Multiplicative martingales and random multifractal functions

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Abstract

Given a real function F on $[0,1]$, for each x in $[0,1]$ we can define the local Holder exponent

$$h_F(x) = \liminf_{r\to 0} \log[O_F(B(x,r))]/\log[r],$$

where $O_F(B(x,r)) := \sup_{s,t\in B(x,r)} |F(s) - F(t)|$ is the oscillation of F over ball $B(x,r)$ centered at x with radius r. According to this $h_F(x)$ we can define the level set

$$E_h := x \in [0,1] : h_F(x) = h.$$

What we are interested in is how big E_h is for each $h = 0$.

In this talk we first give a short introduction about Hausdorff dimension and then we will present a special random function which is generated by sign-changed multiplicative cascades, in this setting we can calculate the multifractal spectrum:

$$d(h) = \dim_H(E_h) \qquad \text{for } h = 0,$$

where $\dim_H(E_h)$ is the Hausdorff dimension of E_h.

Also we calculate the Hausdorff dimension of the subset of graph of F defined as

$$G_h := (x, F(x)) : x \in E_h \qquad \text{for } h > 0.$$