_5+f_8		$\mathcal{E}(f_5+f_8)$
0	f_5+f_8		$\mathcal{E}(f_5+f_8)$
1	~~f_2~~		$\mathcal{E}(f_2)$
0	f_3+f_5		$\mathcal{E}(f_3+f_5)$

- He can remove it completely from the buffer.



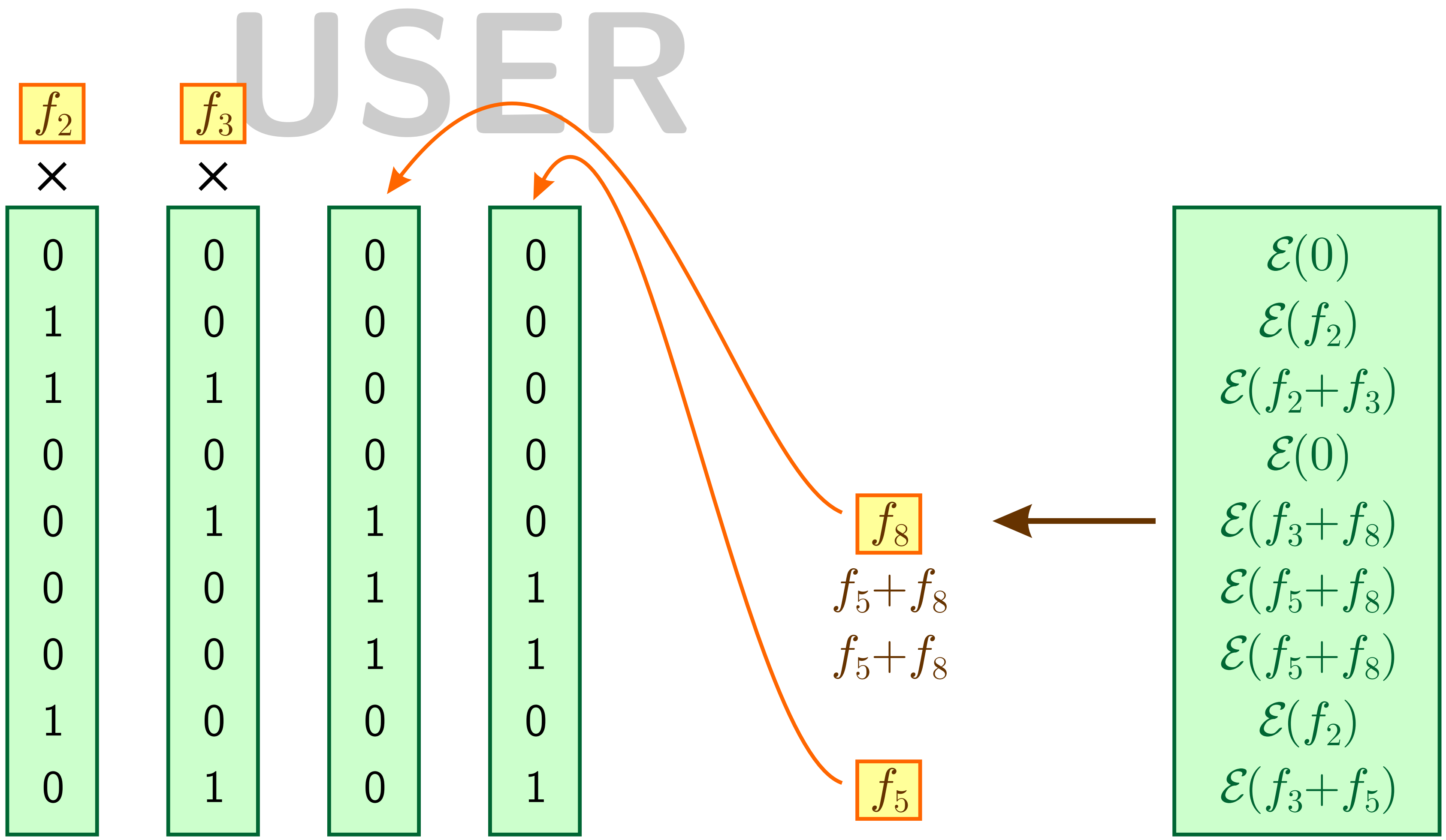
- This uncovers new singletons.

PSS with LDPC codes

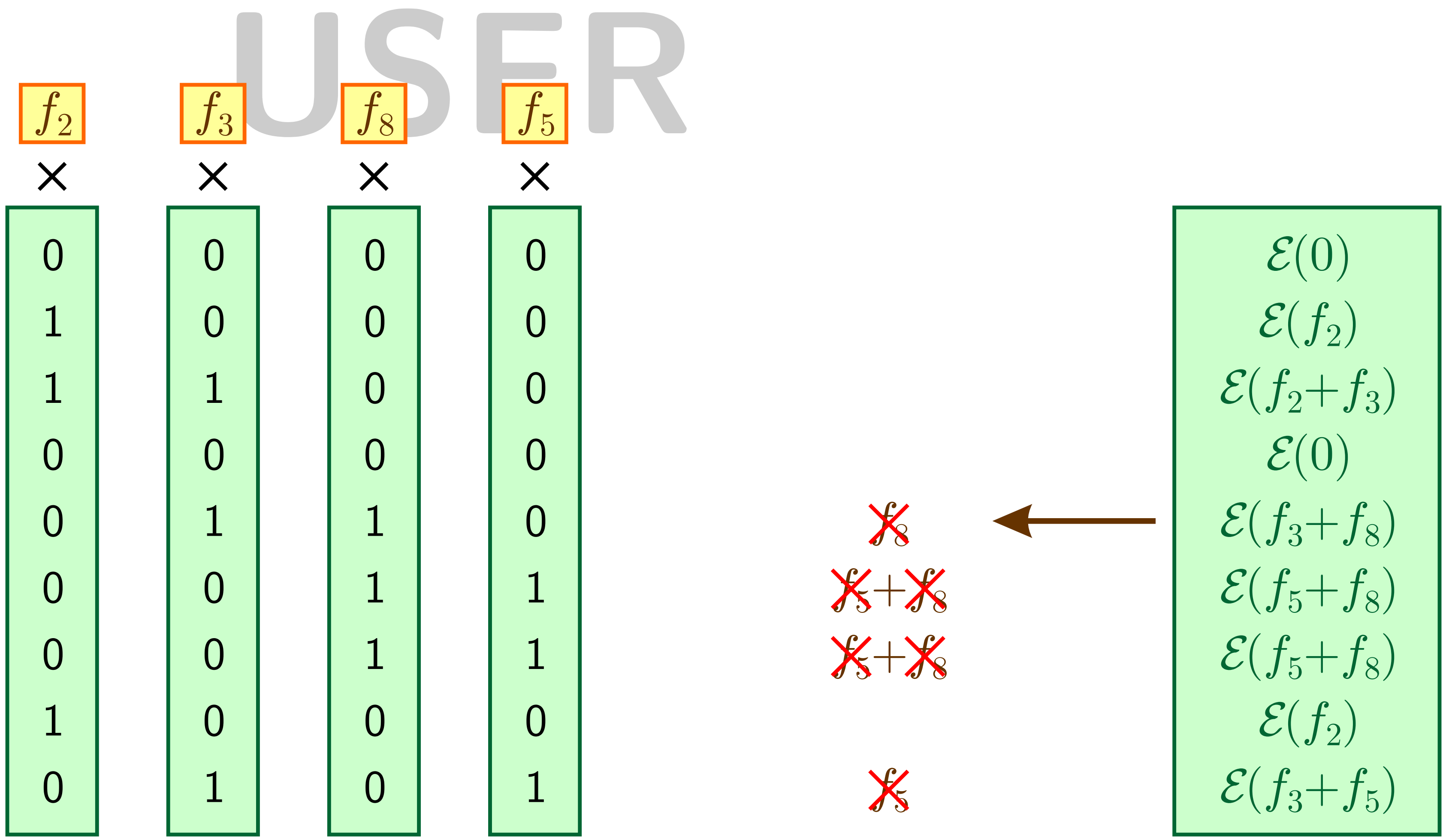


- They can again be stripped from the buffer.

PSS with LDPC codes



PSS with LDPC codes

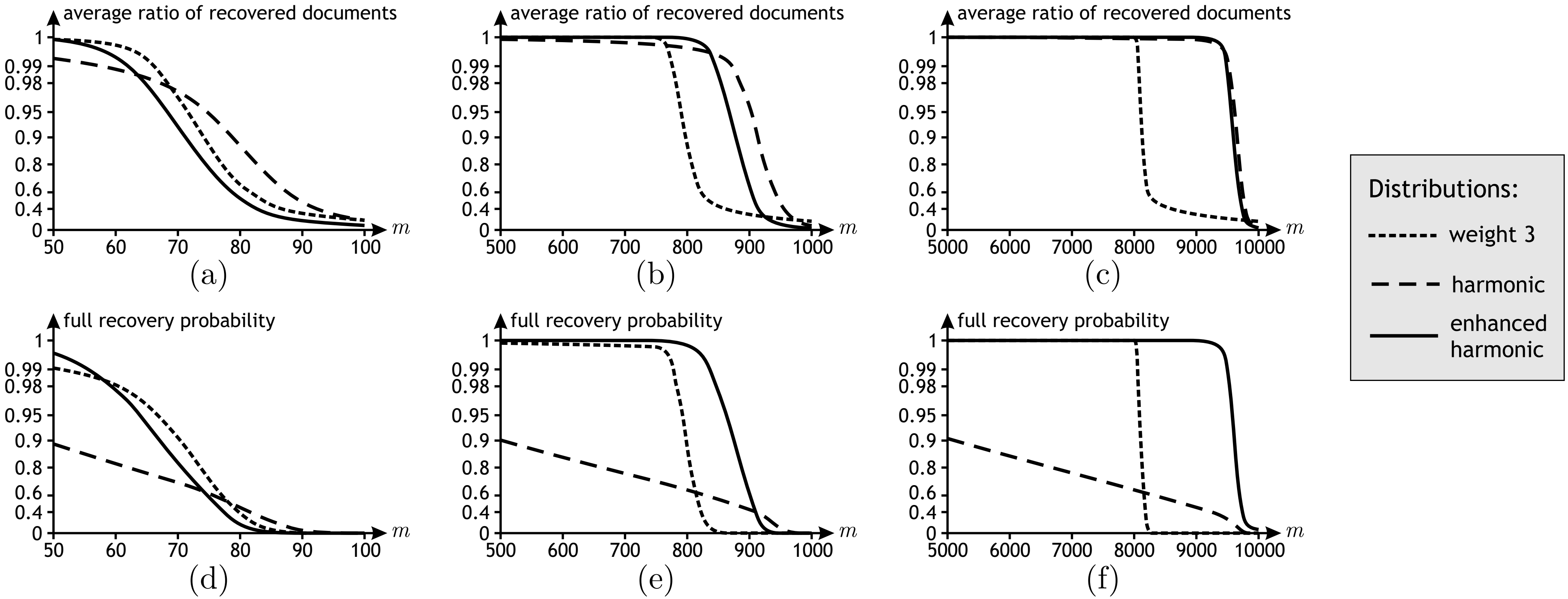


- All documents were recovered when the buffer is 0.

- The whole algorithm is independent of the stream size,
  - the buffer size depends only on the number of matches.
- Computationally very efficient:
  - for the server, one "encryption" per document,
  - for the user, one decryption per buffer position
    → the rest of the decoding is also linear.
- We have full control on the column distribution,
  - possible to use constant weight,
    → not optimal asymptotically,
  - possible to use irregular LDPC codes,
    → use work of Luby, Mitzenmacher, Shokrollahi for asymptotic analysis.

PSS with LDPC codes

Simulation results



- Simulations for buffers of sizes 100, 1 000 and 10 000:
  - for 100, constant weight is as good as irregular,
  - we see the asymptotic limitation of constant weight
    → at least a ratio 1.22 between buffer/matches.

Conclusion

- Our new Private Stream Search scheme:
  - compared to the Ostrovsky-Skeith scheme
    → same computational cost, better communication,
  - compared to a non-private search
    → same asymptotic communication cost, additional computations (especially for the server)
- Is it practical?
  - probably too expensive using Paillier's encryption
    → lighter homomorphic encryption (lattice based?).
  - practical from a communication point of view.
- Would any search engine want to use it?